DELIMITED CONTINUATIONS DEMYSTIFIED

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Lambda Days 2023

HISTORY

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→ Flurry of initial publications, mostly in Scheme.

→ Not much mainstream adoption.

→ Recently: some renewed interest.


Haskell

→ Initial proposal in early 2020; revised version accepted in late 2020.

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- → Started at Tweag last year; patch landed last fall.
- → Finally released this past March in GHC 9.6!

Problem: nobody knows what they are.

DEMYSTIFICATION

TERMINOLOGY

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“continuations”

TERMINOLOGY

~~“continuations”~~

“delimited continuations”

TERMINOLOGY

~~"continuations"~~

"first-class,
delimited continuations"

TERMINOLOGY

~~"continuations"~~

"native, first-class,
delimited continuations"

TERMINOLOGY

"~~continuations~~"

"native, first-class,
delimited continuations"

① continuations

② delimited

③ first-class

④ native

① continuations

② delimited

③ first-class

④ native

A “continuation” is a *concept*,
not a language feature.

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(Like “scope” or “value”.)

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Applies to most programming languages!

Useful for talking about *evaluation*.

```
(1 + 2) * (3 + 4)
```

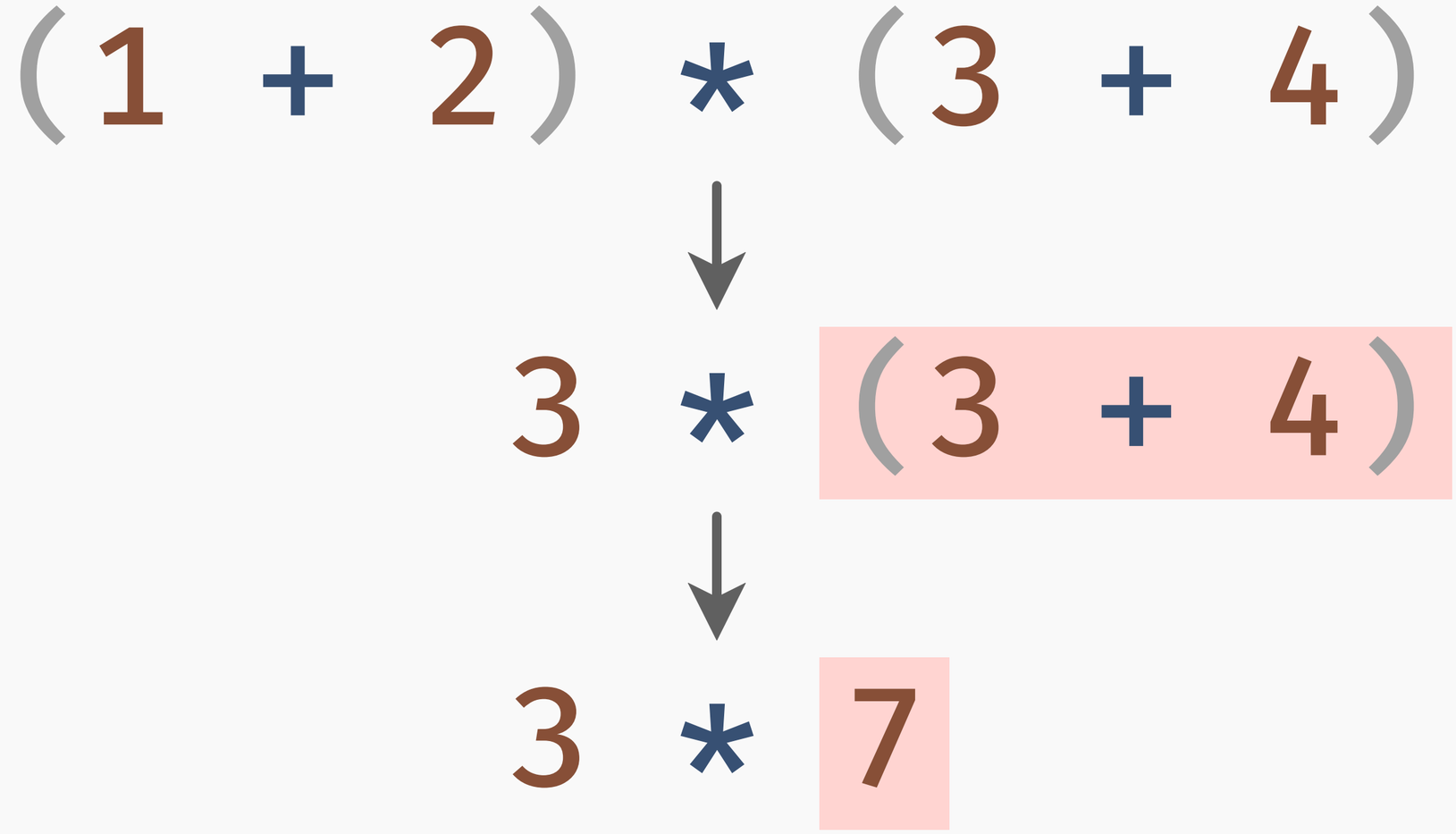
```
(1 + 2) * (3 + 4)
```

```
(1 + 2) * (3 + 4)
```

```
3 * (3 + 4)
```

```
(1 + 2) * (3 + 4)
```

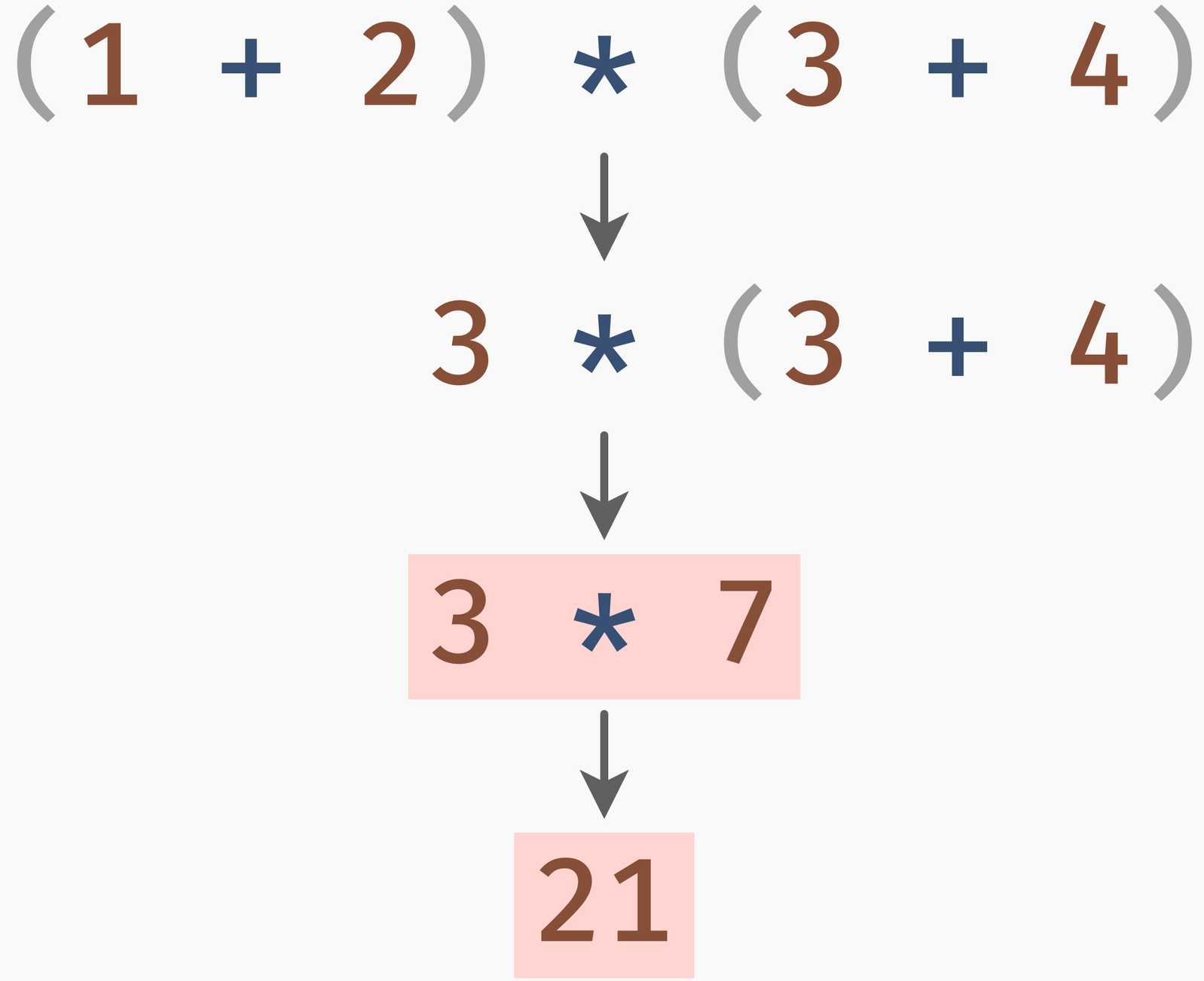
```
3 * (3 + 4)
```

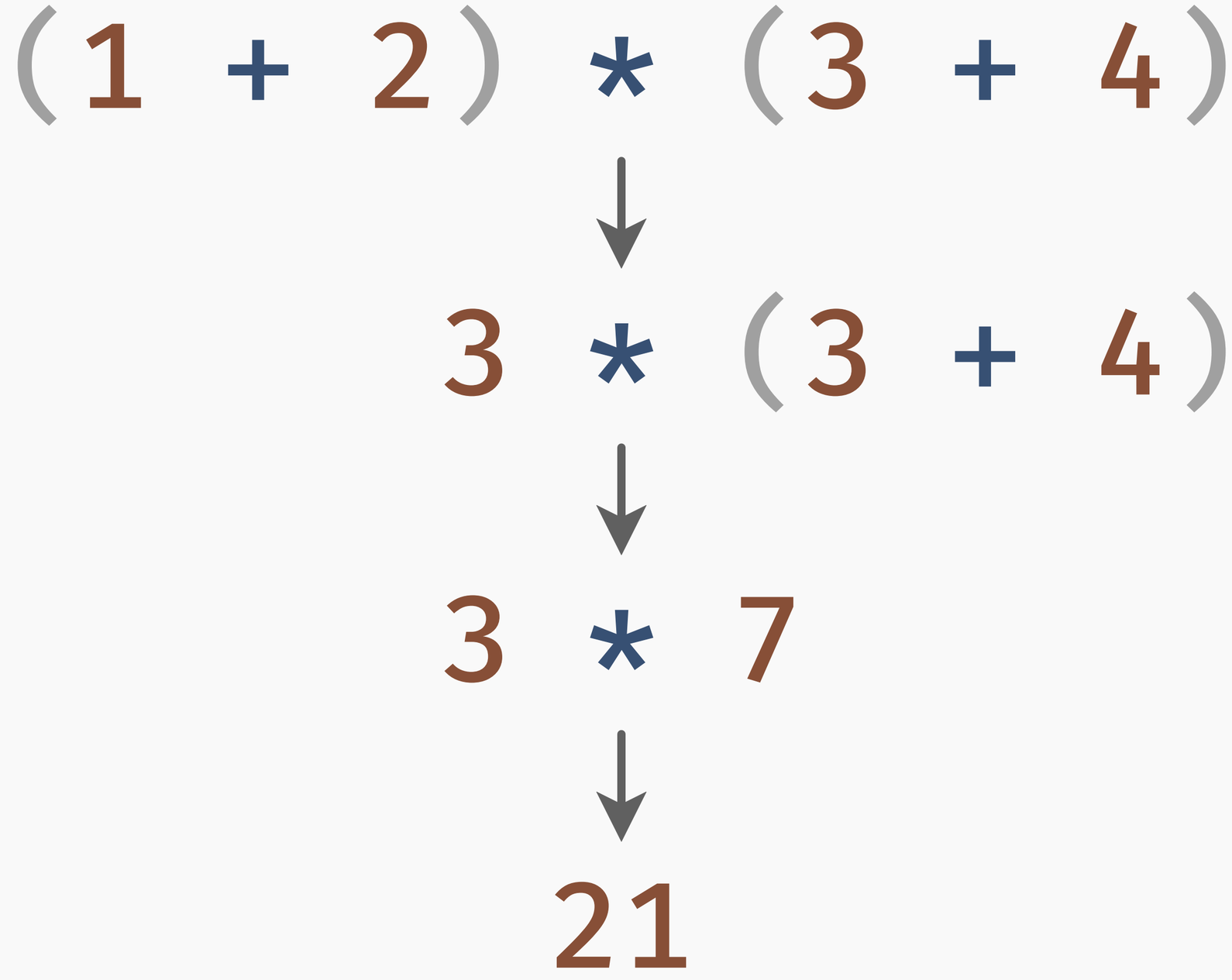

(1 + 2) * (3 + 4)
3 * (3 + 4)
3 * 7

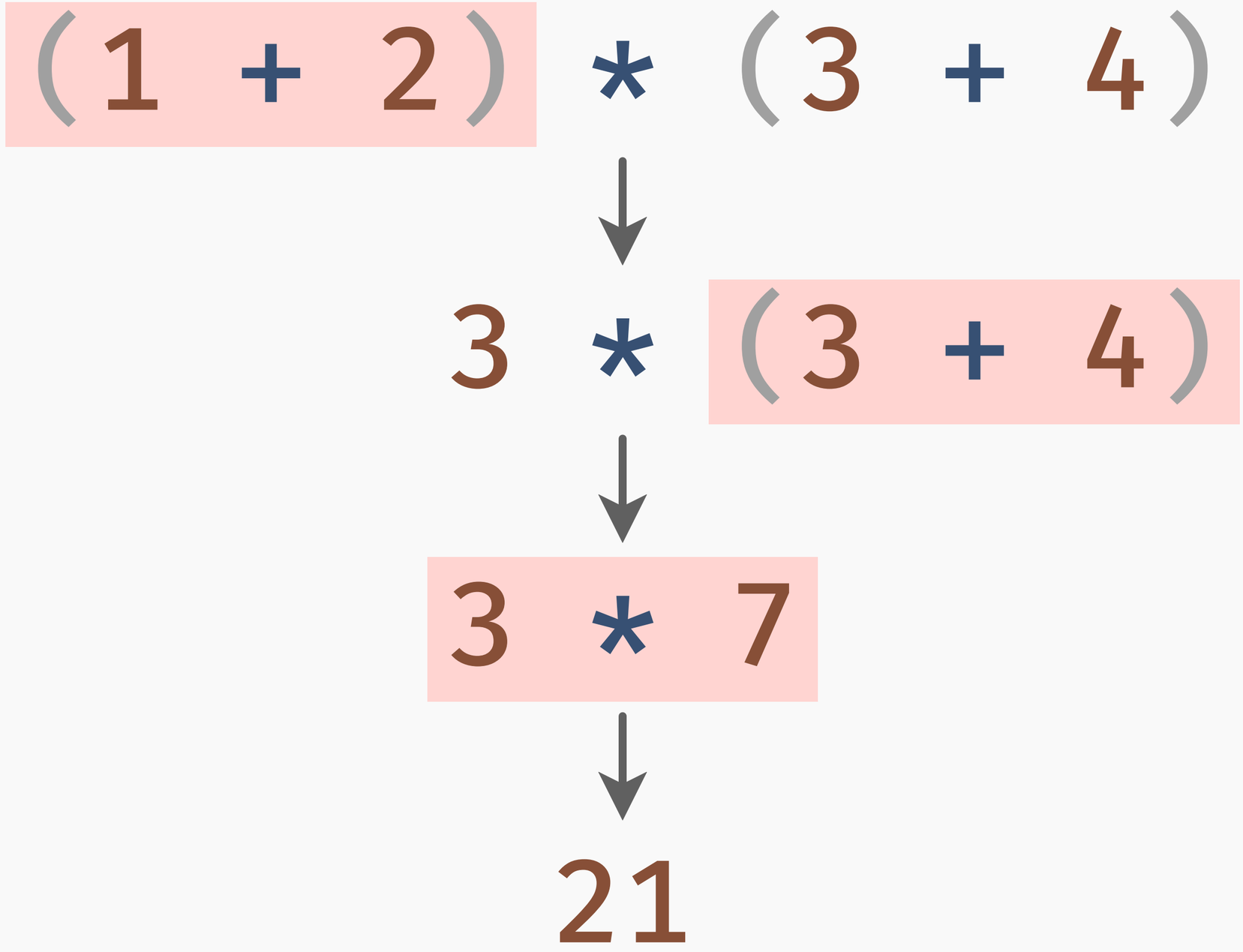
```
(1 + 2) * (3 + 4)
```

```
3 * (3 + 4)
```

```
3 * 7
```


