ECON 3818

Chapter 12

Kyle Butts

28 July 2021

Chapter 12: Introducing Probability

Randomness

What is randomness? A phenomenon is **random** if:

1. Individual outcomes are uncertain
2. Has a distributions of outcomes in a large number of repetitions

Example: A coin toss

Probability

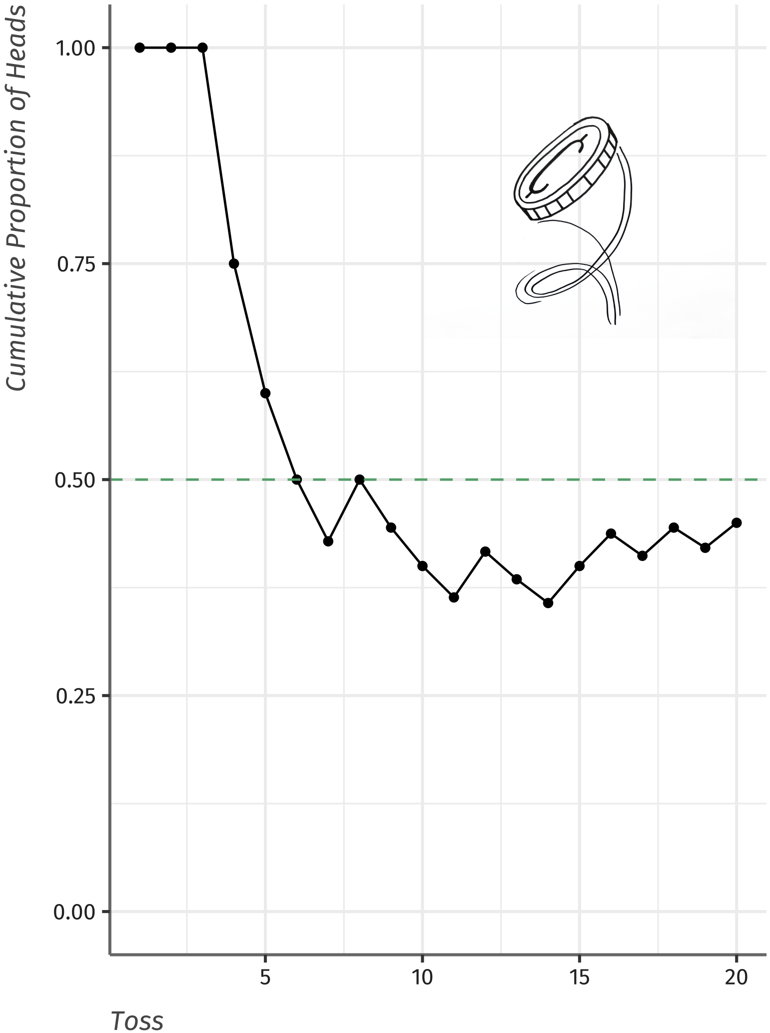
Probability: proportion of times a particular outcome would occur in a very long series of repetitions.

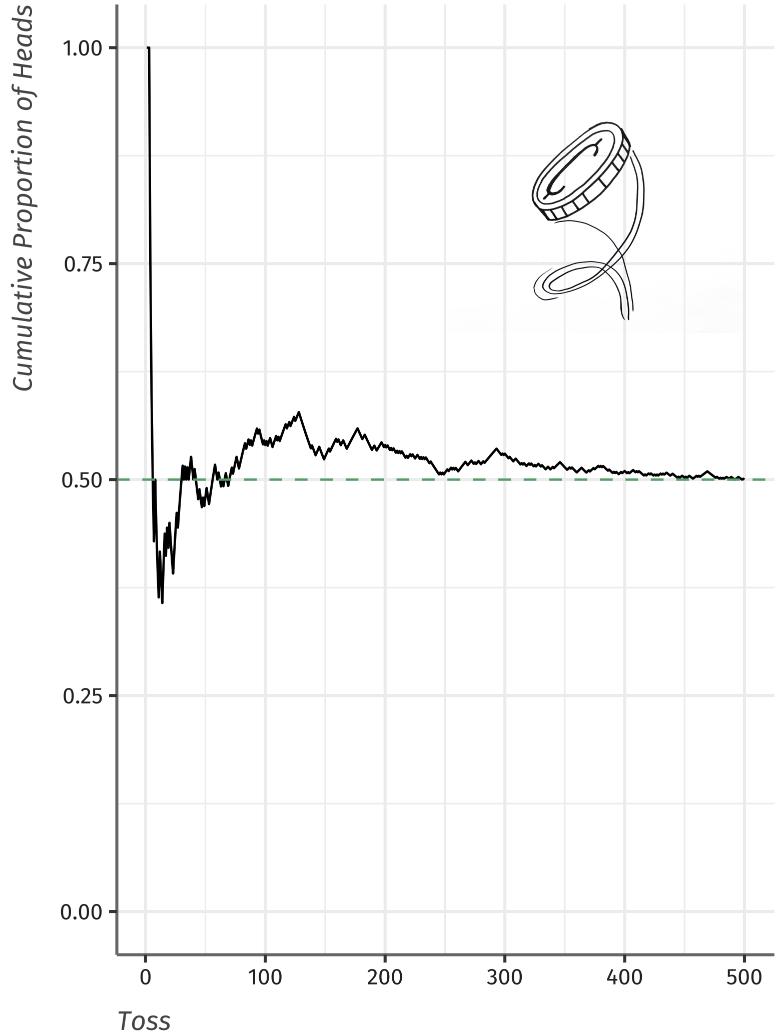
- *Example*: What is the probability of a coin landing on heads?

