ECON 3818

Distributions

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21 July 2021

Distributions

Outline

Discrete Case

- Probability Mass Function
- Calculating probabilities

Continuous Case

- Probability Density Function
- Calculating probabilities

Probability Distribution Functions

Until now we have discussed probability distributions in very loose terms. We will build a formal definition of a **probability distribution function**.

First we consider the discrete case, and then the continuous case.

Discrete

Defining Probability Mass Function

Let X be a discrete random variable defined over sample space S with outcome $x \in S$.

The **probability mass function** (or pmf) of X is a function that assigns a probability value to every possible outcome of X.

We write this as $P_X(x)$, probability of $X = x$.

Example PMF -- Explicitly Given

X is defined as the number of people seated at a random table at a restaurant. The PMF of X is provided below:

Probability Distribution of X

X	P(X = X)
1	0.07
2	0.36
3	0.32
4	0.21
5 or more	0.04

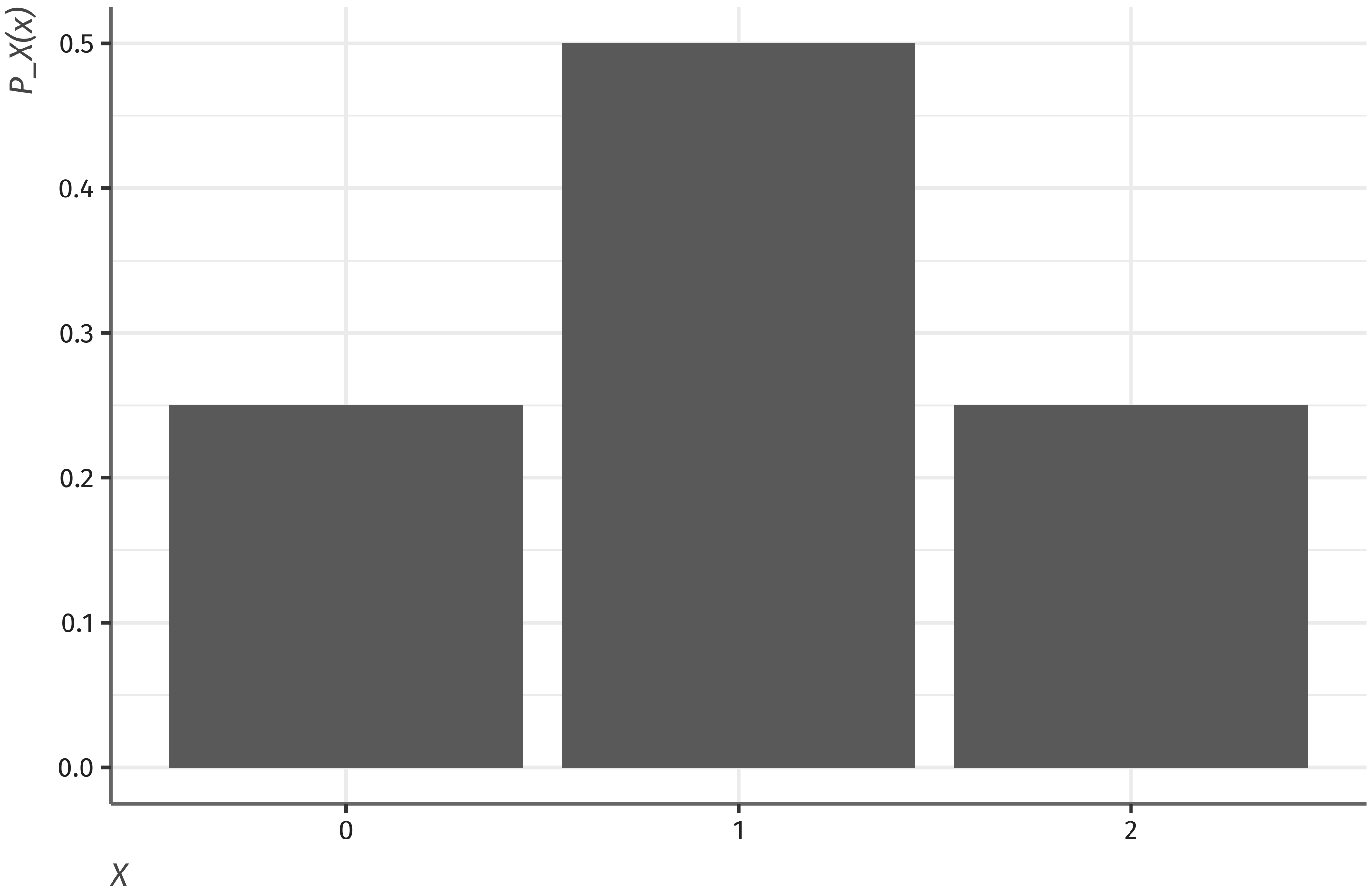
Example PMF -- Based on Scenario

Suppose you flip a fair coin twice. Let X be the number of heads that appear. The pmf of X is