(1 + 2) * (3 + 4)
3 * (3 + 4)
3 * 7
21


(1 + 2) * (3 + 4)
3 * (3 + 4)
3 * 7
21


(1 + 2) * (3 + 4)
3 * (3 + 4)
3 * 7
21

```
(1 + 2) * (3 + 4)
```

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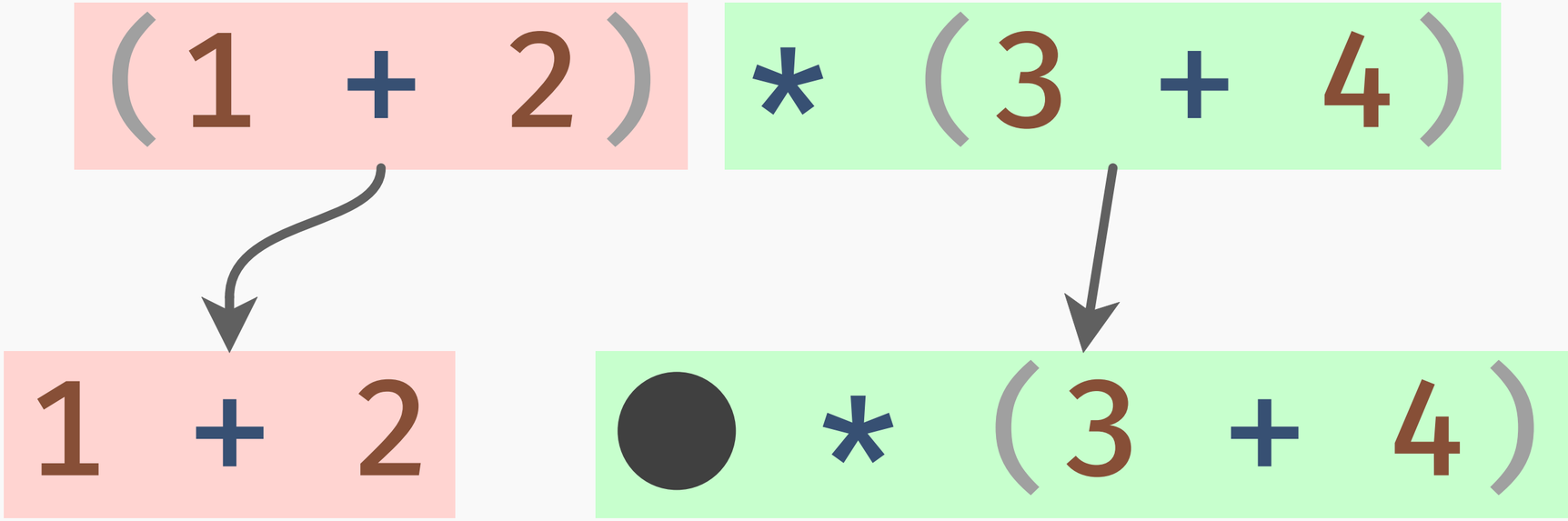
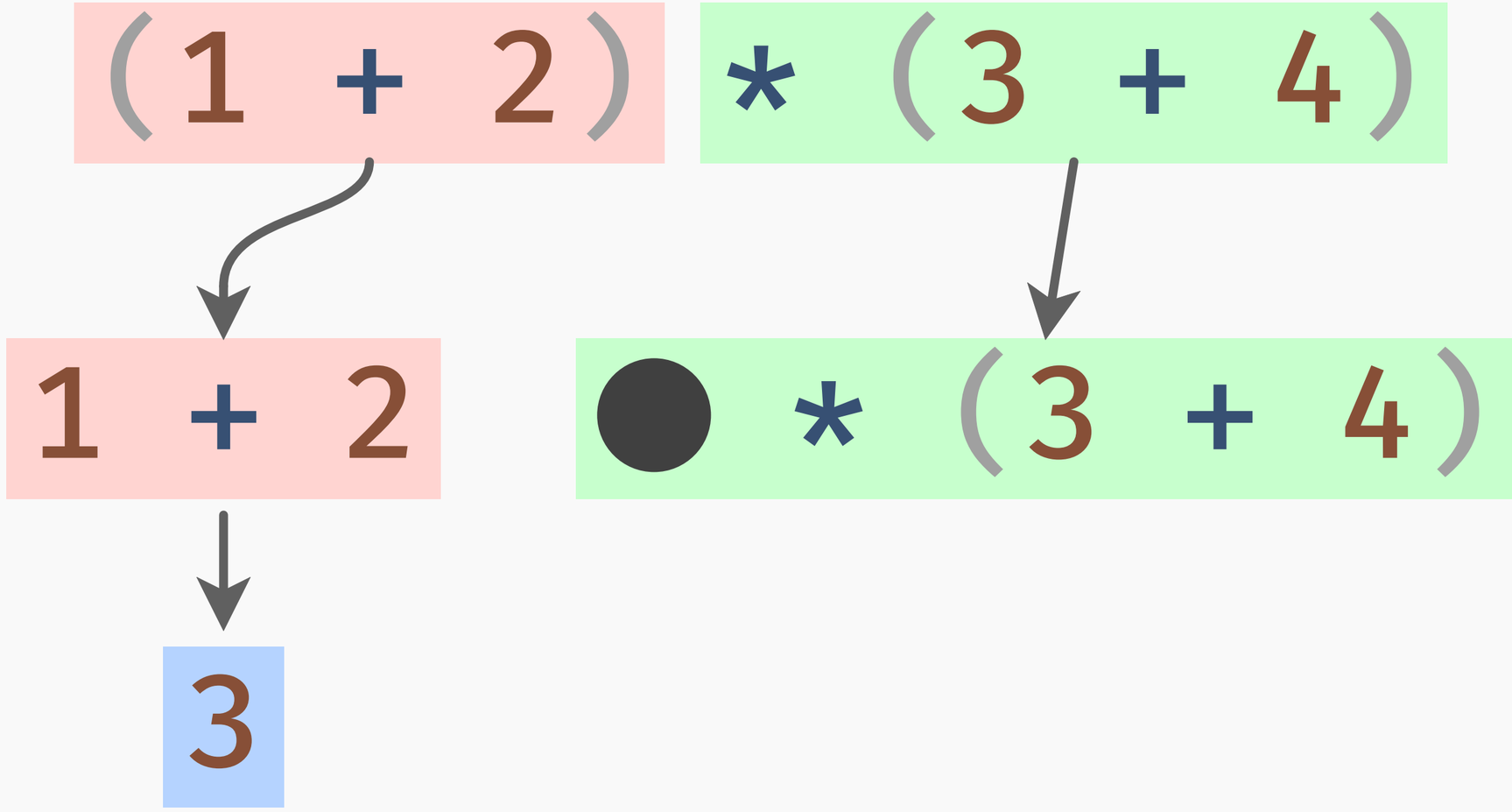
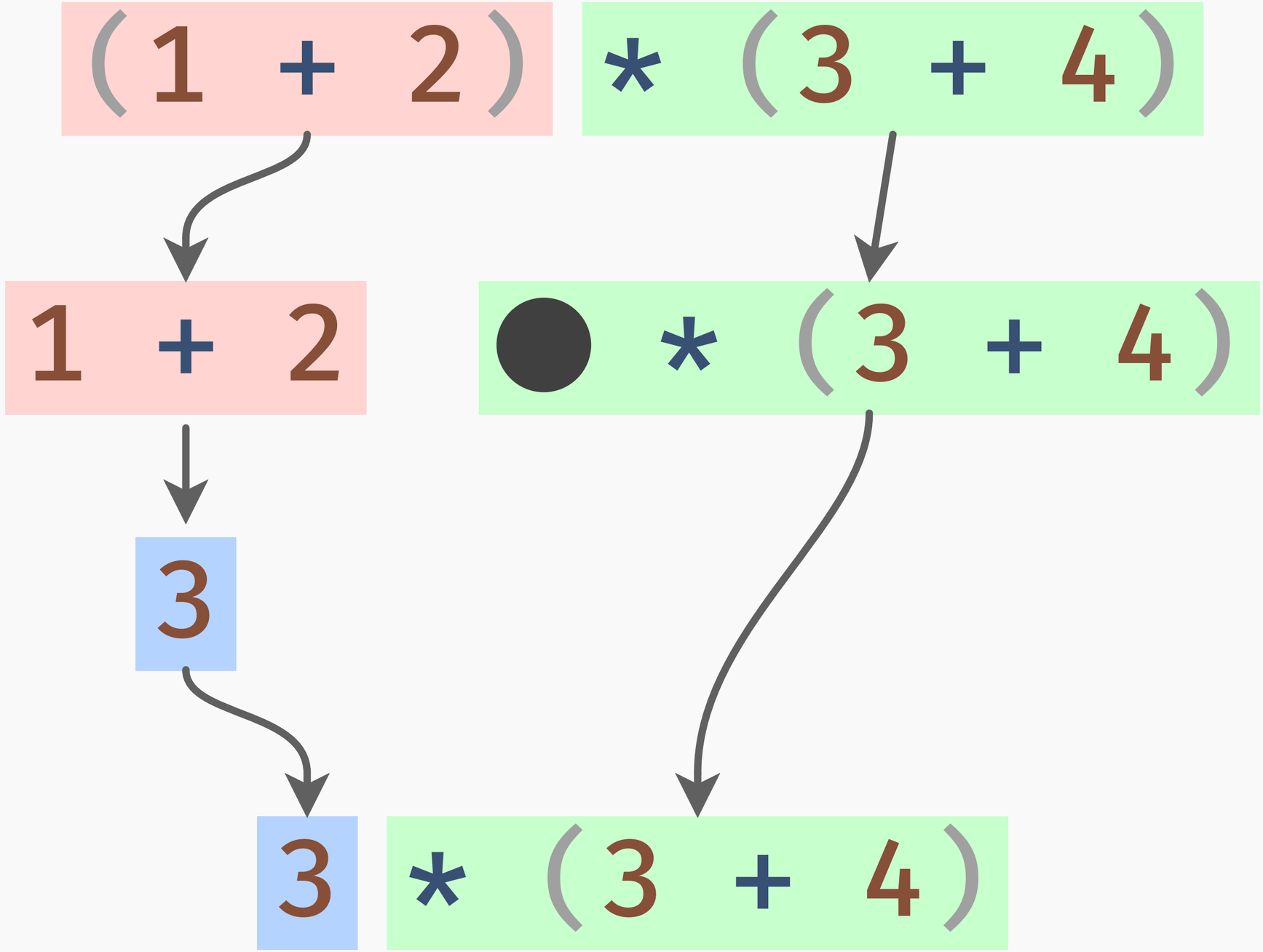
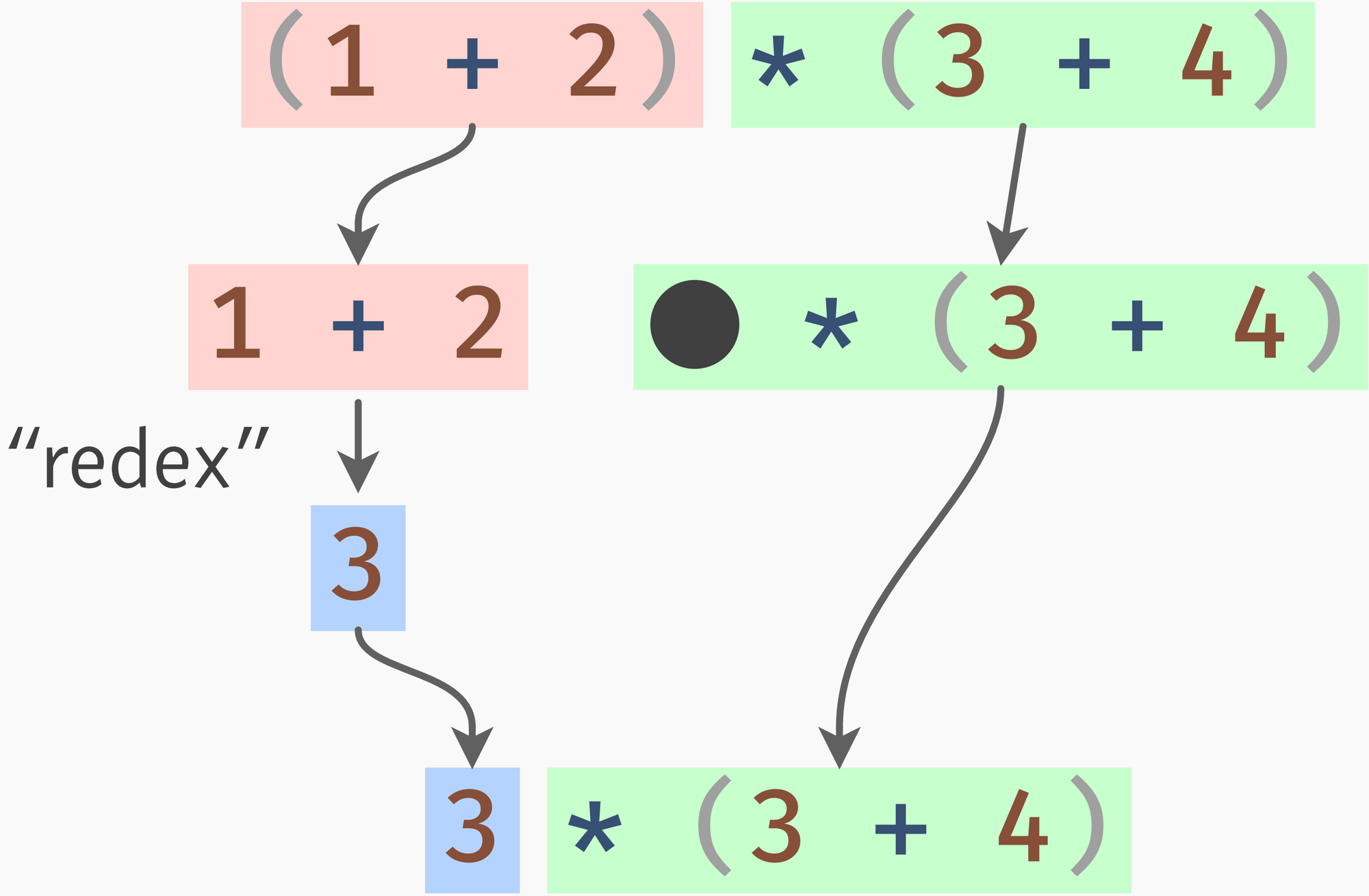
(1 + 2) * (3 + 4)
```

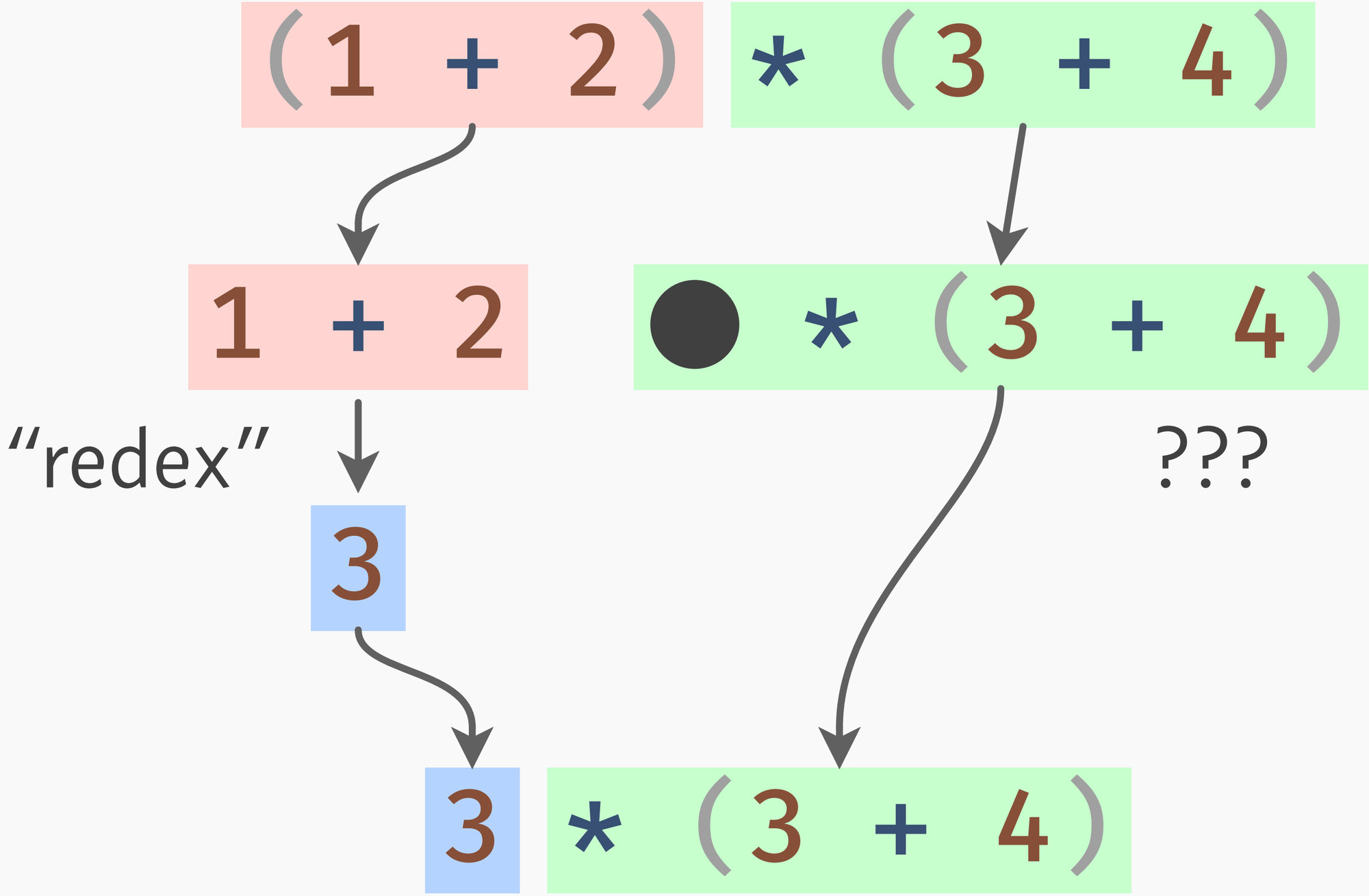
```
(1 + 2) * (3 + 4)
```

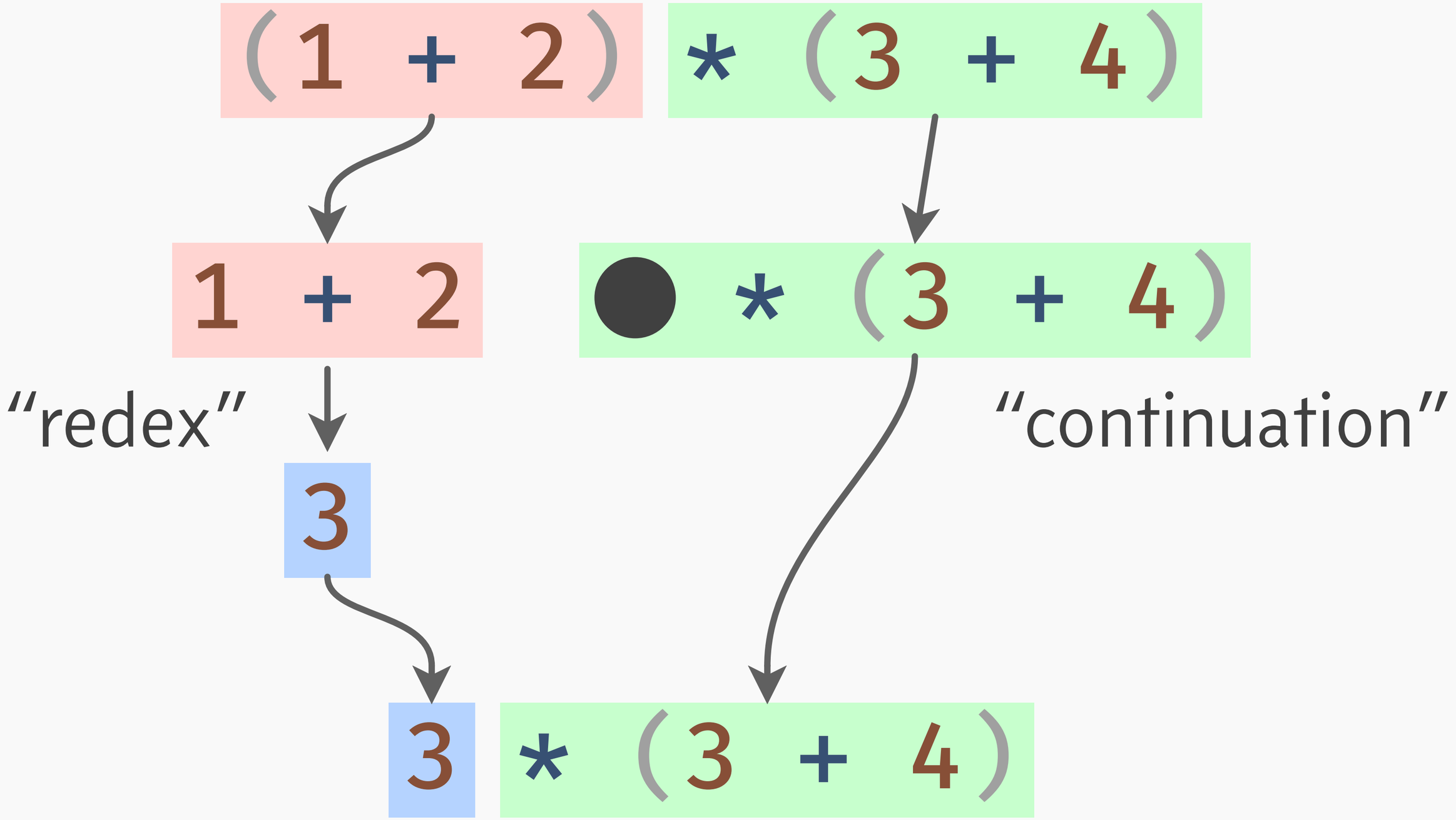

(1 + 2)
* (3 + 4)
1 + 2
● * (3 + 4)


(1 + 2)
* (3 + 4)
1 + 2
● * (3 + 4)
3


(1 + 2)
* (3 + 4)
1 + 2
● * (3 + 4)
3
3
* (3 + 4)

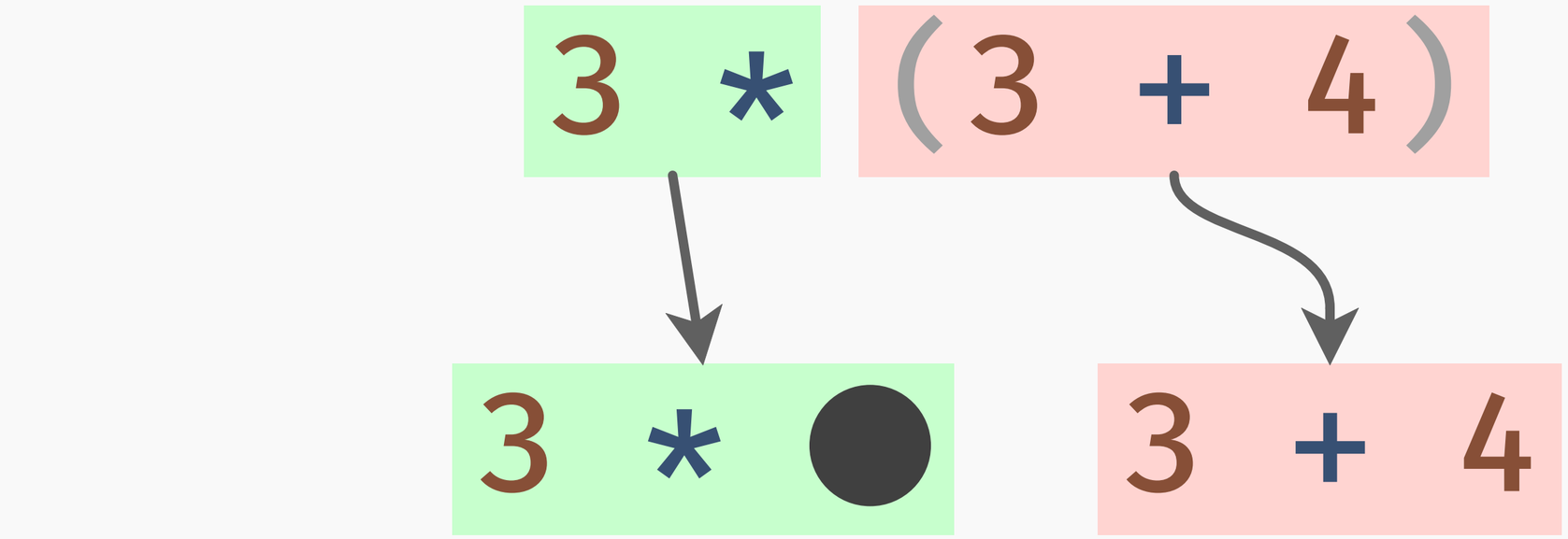

(1 + 2)
* (3 + 4)
1 + 2
● * (3 + 4)
“redex”
3
3
* (3 + 4)

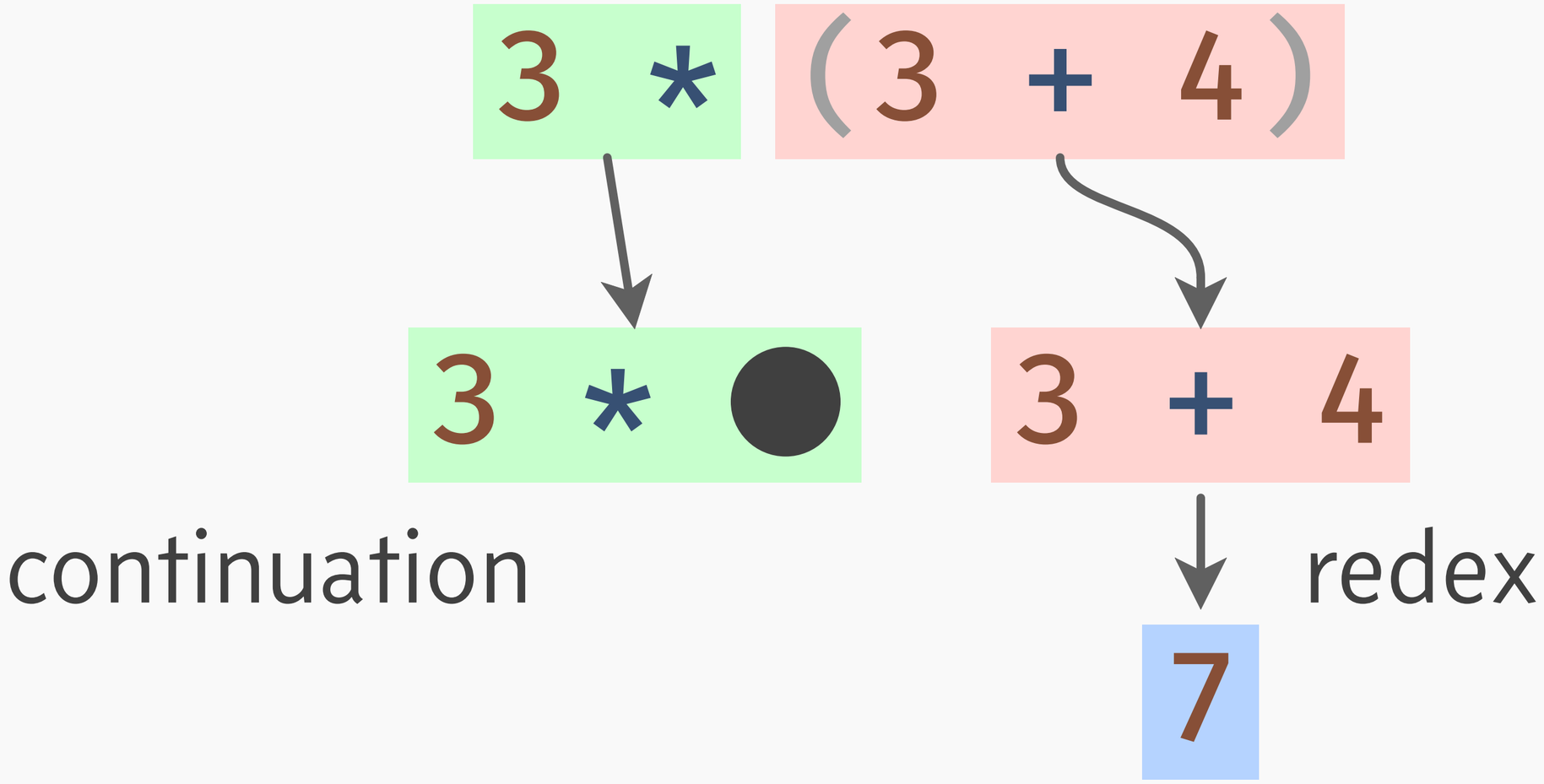

(1 + 2)
* (3 + 4)
1 + 2
● * (3 + 4)
"redex"
???
3
3
* (3 + 4)

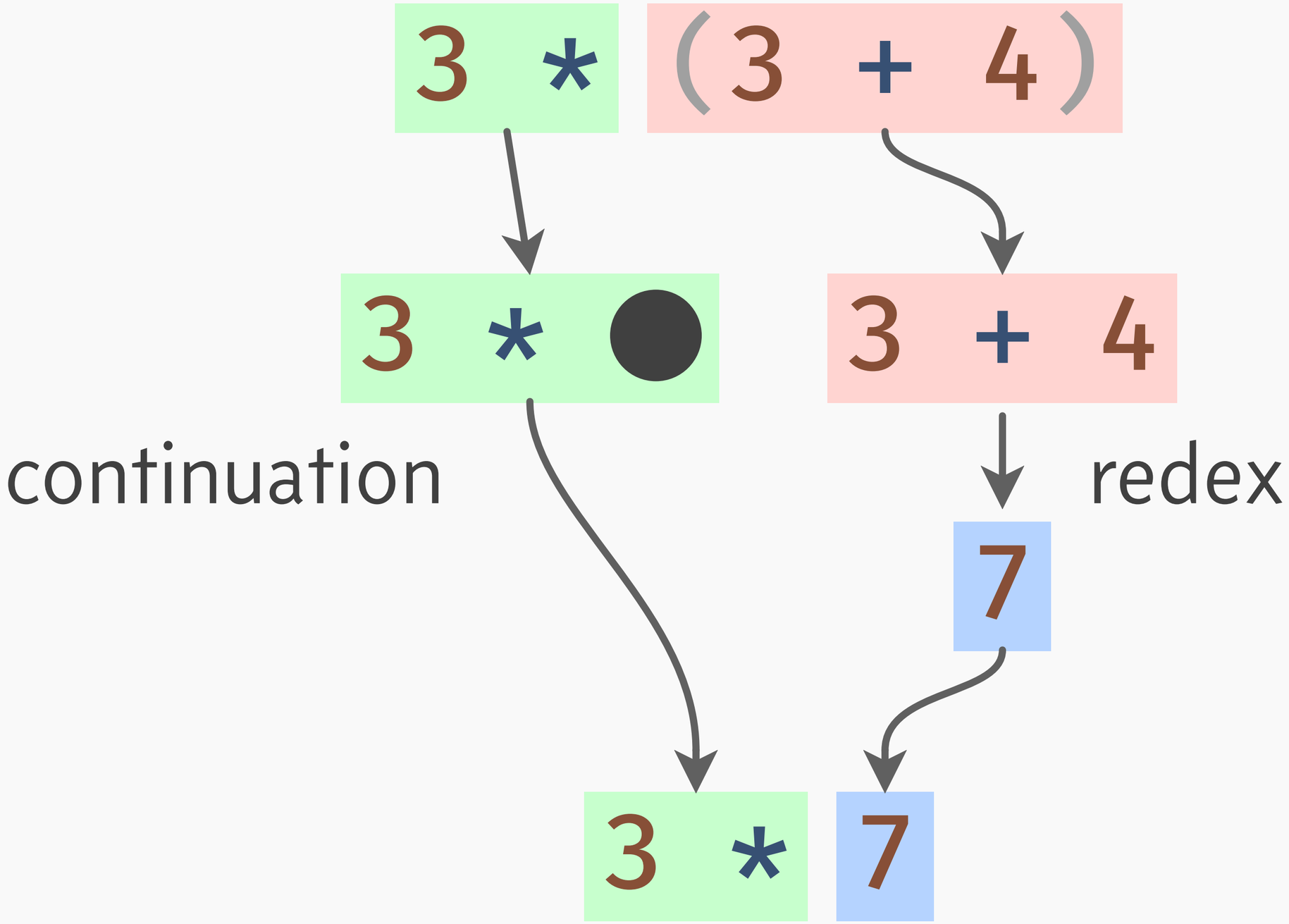

(1 + 2)
* (3 + 4)
1 + 2
● * (3 + 4)
“redex”
“continuation”
3
3
* (3 + 4)