For a given observation, the probability that an event occurs is:

$$\frac{\text{Number of ways event could occur}}{\text{Number of total possible outcomes}}$$

Probability and Randomness





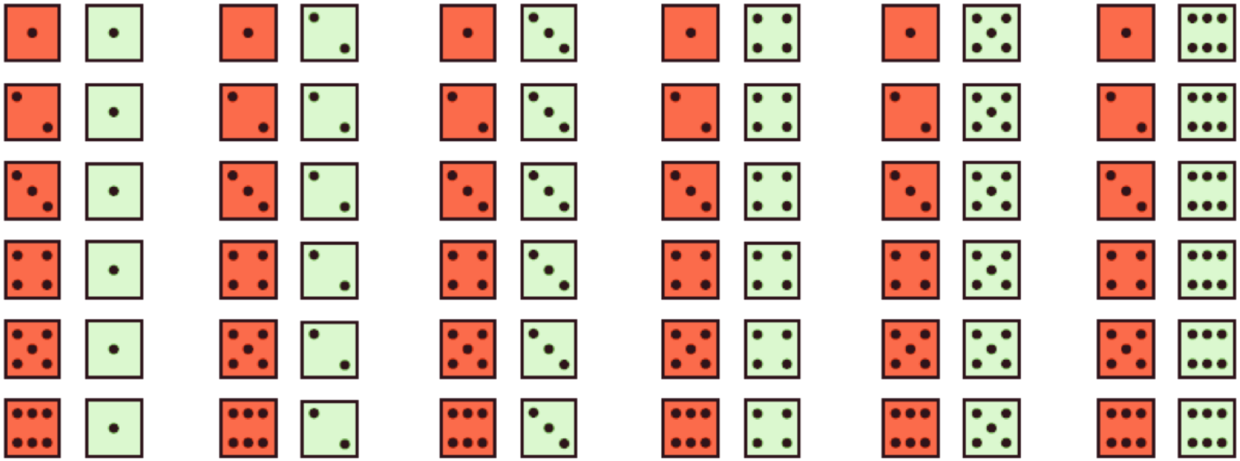
Probability Models

We think of probability utilizing a particular framework, first we define a few useful terms:

- Sample Space: set of all possible outcomes
- Event: outcome (or set of outcomes) of a random phenomenon
    - .coral[Event] is a subset of the .kelly[sample space]
- Probability Model: assigns a probability to every event in the sample space

Probability: Example

Say we roll two six-sided die, the following would be our sample space:



Each outcome is equally likely, specifically each outcome has probability of 1/36

Clicker Question

If I roll two six-sided die, what is the probability I roll a one and a two?

a. 1/36
b. 2/36
c. 3/36
d. 4/36

Set Notation

$A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{1, 2, 3, 4, 5, 6\}$, $D = \{4\}$

$\in$: "belongs to"

- Example: $1 \in A$

$\notin$: "does not belong to"

- Example: $4 \notin A$

$\cup$: Union; combination of two or more sets; "or"

- Example: $A \cup B = \{1, 2, 3, 4, 5\}$

$\cap$: Intersection; overlap of two or more sets; "and"

- Example: $A \cap B = \{3\}$

Set Notation, cont.