Probability Distribution of X

X	P_X(X)
0	0.25
1	0.50
2	0.25

Example PMF



Properties of PMFs

We say a $p_X(x)$ is a **valid** pmf if it satisfies the following:

1. $0 \leq p_X(x) \leq 1$ for all $x \in S$.
2. $\displaystyle\sum_{x \in S} p_X(x) = 1.$

Using PMFs

We can use the PMF to answer questions about cumulative probabilities, for example: Recall the previous example:

Probability Distribution of X

X	P(X = X)
1	0.07
2	0.36
3	0.32
4	0.21
5 or more	0.04

What is the probability a random table at the restaurant has 2 or 3 people seated?

$$P(X = 2) = 0.36 \text{ and } P(X = 3) = 0.32 \implies$$

$$P(X = 2 \text{ or } 3) = 0.36 + 0.32 = .68$$

Clicker Question

Assume there are four outcomes of X: $1, 5, 10$ and 20. Given the following PMF, what is the probability X=20?

Probability Distribution of X

X	P(X = X)
1	0.42
5	0.23
10	0.18
20	?

a. 0.35
b. 0.17
c. 0.40
d. Cannot be determined given the information

Continuous

Defining Probability Density Function

Let Y be a continuous random variable defined over the interval $[a, b]$.

The **probability density function** (or pdf) of Y is a function, $f_Y(y)$, that assigns a probability value to every possible *interval* in $[a, b]$.

