



ECON 3818

Distributions

Kyle Butts

28 July 2021

Distributions

Outline

Discrete Case

- Probability Mass Function
- Calculating probabilities

Continuous Case

- Probability Density Function
- Calculating probabilities

Probability Distribution Functions

Until now we have discussed probability distributions in very loose terms. We will build a formal definition of a **probability distribution function**.

First we consider the discrete case, and then the continuous case.

Discrete

Defining Probability Mass Function

Let X be a discrete random variable defined over sample space S with outcome $x \in S$.

The **probability mass function** (or pmf) of X is a function that assigns a probability value to every possible outcome of X .

We write this as $P_X(x)$, probability of $X = x$.

Example PMF

Explicitly Given

X is defined as the number of people seated at a random table at a restaurant. The PMF of X is provided below:

Probability Distribution of X	
x	$P(X = x)$
1	0.07
2	0.36
3	0.32
4	0.21
5 or more	0.04

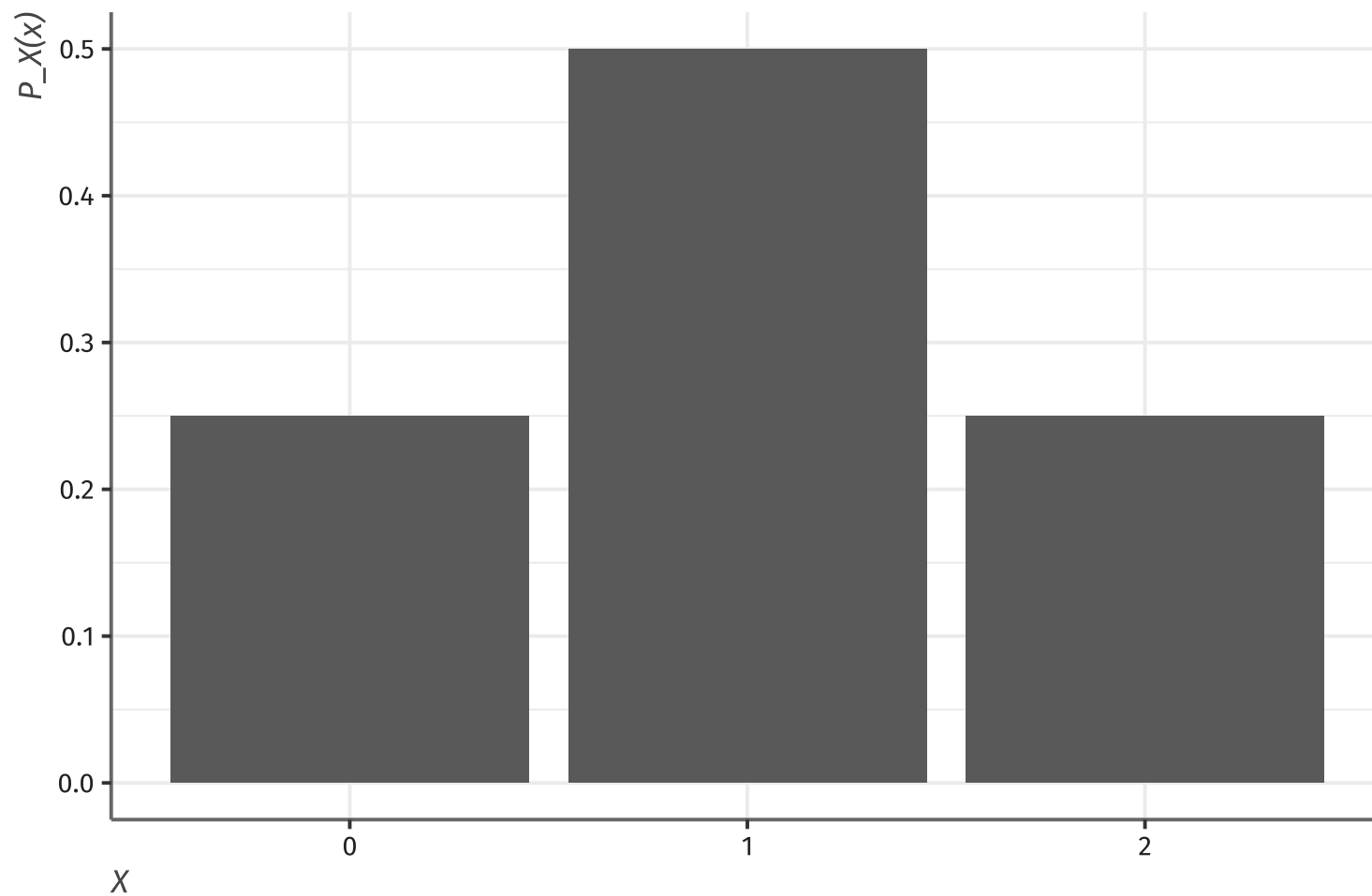
Example PMF

Based on Scenario

Suppose you flip a fair coin twice. Let X be the number of heads that appear. The pmf of X is

Probability Distribution of X	
x	$P_X(x)$
0	0.25
1	0.50
