Air pollution forecasting in Rio de Janeiro

Final assignment on the subject Machine Learning

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Abstract

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1 Introduction

A ir quality is a growing concern and area of research, because most of the cities world-wide have been facing problems with it in the past few decades [8]. The rapid increase of the urban population and the development of the cities is causing environmental pollution, what can give rise to damage on human health.

The emission and transmission of air pollutants, such as, Nitrogen dioxide (NO₂), Carbon monoxide (CO), Ozone (O₃) and, Particulate Matter (PM), result in the ambient air pollution, which can be caused by different factors. The World Health Organization (WHO) explained [19] that PM, O₃, and NO₂ have, respectively, the strongest effects on health of air quality.

In Rio de Janeiro city, the city hall recognized the problem and created the Program MonitorAr-Rio in

2008 [16]. The objective was to monitor the air quality in the city, in order to verify the degree of exposure of the population to the pollutants, and inform the results to the population. Eight fixed stations monitor the main pollutants defined in the legislation, and some meterological conditions, such as, for example, temperature, relative humidity, solar radiation, and wind.

It is important to have updated knowledge and accurate predictions of the air pollutants, in order to help the formulation of public health and environmental policies. This study proposes models for hourly air quality forecasting for the city of Rio de Janeiro.

The text is organized as follows. Section 2 defines the problem clearly and mathematically. Section 3 gives a background on the topic of air quality and air pollution. Section 4 presents the methodology of the work. Section 5 contains a description of the data used in this work and an exploratory data analysis with the data path. Section 6 presents the methods used and the experiments related to each one. Sections 7, 8, and 9 end the text with main results and conclusions.

2 Problem definition

We want to produce time predictions of some pollutants [See Section 3.1 for a detailed description] for the Rio de Janeiro city, considering the weather, location, and time variables. We also want to develop a method for estimating the air quality of not monitored regions by the program based on the monitored ones.

Let Y_i be the random variable indicating the quantity of ith pollutant measured in a specific monitoring station, for instance, the quantity of ozone. If $X \in \mathcal{X} \subseteq \mathbb{R}^p$ is the random vector of meterological conditions measured, t is the time of measurement, and $s_1, ..., s_8$ the positions of the monitoring stations, then it is measured

$$Y_i = f_i(t, s_k, \mathbf{X}) + \epsilon_{t, s_k, i} \tag{1}$$

where $\epsilon_{t,s,i}$ is a random variable with mean zero. After observing, for $t_1 < ... < t_n$,

$$y_i^{t_1,s},...,y_i^{t_n,s}$$

and

$$x^{t_1,s},...,x^{t_n,s},$$

we want to predict $y_i^{\bar{t},s}$ for $\bar{t} > t_{n+1}$ and $y_i^{t,\tilde{s}}$ for $\tilde{s} \notin \{s_k : k = 1, ..., 8\}$.

3 Background of air pollution

A ir pollution is a mixture of particles and gases, often not visible to human eyes. The visible forms are widely known, such as smoke, soot and mold. According to Conselho Nacional do Meio Ambiente (CONAMA) [1, own translation], atmospheric pollution is

"[...] any form of matter or energy with intensity and in quantity, concentration, time or characteristics in disagreement with the established levels, and that make or may make the air: inappropriate, inconvenient, harmful to the environment or harmful to safety."

Different anthropogenic processes emit air pollutants, such as, for instance, fossil fuels (motor traffic and domestic), agriculture, and industries. Besides that, natural disasters are an important gas emitter, not controllable, though.

3.1 Polluting gases

The atmosphere of the Earth is a dynamic and complex system of natural gases, which are necessary to life [17]. The planet has a defense mechanism that absorbs part of them. However, high levels of gases concentration can cause several effects in the living beings. Air pollutants are divided according to their origin [19]:

- **Primary:** those emitted into the atmosphere from a source; or
- **Secondary:** those formed within the atmosphere through a chemical reaction.

