

# Water Balance Model of Wieringermeer Landfill

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The leachates generated from landfills have potential impact on the environment. Understanding the evolution of leachates from history helps us to predict its trend to better evaluate its impact on surrounding environment. In this paper, an existed water balance model was implemented to investigate the dynamic evolution of water in a landfill system at Wieringermeer. The Nash-Sutcliffe efficiency (*NSE*) is used for calibrating the model parameters against the measured data near the landfill. The calibrated model shows good fit to the the measured data. The physical meanings of the model parameters are discussed in detail. Finally, the limitation of the model is stated, and some suggestion is given.

**KEYWORDS**

Landfill model, leachate production, water storage, Nash-Sutcliffe efficiency

## 1 | INTRODUCTION

Landfilling is a common way for the disposal of waste materials. The rapid increase in waste production nowadays leads to more demands of landfills. However, the leachates generated from landfills are always polluting and can cause serious environmental impact without proper treatment. Thereby, a better understanding over the production and prediction of leachates will give great support for the treatment of landfill leachates and contributes to the assessment of the environmental impact.

Since the landfill is a high dynamic system which involves the temporal variability of rain fall, evaporation, water storage and leachate production, it is important to use a proper model to capture and describe the behaviour of landfill leachates. In this paper, a water balance model, which is developed from Benettin's study [3], has been imple-

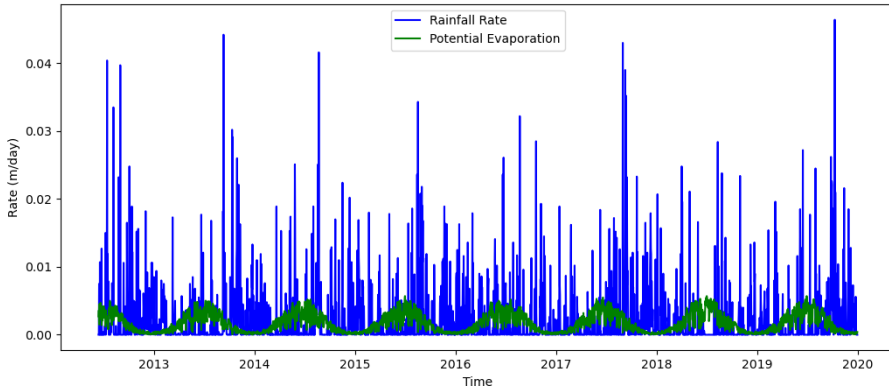
mented in Python to calculate the evolution of leachates from 2013 to 2019. The measured leachate data obtained from Wieringermeer is used for calibrating the parameters of the model. parametric study has been carried out to investigate the effect of each parameter in the model and get a better understanding about its physical meaning.

The following of this paper contains 3 sections. In section 2, the details of the water balance model will be given to explain how the rates of change for the storage of each layer and the leachate production are calculated, the selection of model parameters will also be reported in this section. In section 3, the results obtained from different sets of parameters will be shown and the sensitivity of parameters will be discussed. Finally, conclusions will be drawn in Section 5.

## 2 | METHODS

### 2.1 | Theoretical Formulation

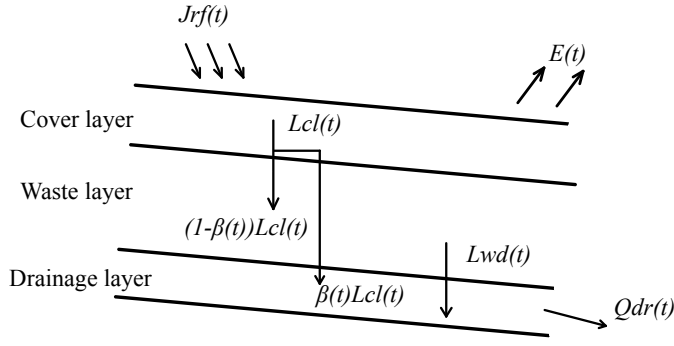
A conceptual model for the water balance of a landfill in Wieringermeer, which is driven by the rainfall, temperature and evapotranspiration and produces leachate, was developed based on Benettin's study [3]. The measured data of rainfall and potential evaporation are shown in *Figure 1*.