```
3 * (3 + 4)
```

```
3 * (3 + 4)
```

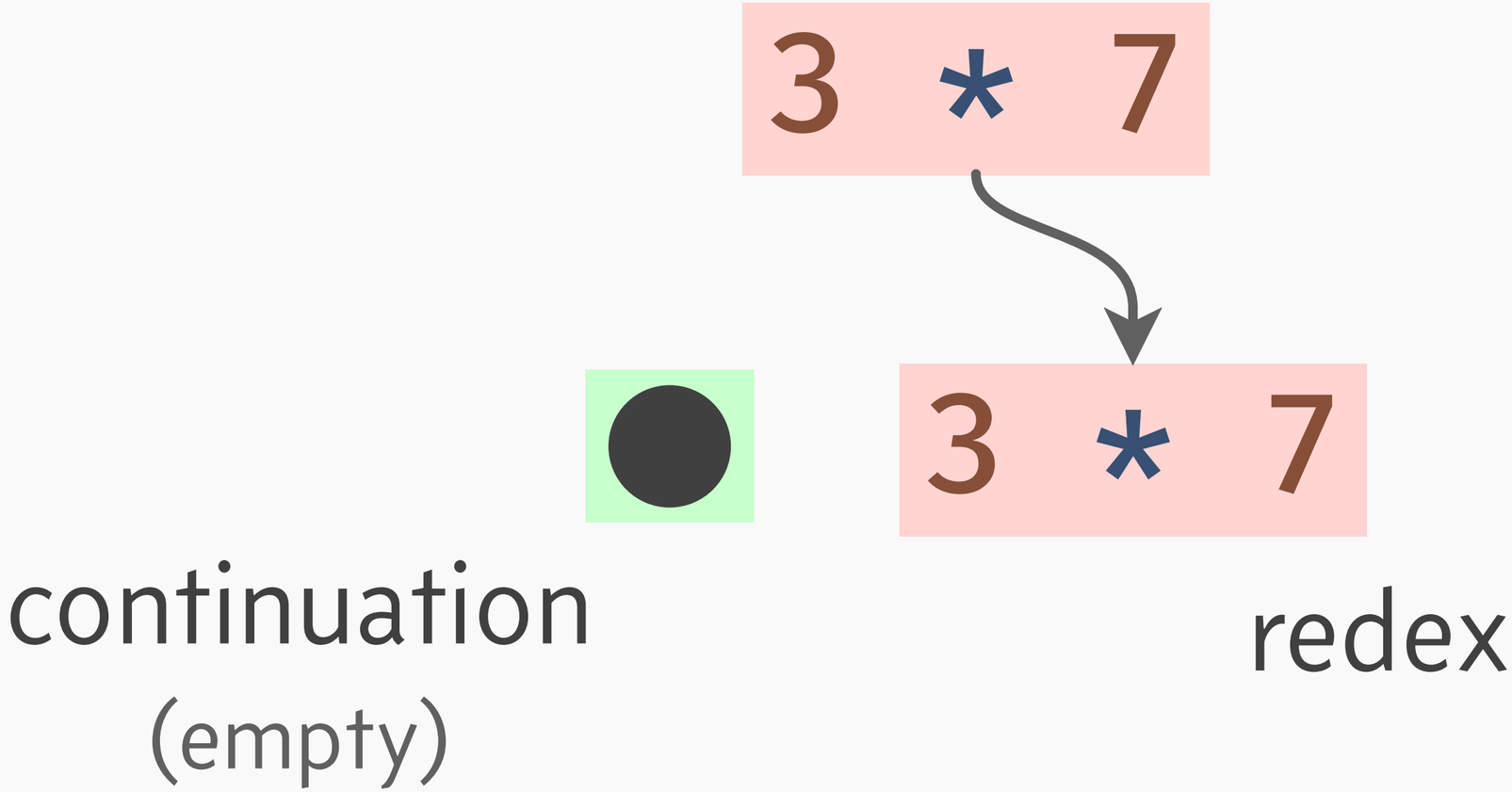

3 *
(3 + 4)
3 *
3 + 4
continuation
redex

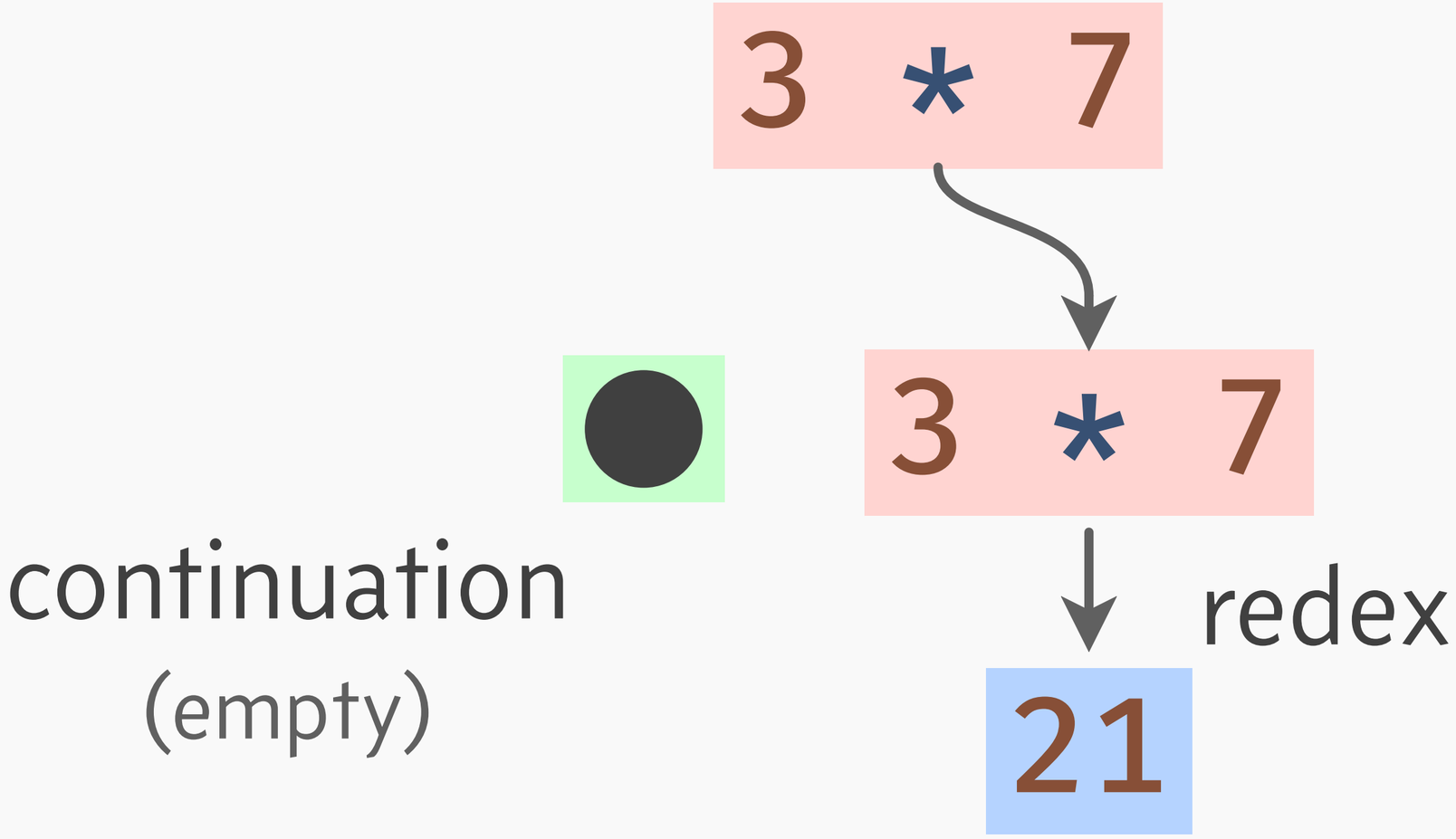

3 * (3 + 4)
3 * ●
3 + 4
continuation
redex
7

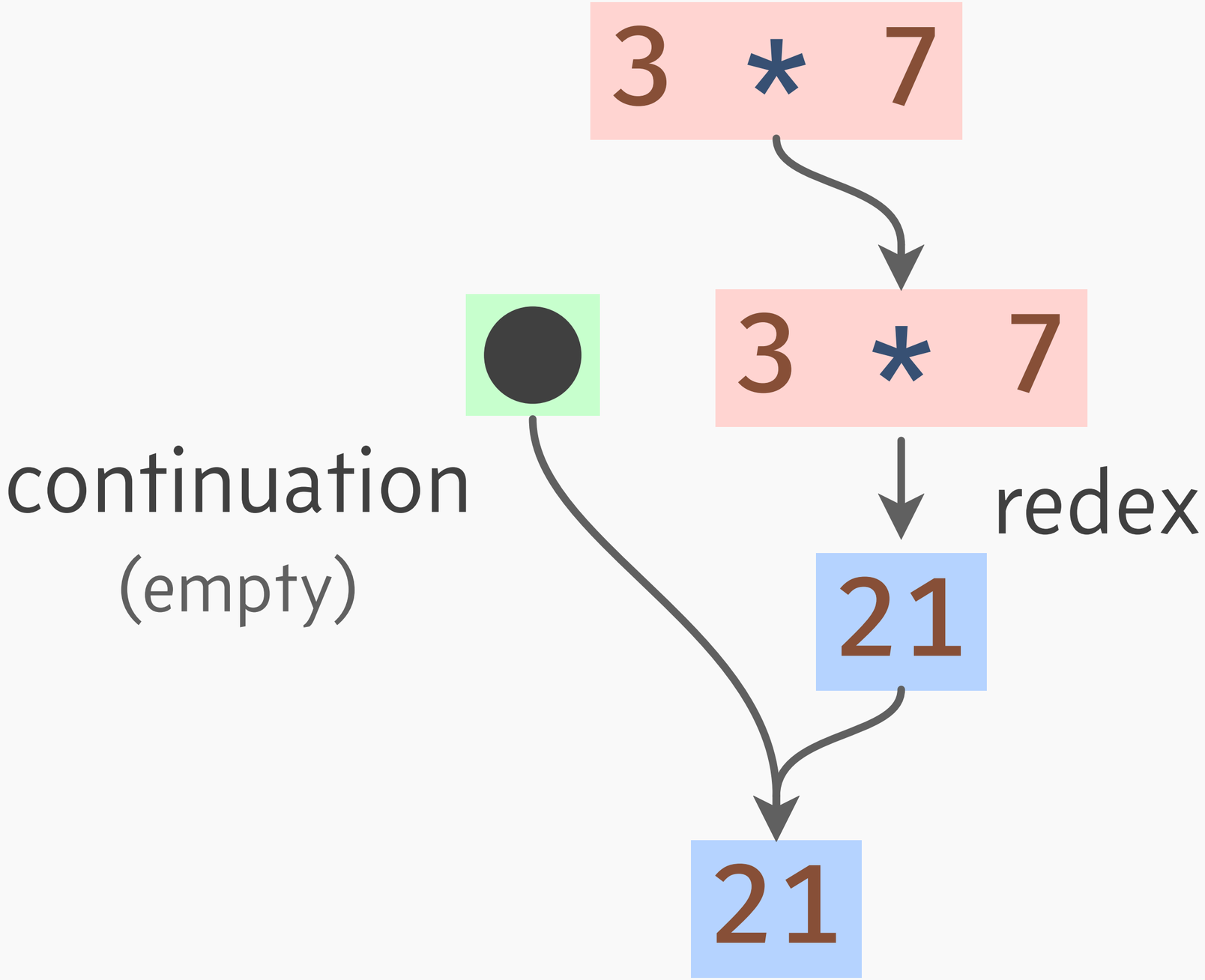

3 * (3 + 4)
3 * ●
3 + 4
continuation
redex
7
3 *
7

3 * 7

```
3 * 7
```


3 * 7
3 * 7
continuation
(empty)
redex


3 * 7
3 * 7
continuation
(empty)
redex
21


3 * 7
3 * 7
continuation
(empty)
redex
21
21

What is the continuation?

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→ The “context” in which the redex is evaluated.

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→ The “context” in which the redex is evaluated.

→ An expression with a hole.

→ The place the redex’s value is “returned to”.

→ “The rest of the program.”

```
let x = 1 + 2
let y = 3 + 4
x * y
```

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x * y
```
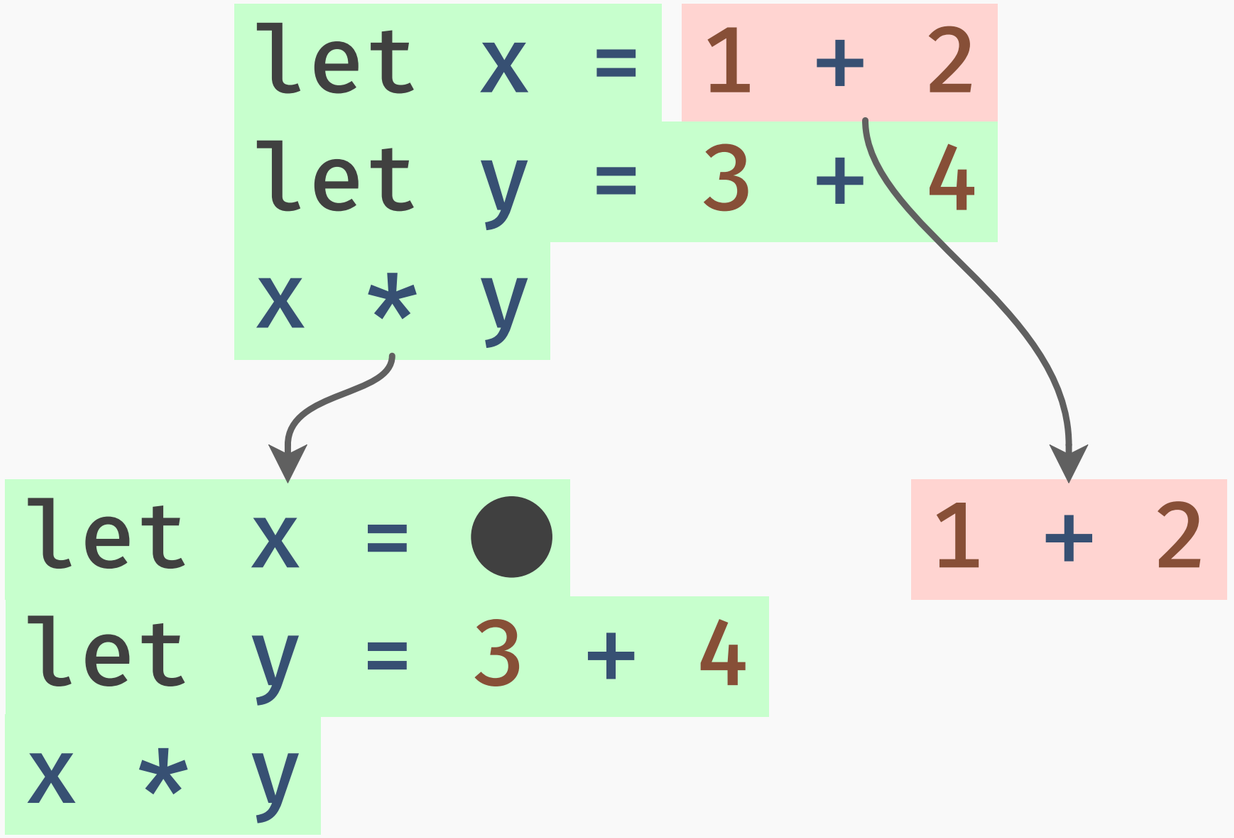
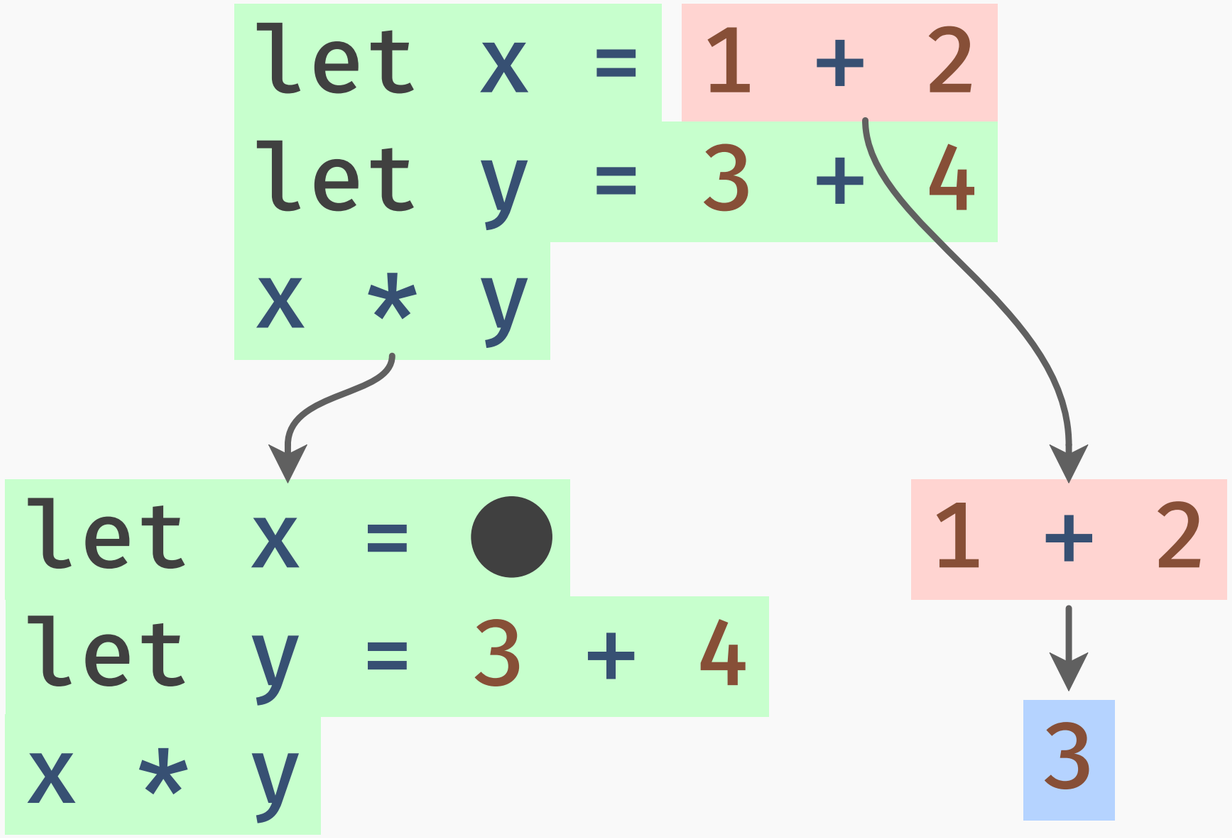
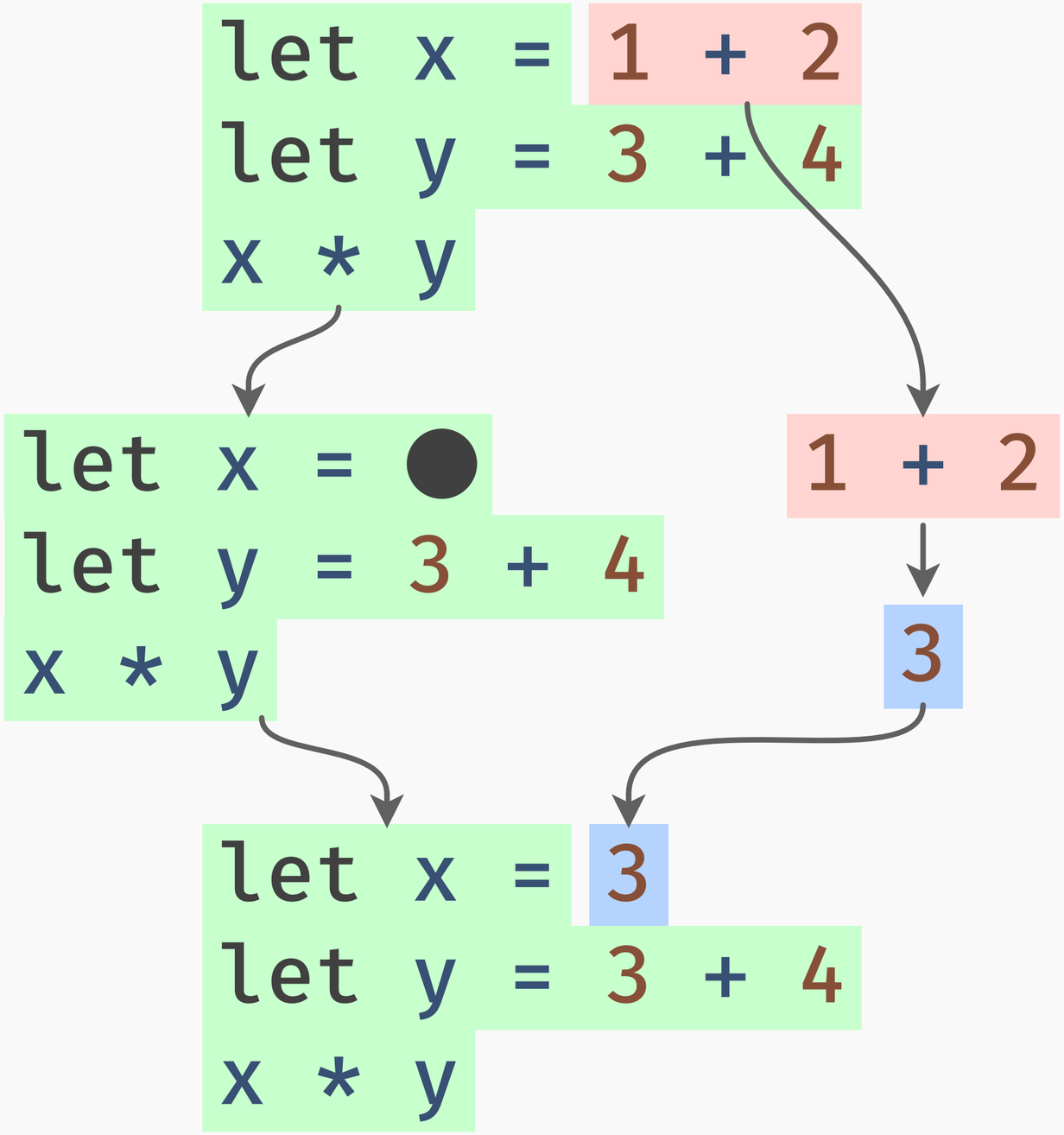


let x = 1 + 2
let y = 3 + 4
x * y
let x = ●
let y = 3 + 4
x * y
1 + 2


let x = 1 + 2
let y = 3 + 4
x * y
let x = ●
let y = 3 + 4
x * y
1 + 2
3


let x = 1 + 2
let y = 3 + 4
x * y
let x = ●
let y = 3 + 4
x * y
1 + 2
3
let x = 3
let y = 3 + 4
x * y

```
let x = 3
let y = 3 + 4
x * y
```

```
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let y = 3 + 4
x * y
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let x = 3
let y = 3 + 4
x * y
```

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let x = 3
let y = 3 + 4
x * y
```

```
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3 * y
```

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```

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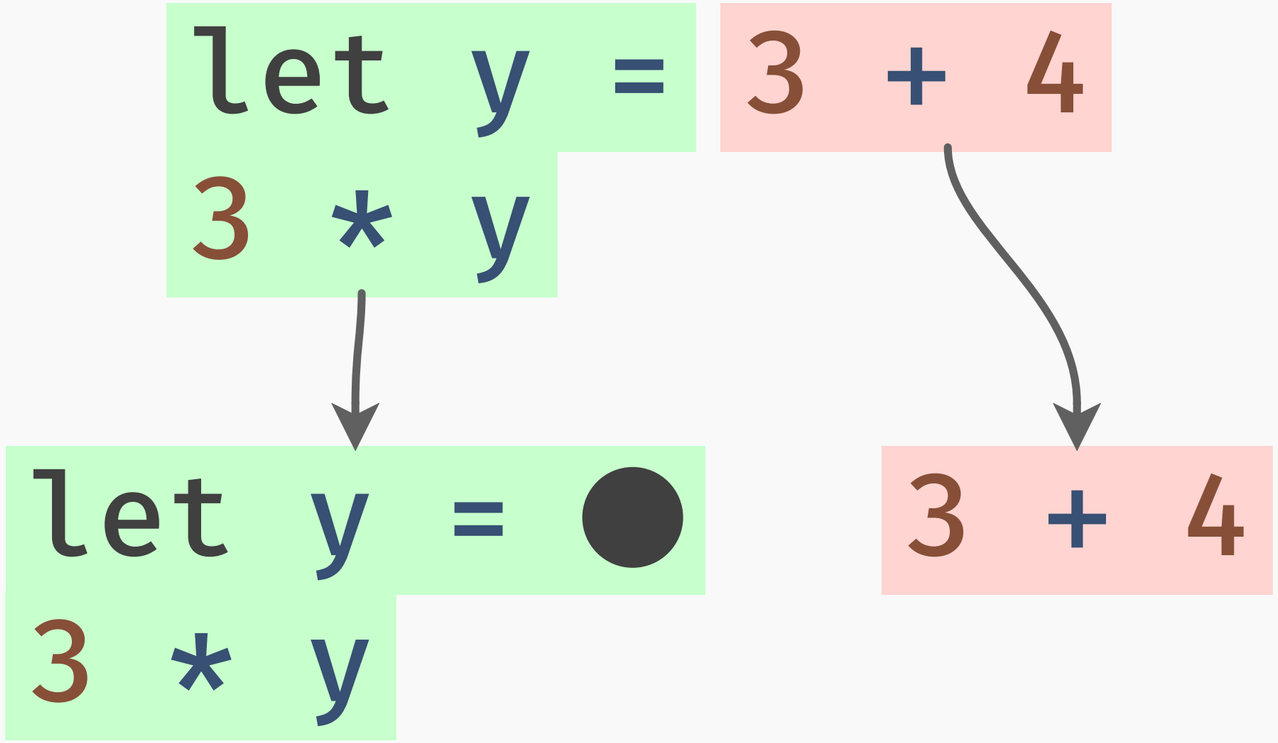
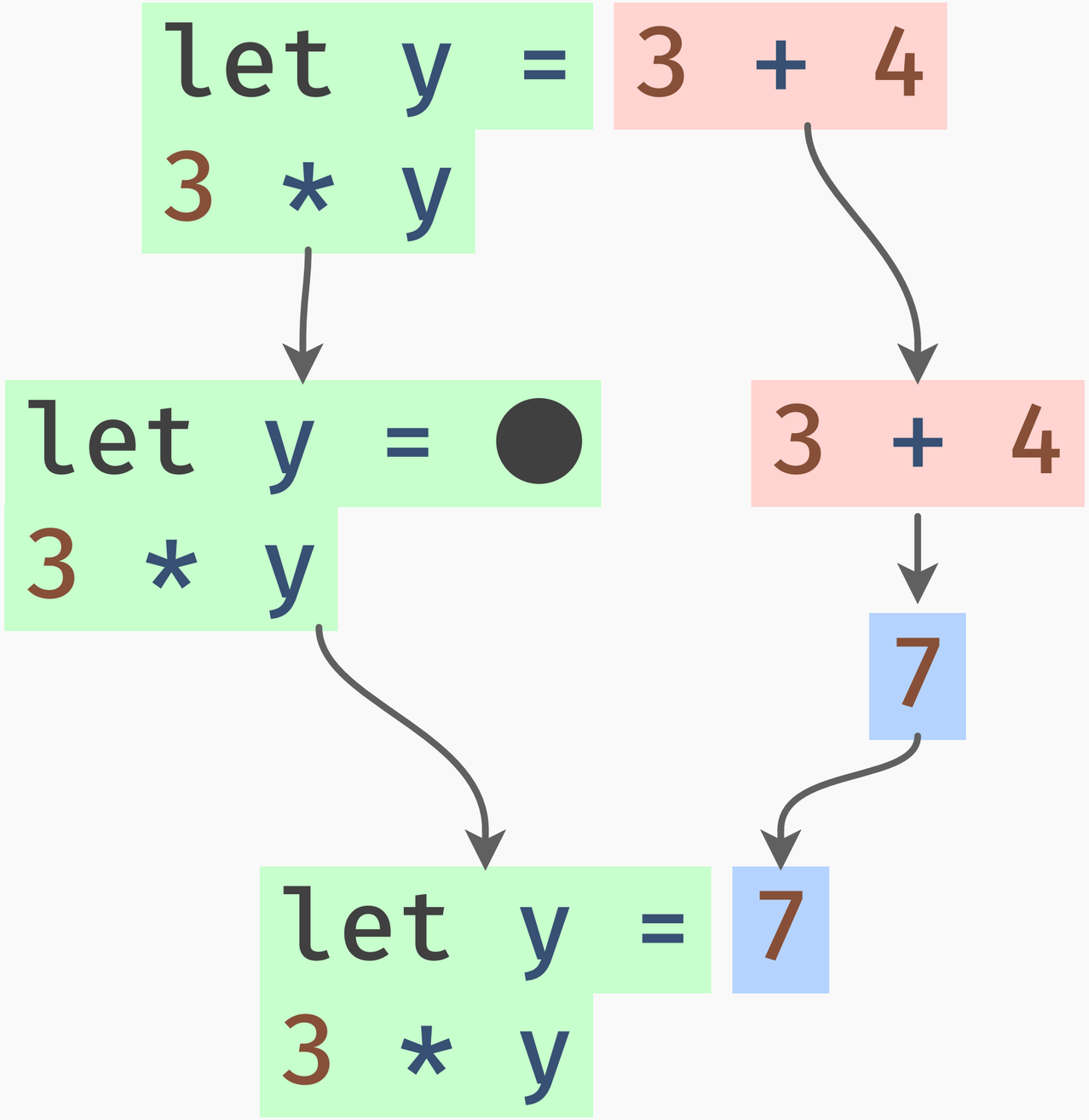
3 * y
```


let y = 3 + 4
3 * y
let y = ●
3 * y
3 + 4


let y =
3 * y
3 + 4
let y =
3 * y
3 + 4
7
let y =
3 * y
7

Why care about continuations?

