

# Leetcode

Hanhee Lee

April 17, 2025

## Contents

<b>1</b>	<b>General</b>	<b>3</b>
1.1	Interviewer Considerations . . . . .	3
1.2	Steps for Success During the Technical Interview . . . . .	3
1.3	Common Mistakes to Avoid . . . . .	3
1.4	Syntax . . . . .	4
<b>2</b>	<b>Arrays and Hashing</b>	<b>5</b>
2.1	When to Use? . . . . .	5
2.2	Hashing . . . . .	5
2.3	Common Problems . . . . .	6
<b>3</b>	<b>Two Pointers</b>	<b>8</b>
3.1	When to Use? . . . . .	8
3.2	Slow and Fast Pointers . . . . .	8
3.2.1	Common Problems . . . . .	8
3.3	Left and Right Pointers . . . . .	9
3.3.1	Common Problems . . . . .	10
<b>4</b>	<b>Sliding Window</b>	<b>12</b>
4.1	Fixed Sliding Window . . . . .	12
4.1.1	Common Problems . . . . .	13
4.2	Dynamic Sliding Window . . . . .	14
4.2.1	Common Problems . . . . .	15
<b>5</b>	<b>Binary Search</b>	<b>17</b>
5.1	When to Use? . . . . .	17
5.1.1	Common Problems . . . . .	18
<b>6</b>	<b>Linked List</b>	<b>19</b>
6.1	When to Use? . . . . .	19
6.2	Singly Linked List . . . . .	19
6.3	Operations . . . . .	19
6.4	Doubly Linked List . . . . .	20
6.5	Operations . . . . .	20
6.6	Circular Linked List . . . . .	21
6.7	Operations . . . . .	21
6.8	Common Problems . . . . .	22
<b>7</b>	<b>Stack and Queue</b>	<b>25</b>
7.1	collections.deque . . . . .	25
7.2	Stack . . . . .	26
7.2.1	When to Use? . . . . .	26
7.2.2	Operations . . . . .	26
7.2.3	Common Problems . . . . .	27
7.3	Queue . . . . .	29
7.3.1	When to Use? . . . . .	29

7.3.2	Operations	29
<b>8</b>	<b>Trees</b>	<b>30</b>
8.1	Binary Search Tree (BST)	30
8.2	Operations	31
8.2.1	Search	31
8.2.2	Insert	31
8.2.3	Delete	31
8.2.4	Find Min	32
8.2.5	Find Max	32
8.2.6	DFS In-order Traversal (Left $\rightarrow$ Root $\rightarrow$ Right)	32
8.2.7	DFS Pre-order Traversal (Root $\rightarrow$ Left $\rightarrow$ Right)	33
8.2.8	DFS Post-order Traversal (Left $\rightarrow$ Right $\rightarrow$ Root)	33
8.2.9	BFS Level-order Traversal (Top $\rightarrow$ Bottom, Left $\rightarrow$ Right)	33
8.2.10	Common Problems	34
8.2.11	BST-based Sets and Maps	35
<b>9</b>	<b>Heaps and Priority Queues</b>	<b>36</b>
9.1	Heap	36
9.2	Heapq	36
9.3	Operations	38
9.3.1	Insert	38
9.3.2	Heapify	38
9.3.3	Extract Min	38
9.3.4	Get Min	39
9.3.5	Build Heap	39
9.3.6	Search	39
9.3.7	Delete	39
9.3.8	Heap Sort	39
9.3.9	Common Problem	41
<b>10</b>	<b>Graphs</b>	<b>43</b>
10.1	Breadth-First Search (BFS)	43
10.1.1	Common Problems	44
10.2	Depth-First Search (DFS)	45
10.2.1	Common Problems	46
10.3	Topological Sort	47
<b>11</b>	<b>Sorting</b>	<b>48</b>
11.1	Stable, In-place, and Divide and Conquer	49
11.2	Merge Sort	49
11.3	Quick Sort	49
<b>12</b>	<b>Images</b>	<b>50</b>
12.1	2D Convolution Operations	50
12.2	Common Problems	50

# 1 General

## 1.1 Interviewer Considerations

### Notes:

- How did the candidate **analyze** the problem?
- Did the candidate miss any special or **edge** cases?
- Did the candidate approach the problem **methodically** and logically?
- Does the candidate have a strong foundation in basic computer science **concepts**?
- Did the candidate produce **working code**? Did the candidate **test** the code?
- Is the candidate's code clean and easy to read and **maintain**?
- Can the candidate **explain** their ideas clearly?

## 1.2 Steps for Success During the Technical Interview

### Summary:

1. **Clarify the question**
  - (a) Understand what the question is asking and gather example inputs and outputs.
  - (b) Clarify constraints such as:
    - i. Can numbers be negative or repeated?
    - ii. Are values sorted or do we need to sort them?
    - iii. Can we assume input validity?
  - (c) Asking clarifying questions shows communication skills and prevents missteps.
2. **Design a solution**
  - (a) Avoid immediate coding; propose an initial approach and refine it.
  - (b) Analyze the algorithm's time and space complexity.
  - (c) Consider and address edge cases.
  - (d) Think aloud to demonstrate logical reasoning and collaboration.
  - (e) Discuss non-optimal ideas to show your thought process.
3. **Write your code**
  - (a) Structure the solution using helper functions.
  - (b) Confirm API details when uncertain.
  - (c) Use your strongest programming language and full syntax.
  - (d) Write complete, working code—not pseudocode.
4. **Test your code**
  - (a) Validate your solution with 1–2 example test cases.
  - (b) Walk through each line using inputs.
  - (c) Do not assume correctness—prove it through testing.
  - (d) Discuss any further optimizations and their trade-offs.

## 1.3 Common Mistakes to Avoid

### Warning:

1. Starting to code without clarifying the problem.
2. Failing to write or discuss sample inputs and outputs.
3. Using pseudocode instead of fully functional code.
4. Misunderstanding the problem or optimizing prematurely.

## 1.4 Syntax

### Summary:

1. `dict.items()`
  - Returns a view object that displays a list of a dictionary's key-value tuple pairs.
2. `sorted(iterable, key=..., reverse=...)`
  - `iterable`: The sequence or collection (e.g., list, dictionary view) to be sorted.
  - `key=...`: A function that extracts a comparison key from each element. Sorting is performed based on the result of this function.
    - `key=lambda x: x[0]`: Sort by the first element of each tuple.
    - `key=lambda x: x[1]`: Sort by the second element of each tuple.
  - `reverse=...`: A boolean value. If `True`, sorted in descending order; otherwise, sorted in ascending order (default is `False`).
3. `collections.Counter(iterable)`
  - Counts the frequency of each unique element in `iterable` and returns a dictionary-like object.
  - **Arguments:**
    - `iterable`: a sequence (e.g., list, string) or any iterable containing hashable elements.

## 2 Arrays and Hashing

### 2.1 When to Use?

#### Summary:

- To count frequencies in  $O(n)$  time.
- To check membership in constant time.
- To map keys to values (e.g., index, count, group).
- To group elements by shared features (e.g., anagrams).
- To detect duplicates efficiently.

### 2.2 Hashing

#### Algorithm:

```
1 def solve_problem(nums):
2     # Step 1: Initialize the hashmap (e.g., for frequency, index, or existence check)
3     hashmap = {}
4
5     # Step 2: Iterate over the array
6     for i, num in enumerate(nums):
7         # Step 3: Define your condition (e.g., check complement, existence, frequency)
8         if some_condition_based_on_hashmap(num, hashmap):
9             # Step 4: Return or process result as needed
10            return result_based_on_condition
11
12        # Step 5: Update the hashmap
13        hashmap_update_logic(num, i, hashmap)
14
15    # Step 6: Handle the case where the condition is never met
16    return final_result_if_needed
17
18 # Helper functions (replace with actual logic based on the problem)
19 def some_condition_based_on_hashmap(num, hashmap):
20     # Example: return (target - num) in hashmap
21     pass
22
23 def hashmap_update_logic(num, i, hashmap):
24     # Example: hashmap[num] = i
25     pass
```

## 2.3 Common Problems

### Summary:

Problem	Description:
217. Contains Duplicate	Given an integer array <code>nums</code> , return <code>true</code> if any value appears at least twice. <ul style="list-style-type: none"> <li>Use a set to store the elements. If an element is already in the set, return <code>True</code>.</li> <li>Otherwise, add it to the set.</li> </ul>
242. Valid Anagram	Given two strings <code>s</code> and <code>t</code> , return <code>true</code> if <code>t</code> is an anagram of <code>s</code> and <code>false</code> otherwise. <ul style="list-style-type: none"> <li>Use a <code>HashMap</code> to count the frequency of each character in <code>s</code> and <code>t</code>.</li> <li>If the frequency maps are equal, return <code>True</code>. Otherwise, return <code>False</code>.</li> </ul>
1. Two Sum	Given an array of integers, return indices of the two numbers s.t. they add up to a specific target. <ul style="list-style-type: none"> <li><b>Tricks:</b> <ul style="list-style-type: none"> <li>Use a <code>HashMap</code> to store the indices of the elements, <code>prevMap[nums[i]] = i</code></li> <li>For each element, check if the <code>target - nums[i]</code> is in the map.</li> <li>If it is, return the index of the <code>target - nums[i]</code> (from <code>prevMap</code>) and <code>i</code>. Otherwise, add <code>target - nums[i]</code>.</li> </ul> </li> </ul>
**49. Group Anagrams	Given an array of strings, group the anagrams together. <ul style="list-style-type: none"> <li>Use a <code>HashMap</code> to store a tuple of count of each char as the key and the list of words as the value.</li> <li>For each word, create a tuple of count of each char and add the word to the list in the map.</li> <li>Finally, return the values of the map.</li> </ul>
**347. Top K Frequent Elements	Given an integer array <code>nums</code> and an integer <code>k</code> , return the <code>k</code> most frequent elements. <ul style="list-style-type: none"> <li>Use a <code>HashMap</code> to count the frequency of each element.</li> <li>Sort the map by frequency and return the top <code>k</code> elements.</li> </ul>
118. Pascal's Triangle	Given an integer <code>numRows</code> , return the first <code>numRows</code> of Pascal's triangle. <ul style="list-style-type: none"> <li><b>Initialize:</b> <code>res = [[1]]</code>.</li> <li><b>Loop from numRows - 1:</b> <ul style="list-style-type: none"> <li><b>Pad the PrevRow:</b> Create <code>dummy_row</code> by padding the last row in <code>res</code> with zeros at both ends.</li> <li><b>Loop 2 from len(prevRow) + 1:</b> For each position <code>i</code>, compute the value <code>dummy_row[i] + dummy_row[i+1]</code> and append it to the new row.</li> </ul> </li> </ul>

**Summary:****Problem**

73. Set Matrix Zeroes

**Description:**

Given an  $m \times n$  integer matrix, if an element is 0, set its entire row and column to 0.

