Team Reference Notes

v18.08
created by Imperez
made in Cuba

*This is the **Team Reference Notes**. This material is just for Formulas and Theorems. If you are looking for some code, please download the **Team Reference Code** from GitHub at https://github.com/ImperezCuba/TeamReferenceAlgorithms/tree/18.08 .*

Index

1. Graph

Cayley's Formula: There are nn−2 spanning trees of a complete graph with n labeled vertices. Example: UVa 10843 - Anne's game.

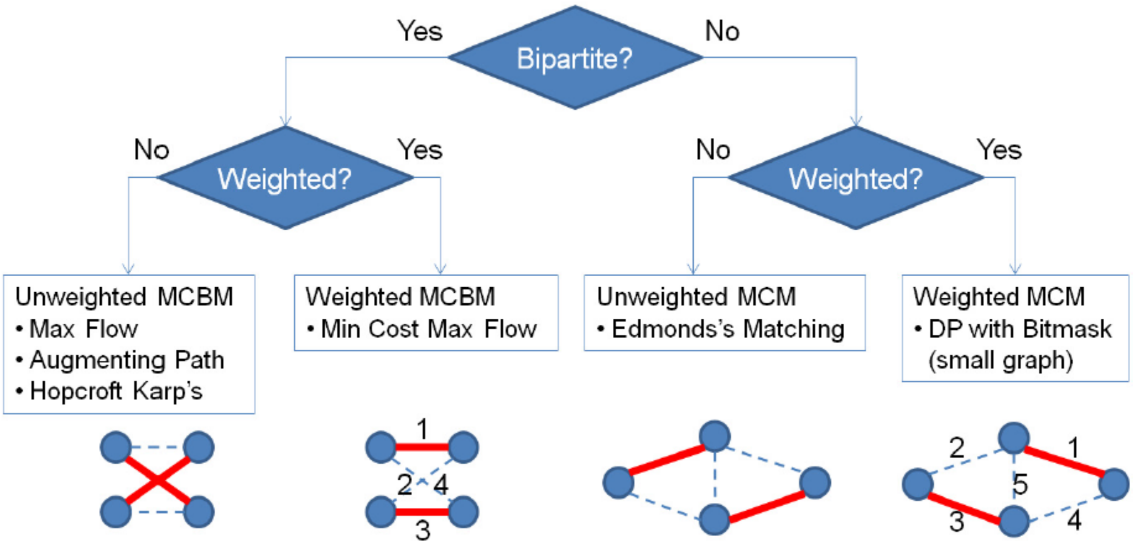
Erdős Gallai's Theorem gives a necessary and sufficient condition for a finite sequence of natural numbers to be the degree sequence of a simple graph. A sequence of nonnegative integers d1 ≥ d2 ≥ . . . ≥ dn can be the degree sequence of a simple graph on n vertices if $f \sum_{i=1}^{n} di$ is even and $\sum_{i=1}^{k} di \leq k \times (k-1) + \sum_{k+1}^{n} \min(di, k)$ holds for $1 \leq k \leq n$. Example : UVa 10720 - Graph Construction.

Euler's Formula for Planar Graph: *V* − *E* + *F* = 2, where *F* is the number of faces7 of the Planar Graph. Example: UVa 10178 - Count the Faces.

The **Number of Spanning Tree** of a complete bipartite graph $K_{n,m}$ is $m^{n-1} \times n^{m-1}$. Example: UVa 11719 - Gridlands Airport.

Graph Matching: Select a subset of edges M of a graph G(V,E) so that no two edges share the same vertex. [1] pp.349

Fig. 1: The Four Common Variants of Graph Matching in Programming Contests [2]



2. Combinatory

Derangement: A permutation of the elements of a set such that none of the elements appear in their original position. The number of derangements der(n) can be computed as follow: $\mathrm{der(n)} = (\mathrm{n}-1) \times (\mathrm{der(n}-1) + \mathrm{der(n}-2))$ where der(0) = 1 and der(1) = 0. A basic problem involving derangement is UVa 12024 - Hats (see Section 5.6).

3. Computational Geometry

Pick's Theorem: Let *I* be the number of integer points in the polygon, *A* be the area of the polygon, and *b* be the number of integer points on the boundary, then

$A = i + \frac{b}{2} - 1$. Example: UVa 10088 - Trees on My Island.

4. Number Theory

Faulhaber's formula: Each sum of the form $\sum_{x=1}^{n} x^k = 1^k + 2^k + 3^k + \cdots + n^k$ where k is a positive integer, has a closed-form formula that is a polynomial of degree k+1.

$$\sum_{x=1}^{n} 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{x=1}^{n} 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Fast Exponentiation: Using built in formula, $a^n = \exp(log\ a * n)\ o\ pow(a,n)$.

Fermat Lite Test: If $2^n mod\ n = 2$ then n has a high probability to be a prime number.

[33] pp.124

Factorial Frequencies: Digits of N! $\boldsymbol{floor}(\frac{\frac{\boldsymbol{log(2*PI*n)}}{\boldsymbol{2}}+\boldsymbol{n*(log(n)-1)}}{\boldsymbol{log(10)}} + \mathbf{1})$

5. Mathematic

Moser's Circle: Determine the number of pieces into which a circle is divided if *n* points on its circumference are joined by chords with no three internally concurrent. Solution: $C_4^n + C_2^n + 1$. Example: UVa 10213 - How Many Pieces of Land?

Carmichael Number: Carmichael number is a number which is not prime but has >= 3 prime factors. You can compute them using prime number generator and prime factoring algorithm. The first 15 Carmichael numbers are:

561, 1105 ,1729 ,2465 ,2821 ,6601 ,8911 ,10585 ,15841 ,29341 ,41041 ,46657 ,52633 ,62745, 63973

[33] pp.93

UVA Problem 10006 - Carmichael Number

6. Bibliografía

[1] N. Nimajneb, The Hitchhiker's Guide to the Programming Contests.

[2] F. H. Steven Halim, Competitive Programming 3, 2013.

[3] A. S. Arefin, Art of Programming Contest, UVA.