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Evaluation is *extremely* regular:

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① Split the redex and continuation.

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④ Repeat.

Why care about continuations?

Evaluation is *extremely* regular:

① Split the redex and continuation.

② Reduce the redex.

③ Substitute the result into the continuation.

④ Repeat.

Why is the continuation itself interesting?

Compiler writers care about the continuation!

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Most programmers don’t have much reason to, most of the time.

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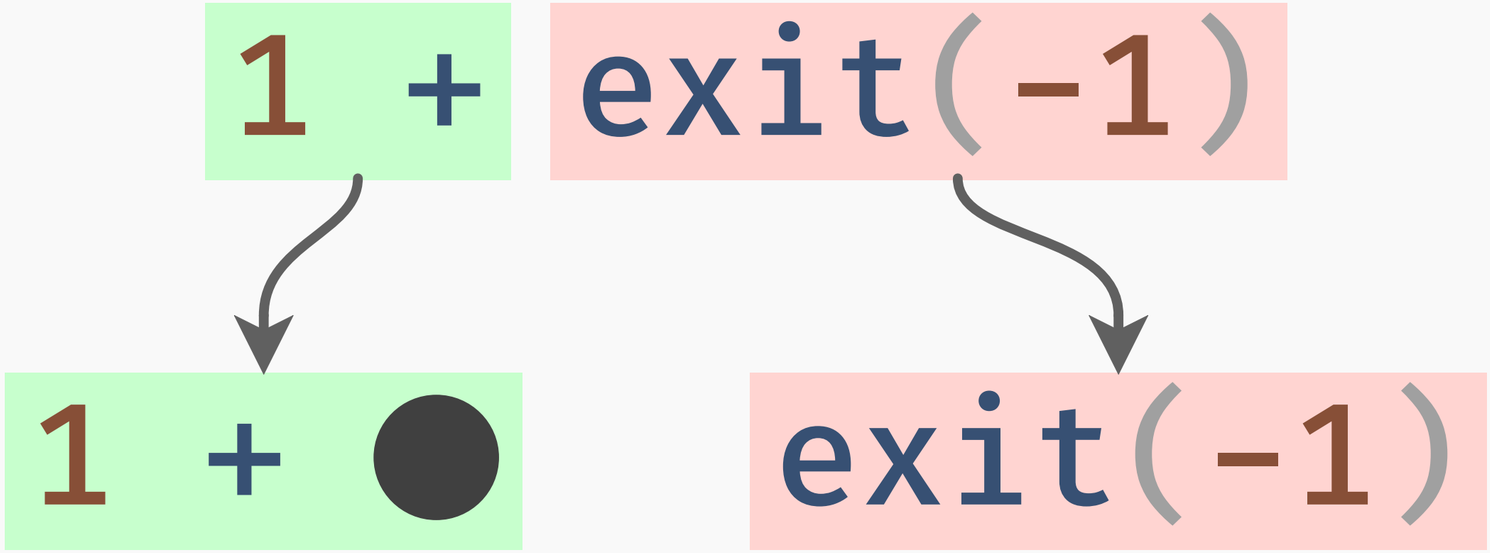
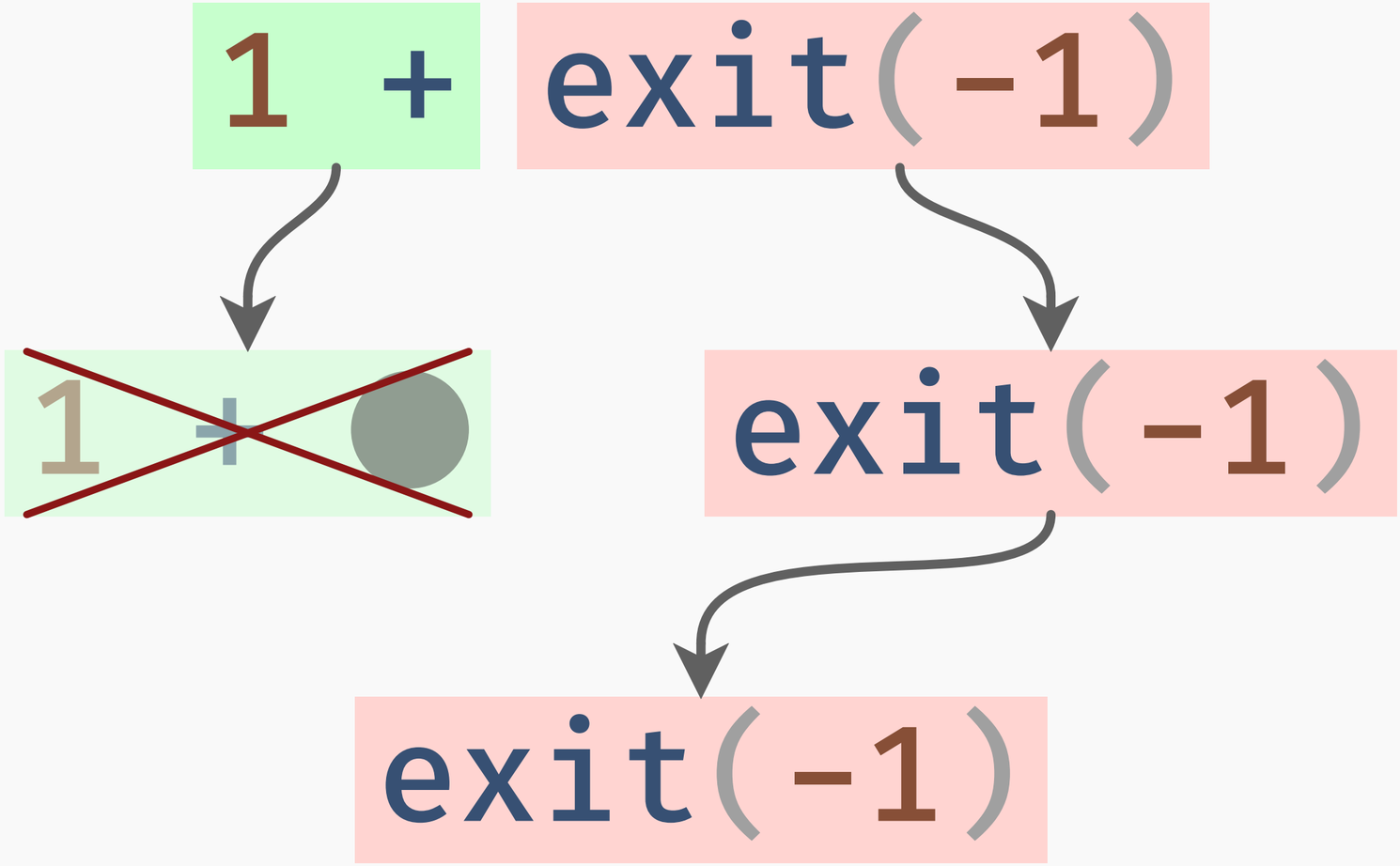
reason to, most of the time.

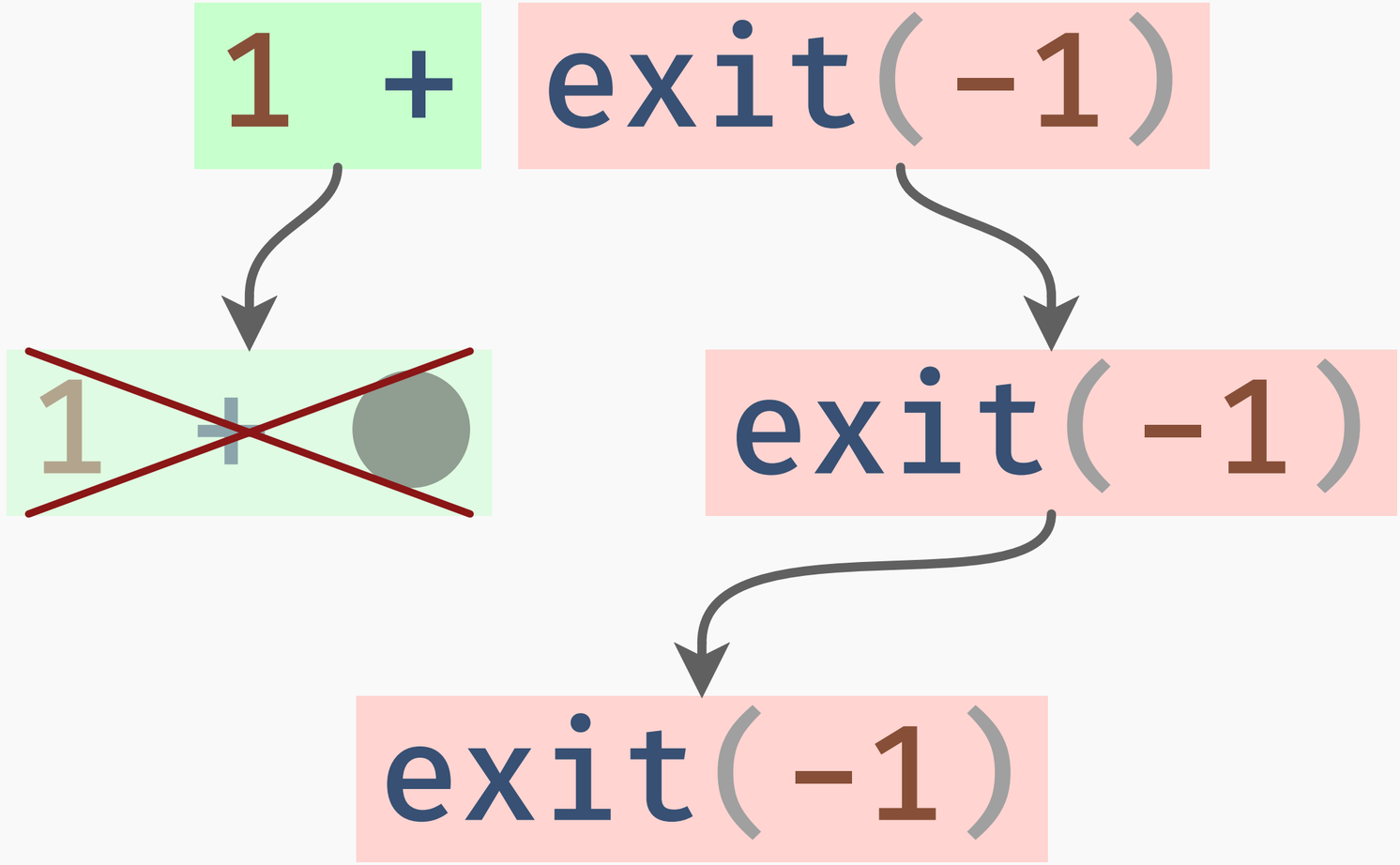
…but what about operators that use different rules?

```
1 + exit(-1)
```

```
1 + exit(-1)
```


1 +
exit(-1)
1 +
exit(-1)


1 +
exit(-1)
1 +
exit(-1)
exit(-1)



Continuation is thrown away!

`exit` is still not terribly interesting.

What about `throw` / `catch`?

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```
throw(exn)
```

What about `throw` / `catch`?

`throw(exn)`

Raises `exn` as an exception.

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Raises `exn` as an exception.

```
catch{body, handler}
```

What about `throw` / `catch`?

`throw(exn)`

Raises `exn` as an exception.

`catch{body, handler}`

Evaluates `body`, and if an exception is raised, evaluates `handler(exn)`.

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + catch{2 * throw(5),
        (n) → 3 * n}
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + (3 * 5)
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + (3 * 5)
```

```
1 + 15
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + (3 * 5)
```

```
1 + 15
```

```
16
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + catch{2 * throw(5),
          (n) → 3 * n}
```

```
1 + catch{2 * throw(5),
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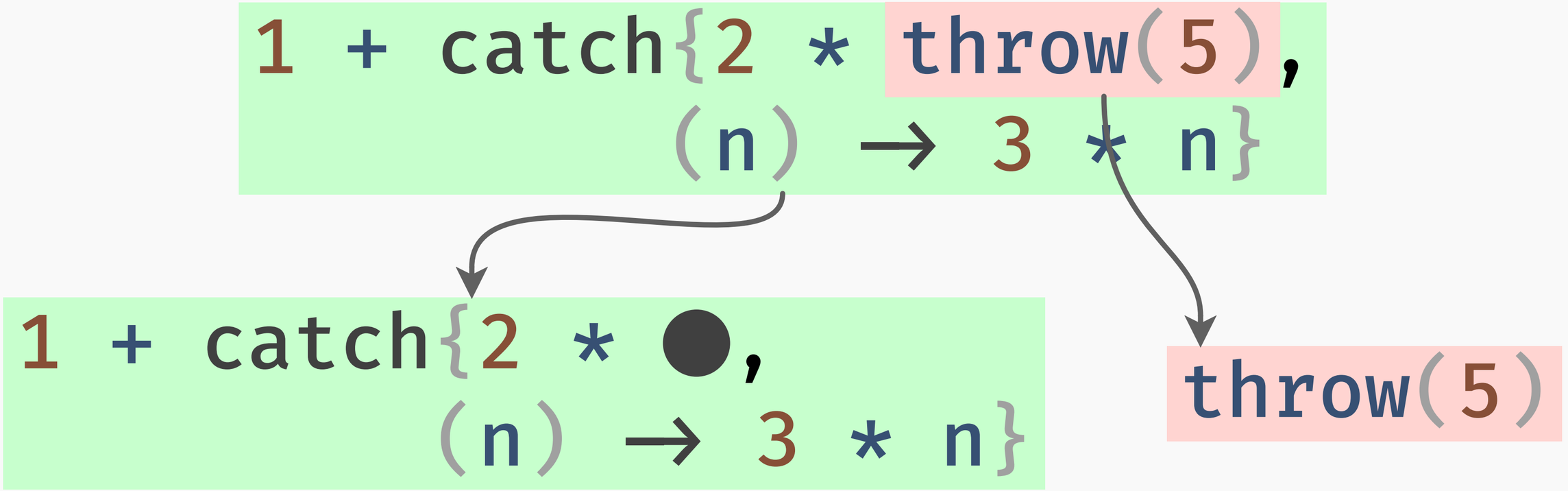
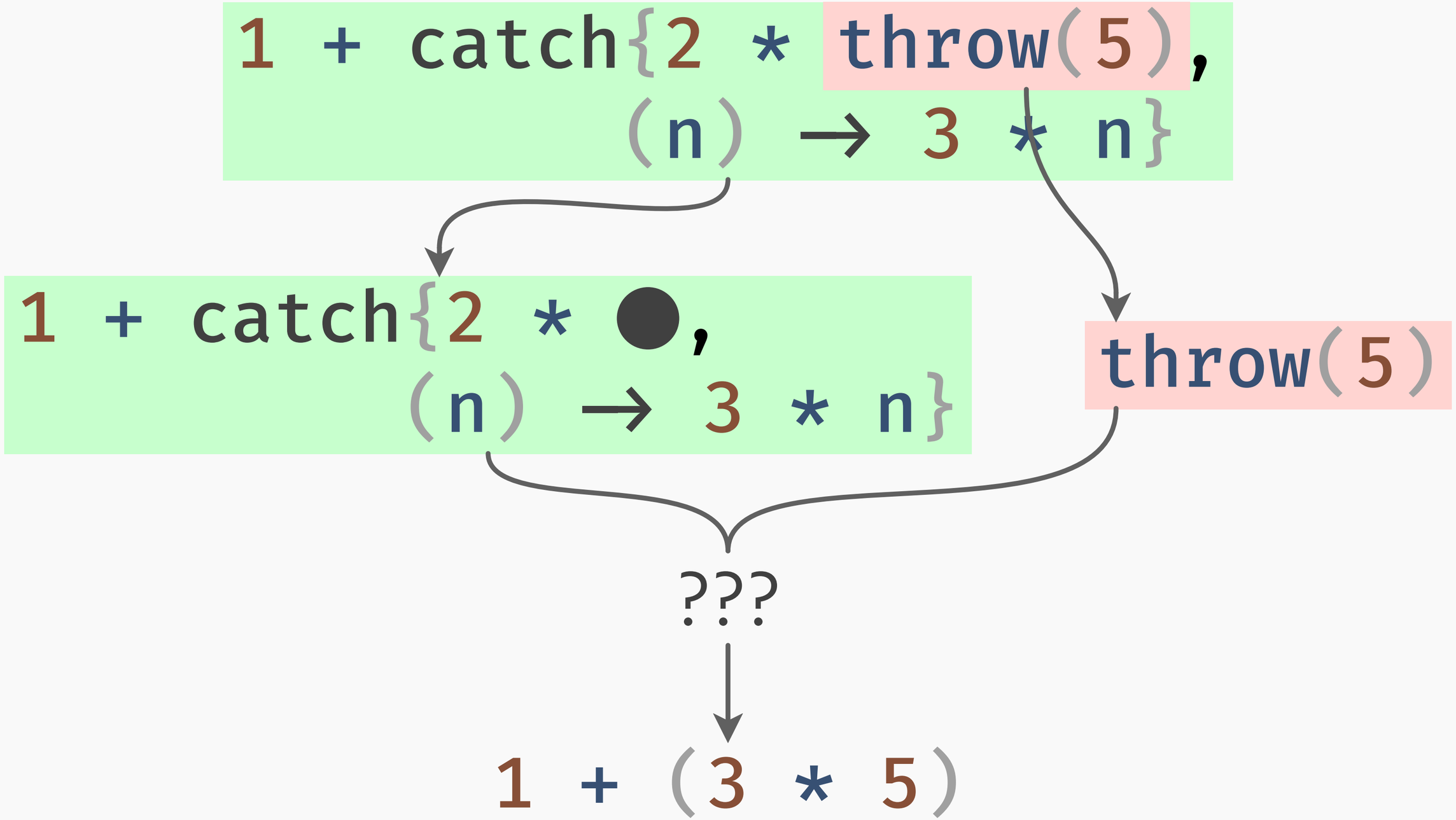
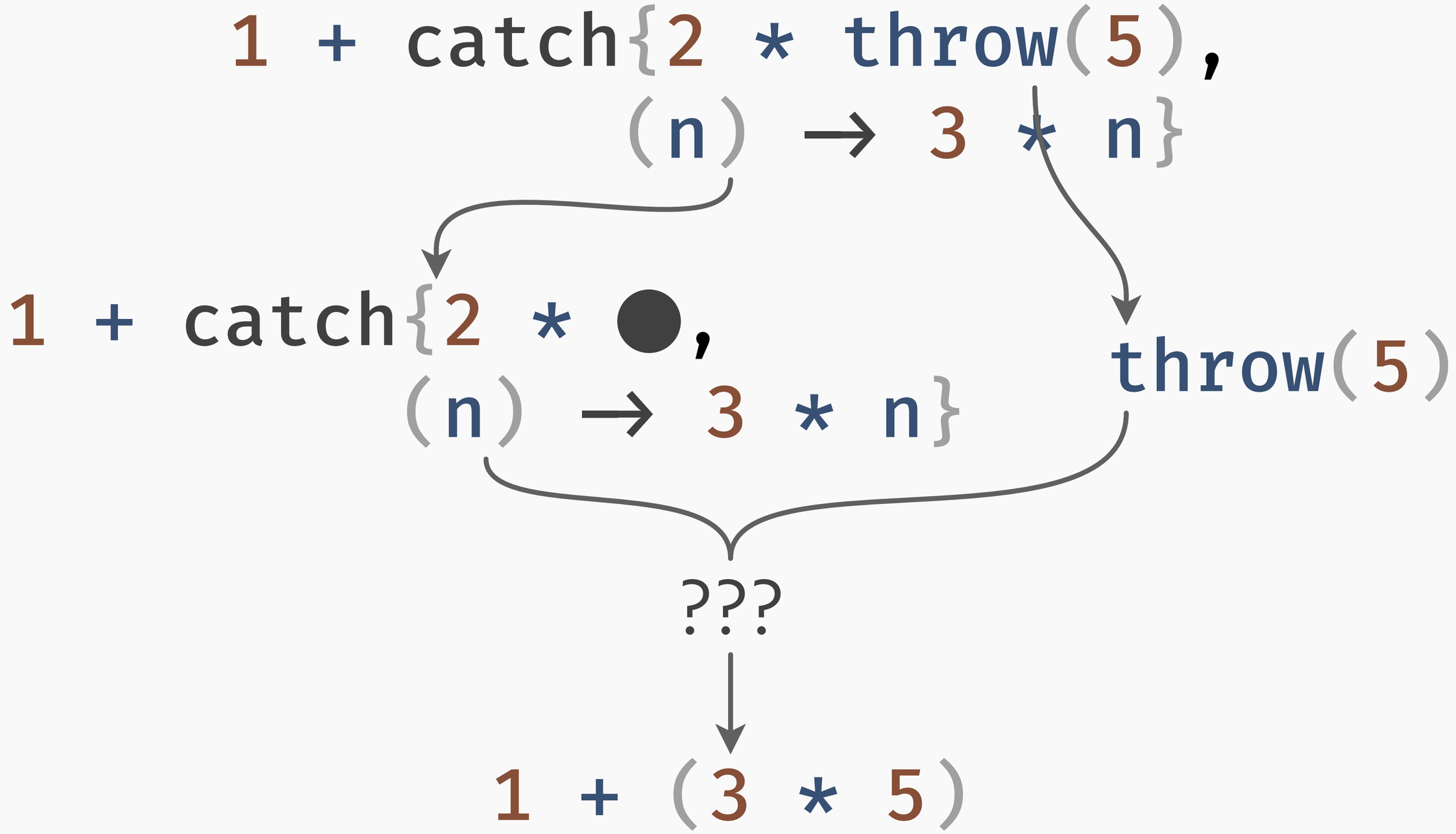
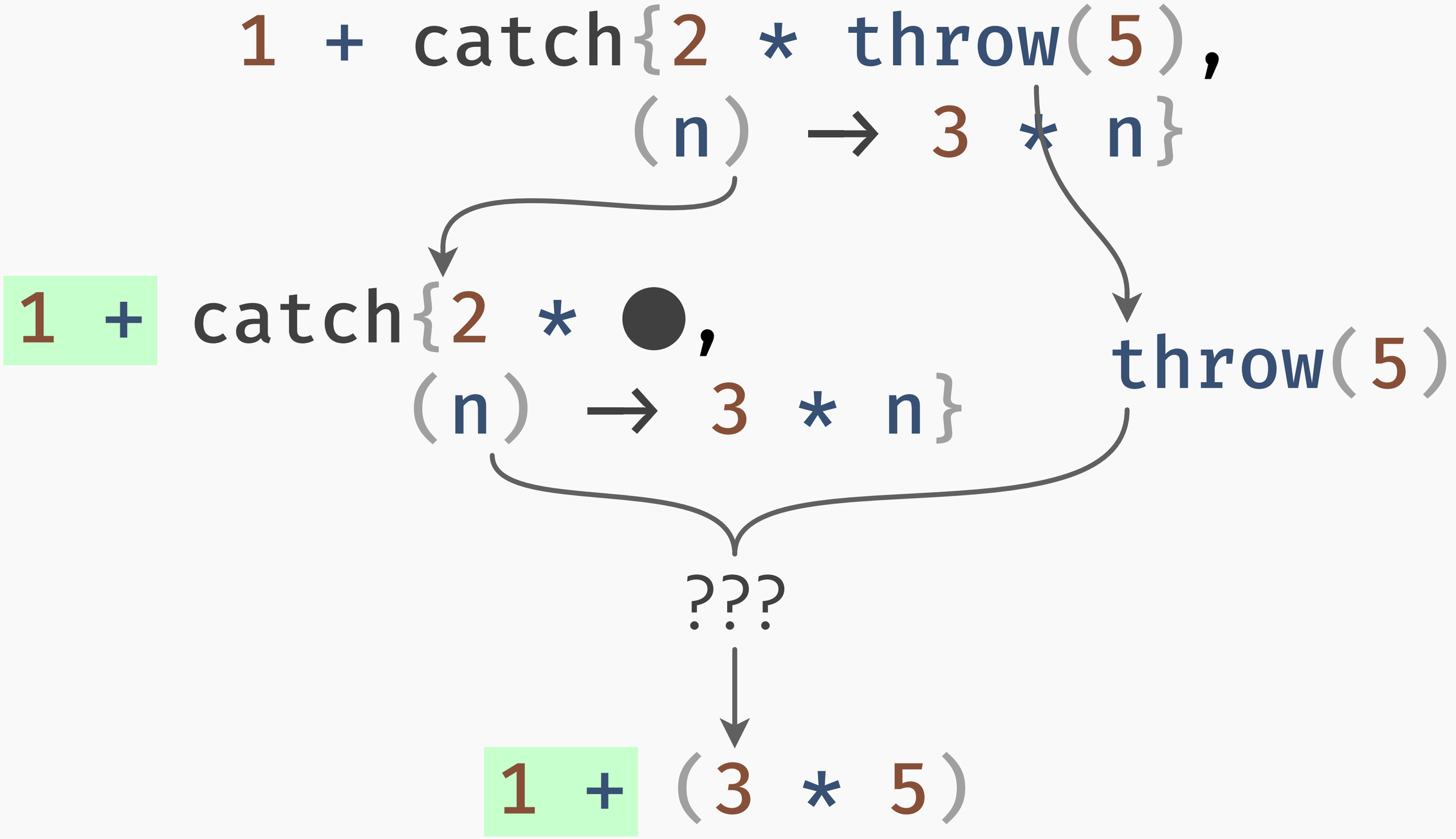
```


