Math Notes for Compound Interest

$$
\begin{aligned}
D_0 &= P \\
D_1 &= (P - p)\left(1 + \frac{r}{n}\right) \\
D_2 &= \left((P - p)\left(1 + \frac{r}{n}\right) - p\right)\left(1 + \frac{r}{n}\right) \\
&= P\left(1 + \frac{r}{n}\right)^2 - p\left(\left(1 + \frac{r}{n}\right)^2 + \left(1 + \frac{r}{n}\right)\right) \\
&\vdots \\
D_{nt} &= P\left(1 + \frac{r}{n}\right)^{nt} - p\sum_{i=1}^{nt}\left(1 + \frac{r}{n}\right)^i
\end{aligned}
$$

$$
\begin{aligned}
p &= qP \\
D_{nt} &= P\left(1 + \frac{r}{n}\right)^{nt} - qP\sum_{i=1}^{nt}\left(1 + \frac{r}{n}\right)^i \\
D_{nt} &= P\left[\left(1 + \frac{r}{n}\right)^{nt} - q\sum_{i=1}^{nt}\left(1 + \frac{r}{n}\right)^i\right]
\end{aligned}
$$

$$D_{nt} = 0$$

$$P\left(1+\frac{r}{n}\right)^{nt} - p\sum_{i=1}^{nt}\left(1+\frac{r}{n}\right)^{i} = 0$$

$$p = \frac{\left(1+\frac{r}{n}\right)^{nt}}{\sum_{i=1}^{nt}\left(1+\frac{r}{n}\right)^{i}}$$

$$\begin{aligned}
x &= \sum_{i=1}^{n} a^{i} \\
\frac{x}{a} &= \sum_{i=0}^{n-1} a^{i} \\
\frac{x}{a} + a^{n} - 1 &= \sum_{i=1}^{n} a^{i} \\
\frac{x}{a} + a^{n} - 1 &= x \\
\left(1-\frac{1}{a}\right)x &= a^{n} - 1 \\
\left(\frac{a-1}{a}\right)x &= a^{n} - 1 \\
x &= \frac{a^{(n+1)} - a}{a-1}
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^{nt}\left(1+\frac{r}{n}\right)^{i} &= \frac{\left(1+\frac{r}{n}\right)^{(nt+1)} - \left(1+\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)-1} \\
&= \frac{n}{r}\left(\left(1+\frac{r}{n}\right)^{(nt+1)} - \left(1+\frac{r}{n}\right)\right) \\
p &= \frac{\left(1+\frac{r}{n}\right)^{nt}}{\frac{n}{r}\left(\left(1+\frac{r}{n}\right)^{(nt+1)} - \left(1+\frac{r}{n}\right)\right)} \\
&= \frac{r}{n\left(\left(1+\frac{r}{n}\right) - \left(1+\frac{r}{n}\right)^{(1-nt)}\right)}
\end{aligned}$$