2	0.25

Example PMF



Properties of PMFs

We say a $p_X(x)$ is a **valid** pmf if it satisfies the following:

1. $0 \leq p_X(x) \leq 1$ for all $x \in S$.

2. $\sum_{x \in S} p_X(x) = 1$.

Using PMFs

We can use the PMF to answer questions about cumulative probabilities, for example: Recall the previous example:

Probability Distribution of X	
x	P(X = x)
1	0.07
2	0.36
3	0.32
4	0.21
5 or more	0.04

What is the probability a random table at the restaurant has 2 or 3 people seated?

$$P(X = 2) = 0.36 \text{ and } P(X = 3) = 0.32 \implies$$

$$P(X = 2 \text{ or } 3) = 0.36 + 0.32 = .68$$

Clicker Question

Assume there are four outcomes of X : 1, 5, 10 and 20. Given the following PMF, what is the probability $X=20$?

Probability Distribution of X	
x	$P(X = x)$
1	0.42
5	0.23
10	0.18
20	?

- a. 0.35
- b. 0.17
- c. 0.40
- d. Cannot be determined given the information

Continuous

Defining Probability Density Function

Let Y be a continuous random variable defined over the interval $[a, b]$.

The **probability density function** (or pdf) of Y is a function, $f_Y(y)$, that assigns a probability value to every possible *interval* in $[a, b]$.

We write

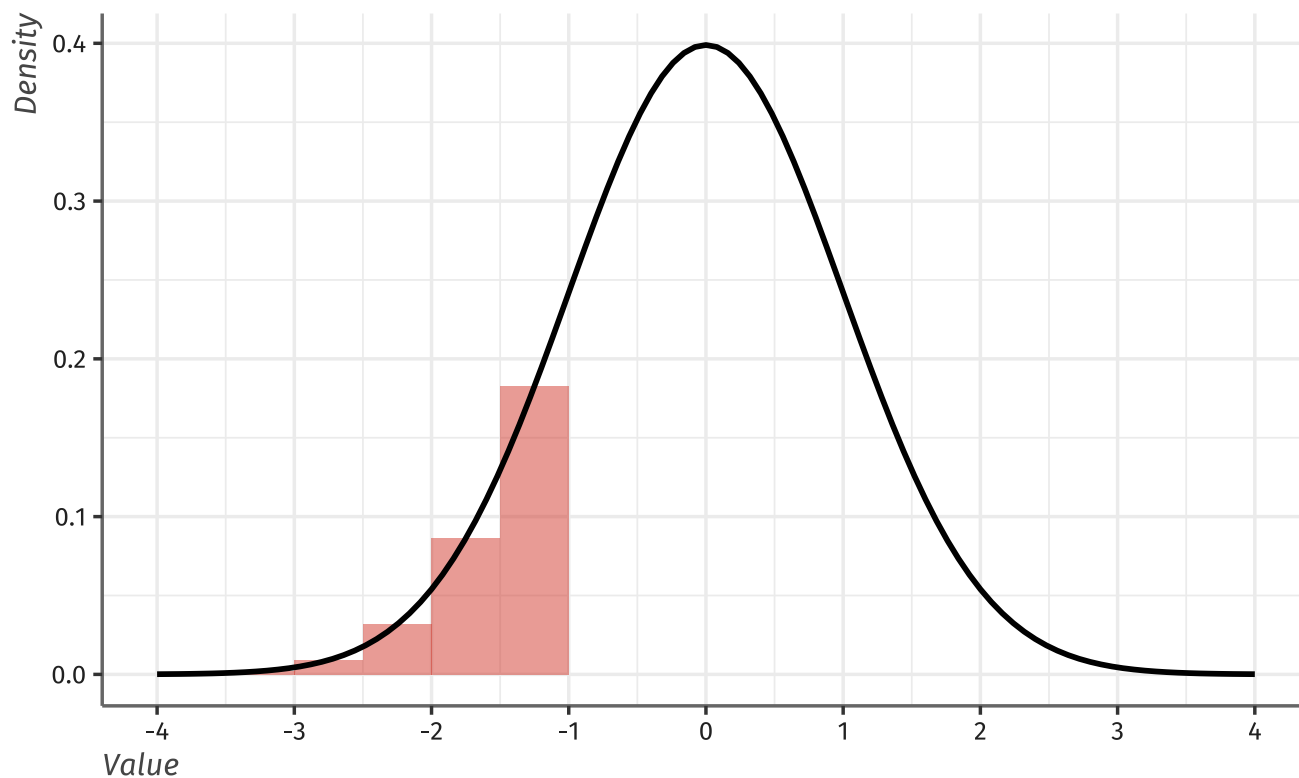
$$Pr(c \leq Y \leq d) = \int_c^d f_Y(y) dy,$$