1 + catch{2 * throw(5),
(n) → 3 * n}
1 + catch{2 * ●,
(n) → 3 * n}
throw(5)


1 + catch{2 * throw(5),
(n) → 3 * n}
1 + catch{2 * ●,
(n) → 3 * n}
throw(5)
???
1 + (3 * 5)


1 + catch{2 * throw(5),
(n) → 3 * n}
1 + catch{2 * ●,
(n) → 3 * n}
throw(5)
???
1 + (3 * 5)


1 + catch{2 * throw(5),
(n) → 3 * n}
1 + catch{2 * ●,
(n) → 3 * n}
throw(5)
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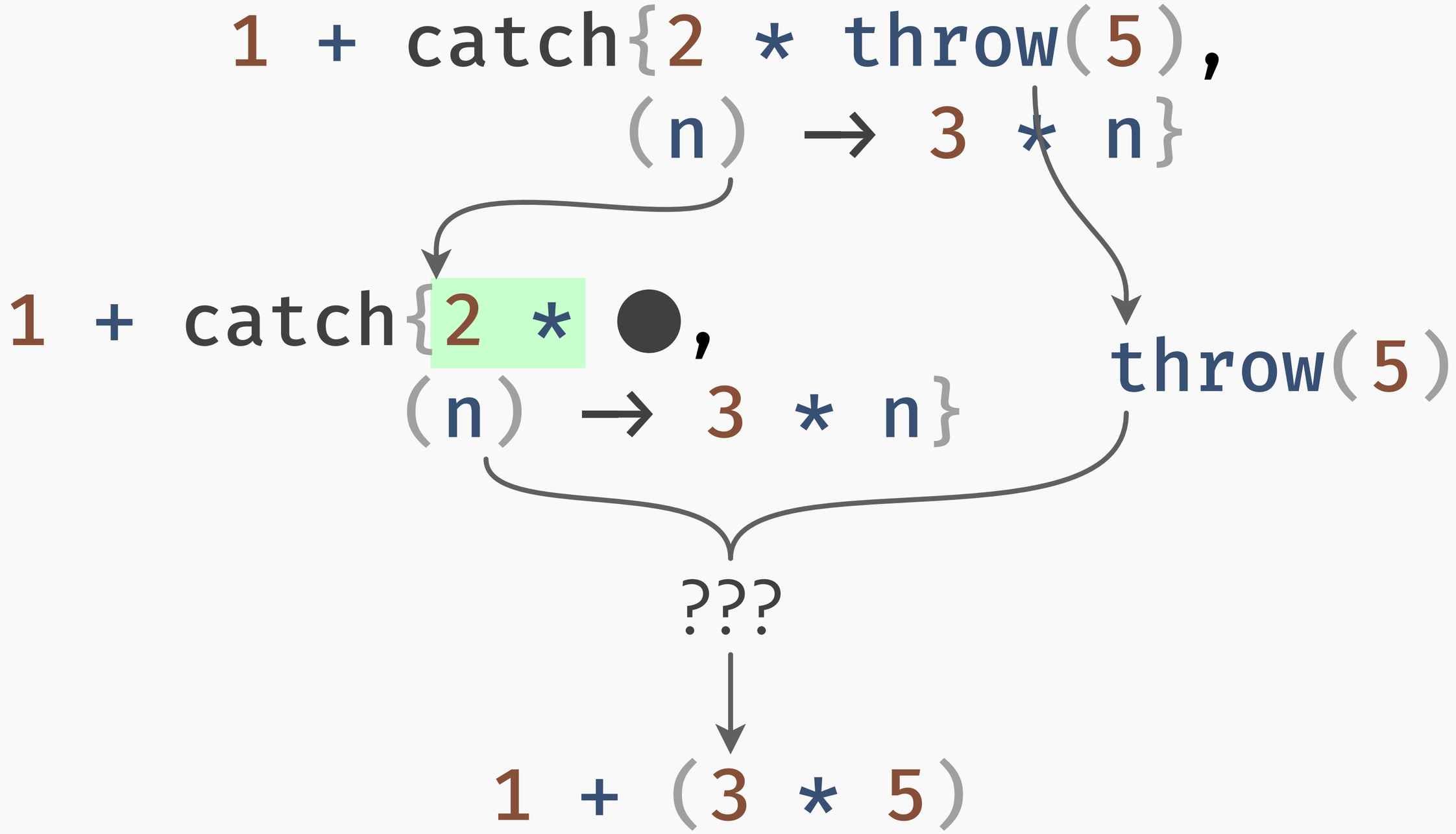
1 + (3 * 5)


1 + catch{2 * throw(5),
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(n) → 3 * n}
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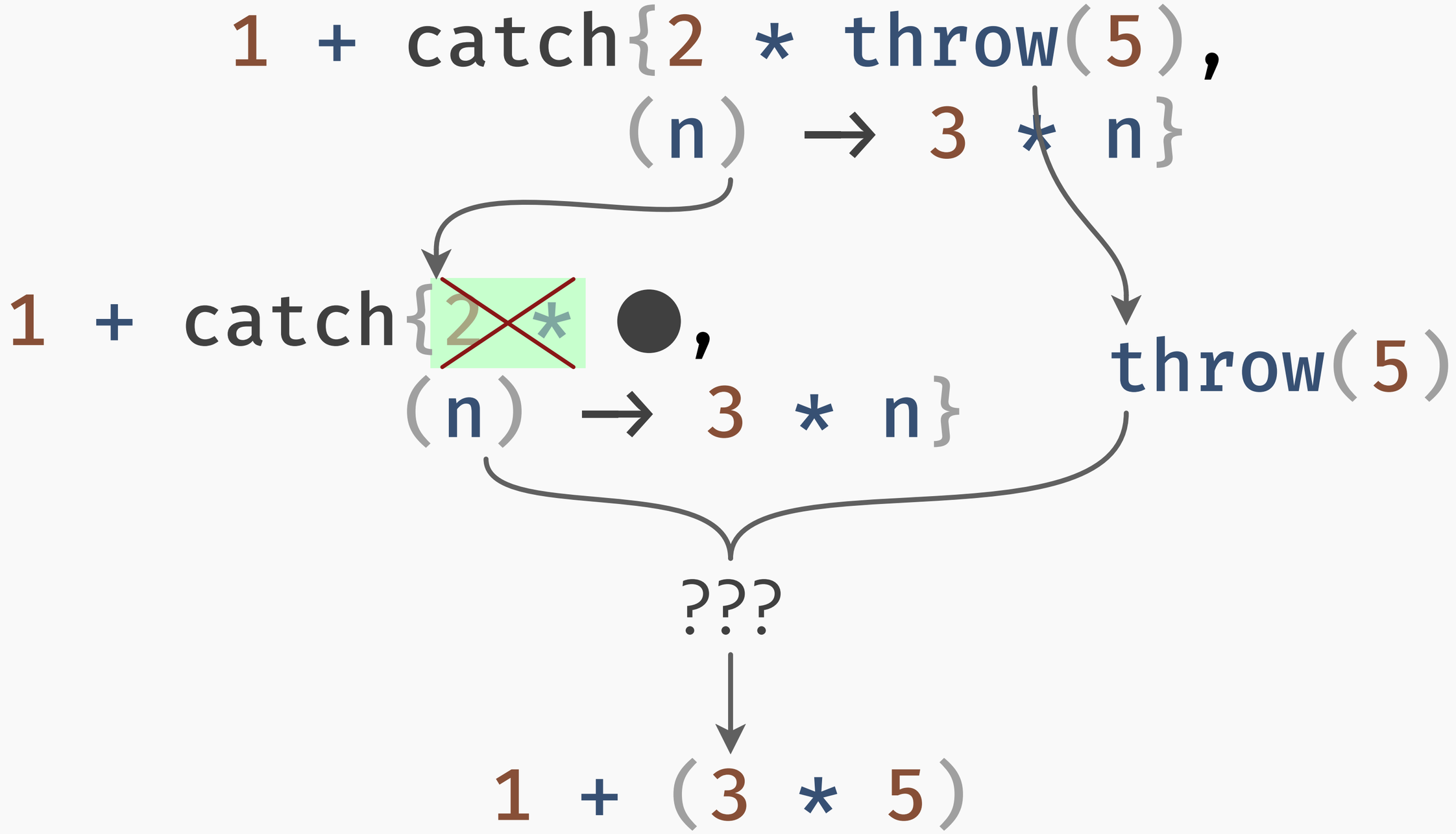
1 + catch{2 * throw(5),
(n) → 3 * n}
1 + catch{2 * ●,
(n) → 3 * n}
throw(5)
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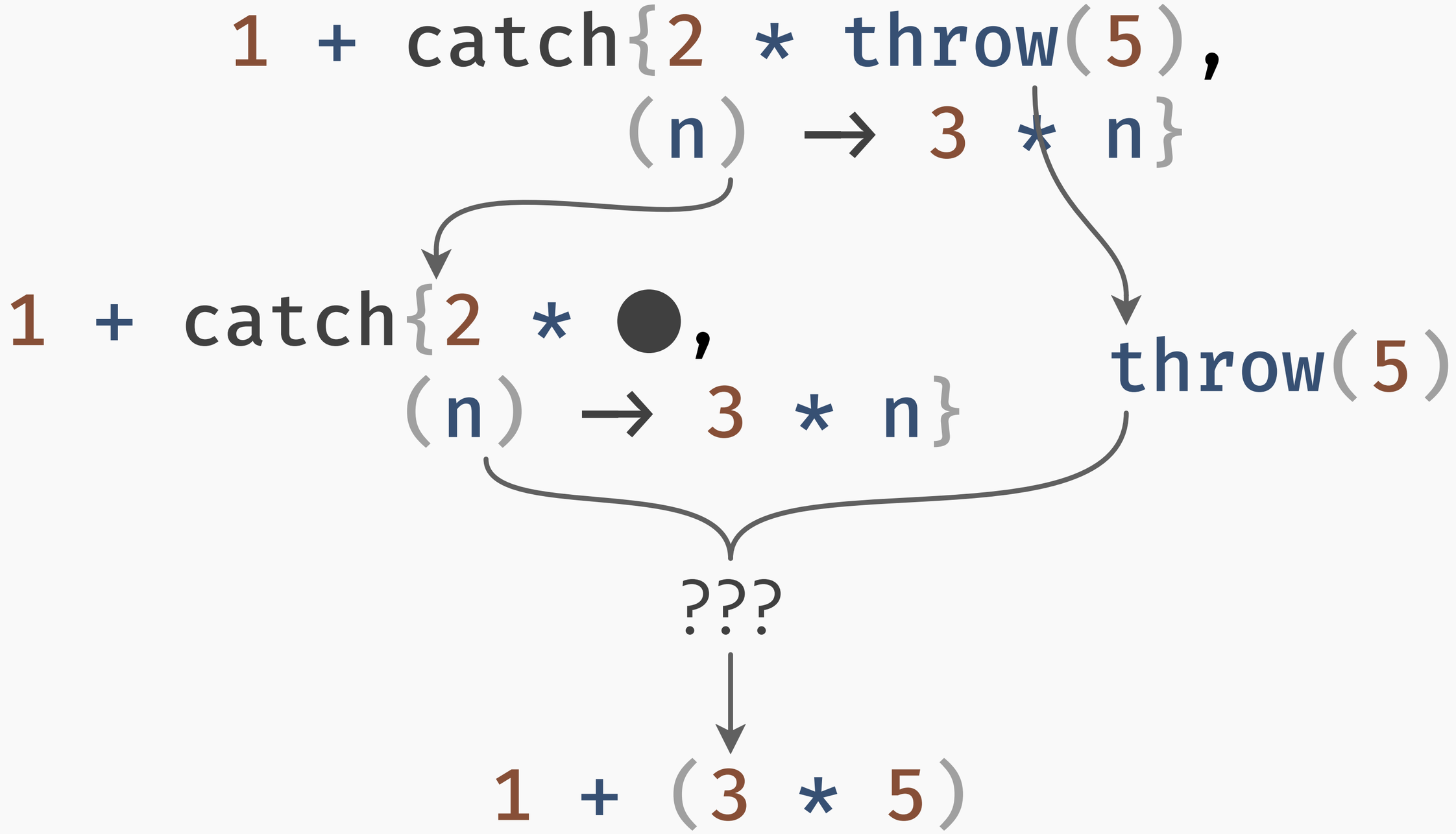
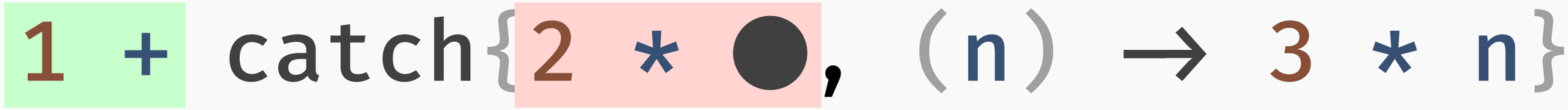
1 + (3 * 5)

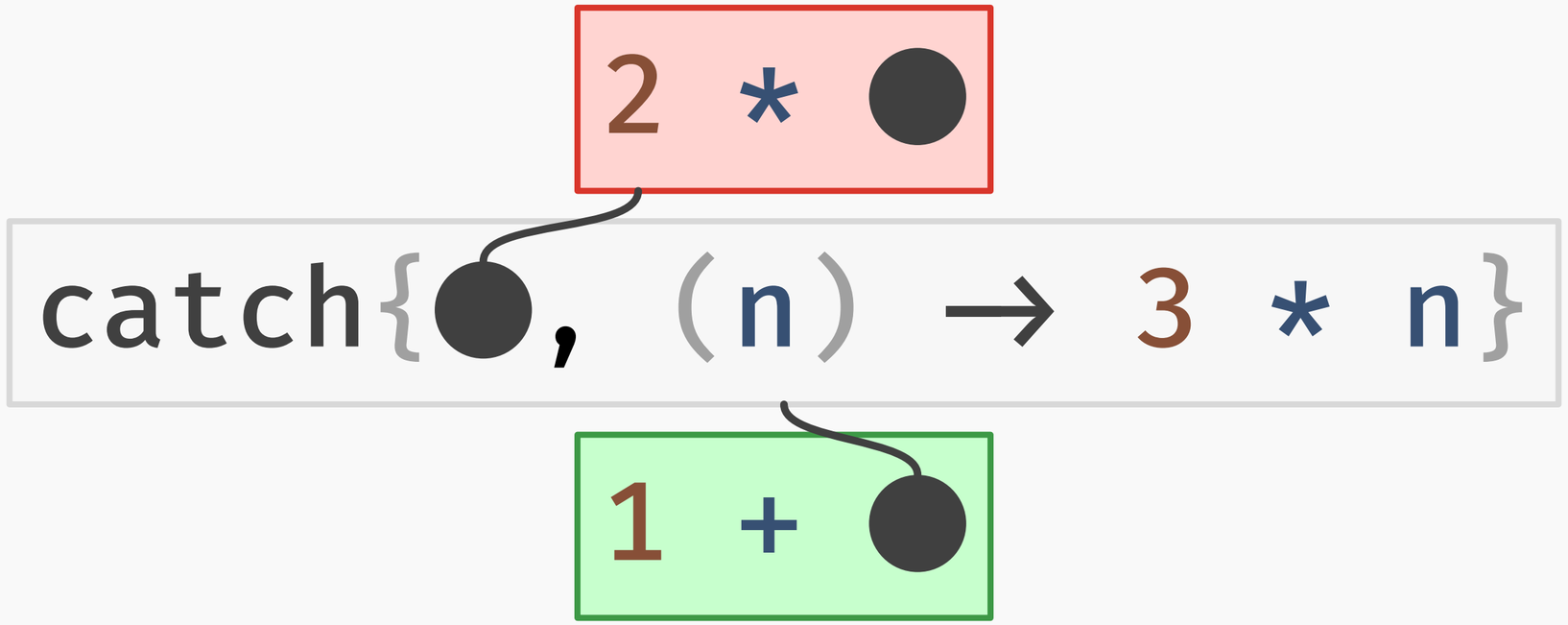

1 + catch{2 * throw(5),
(n) → 3 * n}
1 + catch{2 * ●,
(n) → 3 * n}
throw(5)
???
1 + (3 * 5)

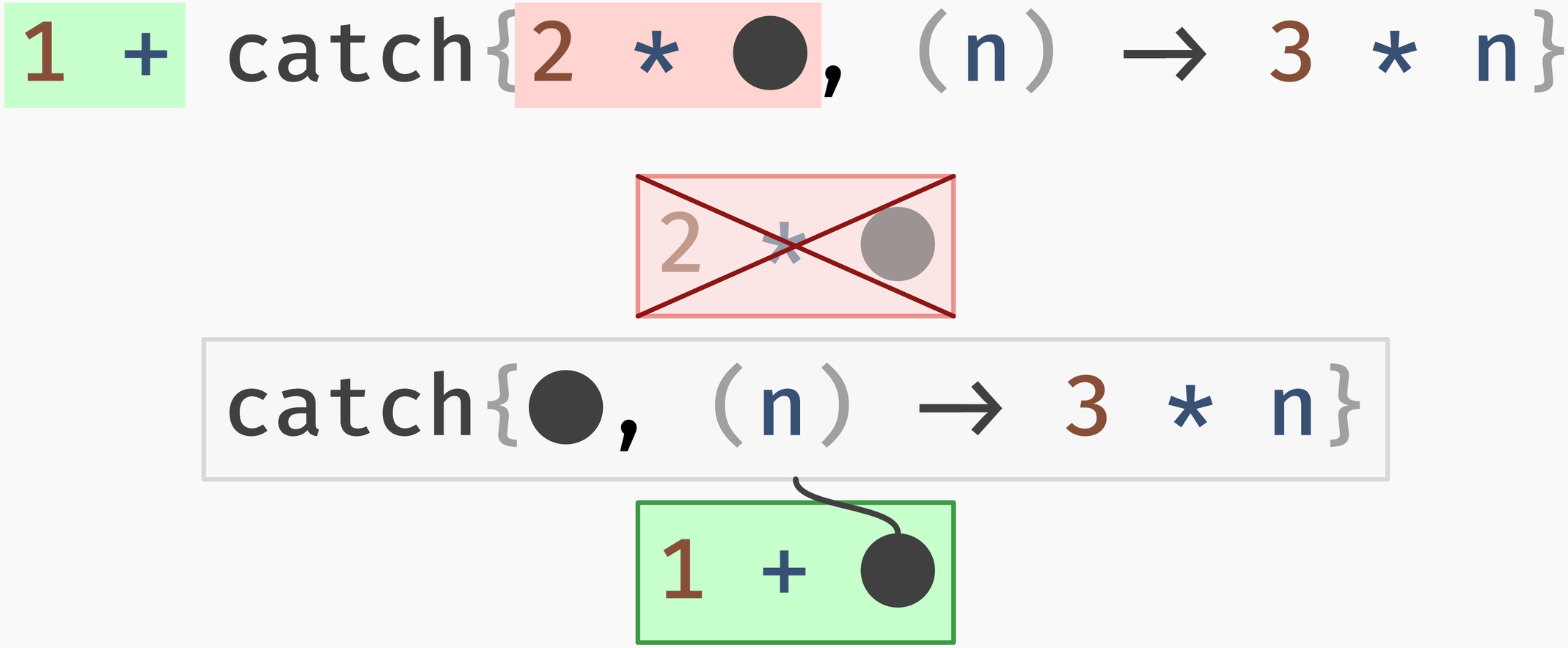

1 + catch{2 * ●, (n) → 3 * n}

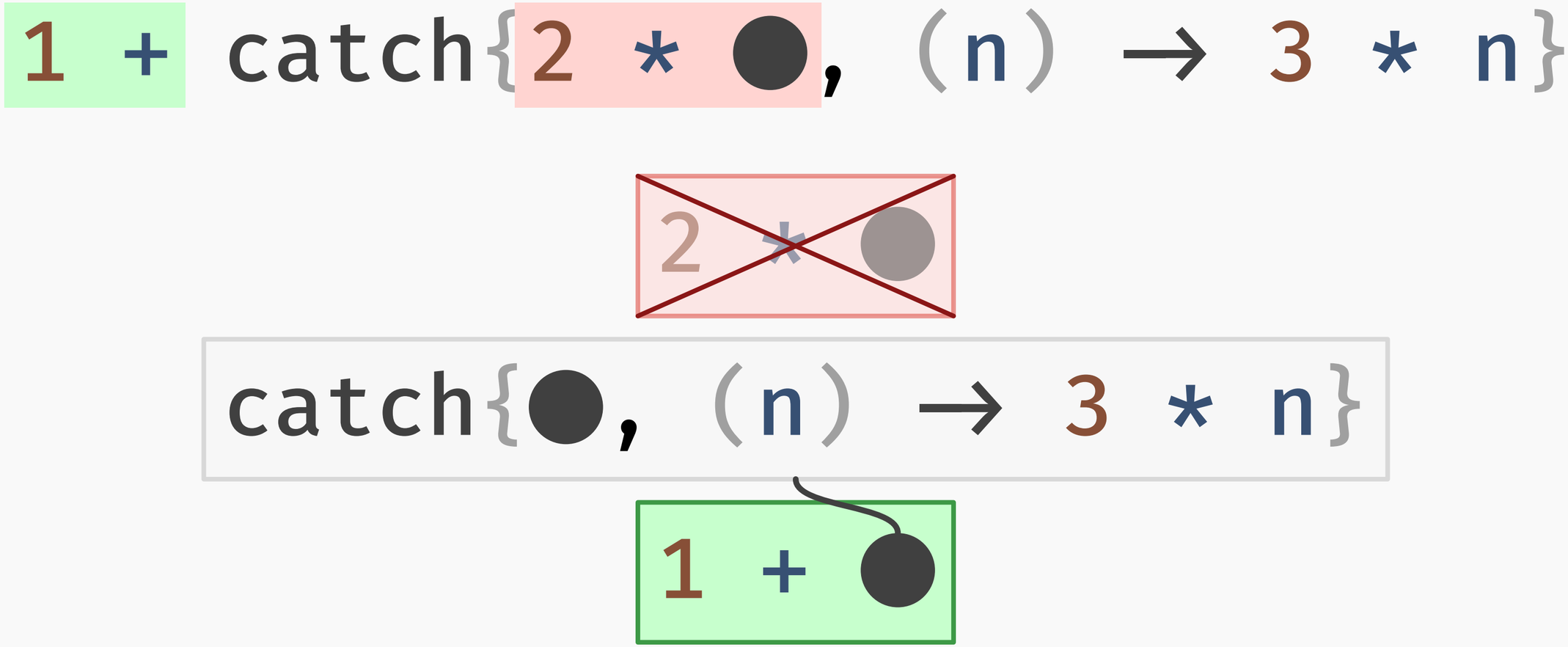

1 + catch{2 * ●, (n) → 3 * n}


1 + catch{2 * ●, (n) → 3 * n}

```
1 + catch{2 * ●, (n) → 3 * n}
```




1 + catch{2 * ●, (n) → 3 * n}
2 *
catch{●, (n) → 3 * n}
1 + ●



`catch` *delimits* the discarded continuation.