**FIGURE 1** Rainfall and evaporation rate

For simplification, the catchment is considered to be a two-layer system with a shallow and deep component (*Figure 2*). The shallow component, namely the cover layer, collects the external rainfall  $J_{rf}(t)$  and produces a certain amount of evaporation  $E(t)$ . The remaining is the leakage part which includes the lateral and vertical subsurface flows. Therefore, the first governing equation representing the changing rate for the storage in the cover layer  $[S_{cl}]$  is obtained (*Equation 1*). A fraction  $\beta(t)$  of the leakage is assumed to flow directly to the drainage layer and the rest is discharged to the waste body. The leaching rate from the waste layer to drainage layer is  $L_{wb}$ . As a result, the rates of change for the storage in the waste layer  $[S_{wb}]$  can be calculated from *Equation 2*. Pumping guarantees the water storage in the drainage system is constant, from which the equation for the storage changing rates in the drainage layer can be deduced (*Equation 3*).

$$\frac{dS_{cl}}{dt} = J_{rf}(t) - L_{cl}(t) - E(t) \quad (1)$$



**FIGURE 2** Conceptual water balance model

$$\frac{dS_{wb}}{dt} = (1 - \beta(t))L_{cl}(t) - L_{wb}(t) \quad (2)$$

$$\frac{dS_{dr}}{dt} = \beta(t)L_{cl}(t) + L_{wb}(t) - Q_{dr}(t) = 0 \quad (3)$$

In these equations,  $J_{rf}(t)$  is the rainfall rate acting on the shallow layer. 'L' terms are the leaching rates from different layers generated by the formulas below.

$$L_{cl} = a_{cl} * \left( \frac{S_{cl} - S_{clmin}}{S_{clmax} - S_{clmin}} \right)^{b_{cl}} \quad (4)$$

$$L_{wb} = a_{wb} * \left( \frac{S_{wb} - S_{wbmin}}{S_{wbmax} - S_{wbmin}} \right)^{b_{wb}} \quad (5)$$

The parameter subscripts  $cl$  and  $wb$  indicate the situations in the cover layer and waste body respectively. Besides,  $a$  is the saturated hydraulic conductivity and  $b$  is a dimensionless empirical parameter.  $S_{max}$  is a maximum achievable storage in the layer,  $S_{min}$  is the minimum storage in the layer where water will still freely drain [6].

$E(t)$  is the evaporation rate which can be calculated as:

$$E(t) = pEv(t) * C_f * f_{red} \quad (6)$$

with

$$f_{red} = \begin{cases} 0 & S_{cl} < S_{Evmin} \\ \frac{S_{cl} - S_{Evmin}}{S_{Evmax} - S_{Evmin}} & S_{Evmin} \leq S_{cl} \leq S_{Evmax} \\ 1 & S_{cl} > S_{Evmax} \end{cases} \quad (7)$$

where  $pEv(t)$  is the potential evaporation [m/day],  $C_f$  is the crop factor and  $f_{red}$  is a reduction factor reducing evapotranspiration under dry soil conditions [6].  $S_{Evmin}$  and  $S_{Evmax}$  are the marginal water storage in the cover layer that are used to determine the reduction effect of evapotranspiration.

$Q_{dr}$  is the leachate production rate from the drainage system and  $\beta(t)$  represents the proportion of leakage flowing directly from the upper layer to the bottom layer and can be computed as the product between a coefficient  $\beta_0$  and the dynamic storage normalized by the root zone pore volume [3], namely:

$$\beta = \beta_0 \left( \frac{S_{cl} - S_{cl_{min}}}{S_{cl_{max}} - S_{cl_{min}}} \right)$$

(8)

Table 1 shown below displays the values of a series of known field parameters that are crucial to set up the model successfully.