- **Record Zero Positions:** Iterate through all elements. If `matrix[i][j] == 0`, append `[i, j]` to list.
- **Row/Column Zeroing:** Set all elements in column `col_ind` to zero and all elements in row `row_ind` to zero using two helpers.

54. Spiral Matrix

Given an  $m \times n$  matrix, return all elements of the matrix in spiral order.

- **Initialize:** Create an empty list `res`, set boundaries: `top`, `bottom`, `left`, `right`, and current pos  $(i, j)$ .
- **Loop:** While `top <= bottom` and `left <= right`. Use helper functions to achieve the following:
  - Traverse from left to right along the top row and adjust top bdy and check if `top > bottom`.
  - Traverse from top to bottom along the right column and adjust right bdy and check if `left > right`.
  - Traverse from right to left along the bottom row and adjust bottom bdy and check if `top > bottom`.
  - Traverse from bottom to top along the left column and adjust left bdy and check if `left > right`.

## 3 Two Pointers

### 3.1 When to Use?

#### Summary:

- If we need to find a pair of elements that satisfy a condition.
- If we need to find a subarray that satisfies a condition.

### 3.2 Slow and Fast Pointers

#### Algorithm:

1.

#### 3.2.1 Common Problems

#### Summary:

Problem	Description:
15. 3Sum	Given an array of integers, return all the triplets [nums[i], nums[j], nums[k]] s.t. $i \neq j$ , $i \neq k$ , and $j \neq k$ .
<ul style="list-style-type: none"> <li>• <b>Tricks:</b></li> </ul>	
125. Valid Palindrome	Given a string, determine if it is a palindrome, considering only alphanumeric characters and ignoring cases.
<ul style="list-style-type: none"> <li>• <code>s_new = ".join(char.lower() for char in s if char.isalnum())</code> to remove non-alphanumeric and lowercase.</li> <li>• Use front and back pointers. If they not equal, return False. If equal move both pointers.</li> </ul>	
167. Two Sum II - Input array is sorted	Given an array of integers that is already sorted in ascending order, find two numbers such that they add up to a target.
<ul style="list-style-type: none"> <li>• Use front and back pointers. If <math>&gt;</math> target, move back pointer left. If <math>&lt;</math> target, move front pointer right.</li> </ul>	



### 3.3 Left and Right Pointers

**Algorithm:**

1. Initialize two pointers. Some common choices:
  - One at the front and one at the back of the array.
  - Both at the front of the array.
  - Both at the back of the array.

### 3.3.1 Common Problems

#### Summary:

Problem	Description:
15. 3Sum	Given an array of integers, return all the triplets [nums[i], nums[j], nums[k]] s.t. $i \neq j$ , $i \neq k$ , and $j \neq k$ .
<ul style="list-style-type: none"> <li>• <b>Tricks:</b></li> </ul>	
125. Valid Palindrome	Given a string, determine if it is a palindrome, considering only alphanumeric characters and ignoring cases.
<ul style="list-style-type: none"> <li>• <code>s_new = ".join(char.lower() for char in s if char.isalnum())</code> to remove non-alphanumeric and lowercase.</li> <li>• Use front and back pointers. If they not equal, return False. If equal move both pointers.</li> </ul>	
167. Two Sum II - Input array is sorted	Given an array of integers that is already sorted in ascending order, find two numbers such that they add up to a target.
<ul style="list-style-type: none"> <li>• Use front and back pointers. If <math>&gt;</math> target, move back pointer left. If <math>&lt;</math> target, move front pointer right.</li> </ul>	
11. Container With Most Water	<p>Given <math>n</math> non-negative integers <math>a_1, a_2, \dots, a_n</math>, where each represents a point at coordinate <math>(i, a_i)</math>. <math>n</math> vertical lines are drawn such that the two endpoints of line <math>i</math> is at <math>(i, a_i)</math> and <math>(i, 0)</math>. Find two lines, which together with x-axis forms a container, such that the container contains the most water.</p>
<ul style="list-style-type: none"> <li>• <b>Initialization:</b> <ul style="list-style-type: none"> <li>– Set <code>left = 0</code>, <code>right = len(height) - 1</code> to have the width as large as possible.</li> <li>– Initialize <code>maxWater = 0</code>.</li> </ul> </li> <li>• <b>Water Area Calculation:</b> Define helper function <code>amtWater(height, left, right)</code>:           <ul style="list-style-type: none"> <li>– Compute <code>height = min(height[left], height[right])</code>.</li> <li>– Compute <code>width = right - left</code>.</li> <li>– Return <code>height * width</code>.</li> </ul> </li> <li>• <b>Two-Pointer Strategy: While <code>left &lt;= right</code>:</b> <ul style="list-style-type: none"> <li>– Compute area between <code>left</code> and <code>right</code>, update <code>maxWater</code>.</li> <li>– Move the pointer at the shorter line inward:               <ul style="list-style-type: none"> <li>* If <code>height[left] &lt;= height[right]</code>, increment <code>left</code>.</li> <li>* Else, decrement <code>right</code>.</li> </ul> </li> </ul> </li> </ul>	

**Summary:**

Problem	Description:
42. Trapping Rain Water	<p>Given n non-negative integers representing an elevation map where the width of each bar is 1, compute how much water it can trap after raining.</p> <ul style="list-style-type: none"><li>• <b>Initialization:</b> If <code>height</code> is empty, return 0.<ul style="list-style-type: none"><li>– Set two pointers: <code>l = 0, r = len(height) - 1</code>.</li><li>– Initialize <code>leftMax = height[l], rightMax = height[r], res = 0</code>.</li></ul></li><li>• <b>Two-Pointer Traversal: While <code>l &lt; r</code>:</b><ul style="list-style-type: none"><li>– If <code>leftMax &lt; rightMax</code>:<ul style="list-style-type: none"><li>* Increment <code>l</code>. Update <code>leftMax = max(leftMax, height[l])</code>.</li><li>* Accumulate water: <code>res += leftMax - height[l]</code>.</li></ul></li><li>– Else:<ul style="list-style-type: none"><li>* Decrement <code>r</code>. Update <code>rightMax = max(rightMax, height[r])</code>.</li><li>* Accumulate water: <code>res += rightMax - height[r]</code>.</li></ul></li></ul></li><li>• <b>Intuition:</b><ul style="list-style-type: none"><li>– Always move the pointer at the side with the smaller maximum, since the water trapped at that point depends on the limiting side.</li><li>– The water accumulated for a column is determined by the height of the column minus the min of the maximum heights on both sides.</li></ul></li></ul>

## 4 Sliding Window

### 4.1 Fixed Sliding Window

#### Summary:

- Find a subarray/substring of a fixed size that satisfies a condition.
- Find the maximum or minimum of a subarray of a fixed size.

#### Algorithm:

```
1 initialize window_sum = 0
2 initialize max_result (or other required value)
3
4 # Set up initial window
5 for i in range(0, k):
6     window_sum += arr[i]
7
8 max_result = window_sum # Initialize result
9
10 # Slide the window
11 for i in range(k, n):
12     window_sum += arr[i] - arr[i - k] # Add new element and remove 1st element of prev window
13     max_result = max(max_result, window_sum) (or other computation)
14
15 return max_result (or other required value)
16
```

## 4.1.1 Common Problems

## Summary:

Problem	Description:
643. Maximum Average Subarray I	Given an integer array <code>nums</code> and an integer <code>k</code> , return the maximum average value of a subarray of length <code>k</code> . <ul style="list-style-type: none"> <li>Follow template.</li> </ul>
567. Permutation in String	Given two strings <code>s1</code> and <code>s2</code> , return true if <code>s2</code> contains a permutation of <code>s1</code> , or false otherwise. <ul style="list-style-type: none"> <li><b>Init:</b> Follow template with <code>window_valid</code>, <code>freqMap_window</code>, <code>freqMap_s1</code>, and fixed size <code>k</code> of <code>len(s1)</code>. Rather than sum, get freq of chars.</li> <li><b>Special Case:</b> If <code>len(s1) &gt; len(s2)</code>, return False.</li> <li><b>For:</b> Since contiguous, slide through <code>s2</code> and update <code>freqMap_window</code> by adding new char and removing old char (make sure to del key if <code>freq = 0</code>).</li> <li><b>Condition:</b> If <code>freqMap_window == freqMap_s1</code>, return True.</li> </ul>
219. Contains Duplicate II	Given an integer array <code>nums</code> and an integer <code>k</code> , return true if there are two distinct indices <code>i</code> and <code>j</code> in the array such that <code>nums[i] == nums[j]</code> and <code>abs(i - j) &lt;= k</code> . <ul style="list-style-type: none"> <li><b>Init:</b> Follow template with <code>window_freq</code> and fixed size <code>k</code>.</li> <li><b>Special Case:</b> If <code>len(nums) &lt; 2</code>, return False.</li> <li><b>Initial window:</b> <code>Range(min(k+1, len(nums)))</code> since first window can be smaller than <code>k</code>.</li> </ul>

## 4.2 Dynamic Sliding Window

### Summary:

- Find longest or shortest subarray/substring that satisfies a condition.