INTERLUDE: NOTATION

$$A \longrightarrow B$$

$$A \longrightarrow B$$

"A reduces to B."

$$A \longrightarrow B$$

"A reduces to B."

```
not(false) ⟶ true
```

$$A \longrightarrow B$$

"A reduces to B."

```
not(false) ⟶ true
not(true)  ⟶ false
```

$$A \longrightarrow B$$

"A reduces to B."

$$\texttt{not(false)} \longrightarrow \texttt{true}$$
$$\texttt{not(true)} \longrightarrow \texttt{false}$$

$$\texttt{if true then } e_1 \texttt{ else } e_2 \longrightarrow e_1$$

$$A \longrightarrow B$$

"A reduces to B."

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$$\texttt{if true then } e_1 \texttt{ else } e_2 \longrightarrow e_1$$
$$\texttt{if false then } e_1 \texttt{ else } e_2 \longrightarrow e_2$$

$$A \longrightarrow B$$

"A reduces to B."

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$$\texttt{if true then } e_1 \texttt{ else } e_2 \longrightarrow e_1$$
$$\texttt{if false then } e_1 \texttt{ else } e_2 \longrightarrow e_2$$

`if not(false) then 1 else 2`?

```
if not(false) then 1 else 2
```

```
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```

```
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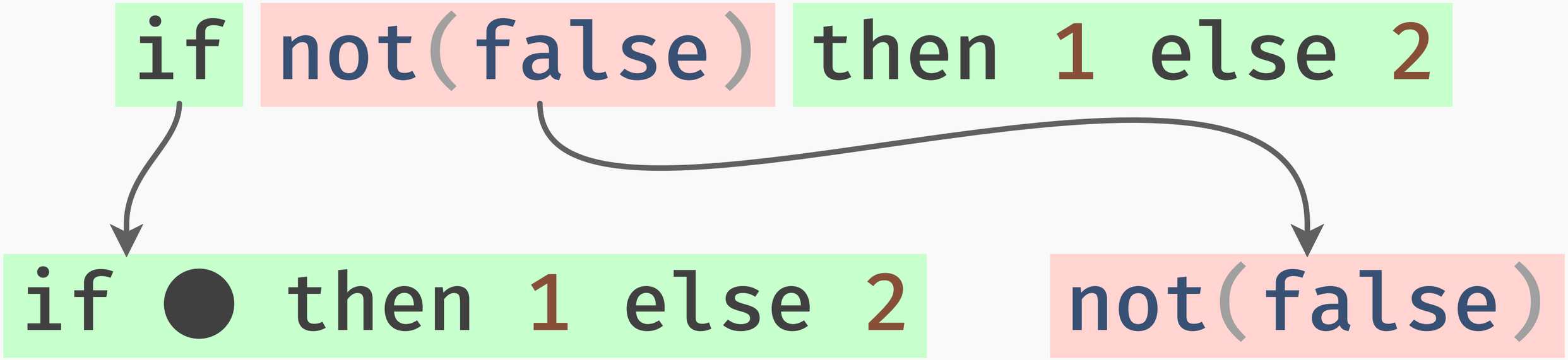
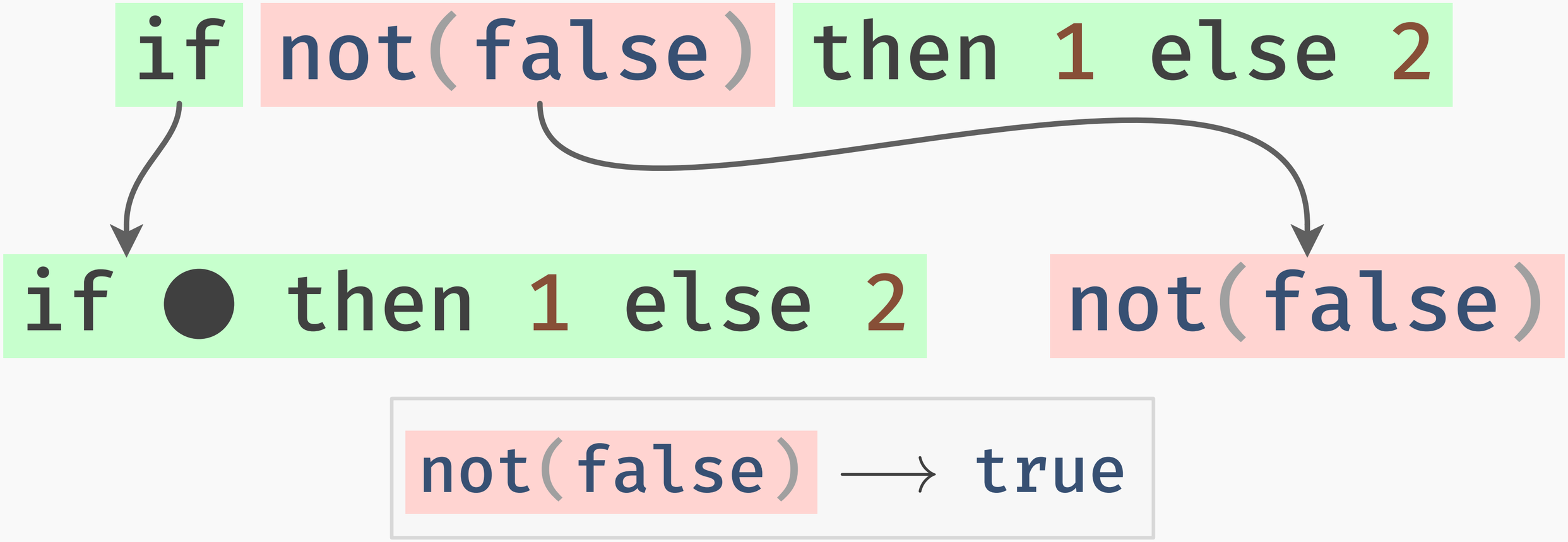
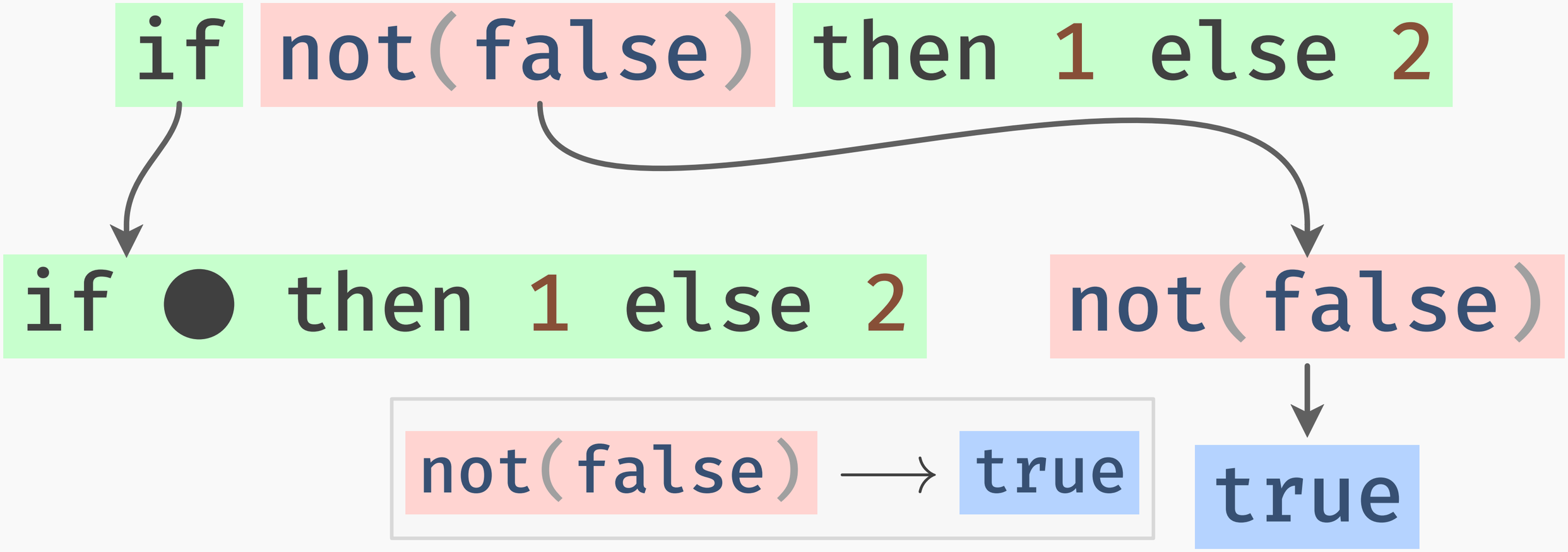
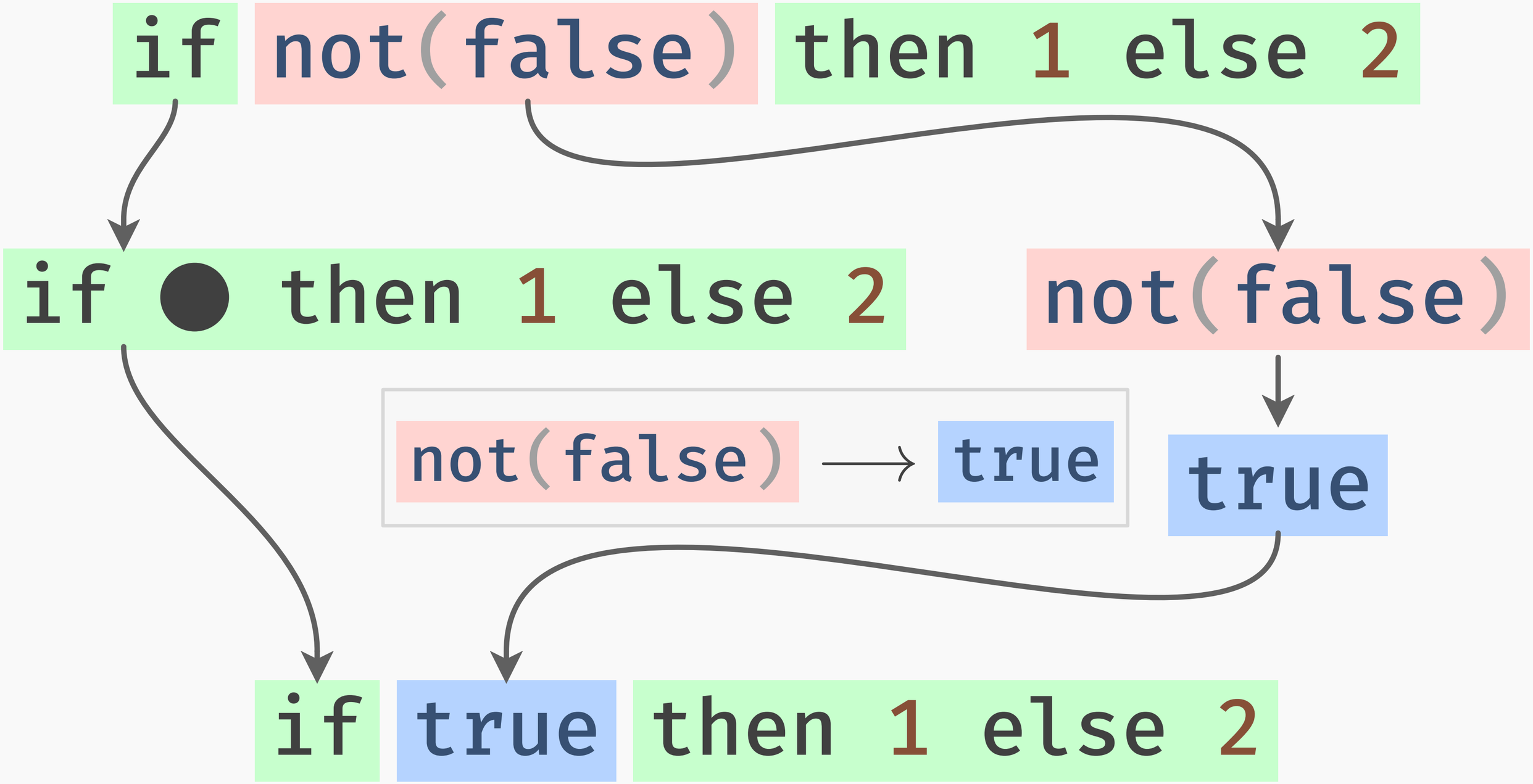
```


if not(false) then 1 else 2
if ● then 1 else 2
not(false)



$$\texttt{not(false)} \longrightarrow \texttt{true}$$


if not(false) then 1 else 2
if ● then 1 else 2
not(false)
not(false) ⟶ true
true


if not(false) then 1 else 2
if ● then 1 else 2
not(false)
not(false) ⟶ true
true
if true then 1 else 2

$$\texttt{not}(\texttt{false}) \longrightarrow \texttt{true}$$

~~`not(false)` $\longrightarrow$ `true`~~

$$E[\texttt{not(false)}] \longrightarrow E[\texttt{true}]$$

~~`not(false)` $\longrightarrow$ `true`~~

$$E[\texttt{not(false)}] \longrightarrow E[\texttt{true}]$$

→ E stands for “some arbitrary continuation”.

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→ E stands for “some arbitrary continuation”.

→ $E[x]$ denotes “plugging the hole” in E with x.

~~`not(false)` $\longrightarrow$ `true`~~

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- E stands for “some arbitrary continuation”.
- $E[x]$ denotes “plugging the hole” in E with x.

$$E = \texttt{if } \bullet \texttt{ then 1 else 2}$$

~~`not(false)` $\longrightarrow$ `true`~~

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→ E stands for “some arbitrary continuation”.

→ $E[x]$ denotes “plugging the hole” in E with x.

$$E = \texttt{if } \bullet \texttt{ then 1 else 2}$$

$$x = \texttt{not(false)}$$

~~`not(false)` $\longrightarrow$ `true`~~

$E[\texttt{not(false)}] \longrightarrow E[\texttt{true}]$

→ E stands for “some arbitrary continuation”.

→ $E[x]$ denotes “plugging the hole” in E with x.

$E = \texttt{if } \bullet \texttt{ then 1 else 2}$

$x = \texttt{not(false)}$

$E[x] = \texttt{if not(false) then 1 else 2}$

Why bother with all of this?

Why bother with all of this?

$$E[\texttt{exit}(v)] \longrightarrow \texttt{exit}(v)$$

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$$E[\texttt{exit}(v)] \longrightarrow \texttt{exit}(v)$$

Why bother with all of this?

$$E[\texttt{exit}(v)] \longrightarrow \texttt{exit}(v)$$

$$E_1[\texttt{catch}\{E_2[\texttt{throw}(v)], f\}] \longrightarrow E_1[f(v)]$$

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Lots of operations can be described this way!

① continuations

② delimited

③ first-class

④ native

① continuations ✓

② delimited

③ first-class

④ native

1. continuations ✓
2. delimited ✓
3. first-class
4. native

① continuations ✓

② delimited ✓

③ first-class

④ native

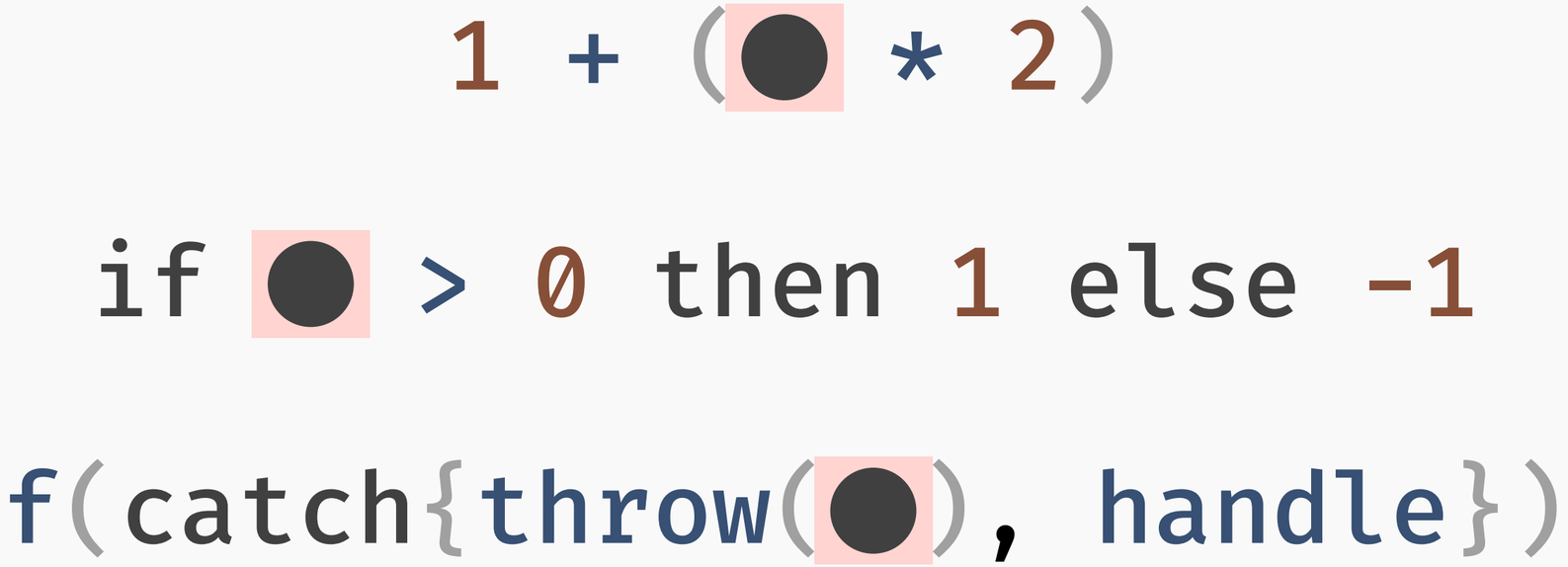
What makes something “first class”?

How could a *continuation* be a *value*?

```
1 + (● * 2)
```

```
if ● > 0 then 1 else -1
```

```
f(catch{throw(●), handle})
```


1 + (● * 2)
if ● > 0 then 1 else -1
f(catch{throw(●), handle})

```
1 + (x * 2)
```

```
if x > 0 then 1 else -1
```

```
f(catch{throw(x), handle})
```

```
(x) → 1 + (x * 2)
```

```
(x) → if x > 0 then 1 else -1
```

```
(x) → f(catch{throw(x), handle})
```

```
(x) → 1 + (x * 2)

(x) → if x > 0 then 1 else -1

(x) → f(catch{throw(x), handle})
```

What is a “first-class continuation”?

What is a “first-class continuation”?

Answer: a continuation reified as a function.

call_cc

`call_cc`

“call with current continuation”

`call_cc`

"call with current continuation"

$$E[\texttt{call\_cc}(f)] \longrightarrow E[f(\texttt{(x)} \rightarrow E[\texttt{x}])]$$

`call_cc`

"call with current continuation"

$$E[\texttt{call\_cc}(\ f\ )] \longrightarrow E[f(\texttt{(x)} \rightarrow E[\texttt{x}])]$$

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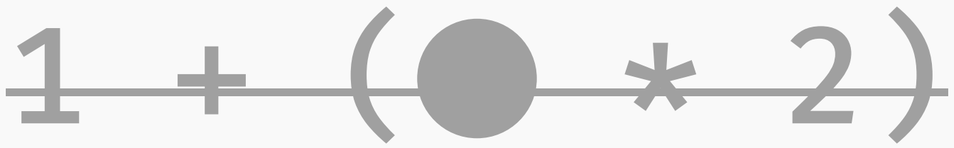
"call with current continuation"

$$E[\texttt{call\_cc}(f)] \longrightarrow E[f(\texttt{(x)} \rightarrow E[\texttt{x}])]$$

This has some problems!



```
print(1 + (● * 2))
shutdown_runtime()
run_libc_atexit()
exit_process()
```

We need more control!

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`prompt` / `control`

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$$E_1[\texttt{prompt}\{E_2[\texttt{control}(f)]\}]$$

$$\longrightarrow E_1[f((\texttt{x}) \rightarrow E_2[\texttt{x}])]$$

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```
1 + prompt{2 * control((k) → k(3) + k(5))}
```

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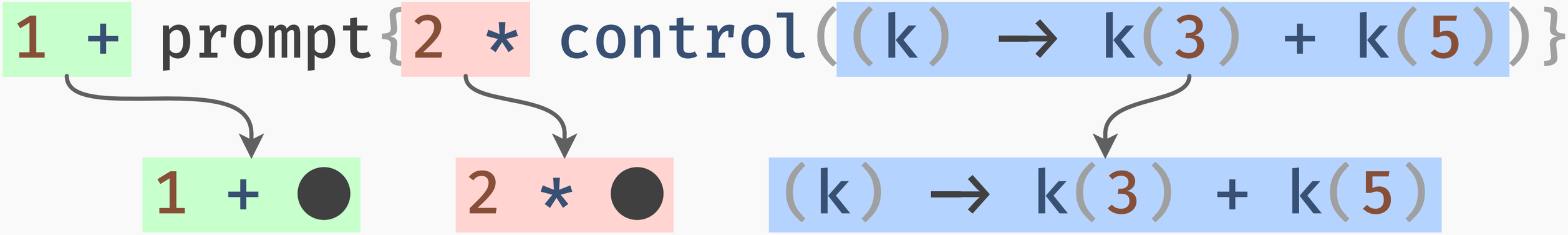
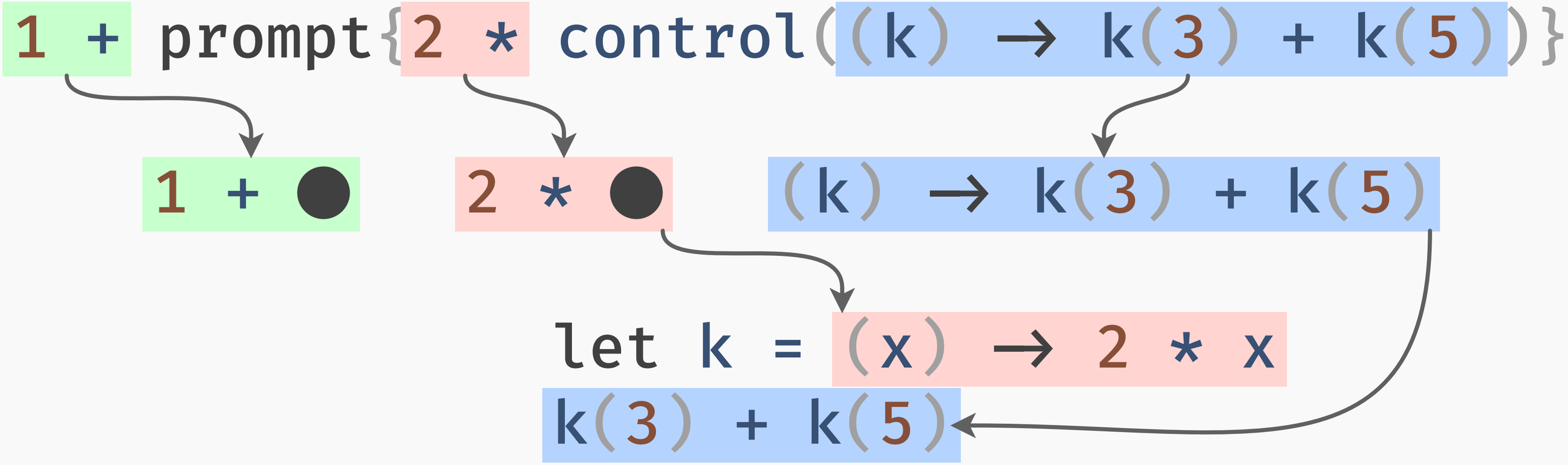
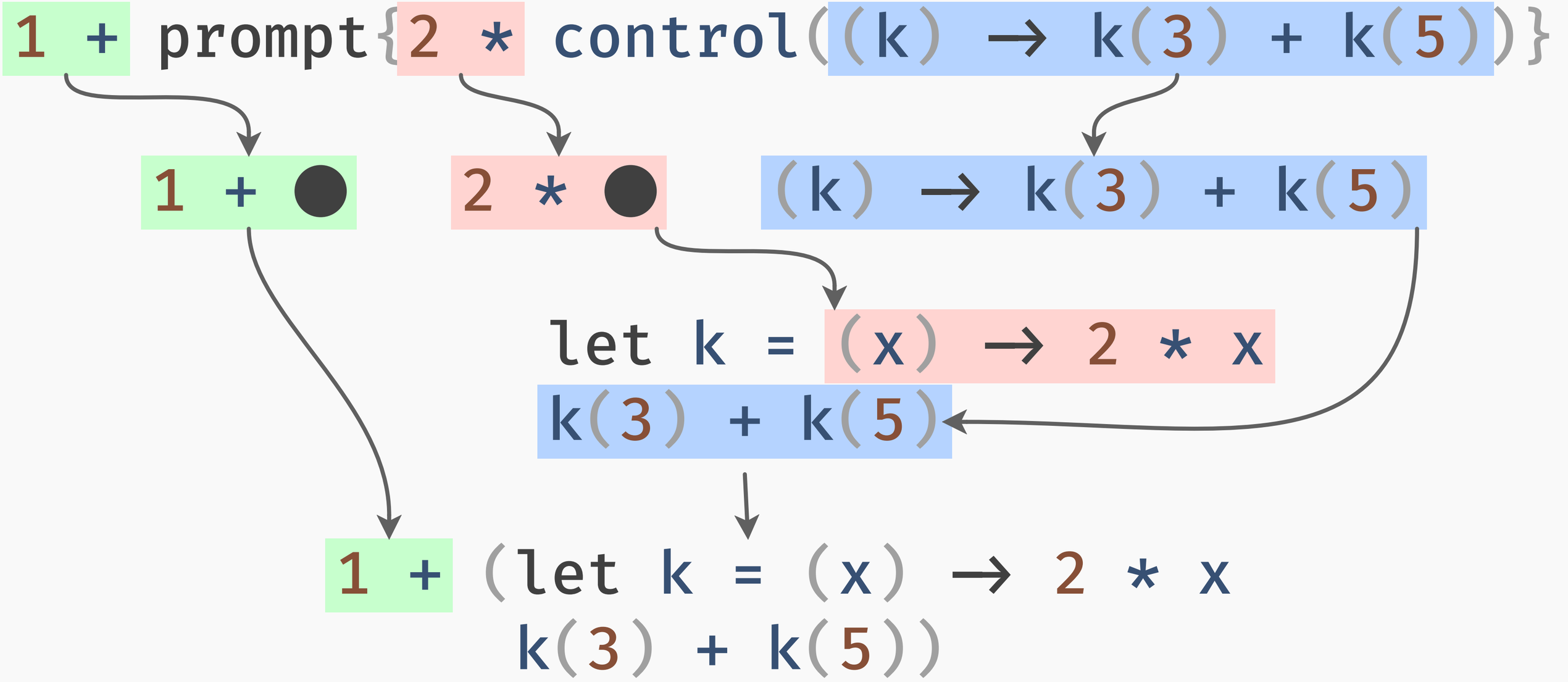

