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COMPRESSIVE SENSING: APPLICATIONS FROM 1-D TO N-D

By

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Abstract

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Chapter 1

Introduction

The recent trend of curiosity-driven human development has caused a surge in the amount of openly accessible data. More often than not, the inflow of information into digital systems happens much faster than the system can process the data. Moore's law implicitly sets a limit to how powerful and how quick our electronic systems can become (barring a significant breakthrough in the field of quantum computing), and the Nyquist-Shannon sampling theorem (NST) limits the range of frequencies a certain device can successfully recover. This study explores the use of compressed sensing (CS)—an emergent sampling theorem that allows recovery of signals from much fewer samples than required by the NST—as a viable sampling method for signals with an arbitrary number of dimensions. In this framework, the computational burden is shifted from the sampling device to the device performing reconstruction/decompression, and as such, there exist many ways to recover a signal from compressive measurements. The use of CS has been applied to simple audio signals containing pure tones [1, 2] and speech [3–5], images [6–8], and videos [9, 10]. Common implementations of CS utilize analytical measurement bases such as the discrete cosine transform (DCT) basis, but recent studies [11–13] have shown that learned bases perform much better on more complex signals. The learning algorithms associated with these bases range from the classical iterative methods, such as matching pursuit (MP) and principal components analysis (PCA), to the more contemporary machine learning methods, most notably recurrent neural networks (RNN) and associative memory neural networks (AMNN). The novelty of this study is to provide a generalization of compressive sensing methods on signals of arbitrary dimensions, and to mathematically bridge the learning phase of neural networks with compressive sensing.

1.1 Background of the study

1.1.1 Compressive sensing

Consider a real-valued signal $\mathbf{x}$ which can be expressed as a vector of length N. Take $\mathbf{\Psi}$ to be an $N \times N$ sparse orthonormal basis, whose column vectors can be expressed as ψ_i. In order to represent the signal $\mathbf{x}$ sparsely for applications in compressive sensing, it may undergo a linear transformation under $\mathbf{\Psi}$ as

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\alpha} = \sum_{i}^{N} \alpha_i \psi_i \tag{1.1}$$

where α_i are the sparse domain coefficients. Sparsity entails that the coefficient sequence of a signal represented in some sparse domain ideally contains very few non-zero coefficients k, and such a signal is referred to as k-sparse. The acquisition and digitization of a signal can also be posed as a linear transformation

$$\mathbf{y} = \mathbf{A}\mathbf{x} \tag{1.2}$$

where $\mathbf{A}$ is referred to as the measurement or sensing matrix, and $\mathbf{y}$ is the acquired signal. According to the Nyquist-Shannon sampling theorem (NST), a periodic signal, which may be composed of a linear superposition of sinusoids of different parameters, has a characteristic bandwidth f_B, or its highest frequency component. The signal can be successfully reconstructed by the acquisition device if it samples the signal uniformly at a frequency f_s which is at least twice f_B; that is $f_s \geq 2f_B$, where $2f_B$ is known as the Nyquist rate [14]. Under the NST, the original signal $\mathbf{x}$ in (1.2) can be recovered by

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{y} \tag{1.3}$$

In signal processing however, the measurement matrix $\mathbf{A}$ is more often then not, ill-posed. This requires us instead to solve the combinatorial minimization problem

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0 \tag{1.4}$$

where $\|\mathbf{x}\|_0$ returns the number of non-zero elements in $\mathbf{x}$. This problem is strictly NP-hard, and it has been shown in [15] that for a sufficiently sparse signal, the convex program

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \tag{1.5}$$

1.2 Related literature

The scope of the methods used in this work can generally be divided into two categories: classical iterative methods, and neural networks.

1.2.1 Classical iterative methods

1.3 Novelty

Chapter 2

Compressed Sensing

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