### Algorithm:

```
1 initialize left = 0
2 initialize window_state (sum, count, frequency map, etc.)
3 initialize min_or_max_result
4
5 for right in range(n):
6     update window_state to include arr[right] # Expand the window
7
8     while window_state violates the condition:
9         update min_or_max_result (if needed)
10        update window_state to exclude arr[left] # Shrink the window
11        move left pointer forward
12
13 return min_or_max_result
```

## 4.2.1 Common Problems

## Summary:

Problem	Description:
121. Best Time to Buy and Sell Stock	<p>Given an array where the <math>i</math>th element is the price of a stock on day <math>i</math>, find the maximum profit you can achieve. You may not engage in multiple transactions.</p> <ul style="list-style-type: none"> <li>• <b>Buy low, sell high principle</b> <ul style="list-style-type: none"> <li>– Use <math>left = buy</math> and <math>right = sell</math>, initialized at 0, 1.</li> <li>– If <math>price[right] \geq price[left]</math>, update max profit. Move right pointer since we can still sell for a profit.</li> <li>– If <math>price[right] &lt; price[left]</math>, move left pointer since we need to find a lower price to buy.</li> <li>– Continue until right pointer reaches the end of the array.</li> </ul> </li> </ul>
3. Longest Substring W/O Repeating Characters	<p>Given a string <math>s</math>, find the length of the longest substring without repeating characters.</p> <ul style="list-style-type: none"> <li>• <b>Init:</b> Follow template and use frequency map of chars for <code>window_state</code>.</li> <li>• <b>While:</b> If a char is repeated, move left pointer to right by 1 and adjust <code>freqMap</code> until current char is unique.</li> <li>• <b>Change:</b> Compare substring length outside of while with <code>max_res = max(max_res, right - left + 1)</code>.</li> </ul>
424. Longest Repeating Character Replacement	<p>Given a string <math>s</math> that consists of only uppercase English letters, you can replace any letter with another letter. Find the length of the longest substr containing the same letter after performing at most <math>k</math> replacements.</p> <ul style="list-style-type: none"> <li>• <b>Init:</b> Follow template and use <code>freqMap</code> of chars for <code>window_state</code>.</li> <li>• <b>While:</b> If the number of replacements needed exceeds <math>k</math>, i.e. <math>(r - l + 1) - \max\_freq &gt; k</math> <ul style="list-style-type: none"> <li>– Move left pointer to right by 1 and adjust <code>freqMap</code> until the condition is satisfied.</li> </ul> </li> <li>• <b>Change:</b> Compare substring length outside of while with <code>max_res = max(max_res, right - left + 1)</code>.</li> </ul>
**76. Minimum Window Substring	<p>Given two strings <math>s</math> and <math>t</math>, return the minimum window substr of <math>s</math> such that every character in <math>t</math> (including duplicates) is included in the window. If there is no such substring, return ""</p> <ul style="list-style-type: none"> <li>• <b>Init:</b> Set <math>left = 0</math>. Initialize <code>count_t</code> as frequency map of <math>t</math>, <code>count_s</code> for current window, and variables <code>have = 0</code>, <code>required = len(count_t)</code>, <code>res = [-1, -1]</code>, and <code>resLen = inf</code>.</li> <li>• <b>For right in range(n):</b> Expand window by adding <math>s[right]</math> to <code>count_s</code>. If relevant char and frequency matches <code>count_t</code>, increment <code>have</code>. <ul style="list-style-type: none"> <li>– <b>While have == required:</b> <ul style="list-style-type: none"> <li>* Update result if current window is smaller w/ coordinates <code>res = [left, right]</code> and length <code>resLen = right - left + 1</code>.</li> <li>* Shrink window by <math>\downarrow count\_s[s[left]]</math>; if below <code>count_t</code>, decrement <code>have</code>; increment <code>left</code>.</li> </ul> </li> </ul> </li> <li>• <b>Return:</b> <code>s[res[0]:res[1]+1]</code> if valid window found, else empty string.</li> </ul>

**Summary:****Problem**

239. Sliding Window Maximum

- Hi

**Description:**

Given an integer array `nums` and an integer `k`,  
return the maximum value in each sliding window of size `k`.



## 5 Binary Search

### Algorithm:

```
1 def binary_search(nums, target):
2     left, right = 0, len(nums) - 1
3
4     while left <= right:
5         mid = (left + right) // 2
6
7         if nums[mid] == target:
8             return mid
9         elif nums[mid] < target:
10            left = mid + 1
11        else:
12            right = mid - 1
13
14    return -1
```

### 5.1 When to Use?

#### Summary:

- Use when the input is **sorted** or can be **monotonically mapped**.
- Common for problems involving **searching for a target**, **finding boundaries**, or **min/max constraints**.
- Works on arrays, answer ranges, or implicit search spaces with  $\mathcal{O}(\log n)$  complexity.

## 5.1.1 Common Problems

## Summary:

Problems	Description
704. Binary Search	Given a sorted array of integers, return the index of the target. If not found, return -1.
<ul style="list-style-type: none"> <li>Implement binary search.</li> </ul>	
74. Search a 2D Matrix	Given a 2D matrix, search for a target value. If found, return its index.
<ul style="list-style-type: none"> <li>Use binary search to find the row by comparing first value of mid row &lt; target (decrement bottom) &amp; last value of mid row &lt; target (increment top).</li> <li>Row is <math>\text{top} + \text{bottom} // 2</math>.</li> <li>Then, use binary search to find the column in that row.</li> </ul>	
875. Koko Eating Bananas	Given an array of piles and an integer h, find the minimum eating speed k such that Koko can eat all bananas in h hours.
<ul style="list-style-type: none"> <li>Use binary search to find the minimum k w/ <math>l = 0, r = \max(\text{piles})</math> since k must be in this range.</li> <li>Check if k is valid by calculating the total hours needed to eat all bananas. <ul style="list-style-type: none"> <li>for p in piles <math>\text{total\_hours} += \text{math.ceil}(p / k)</math></li> <li>Compare total_hours with h. If <math>\text{hours} \leq h</math>, update <math>r = \text{mid} - 1</math> and <math>\text{res} = \text{mid}</math>. Else update <math>l = \text{mid} + 1</math>.</li> </ul> </li> </ul>	
**153. Find Minimum in Rotated Sorted Array	Given a rotated sorted array, find the minimum element.
<ul style="list-style-type: none"> <li>Initialize <code>res = nums[0]</code> as a candidate minimum.</li> <li>Set binary search bounds: <math>l = 0, r = \text{len}(\text{nums}) - 1</math>.</li> <li>While <math>l \leq r</math>: <ul style="list-style-type: none"> <li>If <math>\text{nums}[l] &lt; \text{nums}[r]</math>, subarray is sorted; update <code>res = min(res, nums[l])</code> and break.</li> <li>Compute midpoint <math>m = (l + r) // 2</math>, update <code>res = min(res, nums[m])</code>.</li> <li>If <math>\text{nums}[m] \geq \text{nums}[l]</math>, left half is sorted; search right: <math>l = m + 1</math>.</li> <li>Else, pivot is in left half; search left: <math>r = m - 1</math>.</li> </ul> </li> <li>Return <code>res</code> as the minimum element.</li> </ul>	
**33. Search in Rotated Sorted Array	Given a rotated sorted array, search for a target value. If found, return its index.
<ul style="list-style-type: none"> <li></li> </ul>	

## 6 Linked List

**Summary:** Data structure for storing objects in linear order.

- **Object:** Data and a pointer to the next object.

### 6.1 When to Use?

**Summary:**

- Implement other DS: stacks, queues, hash tables.
- Dynamic memory allocation.

### 6.2 Singly Linked List

**Algorithm:**

```
1 class Node:
2     def __init__(self, data):
3         self.data = data # Value stored in the node
4         self.next = None # Pointer to the next node
5
6 class SinglyLinkedList:
7     def __init__(self, data):
8         self.head = Node(data) # Head of the list
9
10    def operations(self):
11        pass
```

### 6.3 Operations

**Summary:**

Operation	Time Complexity (WC)
Search	$O(n)$
Insert	$O(n)$
Delete	$O(n)$
Access	$O(n)$

## 6.4 Doubly Linked List

### Algorithm:

```
1 class Node:
2     def __init__(self, data):
3         self.data = data # Value stored in the node
4         self.next = None # Pointer to the next node
5         self.prev = None # Pointer to the previous node
6
7 class DoublyLinkedList:
8     def __init__(self, data):
9         self.head = Node(data) # Head of the list
10
11     def operations(self):
12         pass
```

## 6.5 Operations

### Summary:

Operation	Time Complexity (WC)
Search	$O(n)$
Insert	$O(n)$
Delete	$O(n)$
Access	$O(n)$

## 6.6 Circular Linked List

### Algorithm:

```
1 class Node:
2     def __init__(self, data):
3         self.data = data # Value stored in the node
4         self.next = None # Pointer to the next node
5
6 class CircularLinkedList:
7     def __init__(self, data):
8         self.head = Node(data) # Head of the list
9         self.head.next = self.head # Point to itself
10
11     def operations(self):
12         pass
```

## 6.7 Operations

### Summary:

Operation	Time Complexity (WC)
Search	$O(n)$
Insert	$O(n)$
Delete	$O(n)$
Access	$O(n)$