**TABLE 1** Some characteristics of Cell VP-06 of the landfill [6]

Parameter	Unit	Cell VP-06
base area	$m^2$	28355
top area	$m^2$	9100
slope width	$m$	38
waste body height	$m$	12
cover layer height	$m$	1.5
waste (wet) weight	$kg$	281083000
in operation	$year$	1992-1998

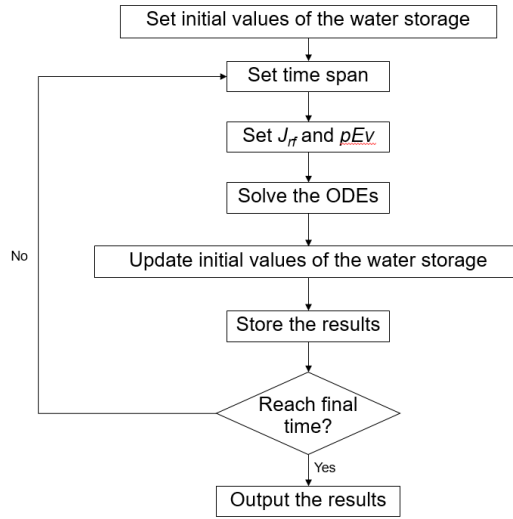
## 2.2 | Numerical Modelling

The theoretical model is implemented in Python. The built-in function solve\_ivp in Module SciPy is used to solve the ordinary differential equations (Equation 1 and 2). Explicit Runge-Kutta method of order 5(4) is chosen as the integration method. Since the rainfall rate and the potential evaporation in Wieringermeer are measured at the meteorological station every day, they are not able to be characterized as continuous functions of time. In the numerical model the values of  $J_{rf}(t)$  and  $pEv(t)$  are resetted at each time step of which the step size is 1 day. The results at one time step are the initial input at next time step until the final time is reached. The flow chart of the algorithm is shown in Figure 3.

## 2.3 | Parameter Description

Within this water balance model, a set of parameters have to be calibrated. Before calibration, a reasonable range of each parameter is derived based on previous literatures.

Parameter  $a$  refers to the saturated hydraulic conductivity of the layers, which should be higher or equal to the maximum outflow rate measured from the drainage layer. The common values of the hydraulic conductivity of saturated soils can vary from  $1 \times 10^{-4}$  to  $300\text{ m/day}$  [4]. Reddy reported that the hydraulic conductivity of MSW in landfills can vary from  $3.2 \times 10^{-5}$  to  $172.8\text{ m/day}$ , which depends on the degrading levels and the vertical stress applied on it [7]. In wet seasons, the rainfall rate is high. The cover layer is nearly saturated, which leads to a high value of  $\beta$  (Equation 8). The leachate production is mainly attributed to the water directly from the cover layer (Equation 3).



**FIGURE 3** Flow chart of the algorithm

As a result, the maximum measured leachate production rate (about  $4.9 \times 10^{-3} \text{ m/day}$ ) in wet seasons should be close to the saturated hydraulic conductivity of the cover layer. Therefore, the estimated range of  $a_{cl}$  is  $5 \times 10^{-3}$  to  $1 \times 10^{-2} \text{ m/day}$  (same order of magnitude as the measured data). In dry seasons, there is little water in the cover layer. The leachate mainly comes from the waste body. As a result, the maximum measured leachate production rate (about  $4.9 \times 10^{-4} \text{ m/day}$ ) in dry seasons should be close to the saturated hydraulic conductivity of the waste body. The estimated range of  $a_{wb}$  is  $5 \times 10^{-4}$  to  $1 \times 10^{-3} \text{ m/day}$ .

Parameter  $b$  links the water storage in each layer with the leaching rate. In other words,  $b$  characterizes the relationship between water content and hydraulic conductivity in unsaturated soils. With progressing drainage, the more pores are drained, the more the hydraulic conductivity decreases, because in addition to higher meniscus forces in pores that are still filled with water, the water-transporting pore surface area becomes smaller [4]. For this reason,  $b$  is supposed to be larger than 1. Benettin et al. showed that the calibrated values of  $b$  for two different soil layers are 7.9 and 28.0 respectively [3]. To investigate the effect of  $b$  in a broader scope, we also consider the cases in which  $b$  is smaller than 1 but larger than 0. In this study, the estimated range for both  $b_{cl}$  and  $b_{wb}$  is 0 to 80.