$A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{1, 2, 3, 4, 5, 6\}$, $D = \{4\}$

A^c: "A complement"

- Example: $A^c = \{x : x \notin A\} = \{4, 5, 6\}$
- Interpreted as "not A"

$\subseteq$: Subset

- Example: $A \subseteq C$, however $C \nsubseteq A$.

$\emptyset$: is the null or empty set

- contains nothing

$A \cap D = \emptyset$: Disjoint

Clicker Question

Given the following sets, $A = \{5, 10, 15, 20\}$ and $B = \{1, 2, 3, 4, 5\}$ Which of the following is true?

a. $A \cup B = \{1, 2, 3, 4, 5, 10, 15, 20\}$
b. $A \cup B = \{5\}$
c. $A \cap B = \{5\}$
d. $A \cap B = \{1, 2, 3, 4, 5, 10, 15, 20\}$
e. Both a. and c.

Axioms of Probability

Let A and B be events, and $P(A)$ and $P(B)$ are the probability of those outcomes. We have a set of rules:

1. Any probability is a number between 0 and 1
2. All possible outcomes together must have the probability of 1
3. If two events are disjoint,
$$P(A \cap B) = 0 \implies P(A \cup B) = P(A) + P(B)$$
4. $P(A^c) = 1 - P(A)$

Clicker Question

Given the three following scenarios:

- A person is randomly selected. A is the event they are under 18. B is the event they are over 18.
- A person is selected at random. A is the event that they earn more than $100,000 per year. B is the event that they earn more than $250,000.
- A pair of dice are tossed. A is the event that one of the die is a 3. B is the event that the sum of two dice is 3.

In which cases are the events, A and B, disjoint?

a. 1 only
b. 2 only
c. 3 only
d. 1 and 2
e. 1 and 3

De Morgan's Law

De Morgan's law of union and intersection. For any two finite sets A and B:

1. $(A \cup B)^c = A^c \cap B^c$
2. $(A \cap B)^c = A^c \cup B^c$

De Morgan's Law Example

Let $S = \{j, k, l, m, n\}$ and $A = \{j, k, m\}$ and $B = \{k, m, n\}$

1. $(A \cup B)^c = (A^c \cap B^c)$
   i. $(A \cup B) = \{j, k, m, n\} \implies (A \cup B)^c = \{l\}$
   ii. $A^c = \{l, n\}$ and $B^c = \{j, l\} \implies A^c \cap B^c = \{l\}$
2. $(A \cap B)^c = A^c \cup B^c$
   i. $(A \cap B) = \{k, m\} \implies (A \cap B)^c = \{j, l, n\}$
   ii. $A^c \cup B^c = \{l, n\} \cup \{j, l\} \implies A^c \cup B^c = \{j, l, n\}$

Random Variables

Random variable: variable whose value is a numerical outcome of a random phenomenon

- **Random variables** can be **discrete** or **continuous**

Example: Coin toss

- X can be defined as the number of heads we see in two tosses:
  - X is a discrete random variable; $X = 0, 1, 2$

Probability distribution: tell us what values random variable X can take, and how to assign probabilities to those values

Example

Flip a coin two times

Sample space:

- {(Head, Tail), (Head, Head), (Tail, Head), (Tail, Tail)}

What is the probability of each event?

Clicker Question

If I toss a coin two times, and X is the number of heads, then what is $P(X = 2)$?

a. 1/4
b. 1/2
c. 3/4
d. 5/4

Additional Examples

Still flipping a coin twice, what is the probability of getting at least one head?

$$P(X = 1) + P(X = 2) = 1 - P(X = 0)$$

- Now I only have to calculate one probability!

Additional Dice Example

What is the probability of rolling a 7, 11, or double when rolling two dice?

- Axiom 3 tells us we can find probabilities simply by adding if the event is disjoint.

Roll a 7: $\{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

Roll a 11: $\{(5,6),(6,5)\}$

Roll a double: $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

- Are all 3 events disjoint?

Additional Dice Example

Roll a 7: $\{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

Roll a 11: $\{(5,6),(6,5)\}$

Roll a double: $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$$P(7) + P(11) + P(\text{doubles}) = \frac{6}{36} + \frac{2}{36} + \frac{6}{36} \approx 0.4$$