## 6.8 Common Problems

### Summary:

Problem	Description:
206. Reverse Linked List	Given the head of a singly linked list, reverse the list and return the reversed list.
<ul style="list-style-type: none"> <li>• <b>Iterative:</b> <ul style="list-style-type: none"> <li>– Init: <math>\underbrace{\text{None}}_{\text{prev}} \rightarrow \underbrace{1}_{\text{cur}} \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5</math></li> <li>– While loop: <ul style="list-style-type: none"> <li>* Temp: <math>\underbrace{\text{None}}_{\text{prev}} \rightarrow \underbrace{1}_{\text{cur}} \rightarrow \underbrace{2}_{\text{temp}} \rightarrow 3 \rightarrow 4 \rightarrow 5</math></li> <li>* Switch link: <math>\underbrace{\text{None}}_{\text{prev}} \leftarrow \underbrace{1}_{\text{cur}} \rightarrow \underbrace{2}_{\text{temp}} \rightarrow 3 \rightarrow 4 \rightarrow 5</math></li> <li>* Swap positions: <math>\underbrace{1}_{\text{prev} = \text{cur}} \rightarrow \underbrace{2}_{\text{cur} = \text{temp}} \rightarrow 3 \rightarrow 4 \rightarrow 5</math></li> </ul> </li> </ul> </li> </ul>	
21. Merge Two Sorted Lists	Given two sorted linked lists, merge them into one sorted list.
<ul style="list-style-type: none"> <li>• <b>Initialize a dummy head:</b> <ul style="list-style-type: none"> <li>– <code>dummy = ListNode()</code>: Placeholder node to simplify edge case handling.</li> <li>– <code>current = dummy</code>: Build the merged list step-by-step.</li> </ul> </li> <li>• <b>Iterate through both lists while neither is empty:</b> <ul style="list-style-type: none"> <li>– <b>Compare current nodes:</b> <ul style="list-style-type: none"> <li>* If <code>list1.val &lt;= list2.val</code>, attach <code>list1</code>'s node to <code>current.next</code>, and move <code>list1</code> forward.</li> <li>Otherwise, do the same for <code>list2</code>.</li> </ul> </li> <li>– Move the <code>current</code> pointer forward.</li> </ul> </li> <li>• <b>Attach remaining nodes (if any):</b> After the loop, only one of <code>list1</code> or <code>list2</code> may still have nodes left. <ul style="list-style-type: none"> <li>– <code>current.next = list1</code> if <code>list1</code> else <code>list2</code> ensures the remainder is attached.</li> </ul> </li> <li>• <b>Return the merged list:</b> return <code>dummy.next</code> returns actual start of the merged list.</li> <li>– <b>Visualization:</b> <ul style="list-style-type: none"> <li>* <b>Initial state:</b> <ul style="list-style-type: none"> <li>· List1: <math>\underbrace{1}_{\text{list1}} \rightarrow 2 \rightarrow 4</math></li> <li>· List2: <math>\underbrace{1}_{\text{list2}} \rightarrow 3 \rightarrow 4</math></li> <li>· Merged List: <math>\underbrace{\text{dummy}}_{\text{start}} \rightarrow</math></li> </ul> </li> <li>* <b>Step 1: Compare list1 and list2 (<math>1 \leq 1</math>), take list1</b> <ul style="list-style-type: none"> <li>· <code>dummy</code> <math>\rightarrow \underbrace{1}_{\text{current}} \rightarrow</math></li> <li>· list1 <math>\rightarrow \underbrace{2}_{\text{list1}} \rightarrow 4</math></li> <li>· list2 unchanged: <math>\underbrace{1}_{\text{list2}} \rightarrow 3 \rightarrow 4</math></li> </ul> </li> <li>* <b>Step 2: Compare 2 and 1 (<math>2 &gt; 1</math>), take list2</b> <ul style="list-style-type: none"> <li>· <code>dummy</code> <math>\rightarrow 1 \rightarrow \underbrace{1}_{\text{current}} \rightarrow</math></li> <li>· list1 remains: <math>\underbrace{2}_{\text{list1}} \rightarrow 4</math></li> <li>· list2 <math>\rightarrow \underbrace{3}_{\text{list2}} \rightarrow 4</math></li> </ul> </li> </ul> </li> </ul>	

## Summary:

Problem	Description:
141. Linked List Cycle	Given a linked list, determine if it has a cycle in it.
<ul style="list-style-type: none"> <li>• <b>Floyd's Cycle Detection Algorithm:</b> <ul style="list-style-type: none"> <li>– Use two pointers (slow and fast) to traverse the list</li> <li>– If they meet, a cycle exists.</li> </ul> </li> <li>• <b>While Condition:</b> While fast and fast.next are not None b/c fast moves twice as fast so will reach the end first if no cycle. <ul style="list-style-type: none"> <li>– <b>fast:</b> Ensures fast is not None, so fast.next is safe.</li> <li>– <b>fast.next:</b> Ensures ttfast.next is not None, so fast.next.next is safe.</li> </ul> </li> </ul>	
143. Reorder List	Given a linked list, reorder it in a specific pattern. Specifically, the pattern is to rearrange the list s.t. the first element is followed by the last element, and so on.
<ul style="list-style-type: none"> <li>• Find middle of list using slow/fast pointers w/ Floyd's algorithm, but have fast start at 2nd node.</li> <li>• Reverse the second half of the list using 206.</li> <li>• Merge the two halves together.</li> </ul>	
19. Remove Nth Node From End of List	Given a linked list, remove the Nth node from the end of the list.
<ul style="list-style-type: none"> <li>• <b>Idea:</b> Use two pointers (slow and fast) to traverse the list by moving fast pointer N steps ahead. <ul style="list-style-type: none"> <li>– <b>Conditional:</b> If fast is None, then this means <math>N = \text{length of list}</math>, therefore, remove head by returning head.next.</li> </ul> </li> <li>• Move both pointers until fast reaches the end. <ul style="list-style-type: none"> <li>– Slow pointer will be at the node to be deleted, so store prev node and do appropriate adjustments.</li> </ul> </li> </ul>	
2. Add Two Numbers	Given two non-empty linked lists representing two non-negative integers, add the two numbers and return a linked list.
<ul style="list-style-type: none"> <li>• <b>Initialize:</b> Create a dummy node, use cur to traverse as cur = dummy. <ul style="list-style-type: none"> <li>– Set next_val = 0 to store carry between digit additions.</li> </ul> </li> <li>• <b>Helper Function:</b> add_two_nodes(n1, n2, next_val) <ul style="list-style-type: none"> <li>– Extract values: val1 = n1.val if n1 else 0, val2 = n2.val if n2 else 0.</li> <li>– Compute sum: <ul style="list-style-type: none"> <li>* rem = (val1 + val2 + next_val) % 10</li> <li>* next_val = (val1 + val2 + next_val) // 10.</li> </ul> </li> <li>– Return updated carry and new node with value rem.</li> </ul> </li> <li>• <b>Traversal Loop:</b> <ul style="list-style-type: none"> <li>– Continue while at least one of l1, l2 exists or next_val != 0.</li> <li>– Call add_two_nodes(l1, l2, next_val) and link result to cur.next.</li> <li>– Advance cur, and move l1, l2 to their next nodes if they exist.</li> </ul> </li> </ul>	
23. Merge K Sorted Lists	Given an array of k linked lists, each list is sorted in ascending order. Merge all the lists into one sorted linked list and return it.
<ul style="list-style-type: none"> <li>• <b>Initialization:</b> Create a min-heap heap to store tuples (node.val, list_index, node). <ul style="list-style-type: none"> <li>– Push the head of each non-empty list in lists into the heap.</li> <li>– Initialize dummy node dummy and pointer cur = dummy.</li> </ul> </li> <li>• <b>Merge Process:</b> While heap is not empty: <ul style="list-style-type: none"> <li>– Pop the smallest element (min_val, list_ind, node) from the heap.</li> <li>– Append node to the result list using cur.next = node, then advance cur.</li> <li>– If node.next exists, push (node.next.val, list_ind, node.next) into the heap.</li> </ul> </li> <li>• <b>Return Result:</b> return dummy.next (skipping dummy head).</li> </ul>	

**Summary:**

Problem	Description:
287. Find the Duplicate Number	Given an array of integers, find the duplicate number.
<ul style="list-style-type: none"><li>• <b>Phase 1: Cycle Detection (Floyd's Tortoise and Hare)</b><ul style="list-style-type: none"><li>– Initialize two pointers: <code>slow = 0, fast = 0</code>.</li><li>– <b>Loop:</b> Move <code>slow = nums[slow]</code>, <code>fast = nums[nums[fast]]</code>.</li><li>– Continue until <code>slow == fast</code>, indicating an intersection point within the cycle.</li></ul></li><li>• <b>Phase 2: Cycle Entrance (Duplicate Finder)</b><ul style="list-style-type: none"><li>– Initialize a new pointer: <code>slow2 = 0</code>.</li><li>– <b>Loop:</b> Move both <code>slow = nums[slow]</code> and <code>slow2 = nums[slow2]</code>.</li><li>– When <code>slow == slow2</code>, return the value at that index as the duplicate.</li></ul></li><li>• <b>Rationale:</b><ul style="list-style-type: none"><li>– Each number is a pointer to the next index; repeated values form a cycle.</li><li>– Detecting the start of the cycle identifies the duplicate.</li></ul></li></ul>	



## 7 Stack and Queue

### 7.1 collections.deque

#### Summary:

```
1 from collections import deque
```

```
1 deque(iterable=None, maxlen=None)
```

- Creates a new deque object initialized with elements from `iterable`.
- **Arguments:** `iterable`: optional iterable of elements; `maxlen`: maximum number of elements.

```
1 d.append(x)
```

- Adds `x` to the right end of the deque.
- **Arguments:** `x`: element to append.

```
1 d.appendleft(x)
```

- Adds `x` to the left end of the deque.
- **Arguments:** `x`: element to append.

```
1 d.pop()
```

- Removes and returns the rightmost element.
- **Raises:** `IndexError` if deque is empty.

```
1 d.popleft()
```

- Removes and returns the leftmost element.
- **Raises:** `IndexError` if deque is empty.

```
1 d.extend(iterable)
```

- Appends elements from `iterable` to the right side.
- **Arguments:** `iterable`: iterable of elements to append.

```
1 d.extendleft(iterable)
```

- Appends elements from `iterable` to the left side (in reverse order).
- **Arguments:** `iterable`: iterable of elements to append.

```
1 d.rotate(n=1)
```

- Rotates the deque `n` steps to the right (left if negative).
- **Arguments:** `n`: number of steps to rotate.

```
1 d.clear()
```

- Removes all elements from the deque.
- **Postcondition:** deque is empty.

```
1 d.count(x)
```

- Counts occurrences of `x` in the deque.
- **Arguments:** `x`: element to count.

```
1 d.remove(value)
```

- Removes the first occurrence of `value`.
- **Raises:** `ValueError` if value is not present.

## 7.2 Stack

**Summary:** Data structure that follows Last-In-First-Out (LIFO) order for inserting and removing elements.

- **Array or Linked List:** Used to maintain the linear order of elements.
- **Top Pointer:** Points to the most recently inserted element.