The maximum storage ( $S_{cl_{max}}$  and  $S_{wb_{max}}$ ) in one soil layer is determined by its porosity. The porosity of common soils varies from 0.3 to 0.65 [4]. The porosity of the soil of the cover layer is assumed to be 0.25 to 0.7 to incorporate a wider range of values for calibration. For MSW, the porosity varies from 0.64 to 0.71 [5]. In this study, the porosity of MSW is assumed to be 0.5 to 0.8. The heights of the cover layer and the waste body are 1.5 m and 12 m. Thus, the range of  $S_{cl_{max}}$  is 0.375 m to 1.05 m, and that of  $S_{wb_{max}}$  is 6 m to 9.6 m. Theoretically the minimum storage in one soil layer is 0 although small amount of residual water may remain in soil. For simplicity,  $S_{cl_{min}}$  and  $S_{wb_{min}}$  are assumed to be 0.

Parameter  $\beta_0$  relates to the certain fraction of water leaching from the cover layer to directly enter the drainage layer when the cover layer is saturated. Its theoretical range is between 0 and 1. But practically, if the cover layer is saturated, more water tends to flow to the drainage layer directly from the cover layer. The value of it should be close to 1.

The evapotranspiration rate is estimated by crop coefficient method. Previous study shows that crop coefficient

varies from 0.2 to 1.2 approximately [1], which is adopted as the range of  $C_f$  in this study. Since the climate is generally wet in the Netherlands, the reduction factor  $f_{red}$  is assumed to be 1, which has to be taken into account when the weather is dry.

The ranges of the model parameters are summarized in *Table 2*, which are then further used for calibration.

**TABLE 2** Model parameters

Parameter	Unit	Max. Value	Min. Value	Mean Value
$a_{cl}$	$m/day$	$1 \times 10^{-2}$	$5 \times 10^{-3}$	$7.5 \times 10^{-3}$
$a_{wb}$	$m/day$	$1 \times 10^{-3}$	$5 \times 10^{-4}$	$7.5 \times 10^{-4}$
$b_{cl}$	-	80	0	40
$b_{wb}$	-	80	0	40
$S_{cl_{max}}$	$m$	1.05	0.375	0.713
$S_{wb_{max}}$	$m$	9.6	6	7.8
$S_{cl_{min}}$	$m$	-	-	0
$S_{wb_{min}}$	$m$	-	-	0
$\beta_0$	-	0	1	0.5
$C_f$	-	0.2	1.2	0.7
$f_{red}$	-	-	-	1

## 2.4 | Calibration Methods

As indicated in *Table 2*, a total amount of ten parameters need to be calibrated, except for  $f_{red}$ , which is assumed to be equal to 1.0. The Nash-Sutcliffe efficiency (*NSE*) is used to evaluate the predictive power of the model by comparing the results of the implemented hydrological model with the observed leachate data. The closer the model efficiency is to 1.0, the more accurate the model is, and normally the value ranging between 0.5 and 0.65 indicates the model has sufficient quality [2]. The expression is shown below.

$$NSE = 1 - \frac{\sum_{t=1}^T (Q_{dr} - Q_{cal})^2}{\sum_{t=1}^T (Q_{dr} - \overline{Q_{dr}})^2}$$

(9)

Where  $T$  is the total number of days counted,  $Q_{dr}$  is observed discharge per day,  $Q_{cal}$  is the calculated leachate production rate and  $\overline{Q_{dr}}$  is the mean value of  $Q_{dr}$ .

Since using the built-in functions for optimization in Python cannot get the results consistent with the physical meaning, a manual method is adopted instead. The optimization of the parameters is suggested to follow a proper sequence and the average value of each parameter is taken as the initial value. The first two parameters to be calibrated are  $S_{cl_{max}}$  and  $S_{wb_{max}}$ , after which, the values of  $S_{cl_0}$  and  $S_{wb_0}$  are determined by dividing the respective maximum value by a multiplier. Considering the landfill being operated for years, the water storage in the two layers has reached a dynamic steady state, therefore, the trends of both  $S_{cl}$  and  $S_{wb}$  over time are supposed to fluctuating in a horizontal range. Followed by  $b_{cl}$  and  $b_{wb}$ , the subsequent parameters to be calibrated are  $a_{cl}$  and  $a_{wb}$ . These