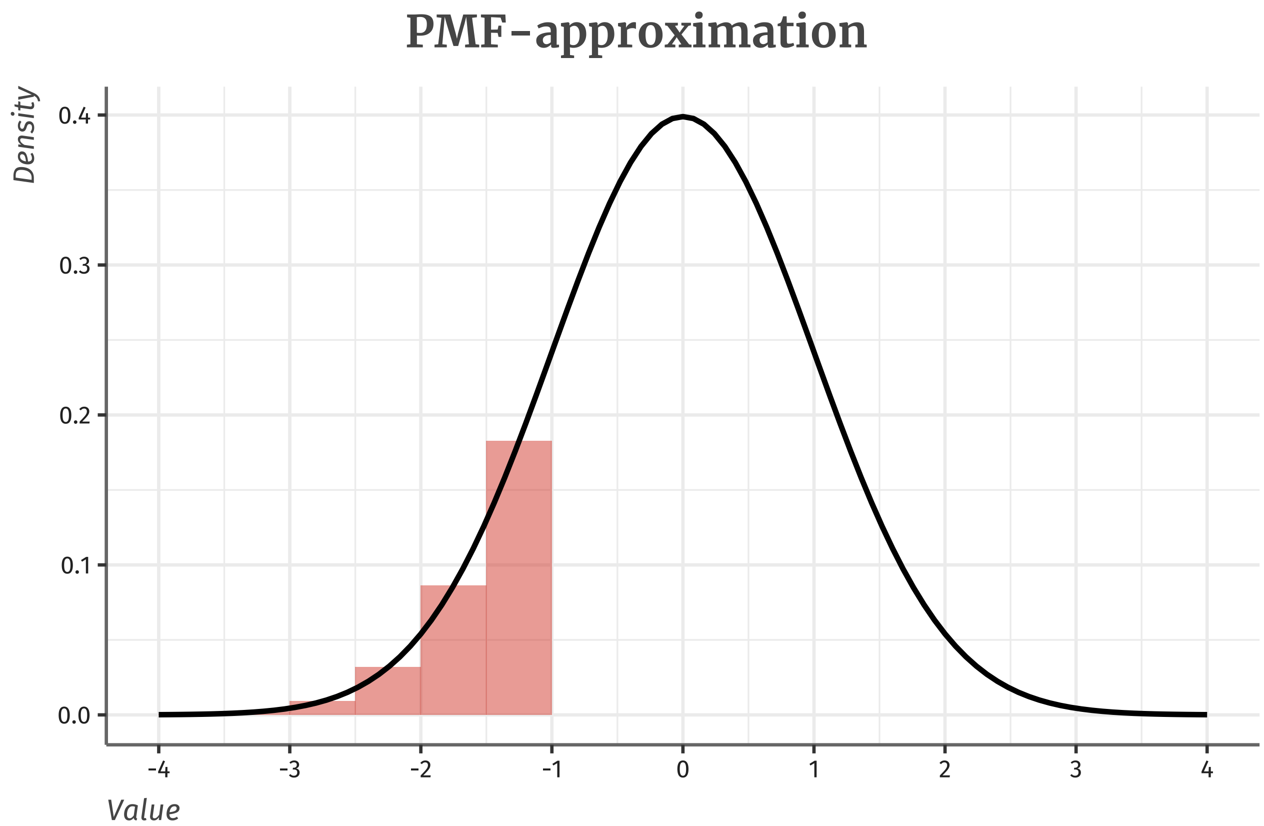
We write

$$Pr(c \leqslant Y \leqslant d) = \int_c^d f_Y(y)dy,$$

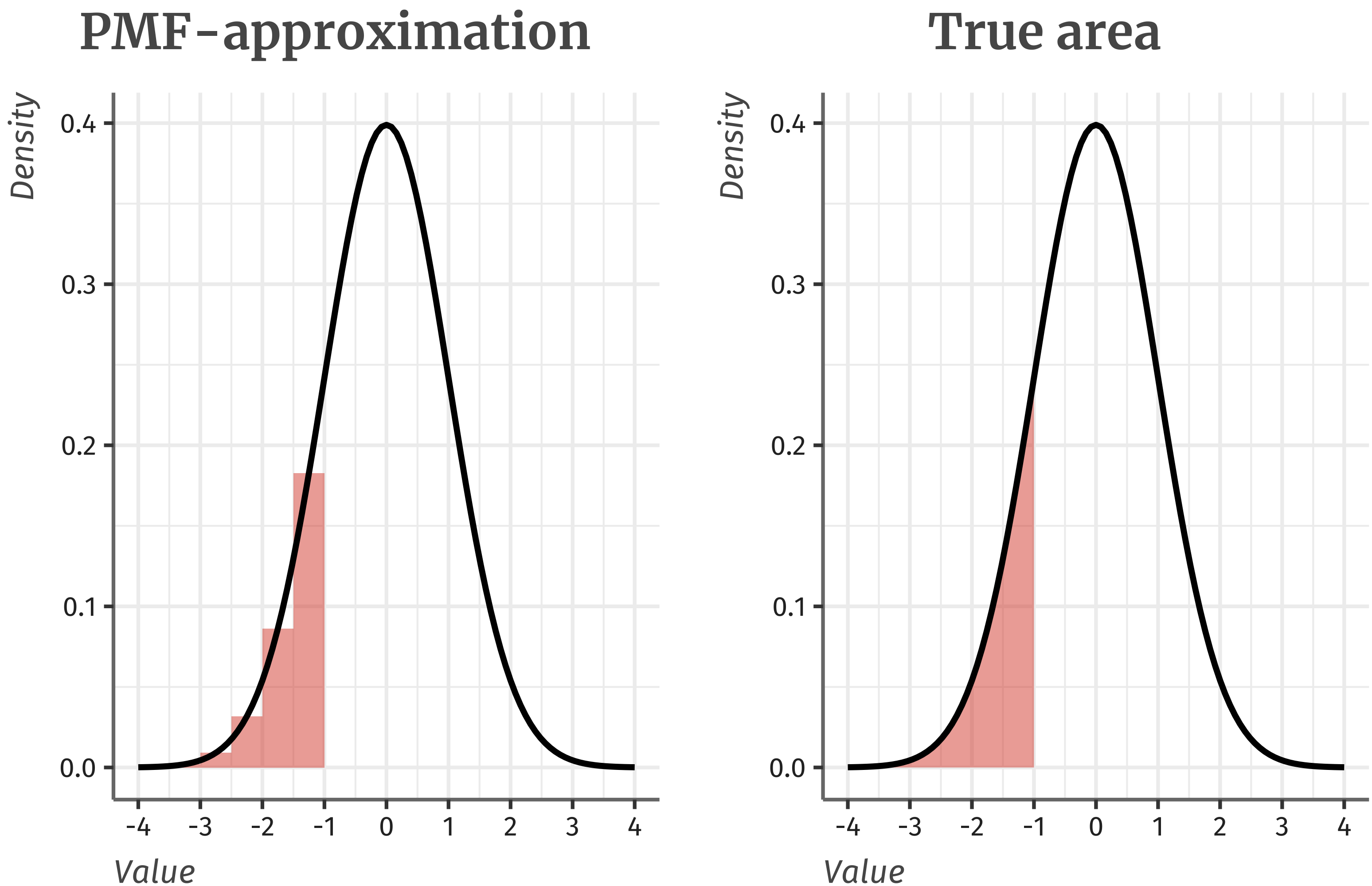
for all $(c, d) \subset [a, b]$.

Example pdf

For $Z \sim N(0, 1)$, find $P(Z <= -1)$.



Integral of PDF = Probability



Example PDF

Suppose that Y is a continuous random variable with pdf $f_Y(y) = 3y^2$ for $0 < y < 1$. What is $P(\frac{1}{4} \leqslant Y \leqslant \frac{1}{2})$?

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Properties of PDFs

We say a $f_Y(y)$ is a **valid** pdf if it satisfies the following:

1. $0 \leqslant \int f_Y(y) \leqslant 1$ for all $y \in [a, b]$.
2. $\displaystyle\int_a^b f_Y(y)dy = 1.$

Note that $Pr(Y = a) = \displaystyle\int_a^a f_Y(y)dy = 0$.

At first this might seem counterintuitive. But imagine trying to stop a stopwatch at exactly 30 seconds. What is the probability of that event?

Clicker Question

Given the pdf, $f(y) = 3y^2$ for $0 < y < 1$. What is the $P(Y < 1/3)$?

a. $\frac{1}{3}$
b. $\frac{1}{9}$
c. $\frac{1}{27}$
d. $\frac{26}{27}$

Midterm Example

Consider the probability distribution for random variable Y:

$$f(y) = 8y,\ 0 \leq y \leq \frac{1}{2}$$

1. Find $P(Y < \frac{1}{3})$
2. Find $P(Y = \frac{1}{4})$
3. Find $P(\frac{3}{4} < Y < 1)$