### 7.2.1 When to Use?

**Summary:**

- Function call management using a call stack.
- Reversing sequences or backtracking algorithms.
- Syntax parsing and expression evaluation.

### 7.2.2 Operations

**Summary:**

Operation	Time Complexity (WC)
Push	$O(1)$
Pop	$O(1)$
Peek	$O(1)$
IsEmpty	$O(1)$

**Algorithm:**

```
1 class Stack:
2     def __init__(self):
3         self.items = [] # Internal array to store elements
4
5     def push(self, item):
6         self.items.append(item) # Add item to top
7
8     def pop(self):
9         if not self.is_empty():
10            return self.items.pop() # Remove and return top element
11
12    def peek(self):
13        if not self.is_empty():
14            return self.items[-1] # Return top element without removing
15
16    def is_empty(self):
17        return len(self.items) == 0
```

### 7.2.3 Common Problems

#### Summary:

Problem	Description
20. Valid Parentheses	Check if parentheses are balanced.
<ul style="list-style-type: none"> <li>• <b>Initialization:</b> <ul style="list-style-type: none"> <li>– Create an empty <b>stack</b> to track unmatched opening brackets.</li> <li>– Define a <b>closeToOpen</b> mapping from closing brackets to corresponding opening brackets.</li> </ul> </li> <li>• <b>String Traversal:</b> <ul style="list-style-type: none"> <li>– For each character: <ul style="list-style-type: none"> <li>* If it is a closing bracket: <ul style="list-style-type: none"> <li>· Check if the stack is non-empty and the top of the stack matches the corresponding opening bracket. If yes, pop the stack; otherwise, return <b>False</b>.</li> </ul> </li> <li>* If it is an opening bracket, push it onto the stack.</li> </ul> </li> </ul> </li> <li>• <b>Final Check:</b> Return <b>True</b> if the stack is empty (all brackets matched), else <b>False</b>.</li> <li>• <b>Key Insight:</b> The stack ensures that brackets are matched in the correct type and order, guaranteeing validity through last-in, first-out (LIFO) behavior.</li> </ul>	
150. Evaluate Reverse Polish Notation	Evaluate expression in postfix notation.
<ul style="list-style-type: none"> <li>• <b>Initialization:</b> Create an empty <b>stack</b> to store intermediate operands and results.</li> <li>• <b>Token Traversal:</b> <ul style="list-style-type: none"> <li>– For each token in the input list: <ul style="list-style-type: none"> <li>* If the token is an operator (+, -, *, /): <ul style="list-style-type: none"> <li>· Pop the top two elements from the stack.</li> <li>· Apply the operator in the correct order (note: for subtraction and division, order matters).</li> <li>· Push the result back onto the stack.</li> </ul> </li> <li>* If the token is a number: <ul style="list-style-type: none"> <li>· Convert it to an integer and push it onto the stack.</li> </ul> </li> </ul> </li> </ul> </li> <li>• <b>Final Result:</b> Return the top element of the stack, which contains the final evaluated value.</li> <li>• <b>Key Insight:</b> Reverse Polish Notation (postfix notation) allows expressions to be evaluated using a stack without parentheses by always applying operations to the two most recent operands.</li> </ul>	
22. Generate Parentheses	Generate all combinations of valid parentheses.
<ul style="list-style-type: none"> <li>• <b>Initialization:</b> Create <b>stack</b> to build the current sequence and <b>res</b> list to store valid sequences.</li> <li>• <b>Recursive Backtracking:</b> Define a recursive function <b>backtrack(openN, closedN)</b>: <ul style="list-style-type: none"> <li>– If both <b>openN</b> and <b>closedN</b> equal <b>n</b>, a complete valid sequence is formed. Append it to <b>res</b>.</li> <li>– If <b>openN</b> &lt; <b>n</b>, add an opening bracket "(", recurse, then backtrack by removing it.</li> <li>– If <b>closedN</b> &lt; <b>openN</b>, add a closing bracket ")", recurse, then backtrack by removing it.</li> </ul> </li> <li>• <b>Initial Call:</b> Start the recursion with <b>openN</b> = 0, <b>closedN</b> = 0.</li> <li>• <b>Return Result:</b> Return the list <b>res</b> containing all valid combinations.</li> <li>• <b>Key Insight:</b> Maintain the constraint that at any point, the number of closing brackets must not exceed the number of opening brackets to ensure sequence validity.</li> </ul>	

**Summary:**

Problem	Description
739. Daily Temperatures	Find days until a warmer temperature. <ul style="list-style-type: none"><li>• <b>Initialization:</b><ul style="list-style-type: none"><li>– Create a result list <b>res</b> initialized with zeros, having the same length as <b>temperatures</b>.</li><li>– Create an empty <b>stack</b> to store pairs of (temperature, index).</li></ul></li><li>• <b>Array Traversal:</b><ul style="list-style-type: none"><li>– Iterate through each temperature and its index:<ul style="list-style-type: none"><li>* While the stack is not empty and the current temperature is greater than the temperature at the top of the stack:<ul style="list-style-type: none"><li>· Pop the stack, and for the popped index, set the result as the difference between the current index and the popped index.</li></ul></li><li>* Push the current temperature and index onto the stack.</li></ul></li></ul></li><li>• <b>Return Result:</b> Return the result list <b>res</b>, which contains the number of days to wait for a warmer temperature for each day.</li><li>• <b>Key Insight:</b> A monotonic decreasing stack is used to efficiently find, for each day, the next day with a higher temperature, achieving linear <math>O(n)</math> time complexity.</li></ul>

## 7.3 Queue

**Summary:** Data structure that follows First-In-First-Out (FIFO) order for inserting and removing elements.

- **Array or Linked List:** Used to store elements in sequence.
- **Front and Rear Pointers:** Track the ends for dequeue and enqueue operations.

### 7.3.1 When to Use?

**Summary:**

- Scheduling processes in operating systems.
- Handling asynchronous data (e.g., IO Buffers, Event Queues).
- Breadth-First Search in graphs or trees.

### 7.3.2 Operations

**Summary:**

Operation	Time Complexity (WC)
Enqueue	$O(1)$
Dequeue	$O(n)$
Peek	$O(1)$
IsEmpty	$O(1)$

**Algorithm:**

```

1 class Queue:
2     def __init__(self):
3         self.items = [] # Internal array to store elements
4
5     def enqueue(self, item):
6         self.items.append(item) # Add item to the rear
7
8     def dequeue(self):
9         if not self.is_empty():
10            return self.items.pop(0) # Remove and return the front element
11
12    def peek(self):
13        if not self.is_empty():
14            return self.items[0] # Return front element without removing
15
16    def is_empty(self):
17        return len(self.items) == 0

```

## 8 Trees

### 8.1 Binary Search Tree (BST)

**Summary:**

- A binary tree where for each node, left subtree values are smaller, and right subtree values are larger.
- **Balanced vs. Unbalanced:**

$$O(\log(n)) \text{ (balanced)} \leq O(h) \leq O(n) \text{ (unbalanced)}$$

**Algorithm:**

```
1 class Node:
2     def __init__(self, key):
3         self.val = key
4         self.left = None
5         self.right = None
6
7 class BST:
8     def __init__(self):
9         self.root = None
10
11     def operations(self, _):
12         pass
```

## 8.2 Operations

### Summary:

Operation	Time Complexity
Search	$O(h)$
Insert	$O(h)$
Delete	$O(h)$
Find Min/Max	$O(h)$
In-order Traversal	$O(n)$
Pre-order Traversal	$O(n)$
Post-order Traversal	$O(n)$
Level-order Traversal	$O(n)$

### 8.2.1 Search

#### Algorithm:

```

1 def search(self, key):
2     current = self.root
3     while current:
4         if key == current.val:
5             return current
6         elif key < current.val:
7             current = current.left
8         else:
9             current = current.right
10    return None

```

### 8.2.2 Insert

#### Algorithm:

```

1 def insert(self, key):
2     def _insert(node, key):
3         if node is None:
4             return TreeNode(key)
5         if key < node.val:
6             node.left = _insert(node.left, key)
7         elif key > node.val:
8             node.right = _insert(node.right, key)
9         return node
10    self.root = _insert(self.root, key)

```

### 8.2.3 Delete

#### Algorithm:

```

1 def delete(self, key):
2     def _delete(node, key):
3         if node is None:
4             return None
5         if key < node.val:
6             node.left = _delete(node.left, key)
7         elif key > node.val:

```

```
8         node.right = _delete(node.right, key)
9     else:
10         # Node with one child or no child
11         if node.left is None:
12             return node.right
13         elif node.right is None:
14             return node.left
15         # Node with two children
16         temp = self._find_min(node.right)
17         node.val = temp.val
18         node.right = _delete(node.right, temp.val)
19     return node
20 self.root = _delete(self.root, key)
```

#### 8.2.4 Find Min

##### Algorithm:

```
1 def find_min(self, node):
2     while node.left is not None:
3         node = node.left
4     return node
```

#### 8.2.5 Find Max

##### Algorithm:

```
1 def find_max(self, node):
2     while node.right is not None:
3         node = node.right
4     return node
```

#### 8.2.6 DFS In-order Traversal (Left → Root → Right)

**Definition:** Visit the left subtree, then the root, and finally the right subtree.

- Used for retrieving elements in sorted order from a BST.

##### Algorithm:

```
1 def inorder(node):
2     if node:
3         inorder(node.left)
4         print(node.val)
5         inorder(node.right)
```



### 8.2.7 DFS Pre-order Traversal (Root $\rightarrow$ Left $\rightarrow$ Right)

**Definition:** Visit the root first, then the left subtree, and finally the right subtree.

- Useful for copying or serializing the tree.

#### Algorithm:

```
1 def preorder(node):
2     if node:
3         print(node.val)
4         preorder(node.left)
5         preorder(node.right)
```

### 8.2.8 DFS Post-order Traversal (Left $\rightarrow$ Right $\rightarrow$ Root)

**Definition:** Visit the left subtree, then the right subtree, and finally the root.

- Useful for deleting or freeing nodes in memory.