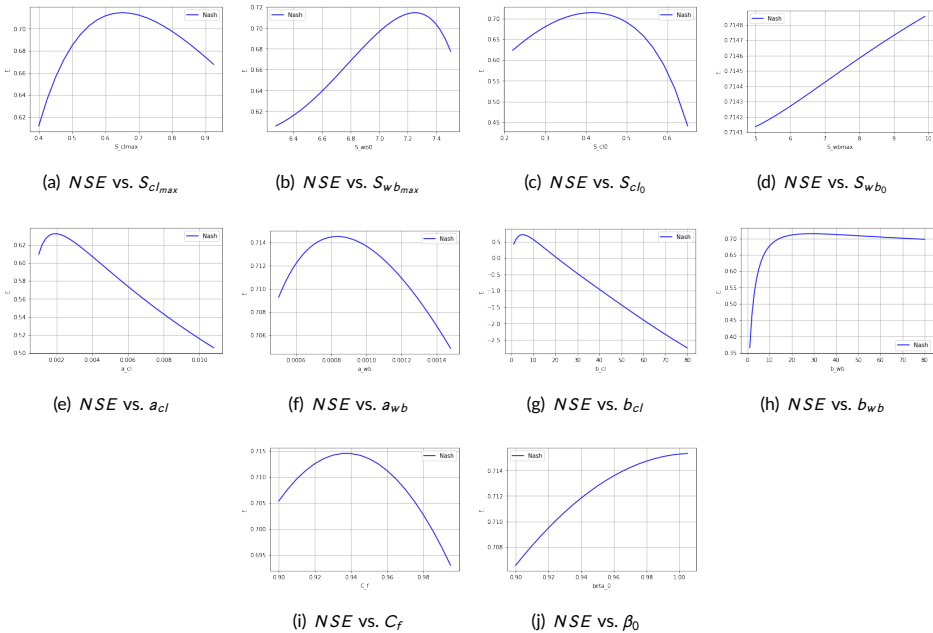
four parameters are determined by the material properties of the soil itself. Afterwards, the calibration value of the crop coefficient  $C_f$  will range from 0.2 to 1.2 and finally, the calibration of  $\beta_{a0}$  is conducted.

The majorized value of each parameter is obtained when the corresponding  $NSE$  reaches its maximum value without overflowing the assigned parameter ranges, and the modified parameter value should be updated in the model for the next parameter calibration step. It is observed that the accuracy of the model is affected by the joint effects of parameters. For example, both  $\beta_0$  and  $S_{cl}$  will contributed to the ratio of water flowing directly to the drainage layer. As a result, several iterations are required to gradually reduce the reasonable range of parameters until all the parameters are convergent, which means that a small variation of a parameter will not influence the stability and effectiveness of the model significantly.

### 3 | RESULTS AND DISCUSSION

#### 3.1 | Calibration Results

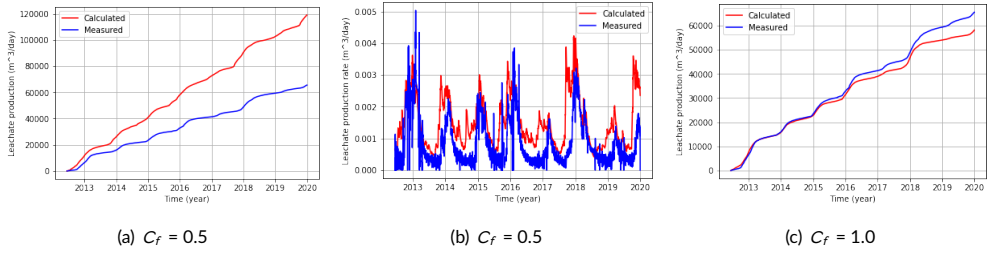
Calibrating two or more parameters at the same time will cost extremely long time. Plotting of the relationships between the value of each parameter and  $NSE$  (Figure 4), the results indicate that within the parameter range, the value of  $NSE$  decreases monotonically on both sides of the maximum point. Therefore the way to calibrate each parameter separately and repeating iteration to capture the optimized model is proven to be rational at a preliminary stage.