Other important distinctions are related to their chemical class - organic or inorganic - and related to their physical state - gaseous or particulate. The selected pollutants for this work are:

- Carbon monoxide (CO): results of incomplete combustion of matter with carbon and it does not present smell or color. Its concentration level is strongly related to car traffic, in addition to agricultural and forest fires. When breathed, it reduces the ability of oxygen transport by the hemoglobins.
- Ozônio (O₃): It exists naturally in the atmosphere, where it has the function of absorbing ultraviolet radiation from the sun and of reducing its impact on the planet Earth surface. Nitrogen

oxides and organic compounds, with oxygen and high temperatures form the ozone. However, in the troposphere, it is toxic and even explosive. Its effect on health can be drastic.

• Particulate matter (PM₁₀): It is composed by particles of solid or liquid matter suspended in the air, with 10 micrometers of less. Combustion is one of the major human sources of particulate matter. Its effects include respiratory tract infections and damages to the environment. When the aerodynamic diameter is between 2.5 and 10 micrometers, the name is PM₁₀, and it is the most common among the suspended particles measured.

Other measured pollutants that we will not study by computational and data limitations are Sulfur Dioxide (SO₂), Nitrogen Oxides (NOx), and Hydrocarbons (HC).

3.2 Urban air quality problems

Air pollution episodes, such as, for example, the 1930 Meuse Valley in Belgium [10], the 1952 Great Fog of London [13], and the 2006 Southeast Asian Haze [6] raised questions and concerns about high levels of pollutants in urban ambients, which caused several deaths and hospitalizations. The rapidly expanding populations and cities also increase the exposure to them.

In Brazil, in the last decade, more than 60,000 died on average per year due to air pollution [5]. Household air pollution from solid fuels caused most of the cases, followed by ozone pollution, and particulate matter pollution.

3.3 Air quality index

The air quality index is a synthetic indicator what simplifies the divulgation and the communication to the population, private and public sectors, ONGs, among others. Its divulgation is made through diary reports for each monitoring station¹ as shown in Figure 1.

The index considers the pollutants PM_{10} , $PM_{2.5}$, O_3 , CO, NO_2 , and SO_2 . For each one, The AQIr is calculated as follows,

$$AQIr = I_{ini} + \frac{I_{fin} - I_{ini}}{C_{fin} - C_{ini}} \times (C - C_{ini}), \qquad (2)$$

such that I_{ini} (I_{fin}) is the value of the index that corresponds to the initial (final) concentration of the range; C_{ini} (C_{fin}) is the initial (final) concentration of the range in which the measured concentration is located, and C is the concentration measured. The value informed is the maximum AQIr of the pollutants.

After calculating the AIQ, we classify air quality at five levels: N1 (0-40), N2 (41-80), N3 (81-120), N4 (121-200), and N5 (> 200). For more details, consult the technical guide from the Ministry of the Environment [9]. In this work, we only consider the pollutants described on Section 3.1 for the forecasting of the air quality index.

4 Methodology

Exploratory data analysis is the first step into the project, after identifying the problem. The proposed EDA includes visualization and descriptive statistics to summarize the most relevant information to have insights. After this, we make data preprocessing, which includes missing data imputation, outlies, and feature engineering.

The evaluation methods for the algorithms are the mean absolute error (MAE), the root mean squared error (RMSE), and the rsquared (R²). The methods were trained using 70% of the available data, considering the first years. The software used for performing this experimental phase was developed in Python (v. 3.9), mainly using the Pandas [11, 18], Numpy [4], Scikit-learn [12], and Statsmodels [15].

5 Exploratory data analysis

5.1 Data description

The dataset used in this study was extracted from the project Monitor Ar-Rio [14]. The table con-

¹http://jeap.rio.rj.gov.br/je-metinfosmac/boletim

MonitorAR Rio Programa de Monitorameto de Qualidade de Ar

BOLETIM DE QUALIDADE DO AR

01/03/2018 15:00H Ouinta-feira

Exibir Boletim Anterior:



	Concentração Máxima Poluentes Monitorados					Índice de		Condições Meteorológicas observadas no período:			
Estação	Material Particulado (MP ₁₀) [µg/m³]	Ozônio (O ₃) [µg/m³]	Monóxido de Carbono (CO) [ppm]	Dióxido de Nitrogênio (NO ₂) [µg/m³]	Dióxido de Enxofre (SO ₂) [µg/m³]	Qualidade do Ar (IQA)	Classificação	A atuação de áreas de instabilidade ocasionou predominio de céu parcialmen nublado, porém sem registros de chuva sobre a Cidade. Desta forma, observou-se manutenção das concentrações dos poluentes em relação ao dia anterior, em que			
Centro	25,8	72,9	ND	NM	NM	46	Boa	qualidade do ar ficou classificada como BOA e REGULAR nos locais monitorados.			
Copacabana	<u>46,6</u>	ND	0,1	NM	1,6	47	Boa	Tendência da Qualidade do Ar para as Próximas 24h: O novo posicionamento do sistema de alta pressão, associado a áreas de instabilidade			
São Cristóvão	19,7	<u>84,5</u>	0,1	NM	8,1	53	Regular	O novo posucioniento do sisetente de ana pressao, associado a areas de instantiador favorecerão nebulosidade variada com possibilidade de chuva fraca em áreas isoladas. Assim, espera-se a manutenção das concentrações dos poluentes, o que deverá deixar a qualidade do ar classificada como BOA e REGULAR nas localidades monitoradas.			
Tijuca	26,3	<u>71,9</u>	0,6	65,9	2,4	45	Boa	a quantata ao a one en e			
Irajá	29,0	66,3	0,4	ND	ND	41	Boa				
Bangu	37,6	126,3	0,7	44,8	3,9	79	Regular				
Campo Grande	30,2	115,9	0,6	52,2	ND	73	Regular				
Pedra de Guaratiba	34,6	<u>75,2</u>	NM	NM	NM	47	Boa				
Unidade Móvel Recreio	47,1	82,6	0,2	NM	ND	52	Regular				

Figure 1: Report from March 1^{st} , 2018.

tains hourly data observations, separated by pollutant, weather condition, and monitoring stations' characteristics from the city of Rio de Janeiro. Table 1 informs the most important variables used, and Table 2 indicates the measured pollutants per monitoring station. The events start on January 1, 2011, and end on March 31, 2021, totalizing 661,662 records.

The missing values (Table 3) for each of the main variables used in this work are around 10%. Some meteorological variables have more than 10% of missing values, which is a lot. The CO gas has more missing values than the other gases because Pedra de Guaratiba does not measure it. When this kind of absent value is disregarded, CO has around 6%. Imputation methods in Section 5.2.

Almost 91% of the values in the Chuva column and 26% of the RS column are zero. If we consider the accumulated monthly amount of rain, it seems to make sense, and it is comparable to other sources.

We did not observe any pattern of missing values per year or monitoring station. In 2012, over half of the wind information is missing, however other features are stable. However, after 2016, UR dominates the number of absent data. If we aggregate by time

(hour, day, month, and year), and sum up the values of the features of each station, there are less missing data, proportionally, what. This implies other stations can provide useful information.

Table 4 contains the summary statistics of the variables. The high skewness and kurtosis from Chuva column reaffirm the heavy tails of its distribution, or the presence of outliers. The same occurs with Press. This is interesting because, it is common to see days with extreme values in meterological variables. In particular, the only hours with precipitation greater than 100 mm were in May 2020 in Tijuca, what is confirmed by weather references [2]. CO and PM_{10} have high kurtosis also and, therefore, can have outliers.

The time series of the three gases are presented in Figure 2. It indicates an increase in ozone levels during the years and a season effect, what corroborates on the way ozone is formed. It also seems there is a reduction in variability in CO and PM_{10} levels. The year of 2020 show a reduction in CO apparently, what is explained by the Coronavirus pandemic.

We confirm the tendencies of PM_{10} and O_3 in Figure 3. It is important to note that 2021 is not fin-

	Name	Type	Description
	Chuva	float	Rainfall (mm)
	Pres	float	Atmospheric Pressure (mbar)
	RS	float	Solar radiation (w/m2)
Meterological conditions	Temp float Temperat		Temperature (°C)
	UR	float	Relative humidity (%)
	Dir_Vento	float	Wind direction (°)
	Vel_Vento	float	Wind speed (m/s)
	Data	datetime	Measurement date and hour
	CodNum	integer	Number of the monitoring station
Measurement conditions	Estação	string	Name of the monitoring station
	Lat	float	Latitude position of the station
	Lon	float	Longitude position of the station

Table 1: Measured parameters by the program MonitorAr.