#### Algorithm:

```
1 def postorder(node):
2     if node:
3         postorder(node.left)
4         postorder(node.right)
5         print(node.val)
```

### 8.2.9 BFS Level-order Traversal (Top $\rightarrow$ Bottom, Left $\rightarrow$ Right)

**Definition:** Visit nodes level-level from top to bottom & left to right.

- Useful for finding shortest paths or visualizing layers of a tree.

#### Algorithm:

```
1 from collections import deque
2
3 def level_order(root):
4     if not root:
5         return
6
7     queue = deque([root])
8     while queue:
9         node = queue.popleft()
10        print(node.val)
11
12        if node.left:
13            queue.append(node.left)
14        if node.right:
15            queue.append(node.right)
```

## 8.2.10 Common Problems

## Summary:

**Problem**

\*\*226. Invert Binary Tree

**Description:**

Given a binary tree, invert it.

- Base case: If the node is None, return.
- Swap left and right children of the current node.
- Recursively call the function on left and right children.

\*\*104. Maximum Depth of Binary Tree    Given a binary tree, find its maximum depth.

• **Recursive DFS:**

- Base case: If the node is None, return 0.
- Recursively find the maximum depth of left and right subtrees.
- Return the maximum of the two depths plus one for the current node.

• **Iterative BFS:**

- Initialize an empty queue  $q$ .
- Append  $root$  to  $q$  and set  $level \leftarrow 0$  unless root is None.
- While  $q$  is not empty:
  - \* For each node in the current level ( $\text{len}(q)$  iterations):
    - Pop the front node from  $q$ .
    - If the node has a left child, append it to  $q$ .
    - If the node has a right child, append it to  $q$ .
  - \* Increment  $level$  after processing all nodes in the current level.
- Return  $level$  as the maximum depth of the tree.

\*\*543. Diameter of Binary Tree

Given a binary tree, find its diameter.

- The diameter is the longest path between any two nodes in the tree.
- Use DFS to calculate the height of each subtree and update the diameter.
- The diameter at each node is the sum of the heights of its left and right subtrees.

110. Balanced Binary Tree

Given a binary tree, check if it is height-balanced.

- A tree is balanced if the heights of the two child subtrees of any node differ by no more than one.
- Use DFS to calculate the height of each subtree and check the balance condition.

### 8.2.11 BST-based Sets and Maps

#### Summary:

- **BST Set:** Stores unique values in sorted order. Supports insert, search, delete.
- **BST Map:** Associates keys with values, maintaining keys in sorted order.
- Can be implemented using self-balancing trees (e.g., AVL, Red-Black Tree) for  $O(\log n)$  operations.
- Useful for range queries, floor/ceiling lookups, and ordered iteration.

#### Algorithm:

```
1 class BSTSet:
2     def __init__(self):
3         self.root = None
4
5     def add(self, val):
6         self.root = insert_bst(self.root, val)
7
8     def contains(self, val):
9         return search_bst(self.root, val) is not None
10
11    def remove(self, val):
12        self.root = delete_bst(self.root, val)
13
14 class BSTMap:
15     def __init__(self):
16         self.root = None
17
18     def put(self, key, value):
19         self.root = self._put(self.root, key, value)
20
21     def _put(self, node, key, value):
22         if not node:
23             return TreeNode((key, value))
24         if key < node.val[0]:
25             node.left = self._put(node.left, key, value)
26         elif key > node.val[0]:
27             node.right = self._put(node.right, key, value)
28         else:
29             node.val = (key, value)
30         return node
31
32     def get(self, key):
33         node = self.root
34         while node:
35             if key < node.val[0]:
36                 node = node.left
37             elif key > node.val[0]:
38                 node = node.right
39             else:
40                 return node.val[1]
41         return None
```

## 9 Heaps and Priority Queues

### 9.1 Heap

#### Summary:

- A **heap** is a complete binary tree where each node follows the heap property:
  - **Max heap**: Largest key at root, where every parent node is **greater than or equal to** its children.
  - **Min heap**: Smallest key at root, where every parent node is **less than or equal to** its children.
- **Balanced Tree**:  $h = \log n$
- **Indexing**: Given a node at index  $i$  in the array, assuming 1-based indexing:
  1. **Parent**:  $\text{parent}(i) = \left\lfloor \frac{i}{2} \right\rfloor$
  2. **Left child**:  $\text{leftchild}(i) = 2i$
  3. **Right child**:  $\text{rightchild}(i) = 2i + 1$

### 9.2 Heapq

#### Summary:

```
1 heapq.heapify(x)
```

- Transforms a list `x` into a valid min-heap in-place.
- **Arguments**: `x`: list to be heapified.

```
1 heapq.heappush(heap, item)
```

- Inserts `item` into `heap` while maintaining the heap invariant.
- **Arguments**: `heap`: list representing a heap; `item`: element to insert.

```
1 heapq.heappop(heap)
```

- Removes and returns the smallest element from the heap.
- **Arguments**: `heap`: non-empty list representing a valid heap.

```
1 heapq.heappushpop(heap, item)
```

- Pushes `item` onto the heap, then pops and returns the smallest element.
- **Arguments**: `heap`: valid heap; `item`: element to insert.

```
1 heapq.heapreplace(heap, item)
```

- Pops and returns the smallest element, then inserts `item` into the heap.
- **Arguments**: `heap`: non-empty valid heap; `item`: element to insert.

```
1 heapq.nlargest(n, iterable)
```

- Returns the `n` largest elements from `iterable` in descending order.
- **Arguments**: `n`: number of elements; `iterable`: list or other iterable.

```
1 heapq.nsmallest(n, iterable)
```

- Returns the `n` smallest elements from `iterable` in ascending order.
- **Arguments**: `n`: number of elements; `iterable`: list or other iterable.

**Algorithm:**

```
1 class MinHeap:
2     def __init__(self):
3         self.heap = []
4
5     def parent(self, i):
6         return (i - 1) // 2
7
8     def left(self, i):
9         return 2 * i + 1
10
11    def right(self, i):
12        return 2 * i + 2
13
14    def operations(self, _):
15        pass
```

## 9.3 Operations

### Summary:

Operation	Time Complexity
Insert	$O(\log n)$
Extract Min	$O(\log n)$
Get Min	$O(1)$
Heapify	$O(n)$
Build Heap	$O(n)$
Search	$O(n)$
Delete	$O(n)$
Heap Sort	$O(n \log n)$

### 9.3.1 Insert

#### Algorithm:

```

1 def insert(self, key):
2     self.heap.append(key)
3     i = len(self.heap) - 1
4     while i != 0 and self.heap[self.parent(i)] > self.heap[i]:
5         self.heap[i], self.heap[self.parent(i)] = self.heap[self.parent(i)], self.heap[i]
6         i = self.parent(i)

```

### 9.3.2 Heapify

**Algorithm:** Restores the heap property by moving a node down the tree to its correct position.

```

1 def heapify(self, i):
2     smallest = i
3     l = self.left(i)
4     r = self.right(i)
5
6     if l < len(self.heap) and self.heap[l] < self.heap[smallest]:
7         smallest = l
8     if r < len(self.heap) and self.heap[r] < self.heap[smallest]:
9         smallest = r
10
11     if smallest != i:
12         self.heap[i], self.heap[smallest] = self.heap[smallest], self.heap[i]
13         self.heapify(smallest)

```

### 9.3.3 Extract Min

#### Algorithm:

```

1 def extract_min(self):
2     if not self.heap:
3         return None
4     if len(self.heap) == 1:
5         return self.heap.pop()
6     root = self.heap[0]
7     self.heap[0] = self.heap.pop()
8     self.heapify(0)

```

```
9 return root
```

### 9.3.4 Get Min

**Algorithm:** Min-heap always has the smallest element at the root.

```
1 def get_min(self):  
2     return self.heap[0] if self.heap else None
```

### 9.3.5 Build Heap

**Algorithm:**

```
1 def build_heap(self, arr):  
2     self.heap = arr[:]  
3     for i in range(len(self.heap) // 2 - 1, -1, -1):  
4         self.heapify(i)
```

### 9.3.6 Search

**Algorithm:**

```
1 def search(self, key):  
2     return key in self.heap
```

### 9.3.7 Delete

**Algorithm:**

```
1 def delete(self, key):  
2     try:  
3         index = self.heap.index(key)  
4         self.heap[index] = self.heap[-1]  
5         self.heap.pop()  
6         if index < len(self.heap):  
7             self.heapify(index)  
8             parent = self.parent(index)  
9             while index > 0 and self.heap[parent] > self.heap[index]:  
10                self.heap[parent], self.heap[index] = self.heap[index], self.heap[parent]  
11                index = parent  
12                parent = self.parent(index)  
13     except ValueError:  
14         pass
```

### 9.3.8 Heap Sort

**Algorithm:**

```
1 def heap_sort(self):  
2     sorted_list = []  
3     original = self.heap[:]  
4     while self.heap:  
5         sorted_list.append(self.extract_min())  
6     self.heap = original
```

```
7 | return sorted_list
```



### 9.3.9 Common Problem

#### Summary:

Problem	Description
703. Kth Largest Element in a Stream	Design a class to find the kth largest element in a stream of numbers.
<ul style="list-style-type: none"> <li>Implement a heap that keeps track of the k largest elements using a min-heap so that the kth largest element is always at the root.</li> <li><b>Process:</b> <ol style="list-style-type: none"> <li>Change list into heap. If more than <math>k</math> elements, pop until only <math>k</math> elements remain.</li> <li>Add: Push new element into heap. If size exceeds <math>k</math>, pop the smallest element. Return the root.</li> </ol> </li> </ul>	
1046. Last Stone Weight	You are given an array of integers. Each integer represents the weight of a stone.
<ul style="list-style-type: none"> <li>Convert the list into a max-heap using heapq with negation.</li> <li>While there are at least 1 stone in heap: <ul style="list-style-type: none"> <li>If they are equal, both stones are destroyed. If not, new stone is created and pushed back into the heap.</li> </ul> </li> <li>Return the weight of the last remaining stone or 0 if there are no stones left.</li> </ul>	
973. K Closest Points to Origin	Given an array of points, find the k closest points to the origin.
<ul style="list-style-type: none"> <li>Use a min-heap to store the points based on their distance using a tuple of (distance, point).</li> <li>While <math>\text{len}(\text{heap}) &gt; k</math>, pop the largest element.</li> <li>Return the k closest points by using <code>heapq.nsmallest</code>.</li> </ul>	
215. Kth Largest Element in an Array	Find the kth largest element in an unsorted array.
<ul style="list-style-type: none"> <li>Use a min-heap to keep track of the k largest elements.</li> <li>If the heap size exceeds k, pop the smallest element.</li> <li>Return the root of the heap.</li> </ul>	
621. Task Scheduler	Given a list of tasks and a cooldown period, find the least time to finish all tasks.
<ul style="list-style-type: none"> <li>Use <b>Counter</b> to count the frequency of each task.</li> <li>Use a max-heap to store tasks by frequency: <code>(-freq, task)</code>.</li> <li>Use a queue to track cooldowns: <code>(-freq, ready_time)</code>.</li> <li>While either heap or queue is non-empty: <ul style="list-style-type: none"> <li>Increment <b>time</b>.</li> <li>If heap is non-empty, pop task, decrement frequency, and if not 0, add to queue with <b>time + n</b>.</li> <li>If the front of the queue is ready (<b>ready_time == time</b>), pop and push it back into the heap.</li> </ul> </li> </ul>	