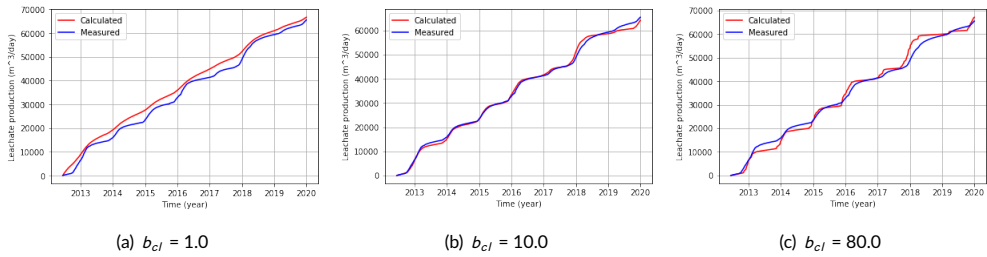
**FIGURE 4** The changes of  $NSE$  with different parameter values

The curve with large slope change in a small scale indicates that the sensitivity of that parameter is comparatively

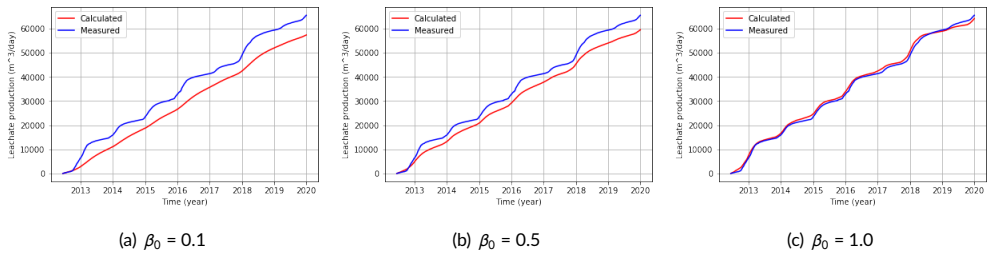
high, and a small fluctuation will significantly change the matching degree between the landfill model and the actual situation.  $C_f$ ,  $b_{cl}$  and  $\beta_0$ , which can have a large changing range and are related to the properties of the cover layer, dominate the effects to leachate production when the model is convergence. Additionally, according to the diagrams of relationship between measured and calculated leachate production,  $C_f$  will influence the slope of the calculated curve, which contributes to the overall shift in the value of leachate production per day (Figure 5).  $b_{cl}$  and  $\beta_0$  mainly only affects the fluctuation degree of the curve (Figure 6 and 7), in other words, they determine the amplitudes of variation of leachate production rate. Moreover, at each iteration,  $b_{cl}$  and  $b_{wb}$  show the least sensitivity to the change of other parameters.



**FIGURE 5**  $C_f$  impacts greatly to the measured and calculated leachate production



**FIGURE 6** Measured and calculated leachate production under different  $b_{cl}$



**FIGURE 7** Measured and calculated leachate production under different  $\beta_0$



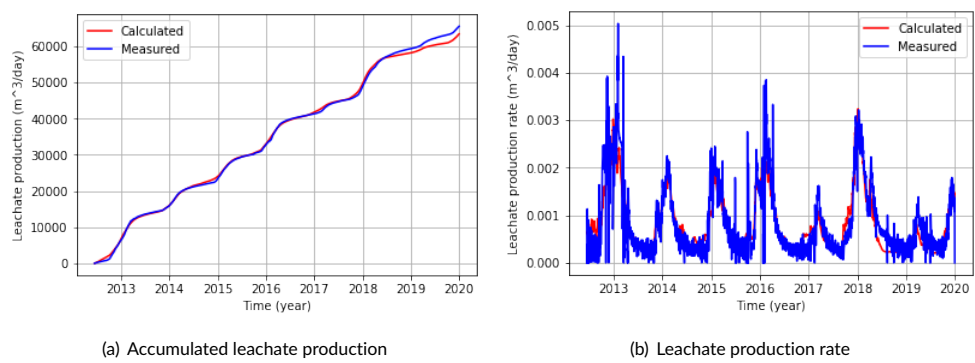
After several iterations of calibrating parameters, the model has achieved a relatively high accuracy, with  $NSE$  equals to approximately 0.715. The final values of each model parameter are shown in the following *Table 3*. Noticed that  $NSE$  grows with the increase of  $S_{wb_{max}}$ , while a smaller value of  $S_{wb_{max}}$  will be in accordance with the actual situation better, which is the reason that the value is taken as 7.5. Also,  $\beta_0$  that is equal to 1.0 is beyond reality, therefore it is chosen to be 0.975.