Monitoring station	Measured gases/particulates
Centro (CA)	O_3 , CO , PM_{10}
Copacabana (AV)	SO_2, O_3, CO, PM_{10}
São Cristóvão (SC)	SO_2, O_3, CO, PM_{10}
Tijuca (SP)	SO_2 , NOx , O_3 , CO , PM_{10}
Irajá (IR)	SO_2 , NOx , O_3 , CO , HC , $PM_{2.5}$, PM_{10}
Bangu (BG)	SO_2 , NOx , O_3 , CO , HC , PM_{10}
Campo Grande (CG)	SO_2 , NOx , O_3 , CO , HC , PM_{10}
Pedra de Guaratiba (PG)	O_3 , PM_{10}

Table 2: Pollutant data measured by each monitoring station. CO and HC are measured in (ppm), while the others are measured in $(\mu g/m^3)$.

Variable	Missing values				
Chuva	15812 (2.38 %)				
Pres	15294 (2.31 %)				
RS	48260 (7.29 %)				
Temp	70617 (10.6 %)				
UR	110619 (16.7 %)				
Dir_Vento	90498 (13.6 %)				
Vel_Vento	90743 (13.7 %)				
CO	114179 (17.2 %)				
O3	37133 (5.61 %)				
PM10	36142 (5.46 %)				

Table 3: Missing data in absolute and proportional values of the main variables. Data, CodNum, Lat, and Lon do not have nan values.

ished, so season effects are not complete yet.

By Figure 4, years 2011 and 2021 have less observation than the others. The year 2021 did not end as previously mentioned and the year 2011 had less monitoring stations.

Figure 5 shows an overview of the correlations between different features of the data to identify possible linear relations. The scatter plots of two to two features represents it with more details, but there much data, and the image has a large size. It does not present anything much different, though. The variables Temp, UR, and RS are strongly linearly related with absolute correlation greater than 0.6.

Figure 6 shows an interesting behavior of the ozone during the day. For each hour of the day, we represent the distribution of ozone in the respective hour. We

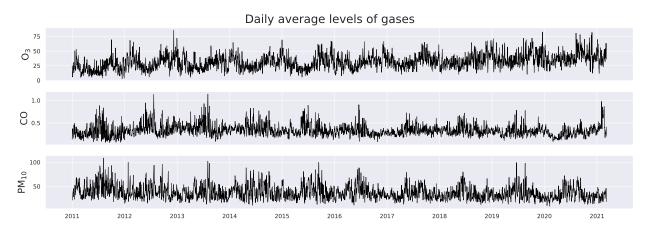


Figure 2: Time series with diary average levels.

	Chuva	Pres	RS	Temp	UR	Dir	Vel	CO	O3	PM10
Mean	0.13	1014.65	152.82	26.12	70.90	163.73	1.21	0.34	31.98	36.91
Std	1.64	5.68	244.37	4.90	18.35	73.45	1.00	0.28	29.81	23.52
Min	0.00	800.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25%	0.00	1011.12	0.00	22.67	58.39	100.00	0.55	0.14	8.68	21.00
50%	0.00	1014.30	6.17	25.54	72.75	166.17	0.92	0.29	24.52	32.00
75%	0.00	1018.02	224.00	28.99	85.08	222.50	1.55	0.46	46.89	47.00
Max	426.60	1036.48	1864.67	49.08	100.00	358.83	25.50	12.08	355.45	994.00
Skew	114.55	-7.32	1.61	0.55	-0.44	0.04	3.74	2.75	1.56	2.72
Kurt	23177.40	282.90	1.48	0.33	-0.40	-0.97	47.30	24.85	3.71	38.67

Table 4: Statistics of the meterological variables and gases.

observe that (1) the pollutants have heavy tail, or this data has a lot of outliers; and (2) the medians go along with the movement of the sun.

