



ECON 3818

Chapter 12

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Chapter 12: Introducing Probability

Randomness

What is randomness? A phenomenon is **random** if:

1. Individual outcomes are uncertain
2. Has a distributions of outcomes in a large number of repetitions

Example: A coin toss

Probability

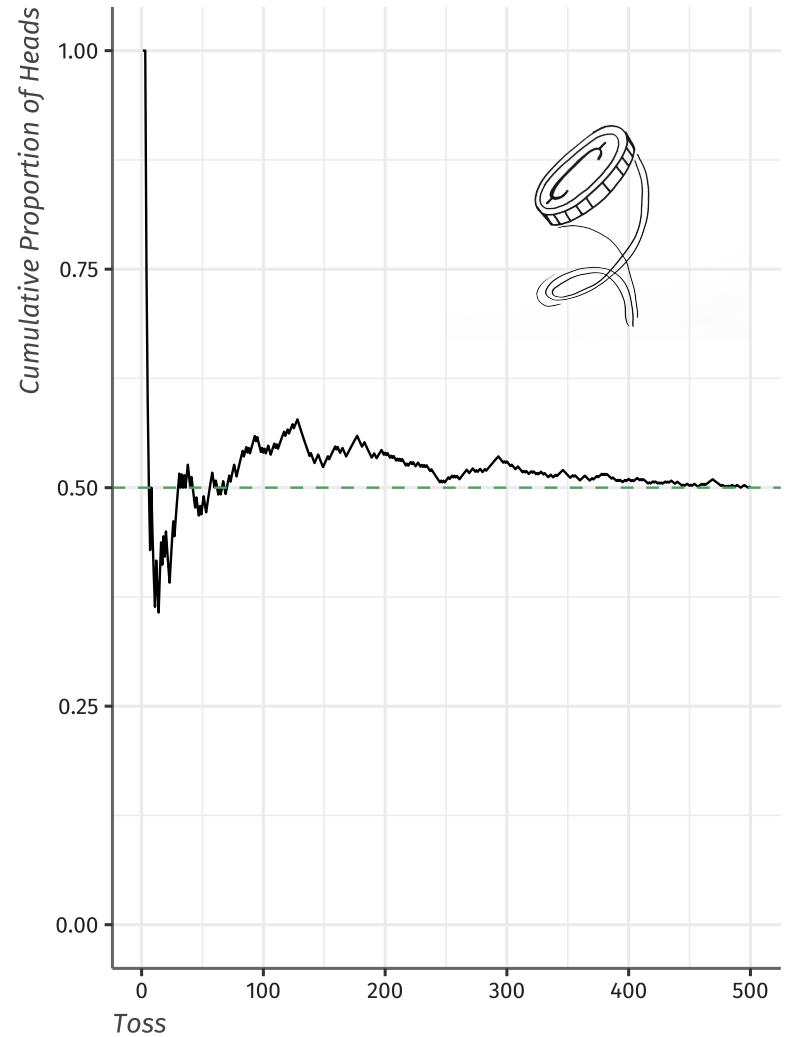
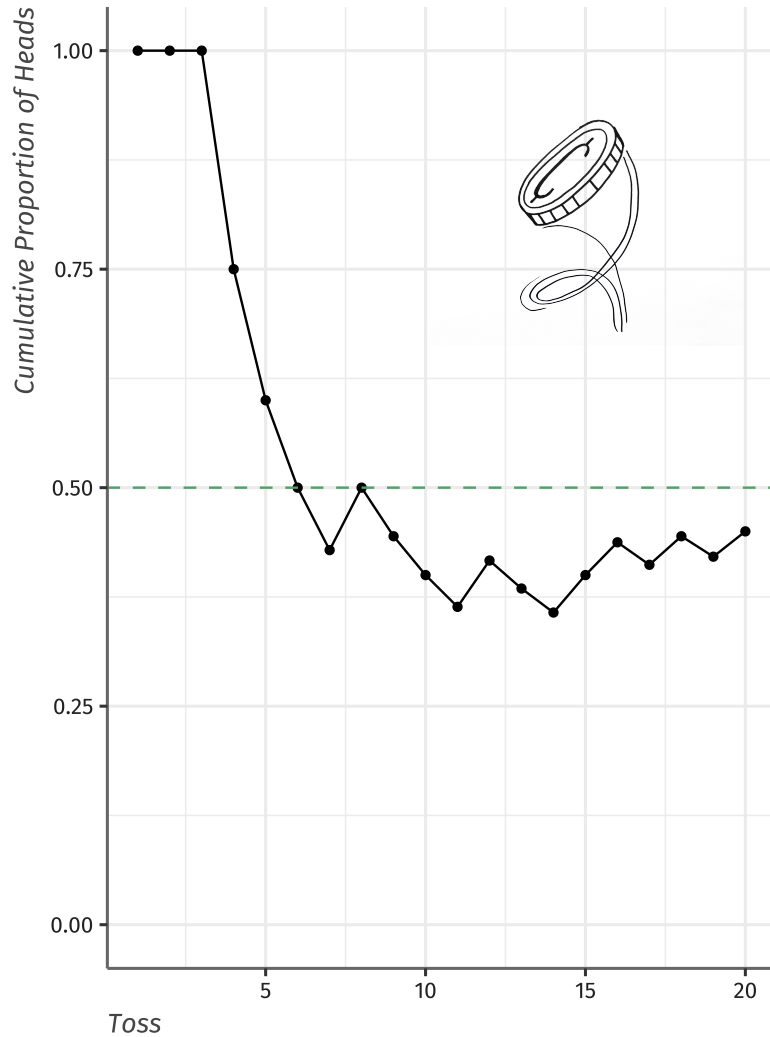
Probability: proportion of times a particular outcome would occur in a very long series of repetitions.