**TABLE 3** Calibration results

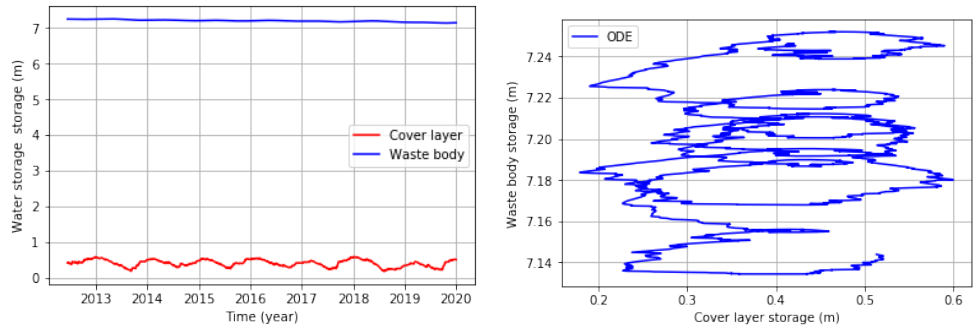
Parameter	Unit	Calibrated value
$S_{cl_{max}}$	$m$	0.65
$S_{wb_{max}}$	$m$	7.5
$S_{cl_0}$	$m$	$S_{cl_{max}}/1.547$
$S_{wb_0}$	$m$	$S_{cl_{max}}/1.035$
$a_{cl}$	$m/day$	0.005
$a_{wb}$	$m/day$	0.00082
$b_{cl}$	-	5.0
$b_{wb}$	-	30.0
$\beta_0$	-	0.975
$C_f$	-	0.94

### 3.2 | Modelling Results

Eventually, the aligned landfill model is developed based on the parameters above *Table 3*. The measured and calculated leachate production accumulation values and rates are shown in *Figure 8* and the evolution of water storage in the cover layer and waste body are shown in *Figure 9*. The calculated model has a nice agreement with the measured results and the water storage in the two layers is basically in equilibrium.



**FIGURE 8** Measured and calculated leachate production



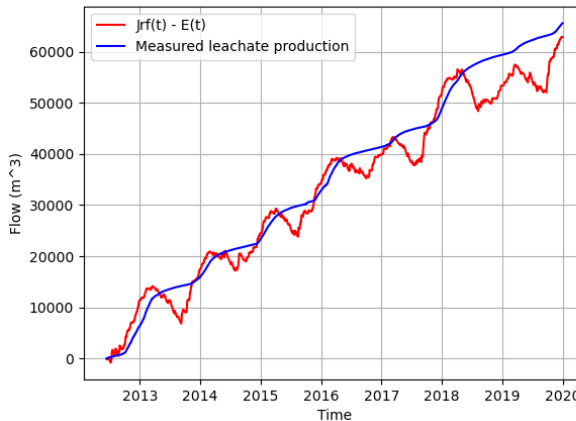
(a) Water storage in the cover layer and waste body over time (b) Relationships between water storage in the cover layer and waste body

**FIGURE 9** Evolution of water storage in the cover layer and waste body

### 3.3 | Discussion

The calibration results reveal influence of the parameters on the accuracy of the model. The selection of the calibrated values of the parameters must match the physical meanings, which has been initially discussed in Section 2.3. This section is going to further unravel the relationship of some parameters and the physical mechanism.

The crop factor has most significant influence on the modelling results (*Figure 4*). Since the landfill has been operating for a long period (*Table 1*), the waste body is not able to produce a lot of water in 2012 to 2020. The leachate production should be equal to the difference between rainfall and evapotranspiration which is highly affected by  $C_f$ . *Figure 10* shows that the trend of calculated water inflow into the system agrees with that of leachate outflow, validating the calibrated values of  $C_f$ .