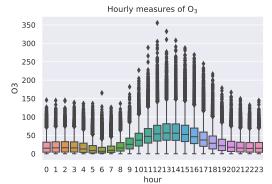


Figure 6: Boxplot of hourly ozone measurements.

5.2 Data preprocessing

The data preprocessing is an important step before the usage of machine learning algorithms, in order to report robust and neat results. Data from year 2020 and 2021 will be removed given the pandemic in the world.

5.2.1 Seasonal and time features

From the variable Data, it is extracted the variables year, month, day, hour, a boolean variable indicating the weekend, and a season variable. In order to consider the hourly seasonality, it is created the variables hour_sin = $\sin(2\pi \text{ hour}/24)$ and hour_cos = $\cos(2\pi \text{ hour}/24)$.

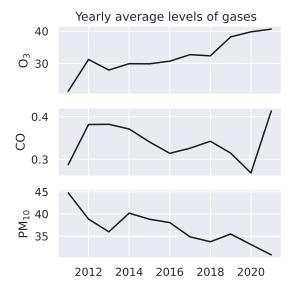


Figure 3: Time series with yearly average levels.

5.2.2 Missing data imputation

In this dataset, there is two types of missing data: (1) monitoring stations do not measure all pollutants by construction. For instance, it is not measured CO in Pedra de Guaratiba; and (2) monitoring stations did not measure in a period for some reason. For the first case, missing values remain in the dataset and the prediction is not performed. For the second case, three methods were compared:

- 1. Simple mean imputation: For each year and each column, the algorithm imputes the mean in the NaN values.
- 2. **Location:** For each time period, the missing values are replaced by the average among the others measuring stations. For example, if CO is missing at 6h on 01/01/2011 at Centro station, it is replaced by the mean among the other stations in the same time.
- 3. **k-NN:** For each year, if a feature is missing in row i, the algorithm selects the k closer points according to the nan euclidean measure. This measure calculates the euclidean distance among

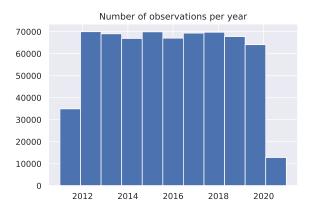


Figure 4: Number of hourly measurements per year. In 2011, only half of the monitoring stations worked.

non NaN entries and rescale depending on the number of them. The years are separated to reduce the number of rows.

The data values are normalized to the range of [0,1], because k-NN is based on a distance measure. To evaluate these methods, we develop a sample strategy similar to the Bootstrap method. In each simulation, we sample 20% of rows with non NaN values and randomly choose 7% of the cells to be removed (this value was chosen because this was observed across the entire dataset). From the simulated dataset with missing values, the methods impute as explained before. We compare the imputed sample after imputation with the sample before removal and calculate the mean squared error. The results for the year 2016 are above 5.

Método	MSE
5-NN	6.504e-04
10-NN	6.504 e-04
30-NN	6.504e-04
50-NN	6.504e-04
100-NN	6.504e-04
Location	1.077e-03
Simple Imputation	1.636e-03

Table 5: Results from the imputation

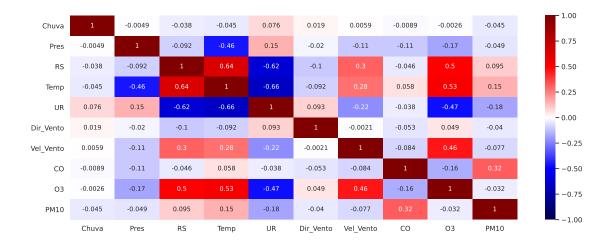


Figure 5: Correlation heat map comparing the data features.

For that reason, we apply the 5-NN in the dataset per year to impute data. It is important to note that the computation was a real barrier for a more deep analysis.

5.2.3 Data transformation

We applied the Yeo-Johnson power transform [20] on continuous variables in order to approximate the data distribution to a Gaussian distribution and to decrease the heteroscedasticity, as suggested by [3]. In figure 7, the gases distribution (disregarded missing data imputation) are shown before and after the transformation. The selected λ for each feature was estimated through maximum likelihood. Following the order from table 4, the values were, approximately, -20.16, 12.58, -0.1, 0.29, 1.59, 0.81, -0.79, -1.69, 0.28, and 0.27.