**Summary:**

Problem	Description
239. Sliding Window Maximum	Given an array and a window size, find the maximum in each sliding window. <ul style="list-style-type: none"><li>• <b>Initialization:</b><ul style="list-style-type: none"><li>– Create an empty max-heap <code>heap</code> using negated values: <code>(-value, index)</code>.</li><li>– Fill the initial window with the first <code>k</code> elements.</li><li>– Append the maximum (top of the heap) to result: <code>res.append(-heap[0][0])</code>.</li></ul></li><li>• <b>Sliding the Window:</b><ul style="list-style-type: none"><li>– For each new index <code>i</code>, push <code>(-nums[i], i)</code> into the heap.</li><li>– <b>Lazy Removal:</b> While the top element's index is outside the window (<code>heap[0][1] ≤ i - k</code>), remove it using <code>heapq.heappop()</code>.<ul style="list-style-type: none"><li>* Heap may contain elements outside the current window, but they are ignored. Only care about root element.</li></ul></li><li>– Append the current maximum to the result: <code>res.append(-heap[0][0])</code>.</li></ul></li><li>• <b>Return:</b> The list <code>res</code> containing all sliding window maximums.</li></ul>

## 10 Graphs

### Summary:

Algorithm	Time Complexity	Space Complexity
BFS	$O(V + E)$	$O(V)$
DFS	$O(V + E)$	$O(V)$
Topological Sort (DFS)	$O(V + E)$	$O(V)$

### 10.1 Breadth-First Search (BFS)

#### Summary:

- Use when exploring nodes layer-by-layer, typically in unweighted graphs or grids.
- Ideal for finding the shortest path, level order traversal, or minimum number of steps.
- Queue-based traversal ensures nodes are visited in order of increasing distance from the source.

#### Algorithm:

```
1 from collections import deque
2
3 def bfs(start, graph):
4     visited = set()
5     queue = deque([start])
6     visited.add(start)
7
8     while queue:
9         node = queue.popleft() # FIFO (BFS)
10
11         for neighbor in graph[node]:
12             if neighbor not in visited:
13                 visited.add(neighbor)
14                 queue.append(neighbor)
```

## 10.1.1 Common Problems

## Summary:

Problem	Description:
994. Rotting Oranges	<p>Given a <math>m \times n</math> grid of oranges, where 0 = empty cell, 1 = fresh orange, and 2 = rotten orange, and where fresh oranges rot if adjacent to a rotten orange, return the minimum time required for all oranges to become rotten.</p> <ul style="list-style-type: none"> <li>• <b>Initialization:</b> <ul style="list-style-type: none"> <li>– Create a queue <code>q</code> to store coordinates of initially rotten oranges.</li> <li>– Count total fresh oranges <code>fresh = 0</code>, and set <code>time = 0</code>.</li> <li>– Traverse <code>grid</code>: <ul style="list-style-type: none"> <li>* If <code>grid[r][c] == 1</code>, increment <code>fresh</code>.</li> <li>* If <code>grid[r][c] == 2</code>, append <code>(r, c)</code> to <code>q</code>.</li> </ul> </li> </ul> </li> <li>• <b>BFS Propagation:</b> While <code>fresh &gt; 0</code> and <code>q</code> is not empty: <ul style="list-style-type: none"> <li>– Go through all rotten oranges in <code>q</code> and check their neighbors to mark it as rotten.</li> <li>– Increment <code>time</code> after each level (i.e. after processing all rotten oranges at the current level).</li> </ul> </li> <li>• <b>Return Result:</b> If <code>fresh == 0</code>, return <code>time</code>; else return <code>-1</code>.</li> </ul>
**417. Pacific Atlantic Water Flow	<p>Given an <math>m \times n</math> matrix of non-negative integers representing the height of each cell, return the coordinates of cells that can flow to both the Pacific &amp; Atlantic oceans.</p> <ul style="list-style-type: none"> <li>• <b>Initialization:</b> <ul style="list-style-type: none"> <li>– Define <code>ROWS</code>, <code>COLS</code> as the dimensions of the input matrix.</li> <li>– Define <code>directions</code> as the 4 possible adjacent moves (up, down, left, right).</li> <li>– Create two 2D boolean matrices: <code>pac</code> and <code>atl</code>, indicating cells reachable by Pacific and Atlantic oceans.</li> </ul> </li> <li>• <b>Construct Ocean Borders:</b> <ul style="list-style-type: none"> <li>– Initialize Pacific ocean border with all top row and leftmost column coordinates.</li> <li>– Initialize Atlantic ocean border with all bottom row and rightmost column coordinates.</li> </ul> </li> <li>• <b>BFS Traversal Function:</b> <ul style="list-style-type: none"> <li>– Perform breadth-first search (BFS) from all coordinates along each ocean's border.</li> <li>– For each visited cell, enqueue adjacent cells that: <ul style="list-style-type: none"> <li>* Are within bounds.</li> <li>* Are not yet marked as reachable.</li> <li>* Have equal or greater height (ensuring water can flow from neighbor to current).</li> </ul> </li> </ul> </li> <li>• <b>Mark Reachable Cells:</b> <ul style="list-style-type: none"> <li>– Call <code>bfs(pacific, pac)</code> and <code>bfs(atlantic, atl)</code> to fill in reachable matrices.</li> </ul> </li> <li>• <b>Collect Intersection Points:</b> Iterate through all cells in the matrix. <ul style="list-style-type: none"> <li>– If a cell is marked <code>True</code> in both <code>pac</code> and <code>atl</code>, append it to the result.</li> </ul> </li> <li>• <b>Return:</b> Return list of coordinates where water can flow to both oceans.</li> </ul>
130. Surrounded Regions	<p>Given a 2D board containing 'X' and 'O', capture all regions surrounded by 'X'.</p> <ul style="list-style-type: none"> <li>• <b>Border Traversal:</b> Iterate through all border cells (first and last rows, first and last columns). <ul style="list-style-type: none"> <li>– Enqueue all 'O's found on the border into the queue <code>q</code>.</li> </ul> </li> <li>• <b>BFS Flood Fill:</b> <ul style="list-style-type: none"> <li>– For each 'O' in the queue, mark it and all connected 'O's as visited by changing them to temporary symbol '#'.</li> <li>– Only move to valid neighbors that are within bounds, are 'O', and unvisited.</li> </ul> </li> <li>• <b>Final Transformation:</b> Traverse the entire board. <ul style="list-style-type: none"> <li>– Change all remaining 'O's (not connected to border) to 'X' and all temporary '#' markers back to 'O'.</li> </ul> </li> <li>• <b>Key Insight:</b> Only 'O's connected to the border should be preserved.</li> </ul>

## 10.2 Depth-First Search (DFS)

### Summary:

- Use when traversing all nodes or paths in **trees**, **graphs**, or **matrices**.
- Ideal for problems involving **backtracking**, **recursion**, or exploring all **connected components**.
- Can be implemented recursively or iteratively with a stack.
- Maintain a **visited** set or matrix to avoid revisiting nodes.
- Useful for **topological sorting**, **cycle detection**, and **pathfinding**.

### Algorithm:

```
1 from collections import deque
2
3 def dfs(start, graph):
4     visited = set()
5     stack = deque([start])
6     visited.add(start)
7
8     while stack:
9         node = stack.pop() # LIFO (DFS)
10
11         for neighbor in graph[node]:
12             if neighbor not in visited:
13                 visited.add(neighbor)
14                 stack.append(neighbor)
```

### Algorithm:

```
1 def dfs(node, visited):
2     if node in visited:
3         return
4
5     visited.add(node)
6
7     for neighbor in graph[node]:
8         dfs(neighbor, visited)
```

### 10.2.1 Common Problems

#### Summary:

Problem	Description:
200. Number of Islands	<p>Given a 2D grid of '1's (land) and '0's (water), count the number of islands.</p> <ul style="list-style-type: none"> <li>• Use DFS or BFS to explore all connected '1's and mark them as visited.</li> <li>• Increment the island count for each unvisited '1'.</li> </ul>
695. Max Area of Island	<p>Given a 2D grid of '1's (land) and '0's (water), find the maximum area of an island.</p> <ul style="list-style-type: none"> <li>• Use DFS or BFS to explore all connected '1's and calculate the area.</li> <li>• Keep track of the maximum area encountered during the traversal.</li> </ul>
79. Word Search	<p>Given a 2D board and a word, check if the word exists in the grid.</p> <ul style="list-style-type: none"> <li>• Use DFS to explore all possible paths in the grid.</li> <li>• Mark cells as visited to avoid revisiting.</li> <li>• Backtrack if the current path does not lead to a solution.</li> </ul>
133. Clone Graph	<p>Given a reference to a node in a connected undirected graph, return a deep copy of the graph.</p> <ul style="list-style-type: none"> <li>• <b>Initialization:</b> Create a hash map <code>oldToNew</code> to store mappings from original nodes to their cloned nodes.</li> <li>• <b>DFS:</b> Define <code>dfs(node)</code> to recursively clone the graph. <ul style="list-style-type: none"> <li>– <b>Base Case:</b> If <code>node</code> already in <code>oldToNew</code>, return the cloned node.</li> <li>– <b>Clone Creation:</b> Create a new <code>Node(node.val)</code>, store in <code>oldToNew</code>.</li> <li>– <b>Neighbor Cloning:</b> For each <code>neigh</code> in <code>node.neighbors</code>, recursively clone and append to <code>copy.neighbors</code>.</li> </ul> </li> <li>• <b>Entry Point:</b> Return <code>dfs(node)</code> if <code>node</code> is not <code>None</code>; otherwise return <code>None</code>.</li> </ul>

## 10.3 Topological Sort

### Summary:

- **Overview:** Produces a total ordering from partial ordering.
- **DAG:**  $G = (V, E)$  must be a DAG to produce a valid topological sorting.
- Given a DAG, create a linear (total) order out of the partial order  $\rightarrow$  "serialize" these events
  - **Intuition:** Arranges the vertices of a DAG in a linear order such that for every directed edge  $u \rightarrow v$ , vertex  $u$  appears before  $v$ .

