ECON 3818

Chapter 14

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Chapter 14: Binomial Distribution

Binomial Distribution

Probability model used when there are two outcomes: success or failure

- **Bernoulli distribution**: one trial.
- **Binomial distribution**: multiple trials.

Examples:

- Flipping a coin, shooting a free throw

Bernoulli Distribution

If X follows a Bernoulli process, then we say that

$$X \sim B(1, p)$$

- 1 tells us the number of trials
- p tells us the probability of success
- $\sim$: "is distributed"

Binomial Distribution

Generally, we focus on situations where we have more than one trial so we use the **binomial distribution**

The Binomial Setting

1. There are a fixed number, n, trials

2. These n trials are all independent

3. Each trial falls into one of just two categories -- "success" or "failure"

4. The probability of success, p, is the same for each observations

Notation: $X \sim B(n, p)$

- where n, p are *parameters*

Binomial Example

Shaquille O'Neal is allowed 5 free throw shots. He has a 60% chance of making each shot. What is the probability he makes 3 out of the 5 shots?

1. There are fixed number of trials
   - ✓ yes, n=5
2. These trials are independent
   - ✓ yes, maybe not (getting nervous) but go with it :-)
3. Each trial falls into one of two categories
   - ✓ yes, miss or make
4. Probability it the same for each trial
   - ✓ yes, p=0.6

$$X\ B(5, 0.6)$$

Clicker Question

For which of the following counts would a binomial probability model be reasonable? If not, why not?

a. the number of phone calls received in a one-hour period
b. the number of hearts you draw when you select 5 cards from a standard deck of 52 cards
c. the number of sevens in a randomly selected set of five digits from a table of random digits

Binomial Example

Important: There is more than one way to make 3 out 5 free throws

1. Make, Make, Make, Miss, Miss
2. Make, Make, Miss, Miss, Make
3. Make Make, Miss, Make, Miss
4. Make, Miss, Miss, Make, Make
5. Make, Miss, Make, Miss, Make
6. Miss, Make, Make, Make, Miss
7. Miss, Make, Make, Miss, Make
8. Miss, Make, Miss, Make, Make
9. Miss, Make, Make, Make, Miss
10. Miss, Miss, Make, Make, Make

Binomial Coefficient

First step to solving a binomial probability is to calculate the number of different ways of getting exactly k success in n observations.

This is **Binomial Coefficient**:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- $\binom{n}{k}$ is read "n choose k" which means "how many different ways to get k successes in n trials" (you can google n choose k to get this)
- $n! = n * (n-1) * (n-2) \ldots (3) * (2) * (1)$

Example: $5! = 5 * 4 * 3 * 2 * 1 = 120$

- Note: 0! = 1

Binomial Example

Lets use this formula to calculate how many ways are there to make 3 out 5 free throws

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{120}{6 * 2} = 10$$

Binomial Formula

Formula tells us the probability of getting k successes in n trials:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

- $\binom{n}{k}$: number of ways to get k successes in n trials
- p^k: probability of success, raised by the number of successes
- $(1 - p)^{n-k}$: probability of failure, raised by the number of failures

Binomial Example

Back to our previous example. Shaq is shooting 5 free throws and has a 60% chance of making each one. What is the probability he makes 3?

$$P(X=3)=\binom{5}{3}*0.6^3*0.4^2=0.3456$$

Clicker Question

What is the probability of making 4 out of 6 penalty kicks if the probability of scoring is 70%?

a. 70%
b. 2.2%
c. 32.4%
d. 65.47%

Example

A local veterinary clinic typically sees 15% of its horses presenting with West Nile virus. If 10 horses are admitted during July, what is the probability that 2 or fewer horses among the 10 horses admitted have been infected with West Nile virus?

Binomial Probabilities

- The binomial formula

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

  only calculates the probability that X is equal to one specific (discrete) number.

In order to calculate the probability that X takes on multiple value means we must use this formula repeatedly

- What if we wanted to know the probability that Shaq makes at least two of the five free throws?

$$P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

Cumulative Binomial Probabilities

Recall the probability rule:

$$P(A^c) = 1 - P(A)$$

We can use this when calculating binomial probabilities. Make sure to pay attention to the wording!

At least is inclusive.

- P(at least 1) = (P(X\geq1)=1-P(X=0))

More than is not inclusive

- P(more than 1)= (P(X>1)=1-P(X=0)-P(X=1))

Cumulative Example

What is the probability that Shaq makes at least two out of his five free throws?

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

- $P(X = 0) = \binom{5}{0} * 0.6^0 * 0.4^5 = 0.010$
- $P(X = 1) = \binom{5}{1} * 0.6^1 * 0.4^4 = 0.077$
- $P(X \geq 2) = 1 - 0.01 - 0.077 = 0.913$

Clicker Question

You are asking someone out on a date. Your probability of success is 35% each time you try. If you ask out 4 people, what are the odds that you get at least one yes?

a. 98.5%
b. 1.5%
c. 82.15%
d. 17.85%

Binomial Mean and Standard Deviation

If $X \sim B(n, p)$, the **mean** and **standard deviation** of X are:

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

Clicker Question (Midterm Example)

If X has a binomial distribution with 20 trials and a mean of 6, then the success probability, p, is:

a. 0.3
b. 0.5
c. 0.75
d. Cannot be determined given the information

Recognizing the Binomial Setting

Which of the three scenarios would it be reasonable to use a binomial distribution for random variable X?

- An auto manufacturer chooses one car from each hour's production for a quality inspection. The variable X is the count of defects in the car's paint
- The pool of potential jurors for a murder case contains 100 people chosen at random. Each person in in the pool is asked whether they oppose the death penalty. X is the number of people who say yes
- Joe buys a ticket in his state's lottery game every week. X is the number of times in that year that he receives a prize.

Example

You're taking an exam consisting of 8 multiple choice questions, each with 4 available answers. You forgot to study, so you have to guess on every question. What is the probability you only get 2 questions wrong?

a. 0.3114
b. 0.0038