for all $(c, d) \subset [a, b]$.

Example pdf

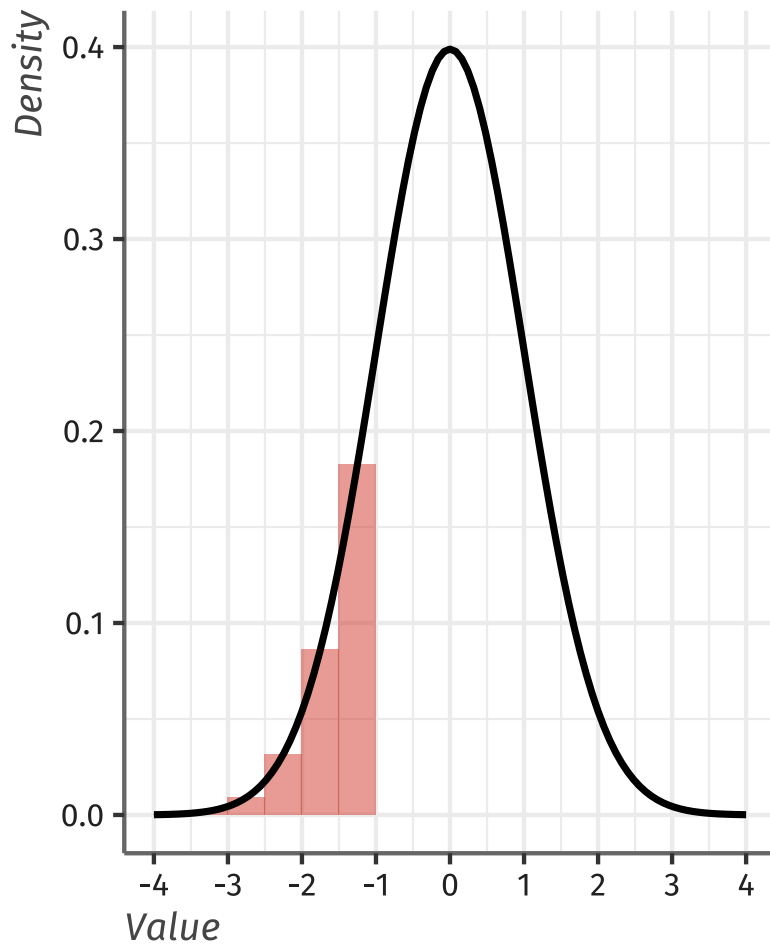
For $Z \sim N(0, 1)$, find $P(Z \leq -1)$.