5.2.4 Feature extraction

The lag is a time gap in the series and are useful to analyse seasonality. In general, besides the influence of another variables, the past values can be helpful to make good predictions. An exploratory study with autocorrelation (ACF) and partial autocorrelation (PACF) to define the number of lag variables. Figure 8 shows interesting patterns: CO has

spikes at 12 multiples. That indicates a high correlation with twelve hours difference. The ozone does not have this quality, but presents the diary spikes. It is interesting that at Bangu station, this is negatively related. Since the PACF has only two spikes in all graphs, there is an autoregressive term in the pollutants series of order two. We add two lag variables for each pollutant and each monitoring station, then. We also add a 24 lag time to catch the diary influence. Besides that, a rolling mean variable with a 24 lag is added to simulate a moving average (MA) term. It is not possible to use the MA model in this framework since they are not observed [7].

Variables to remove: hour since it is highly correlated with hour_sin, and Data, because it has no service anymore. We will have only numeric variables from now on. The total number of features after all the above processes is 32 including meteorological conditions, time-related, lags, moving averages, and pollutants measurements.

Given the high dimensionality (27 independent variables), a principal component analysis (PCA) was applied to compare the results when the whole dataset is used. This is done only to verify if all the variables really help the forecasting.

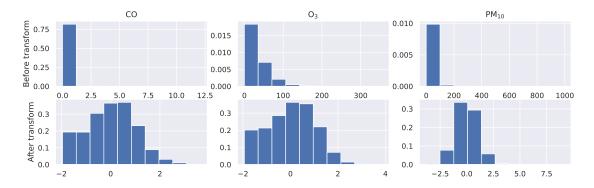


Figure 7: Gases distribution before and after the power transform.

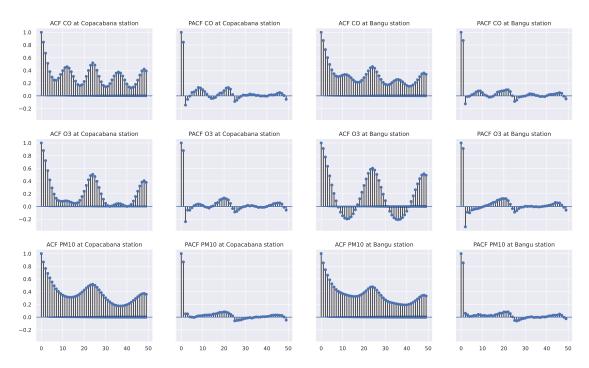


Figure 8: ACF and PACF plots for CO, O_3 , and PM_{10} at Copacabana and Bangu stations.

6 Experiment settings

We have already separated the dataset into training and testing. In this section we describe the methods used for the estimation: Linear Regression, Support Vector Machine, Random Forest, Boosting, and a model that handle missing data simultaneously with the fitting, using the Expectation Maximization.

6.1 Linear Regression

The first attempt was to consider linear regression. In this case, the model supposes that the expected value of the gases quantity (conditioned on the data) is a linear combination of the independent variables.

First we apply the simple OLS on the whole dataset. In general this is not a great model, because it adds variance on the estimation and, for that reason, we consider a regularization term (elastic net) with two parameters: α to control the penalty, and w_{l1} to control the weight given to \mathcal{L}_1 penalty. The parameters are chosen with cross validation.

Finally, to reduce the number of features considered in the model, a Forward Feature Selector with cross validation is performed. It adds features in a greedy fashion. The estimator chooses the best feature based on the cross-validation score, the mean squared error (MSE), for instance.

- 6.2 Support Vector Machine
- 6.3 Random Forest
- 6.4 Boosting
- 6.5 Linear Regression + Expectation Maximization

7 Results

- 1. Quantitative results of my experiments.
- 2. Statistical significance.

Lembrar de

1. Testar estacionaridade de cada série;

- 2. Lembrar de inverter os dados pelo power transformation: ler p2, fazer power transform.
- 8 Discussion and Future work
- 9 Conclusion

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