### Algorithm:

```
1 from collections import defaultdict, deque
2
3 def topological_sort(num_nodes, edges):
4     # Build adjacency list and in-degree count
5     graph = defaultdict(list)
6     in_degree = [0] * num_nodes
7
8     for u, v in edges:
9         graph[u].append(v)
10        in_degree[v] += 1
11
12    # Start with all nodes that have in-degree 0
13    queue = deque([i for i in range(num_nodes) if in_degree[i] == 0])
14    topo_order = []
15
16    while queue:
17        node = queue.popleft()
18        topo_order.append(node)
19
20        for neighbor in graph[node]:
21            in_degree[neighbor] -= 1
22            if in_degree[neighbor] == 0:
23                queue.append(neighbor)
24
25    # If not all nodes are processed, there is a cycle
26    if len(topo_order) != num_nodes:
27        return [] # or raise an error
28
29    return topo_order
```

## 11 Sorting

### Summary:

Name	Space	Time (BC,AC,WC)	Prop. (In-place, Stable, D&C)
<b>Comparison Based</b>			
Merge Sort	$\Theta(n)$	$\Theta(n \log n)$	$\overline{IP}, S, DC$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Auxiliary arrays</li> </ul>			
Quick Sort	$\Theta(1), \Theta(\log n), \Theta(n)$	$\Theta(n \log n), \Theta(n \log n), \Theta(n^2)$	$IP, \overline{S}, DC$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Original array (in-place)</li> <li>• Space complexity depends on implementation.</li> </ul>			
Heap Sort	$\Theta(1)$	$\Theta(n \log n)$	$IP, \overline{S}, \overline{DC}$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Binary heap</li> <li>• <b>Time Complexity:</b> <ul style="list-style-type: none"> <li>– <math>n</math>: Number of elements in the input array that need to be sorted.</li> <li>– <math>h = \log n</math>: Height of the binary heap (can be changed)</li> </ul> </li> <li>• Space complexity doesn't include the input.</li> </ul>			
Bubble Sort	$\Theta(1)$	$\Theta(n), \Theta(n^2), \Theta(n^2)$	$IP, S, \overline{DC}$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Original array (in-place)</li> </ul>			
Selection Sort	$\Theta(1)$	$\Theta(n^2)$	$IP, \overline{S}, \overline{DC}$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Original array (in-place)</li> </ul>			
Insertion Sort	$\Theta(1)$	$\Theta(n), \Theta(n^2), \Theta(n^2)$	$IP, S, \overline{DC}$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Original array (in-place)</li> </ul>			
<b>Non-Comparison Based</b>			
Counting Sort	$\Theta(n + k)$	$\Theta(n + k)$ if $k \gg O(n)$ , $\Theta(n)$ if $k \leq O(n)$	$\overline{IP}, S, \overline{DC}$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Auxiliary arrays</li> <li>• <b>Assumption:</b> Elements are integers ranging from 0 to <math>k</math>.</li> <li>• <math>n</math>: Size of array</li> <li>• <math>k</math>: Range of numbers (i.e. <math>[0, \dots, k]</math>)</li> </ul>			
Radix Sort	$\Theta(n + k)$	$\Theta(d \cdot (n + k))$	$\overline{IP}, S, \overline{DC}$
<ul style="list-style-type: none"> <li>• <b>DS:</b> Auxiliary arrays</li> <li>• <b>Assumption:</b> All elements have <math>\leq d</math>-digits</li> <li>• <b>One pass time complexity:</b> <math>O(n + k)</math> (i.e. counting sort)</li> <li>• <b>Note:</b> If <math>k = d = O(1)</math>, then <math>O(n)</math> time complexity.</li> <li>• <math>n</math>: Size of array</li> <li>• <math>k</math>: Range of numbers</li> <li>• <math>d</math>: Number of digits</li> </ul>			



## 11.1 Stable, In-place, and Divide and Conquer

### Definition:

- **Stable:** Relative order of ties is maintained.
  - e.g.  $[2_a, 3, 2_b, 1] \rightarrow [1, 2_a, 2_b, 3]$
- **In-place sorting:**  $O(1)$  extra space.
- **Lower bound on comparison-based sorting:** No CBS algorithm on **unrestricted** range is better than  $\Omega(n \log n)$

## 11.2 Merge Sort

### Algorithm:

```
1 def merge_sort(arr):
2     if len(arr) <= 1:
3         return arr
4
5     # Divide
6     mid = len(arr) // 2
7     left = merge_sort(arr[:mid])
8     right = merge_sort(arr[mid:])
9
10    # Conquer: Merge two sorted halves
11    return merge(left, right)
12
13 def merge(left, right):
14     result = []
15     i = j = 0
16
17     # Merge two sorted lists
18     while i < len(left) and j < len(right):
19         if left[i] <= right[j]:
20             result.append(left[i])
21             i += 1
22         else:
23             result.append(right[j])
24             j += 1
25
26     # Append remaining elements
27     result.extend(left[i:])
28     result.extend(right[j:])
29     return result
```

## 11.3 Quick Sort

### Algorithm:

```
1 def quick_sort(arr):
2     if len(arr) <= 1:
3         return arr
4
5     pivot = arr[len(arr) // 2] # Choose middle element as pivot
6     left = [x for x in arr if x < pivot]
7     middle = [x for x in arr if x == pivot]
8     right = [x for x in arr if x > pivot]
9
10    # Recursively sort left and right, then concatenate
11    return quick_sort(left) + middle + quick_sort(right)
```

## 12 Images

### 12.1 2D Convolution Operations

Notes:

1. **1. Output Dimensions**

The output height and width of a 2D convolution are given by:

$$\text{out\_height} = \left\lfloor \frac{\text{in\_height} + 2 \cdot \text{padding}_h - \text{effective\_kernel}_h}{\text{stride}_h} \right\rfloor + 1$$

$$\text{out\_width} = \left\lfloor \frac{\text{in\_width} + 2 \cdot \text{padding}_w - \text{effective\_kernel}_w}{\text{stride}_w} \right\rfloor + 1$$

2. **2. Effective Kernel Size (with Dilation)**

The effective kernel size when dilation is applied:

$$\text{effective\_kernel}_h = \text{kernel\_height} + (\text{kernel\_height} - 1) \cdot (\text{dilation}_h - 1)$$

$$\text{effective\_kernel}_w = \text{kernel\_width} + (\text{kernel\_width} - 1) \cdot (\text{dilation}_w - 1)$$

3. **3. Convolution Operation (Batch, Channel-aware)**

The general convolution operation for a batch of input tensors is:

$$\text{output}[b, c_{\text{out}}, h_{\text{out}}, w_{\text{out}}] = \sum_{c_{\text{in}}} \sum_{k_h} \sum_{k_w} (\text{input}[b, c_{\text{in}}, h_{\text{in}} + k_h \cdot \text{dilation}_h, w_{\text{in}} + k_w \cdot \text{dilation}_w] \cdot \text{filter}[c_{\text{out}}, c_{\text{in}}, k_h, k_w])$$

where:

$$h_{\text{in}} = h_{\text{out}} \cdot \text{stride}_h, \quad w_{\text{in}} = w_{\text{out}} \cdot \text{stride}_w$$

### 12.2 Common Problems

**Summary:**

Problem	Description
661. Image Smoother	<p>Given an image represented by a 2D array, smooth the image by averaging the pixel values of each pixel and its neighbors.</p> <ul style="list-style-type: none"> <li>• Loop through the cols and rows of the image, then <ul style="list-style-type: none"> <li>– total sum for each pixel = <math>\sum_{x,y \in \text{neighbours}} \text{image}[x][y] = \sum_{x=i-1}^{i+1} \sum_{y=j-1}^{j+1} \text{image}[x][y]</math> <ul style="list-style-type: none"> <li>* If <math>x</math> or <math>y</math> is out of bounds, ignore it.</li> </ul> </li> <li>– count = <math>\sum_{x=i-1}^{i+1} \sum_{y=j-1}^{j+1} 1</math></li> <li>– average = total sum // count</li> </ul> </li> <li>• result[i][j] = average</li> </ul>
832. Flipping an Image	<p>Given a binary matrix, flip the image horizontally and invert it.</p> <ul style="list-style-type: none"> <li>• Loop through the rows of the image, then use .reverse() to flip the row horizontally.</li> <li>• Double for loop to invert image (change 0 to 1 and 1 to 0).</li> </ul>
48. Rotate Image	<p>Given an n x n 2D matrix, rotate the image 90 degrees clockwise.</p> <ul style="list-style-type: none"> <li>• Transpose the matrix (swap rows and columns) if <math>i &lt; j</math>, then <math>\text{matrix}[i][j] \xleftrightarrow{\text{swap}} \text{matrix}[j][i]</math>.</li> <li>• Reverse each row.</li> </ul>
**835. Image Overlap	<p>Given two images represented by 2D arrays, find the maximum overlap between the two images.</p> <ul style="list-style-type: none"> <li>• Try all possible translations of img1.</li> <li>• For each translation, calculate the overlap with img2.</li> </ul>