1 + prompt{2 * control((k) → k(3) + k(5))}
1 + ●
2 * ●
(k) → k(3) + k(5)


1 + prompt{2 * control((k) → k(3) + k(5))}
1 + ●
2 * ●
(k) → k(3) + k(5)
let k = (x) → 2 * x
k(3) + k(5)


1 + prompt{2 * control((k) → k(3) + k(5))}
1 + ●
2 * ●
(k) → k(3) + k(5)
let k = (x) → 2 * x
k(3) + k(5)
1 + (let k = (x) → 2 * x
k(3) + k(5))

```
1 + prompt{2 * control((k) → k(3) + k(5))}
```

↓

```
1 + (let k = (x) → 2 * x
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```

```
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↓

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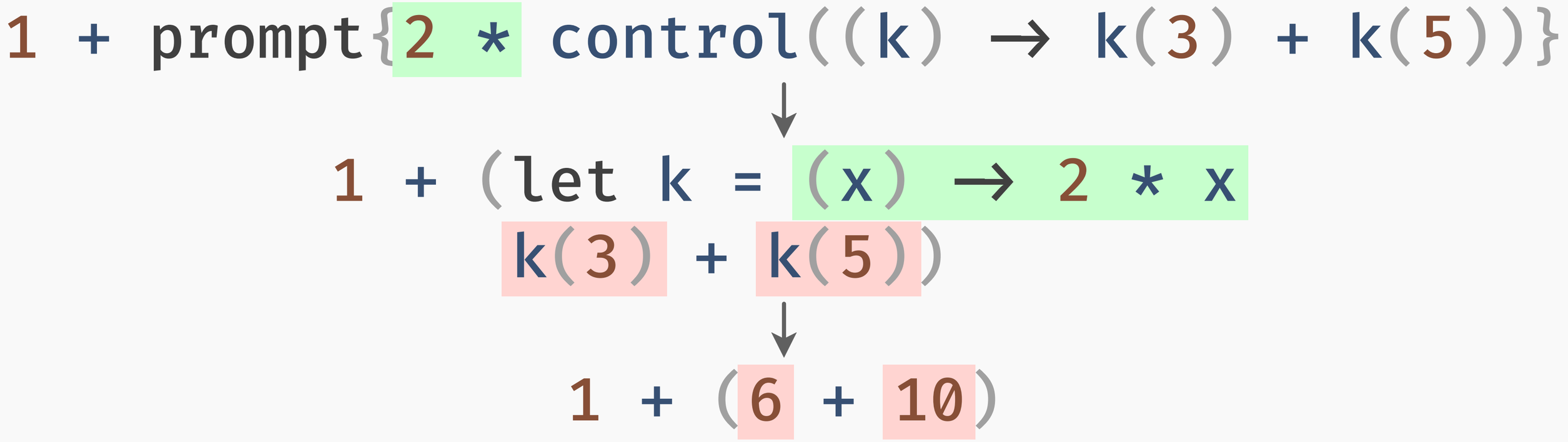
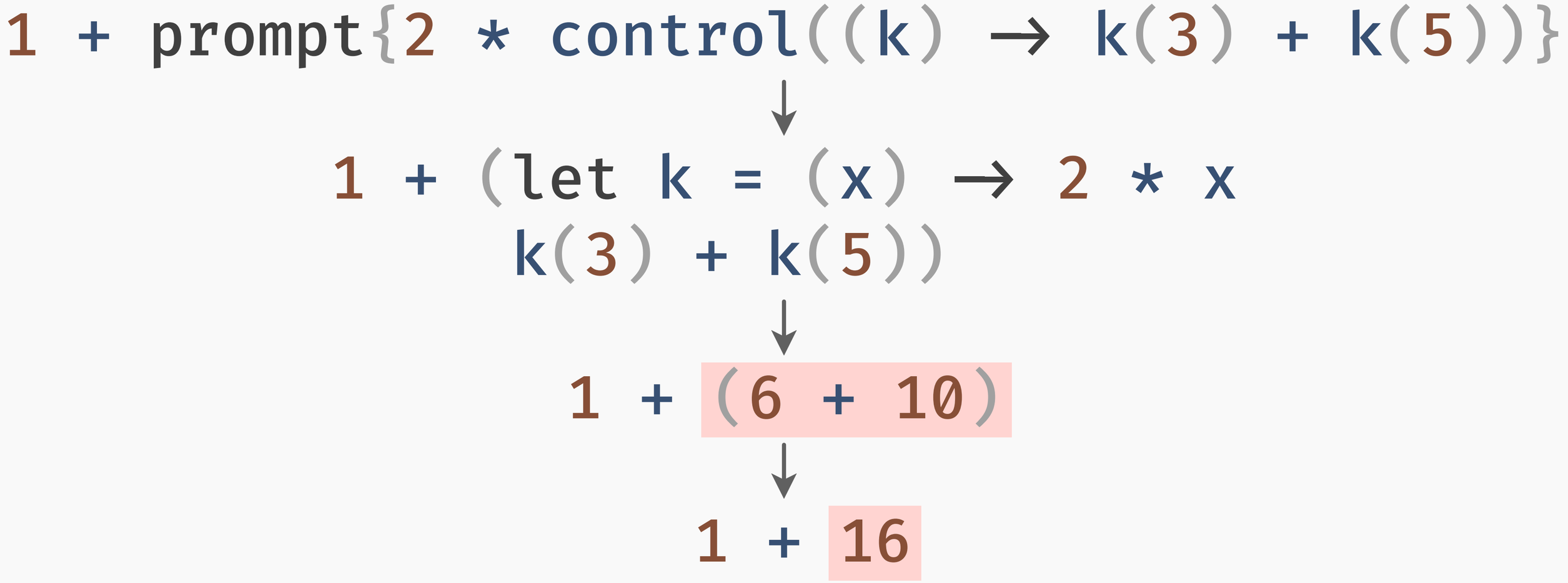
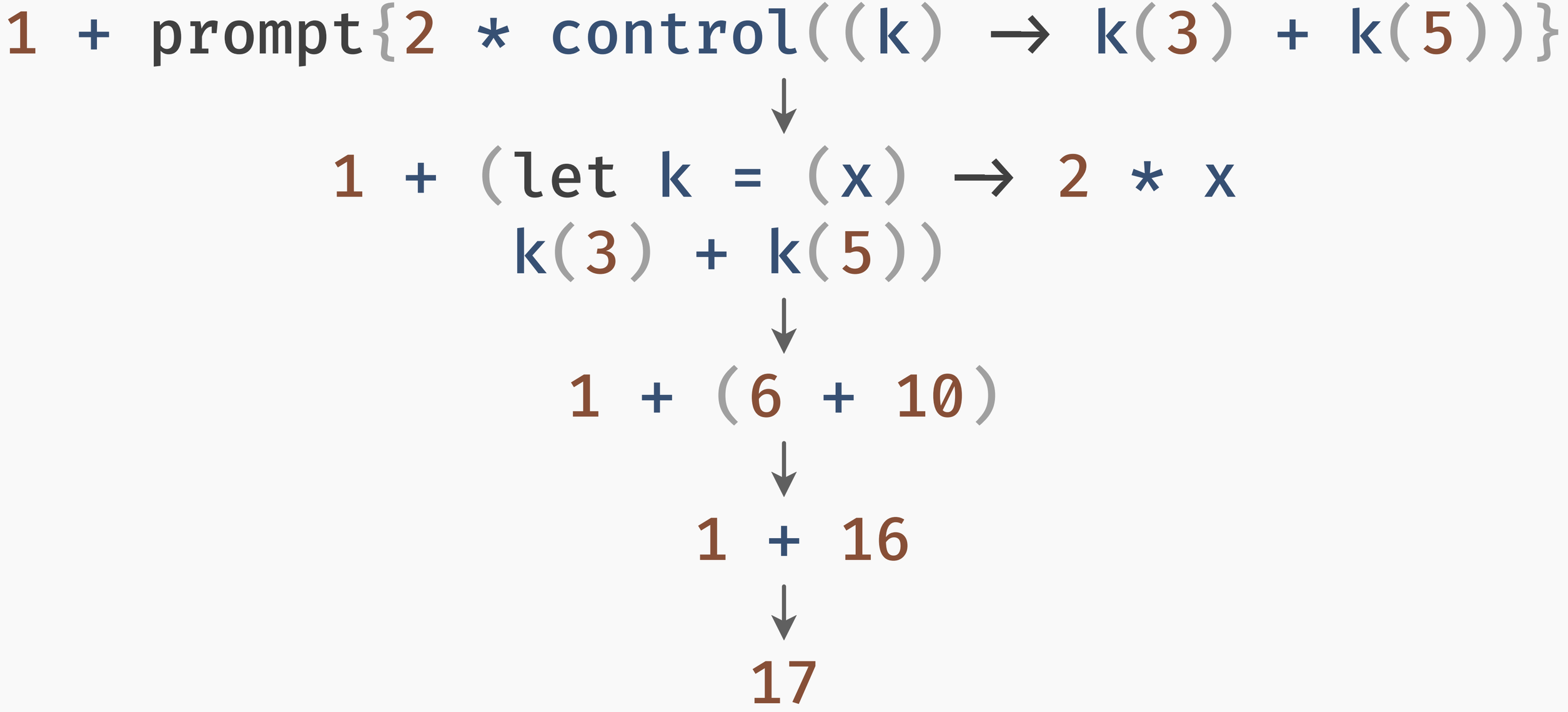

1 + prompt{2 * control((k) → k(3) + k(5))}
1 + (let k = (x) → 2 * x
k(3) + k(5))
1 + (6 + 10)


1 + prompt{2 * control((k) → k(3) + k(5))}
1 + (let k = (x) → 2 * x
k(3) + k(5))
1 + (6 + 10)
1 + 16


1 + prompt{2 * control((k) → k(3) + k(5))}
1 + (let k = (x) → 2 * x
k(3) + k(5))
1 + (6 + 10)
1 + 16
17

Why is this so confusing?

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$$E_1[\texttt{catch}\{E_2[\texttt{throw}(v)],\ f\}] \longrightarrow E_1[f(v)]$$

$$E_1[\texttt{prompt}\{E_2[\texttt{control}(f)]\}] \longrightarrow E_1[f((\texttt{x}) \to E_2[\texttt{x}])]$$

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`delimit` / `yield` provide *resumable exceptions*.

```
1 + delimit{2 * yield(()),
            ((), k) → k(3) + k(5)}
```

```
1 + delimit{2 * yield(()),
             ((), k) → k(3) + k(5)}
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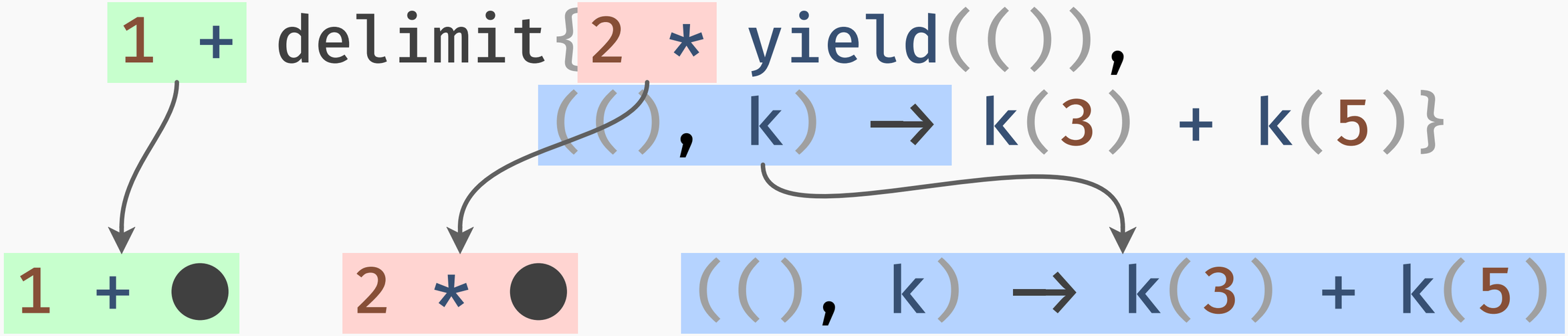
1 + delimit{2 * yield(()),
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1 + ●
2 * ●
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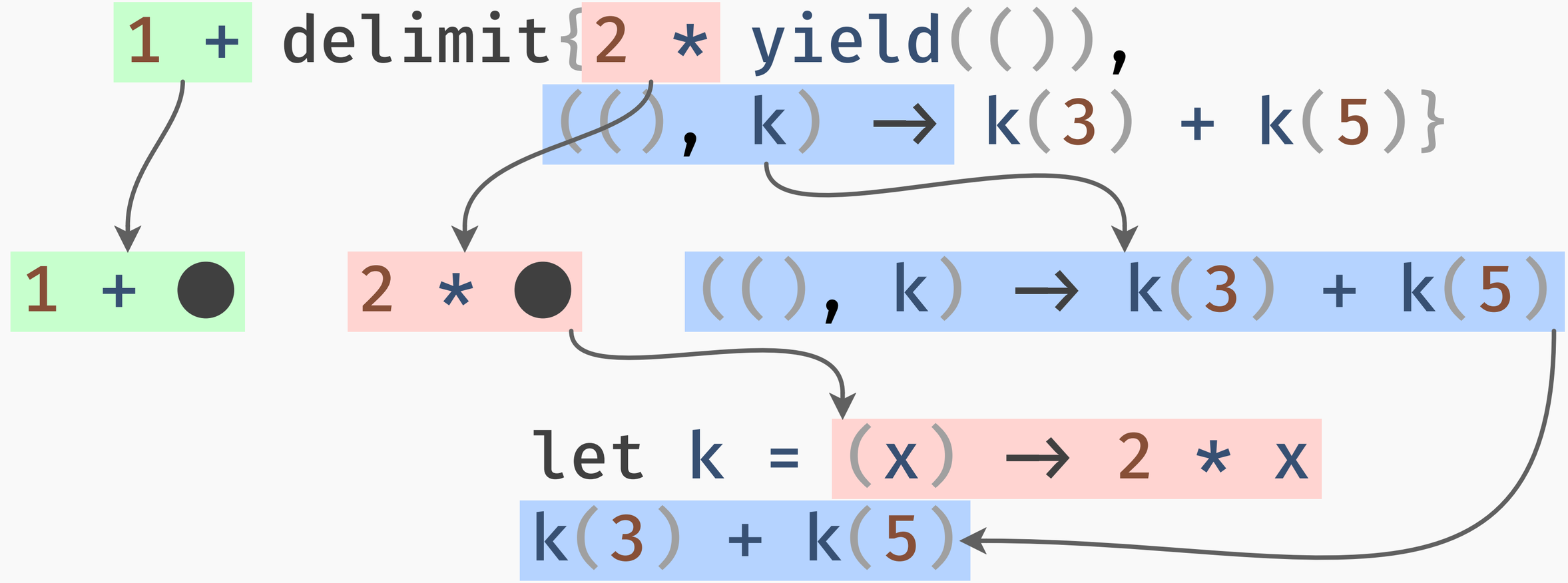
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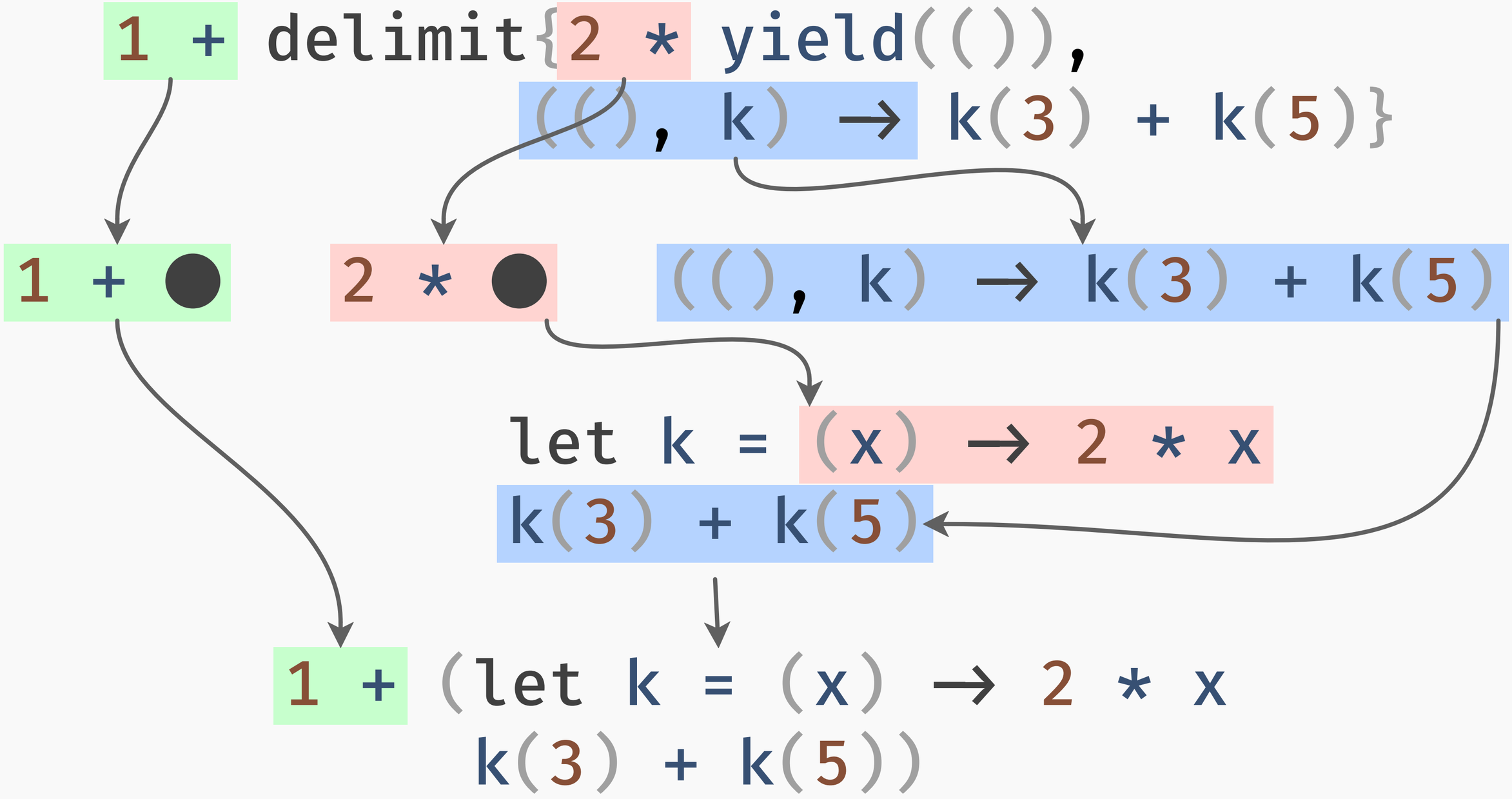
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((), k) → k(3) + k(5)}
1 + ●
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let k = (x) → 2 * x
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Why prompt / control?

Why `prompt` / `control`?

→ In some sense “simpler”.

Why `prompt` / `control`?

→ In some sense “simpler”.

→ Historical relationship to `call_cc`.

Why `prompt` / `control`?

→ In some sense “simpler”.

→ Historical relationship to `call_cc`.

→ Easier to statically type.

TYPES

Even typing exceptions is hard!

Even typing exceptions is hard!

```
throw : Exception → a
```

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```
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```
throw : Exception → a

catch{body, handler} : b
```

Even typing exceptions is hard!

```
throw : Exception → a

catch{body, handler} : b
        body : b
handler : Exception → b
```

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        body : b
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```
yield : DelimiterTag → a
```

```
yield : DelimiterTag → a

delimit{body, handler} : b
```

```
yield : DelimiterTag → a

delimit{body, handler} : b
        body : b
handler : DelimiterTag → (a → b) → b
```

```
yield : DelimiterTag → a

delimit{body, handler} : b
        body : b
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$$E_1[\texttt{delimit}\{E_2[\texttt{yield}(v)],\ f\}]$$
$$\longrightarrow E_1[f(v,\ (\texttt{x}) \to E_2[\texttt{x}])]$$

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delimit{body, handler} : b
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```

```
handler : DelimiterTag → (a → b) → b
```

$$E_1[\texttt{delimit}\{E_2[\texttt{yield}(v)],\ f\}]$$
$$\longrightarrow E_1[f(v,\ (\texttt{x}) \to E_2[\texttt{x}])]$$

```
prompt{body} : b
```

```
prompt{body} : b
     body : b
```

```
prompt{body} : b
     body : b

control : ((a → b) → b) → a
```

$$\texttt{prompt}\{\texttt{body}\} : \texttt{b}$$

$$\texttt{body} : \texttt{b}$$

$$\texttt{control} : ((\texttt{a} \to \texttt{b}) \to \texttt{b}) \to \texttt{a}$$

$$E_1[\texttt{prompt}\{E_2[\texttt{control}(f)]\}]$$

$$\longrightarrow E_1[f((\texttt{x}) \to E_2[\texttt{x}])]$$

```
prompt{body} : b
```

```
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```
control : ((a → b) → b) → a
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$$E_1[\texttt{prompt}\{E_2[\texttt{control}(f)]\}]$$

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$$\texttt{prompt}\{\texttt{body}\} : \texttt{b}$$

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$$E_1[\texttt{prompt}\{E_2[\texttt{control}(f)]\}]$$

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$$E_1[\texttt{prompt}\{E_2[\texttt{control}(f)]\}]$$

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Solution: tagged prompts.