- *Example*: What is the **probability** of a coin landing on heads?

For a given observation, the **probability** that an **event** occurs is:

$$\frac{\text{Number of ways event could occur}}{\text{Number of total possible outcomes}}$$

Probability and Randomness



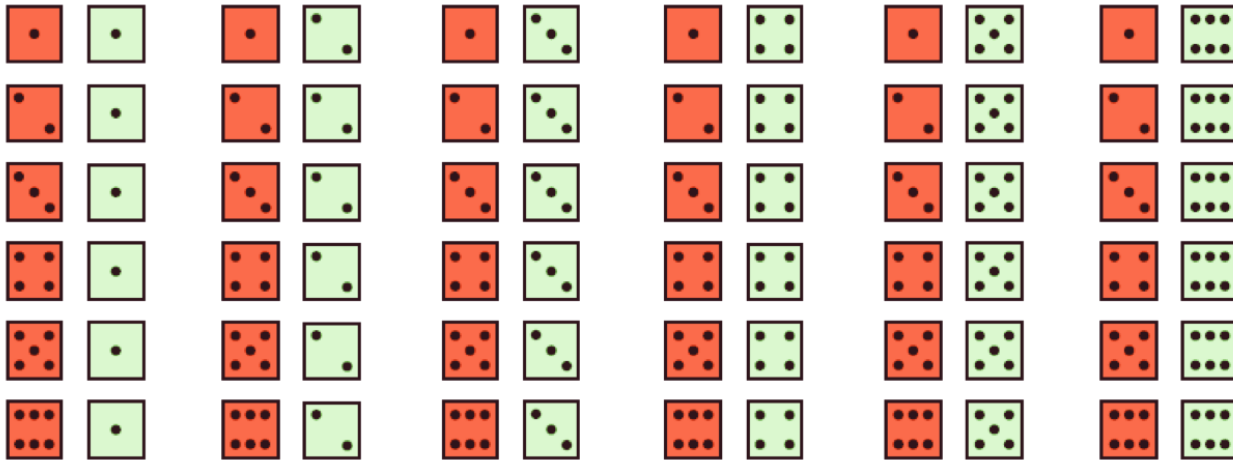
Probability Models

We think of probability utilizing a particular framework, first we define a few useful terms:

- **Sample Space**: set of all possible outcomes
- **Event**: outcome (or set of outcomes) of a random phenomenon
 - `.coral[Event]` is a subset of the `.kelly[sample space]`
- **Probability Model**: assigns a probability to every **event** in the **sample space**

Probability: Example

Say we roll two six-sided die, the following would be our **sample space**:



Each outcome is equally likely, specifically each outcome has **probability** of $1/36$

Clicker Question

If I roll two six-sided die, what is the probability I roll a one and a two?

- a. $1/36$
- b. $2/36$
- c. $3/36$
- d. $4/36$

Set Notation

$$A = \{1, 2, 3\}, B = \{3, 4, 5\}, C = \{1, 2, 3, 4, 5, 6\}, D = \{4\}$$

\in : "belongs to"

- Example: $1 \in A$

\notin : "does not belong to"

- Example: $4 \notin A$

\cup : Union; combination of two or more sets; "or"

- Example: $A \cup B = \{1, 2, 3, 4, 5\}$

\cap : Intersection; overlap of two or more sets; "and"

- Example: $A \cap B = \{3\}$

Set Notation, cont.

$$A = \{1, 2, 3\}, B = \{3, 4, 5\}, C = \{1, 2, 3, 4, 5, 6\}, D = \{4\}$$

A^c : "A complement"

- Example: $A^c = \{x : x \notin A\} = \{4, 5, 6\}$
- Interpreted as "not A"

\subseteq : Subset

- Example: $A \subseteq C$, however $C \not\subseteq A$.