PMF-approximation

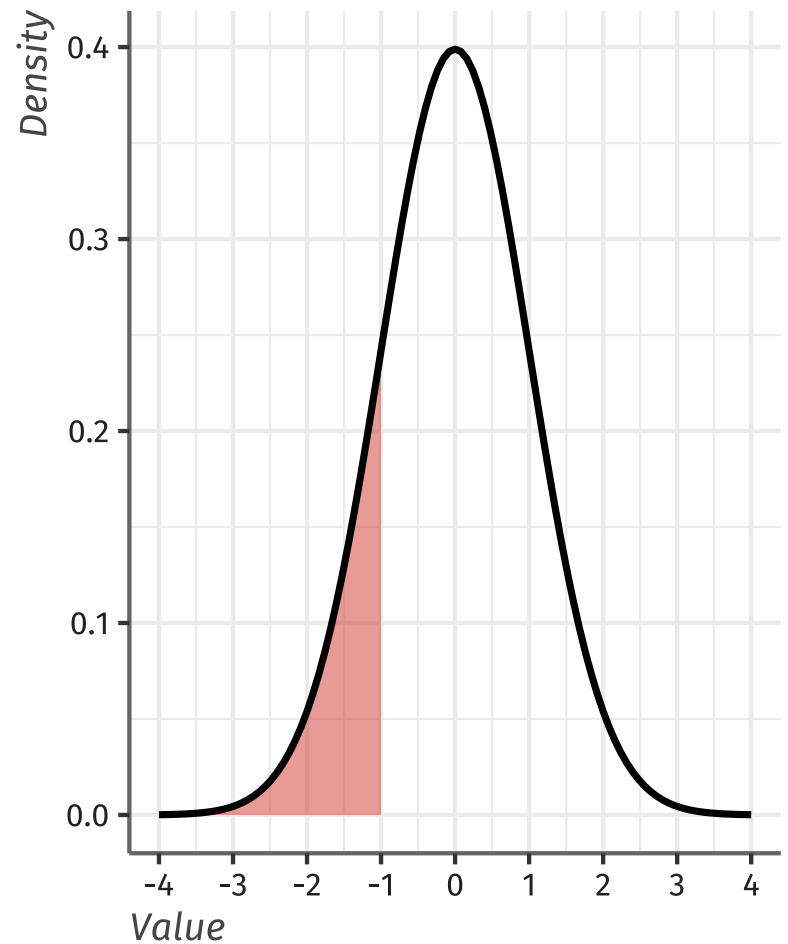


Integral of PDF = Probability

PMF-approximation

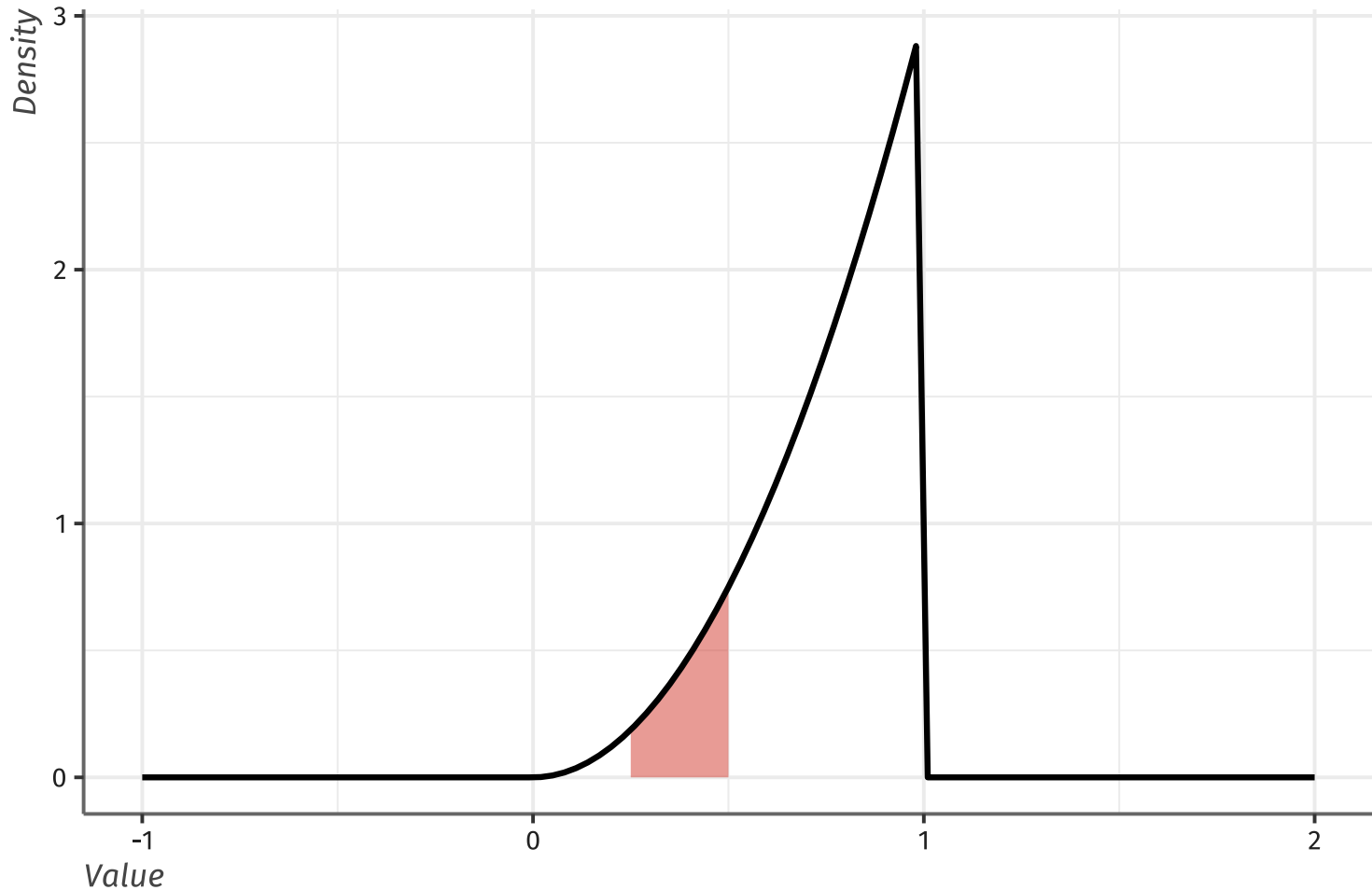


True area



Example PDF

Suppose that Y is a continuous random variable with pdf $f_Y(y) = 3y^2$ for $0 < y < 1$. What is $P(\frac{1}{4} \leq Y \leq \frac{1}{2})$?



Example PDF

Suppose that Y is a continuous random variable with pdf $f_Y(y) = 3y^2$ for $0 < y < 1$. What is $P(\frac{1}{4} \leq Y \leq \frac{1}{2})$?

Properties of PDFs

We say a $f_Y(y)$ is a **valid** pdf if it satisfies the following:

1. $0 \leq f_Y(y) \leq 1$ for all $y \in [a, b]$.

2. $\int_a^b f_Y(y) dy = 1$.

Note that $Pr(Y = a) = \int_a^a f_Y(y) dy = 0$.

At first this might seem counterintuitive. But imagine trying to stop a stopwatch at exactly 30 seconds. What is the probability of that event?

Clicker Question

Given the pdf, $f(y) = 3y^2$ for $0 < y < 1$. What is the $P(Y < 1/3)$?

- a. $\frac{1}{3}$
- b. $\frac{1}{9}$
- c. $\frac{1}{27}$
- d. $\frac{26}{27}$

Midterm Example

Consider the probability distribution for random variable Y :

$$f(y) = 8y, \quad 0 \leq y \leq \frac{1}{2}$$

1. Find $P(Y < \frac{1}{3})$
2. Find $P(Y = \frac{1}{4})$
3. Find $P(\frac{3}{4} < Y < 1)$