```
new_prompt_tag : () → PromptTag<b>
```

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prompt{tag, body} : b
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prompt{tag, body} : b
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prompt{tag, body} : b
  tag : PromptTag<b>
  body : b

control : (PromptTag<b>, ((a → b) → b)) → a
```

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new_prompt_tag : () → PromptTag<b>
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prompt{tag, body} : b
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$$E_1[\texttt{prompt}\{tag,\ E_2[\texttt{control}(tag,\ f)]\}]$$
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new_prompt_tag : () → PromptTag<b>
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prompt{tag, body} : b
  tag : PromptTag<b>
  body : b
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control : (PromptTag<b>, ((a → b) → b)) → a
```

$$E_1[\texttt{prompt}\{tag,\ E_2[\texttt{control}(tag,\ f)]\}]$$
$$\longrightarrow E_1[f((\texttt{x}) \rightarrow E_2[\texttt{x}])]$$

1. continuations ✓
2. delimited ✓
3. first-class
4. native

① continuations ✓

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Problem: slow! (See my talk from ZuriHac 2020.)

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Option two: bake them into the runtime.


1 + prompt{tag, f(true, ●) * 5}

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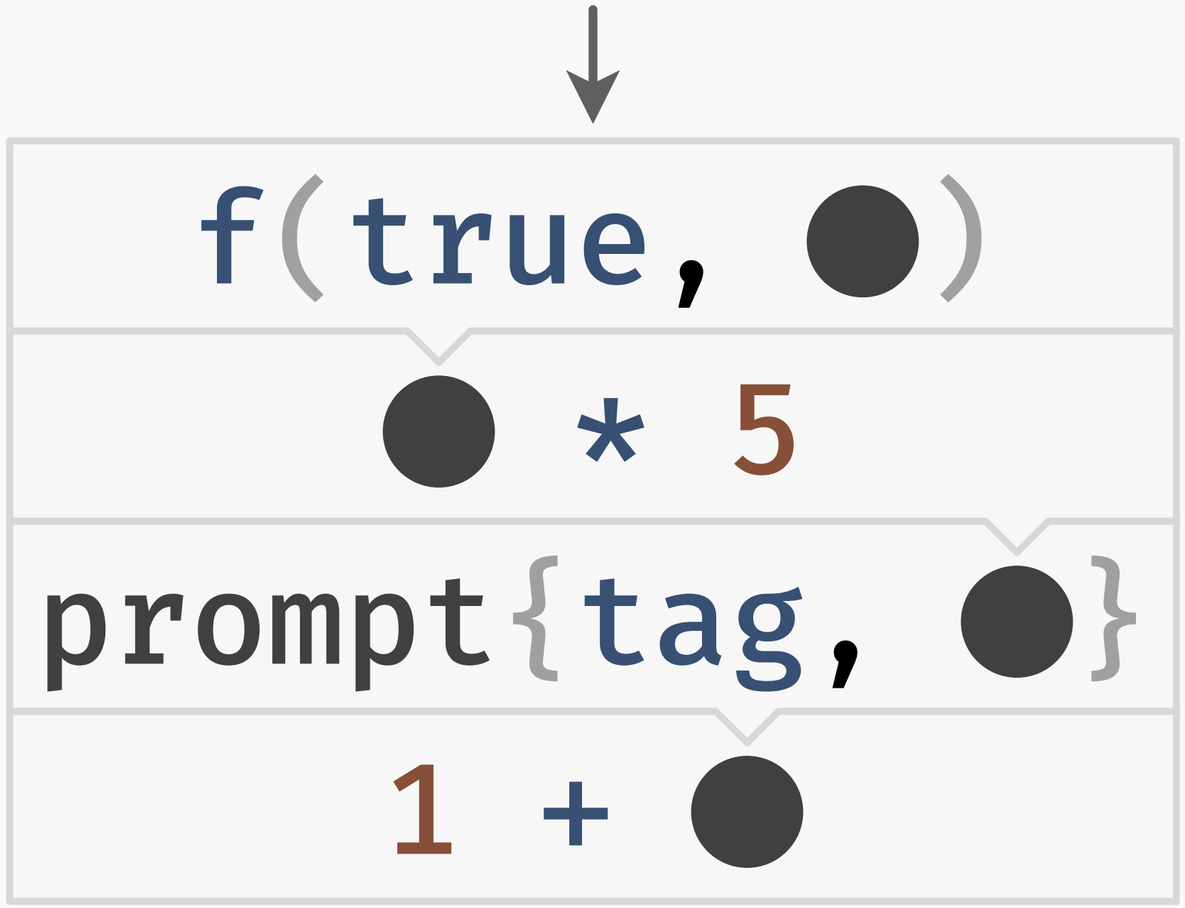

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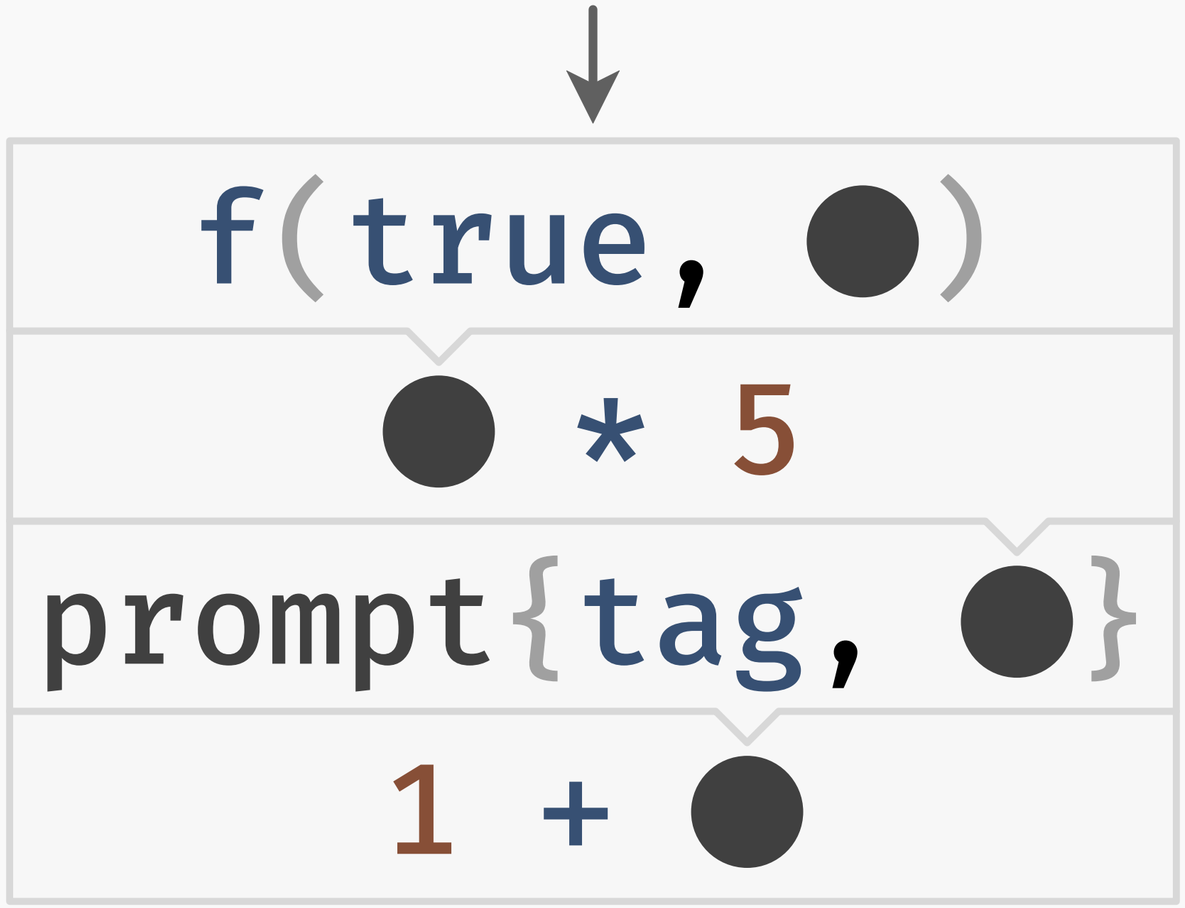

This is a call stack!

redex: `control(tag, g)`

stack:

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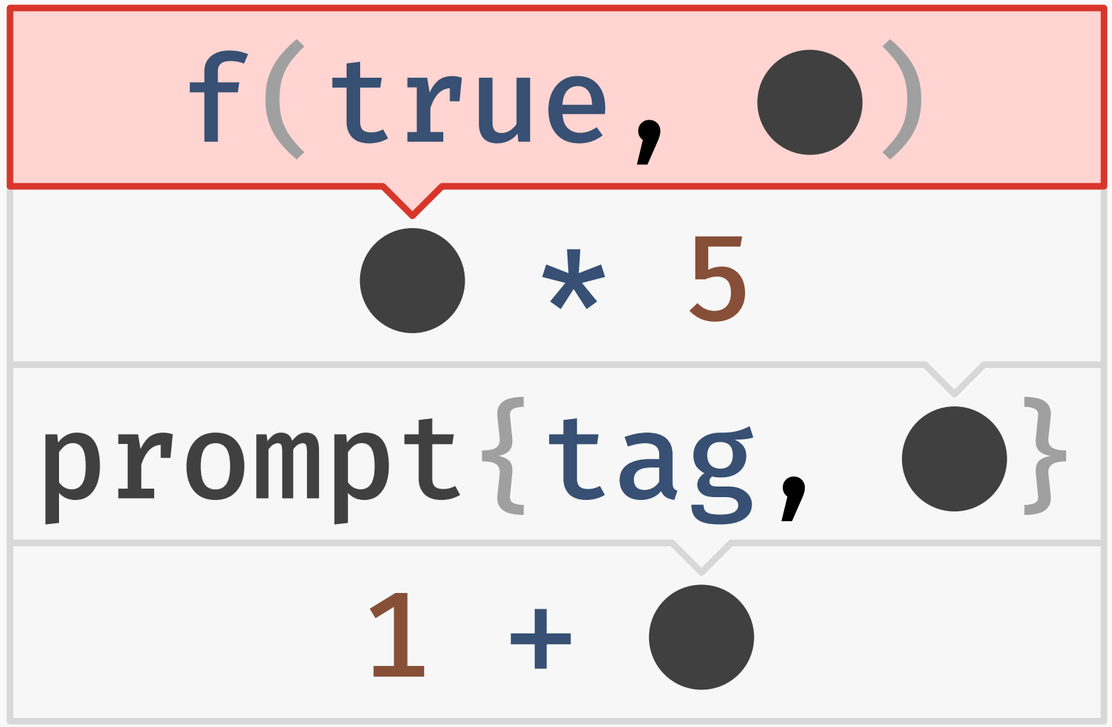
stack:



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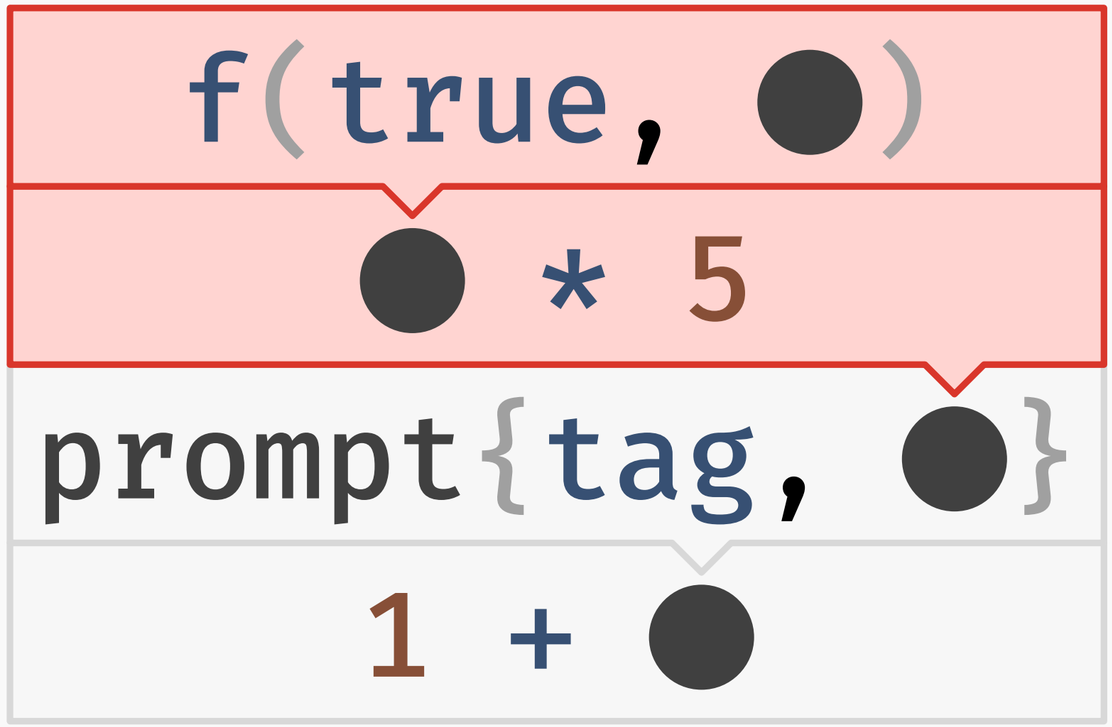

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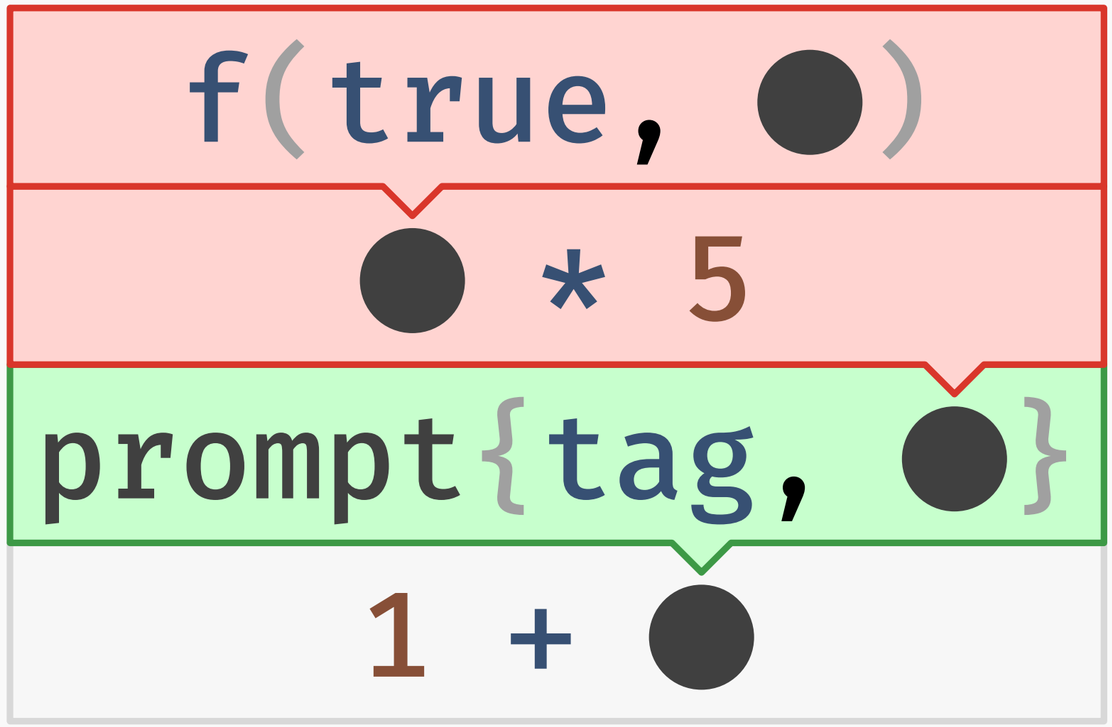

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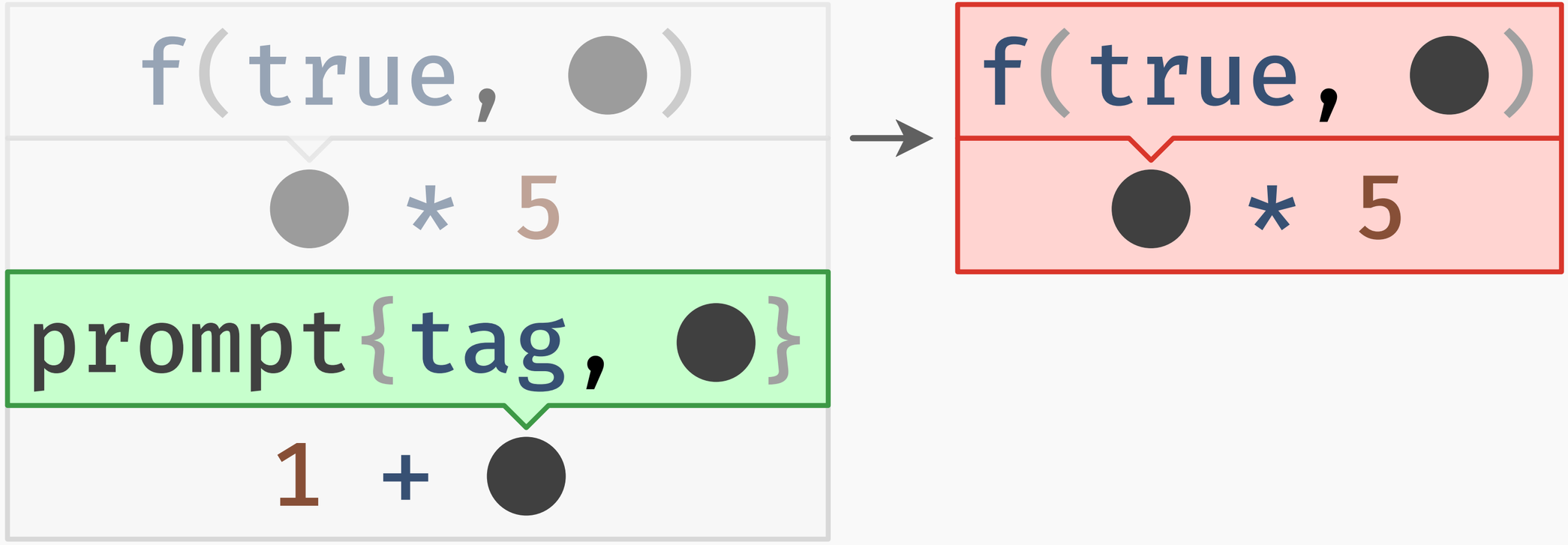

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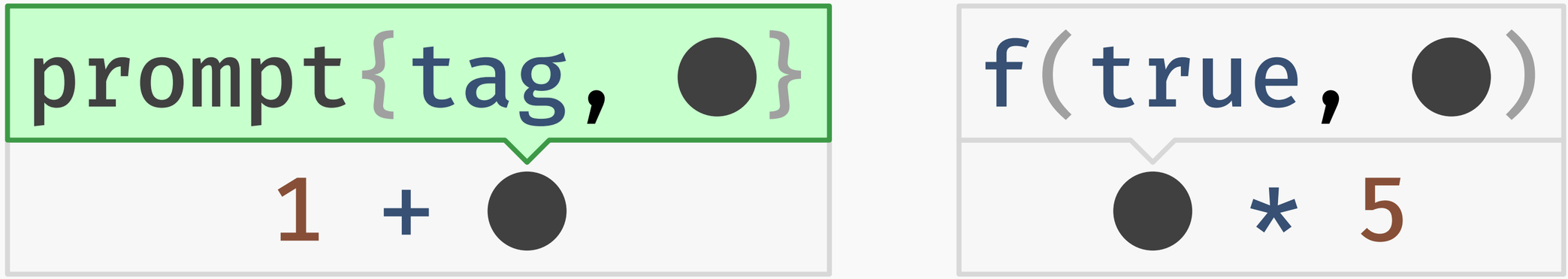

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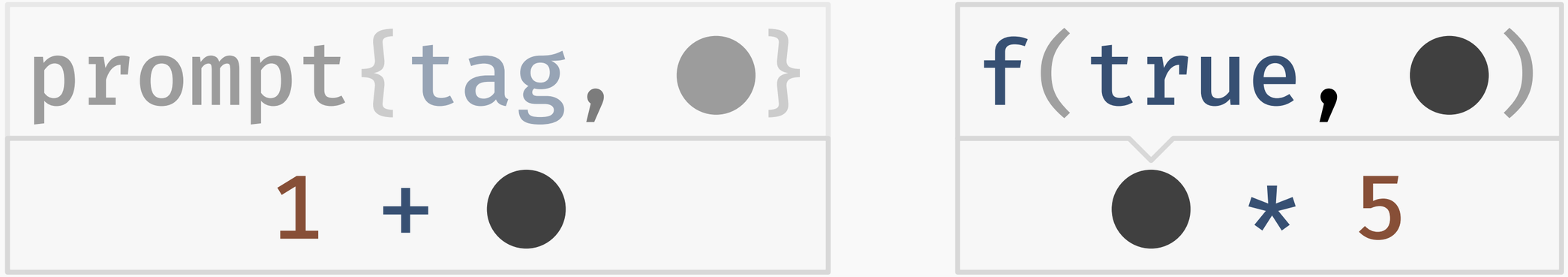
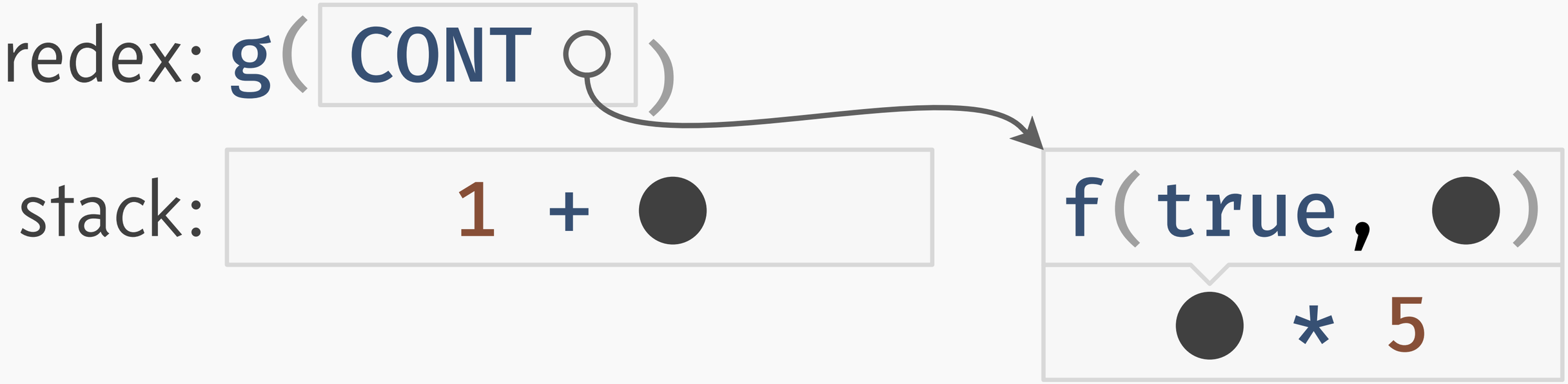


redex: `control(tag, g)`

stack:


redex: g(CONT)
stack:
1 + ●
f(true, ●)
● * 5


redex:
CONT
("hello")
stack:
1 +
f(true,)
* 5

redex: `"hello"`

stack:

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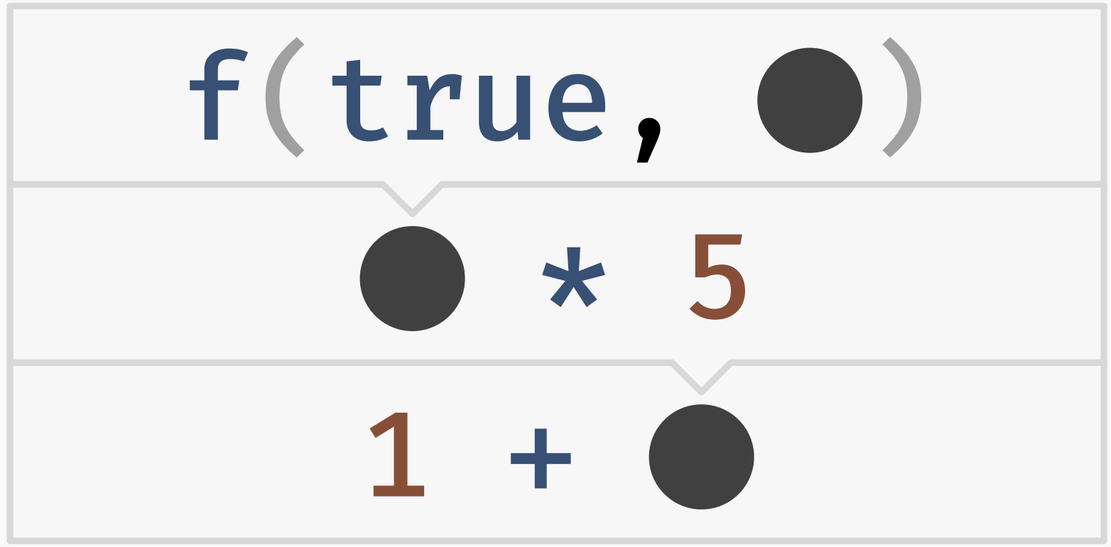

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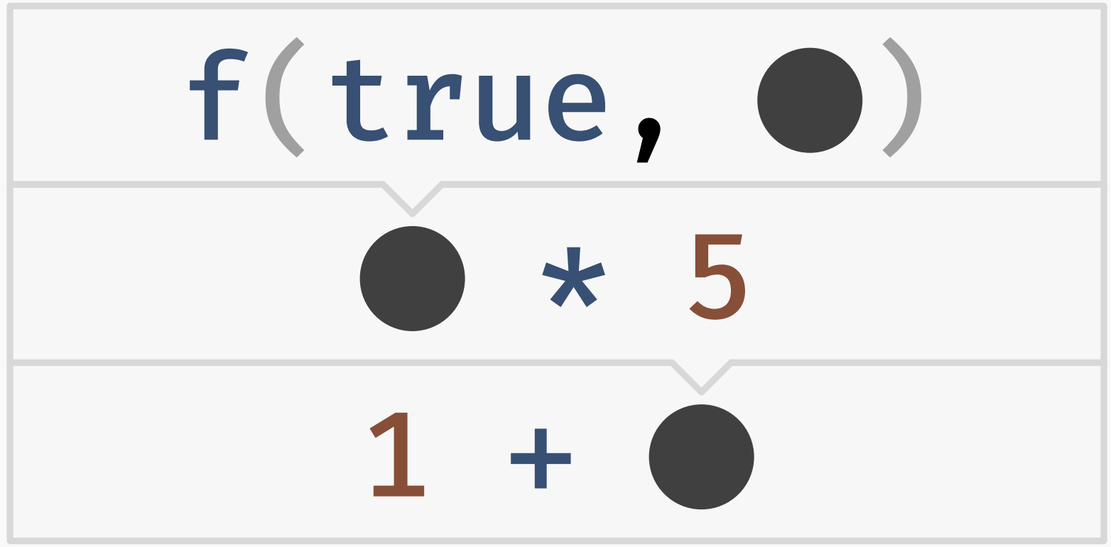

Capture/restore are just `memcpy`!

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② delimited ✓

③ first-class ✓

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- → Can further optimize implementation for specific use cases.
- → Strict monads permit embedding into a lazy language.
- → Reality is always at least a little more complicated (e.g. stack overflow, async exceptions).
- → We sorely lack non-synthetic continuation benchmarks!

The unsung hero of this talk:

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reduction semantics.

① continuations

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④ native

① continuations ✓

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Still extremely useful!

→ Continuations are a concept that arises naturally in evaluation.

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- → First-class continuations allow reifying the continuation as a function.
- → Remarkably, this corresponds to manipulation of the call stack.

Thanks!

me: https://lexi-lambda.github.io/
https://twitter.com/lexi_lambda
Tweag: https://www.tweag.io/