\emptyset : is the null or empty set

- contains nothing

$A \cap D = \emptyset$: Disjoint

Clicker Question

Given the following sets, $A = \{5, 10, 15, 20\}$ and $B = \{1, 2, 3, 4, 5\}$ Which of the following is true?

- a. $A \cup B = \{1, 2, 3, 4, 5, 10, 15, 20\}$
- b. $A \cup B = \{5\}$
- c. $A \cap B = \{5\}$
- d. $A \cap B = \{1, 2, 3, 4, 5, 10, 15, 20\}$
- e. Both a. and c.

Axioms of Probability

Let A and B be events, and $P(A)$ and $P(B)$ are the probability of those outcomes. We have a set of rules:

1. Any probability is a number between 0 and 1
2. All possible outcomes together must have the probability of 1
3. If two events are disjoint,

$$P(A \cap B) = 0 \implies P(A \cup B) = P(A) + P(B)$$

4. $P(A^c) = 1 - P(A)$

Clicker Question

Given the three following scenarios:

- A person is randomly selected. **A** is the **event** they are under 18. **B** is the **event** they are over 18.
- A person is selected at random. **A** is the **event** that they earn more than \$100,000 per year. **B** is the **event** that they earn more than \$250,000.
- A pair of dice are tossed. **A** is the **event** that one of the die is a 3. **B** is the **event** that the sum of two dice is 3.

In which cases are the **events**, A and B, disjoint?

- a. 1 only
- b. 2 only
- c. 3 only
- d. 1 and 2
- e. 1 and 3

De Morgan's Law

De Morgan's law of union and intersection. For any two finite sets **A** and **B**:

$$1. (A \cup B)^c = A^c \cap B^c$$

$$2. (A \cap B)^c = A^c \cup B^c$$

De Morgan's Law Example

Let $S = \{j, k, l, m, n\}$ and $A = \{j, k, m\}$ and $B = \{k, m, n\}$

$$1. (A \cup B)^c = (A^c \cap B^c)$$

$$\text{i. } (A \cup B) = \{j, k, m, n\} \implies (A \cup B)^c = \{l\}$$

$$\text{ii. } A^c = \{l, n\} \text{ and } B^c = \{j, l\} \implies A^c \cap B^c = \{l\}$$

$$2. (A \cap B)^c = A^c \cup B^c$$

$$\text{i. } (A \cap B) = \{k, m\} \implies (A \cap B)^c = \{j, l, n\}$$

$$\text{ii. } A^c \cup B^c = \{l, n\} \cup \{j, l\} \implies A^c \cup B^c = \{j, l, n\}$$

Random Variables

Random variable: variable whose value is a numerical outcome of a random phenomenon

- **Random variables** can be **discrete** or **continuous**

Example: Coin toss

- X can be defined as the number of heads we see in two tosses:
 - X is a discrete random variable; $X = 0, 1, 2$

Probability distribution: tell us what values random variable X can take, and how to assign probabilities to those values

Example

Flip a coin two times

Sample space:

- {(Head, Tail), (Head, Head), (Tail, Head), (Tail, Tail)}

What is the probability of each event?

Clicker Question

If I toss a coin two times, and X is the number of heads, then what is $P(X = 2)$?

- a. $1/4$
- b. $1/2$
- c. $3/4$
- d. $5/4$

Additional Examples

Still flipping a coin twice, what is the probability of getting at least one head?

$$P(X = 1) + P(X = 2) = 1 - P(X = 0)$$

- Now I only have to calculate one probability!

Additional Dice Example

What is the **probability** of rolling a 7, 11, or double when rolling two dice?

- Axiom 3 tells us we can find probabilities simply by adding if the **event** is disjoint.

Roll a 7: $\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$

Roll a 11: $\{(5, 6), (6, 5)\}$

Roll a double: $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

- Are all 3 **events** disjoint?

Additional Dice Example

Roll a 7: $\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$

Roll a 11: $\{(5, 6), (6, 5)\}$

Roll a double: $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$$P(7) + P(11) + P(\text{doubles}) = \frac{6}{36} + \frac{2}{36} + \frac{6}{36} \approx 0.4$$