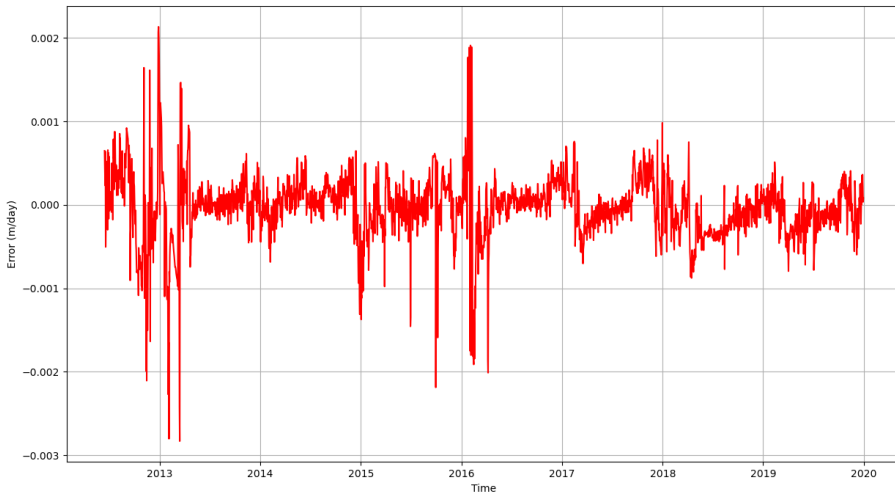
**FIGURE 10** Water inflow and outflow

The modelling results at the initial stage are sensitive to the input of initial storage in both layers. The start date should be taken into account when determining the initial values of storage. The storage of each layer has reached a stable state which fluctuates around a certain value. The start date of the observation in this study is in dry summer in 2012 (*Figure 1*). For the cover layer of which the storage depends on the magnitude of rainfall, the initial value of the storage should be close to that in dry seasons. For the waste body of which the storage less depends on the magnitude of rainfall, the initial value of the storage should be close to the stable value.

Parameter  $a_{cl}$  controls the magnitude of the leaching rates in the cover layer and should be larger than the maximum leachate production rate. However, the maximum leachate production rate (about  $4.9 \times 10^{-3} \text{ m/day}$ ) which is adopted in this study only occurs once in the summer in 2012. The maximum leachate production rate in other years is about  $2.8 \times 10^{-3} \text{ m/day}$ . The lower boundary of  $a_{cl}$  may be overestimated.

Parameter  $a_{wb}$  can also be evaluated by *Equation 2*. Since  $S_{wb}$  varies very slightly with time (*Figure 9(a)*), its changing rate is nearly zero. This leads to  $L_{wb}(t)$  equal to  $(1 - \beta(t))L_{cl}(t)$ . In wet seasons,  $\beta(t)$  is about 0.9, and  $L_{cl}(t)$  is equal to  $a_{cl}$  which is  $5 \times 10^{-3} \text{ m/day}$ . And  $L_{wb}(t)$  is equal to about 0.3  $a_{wb}$  when considering  $S_{wb}/S_{wbmax}$  and  $b_{wb}$  as 0.96 and 30 respectively. It leads to  $a_{wb}$  equal to  $1.67 \times 10^{-3} \text{ m/day}$  which is close to the calibrated value, validating the calibration outcome of  $a_{wb}$ .

### 3.4 | Limitation and Improvements



**FIGURE 11** Error between modeling results and measured data

Although the model generally shows satisfactory results with respect to the observed data in Wieringermeer landfill, some apparent errors cannot be ignored. *Figure 11* shows that in the winters in 2013 and 2016 the errors are large and changes violently between positive and negative values, while in other winters such phenomenon is not observed. This is because the pump that was used in the leachate collection pit was switched for maintenance on those days. As a result, no flow was recorded. When the pump was switched back on, the pump switched on and

quickly pumped out the excess leachate leading to and overestimation in flow.

Except for these two winters, the errors display a periodic pattern, especially from 2016 to 2020. These periodic errors are probably brought by some variables that also change periodically and are not incorporated in the model. Simplified assumptions have been made when establishing the model. However, the water balance processes in landfill are complex, involving biochemical reactions, soil mechanics and biological activities. These will lead to the fact that the observed leachate production could be effected by other processes which are not captured by the models.

Another limitation in this study lies in the calibration method. Although the calibration method is time-saving, the influence that the parameters have on each other and the final results is not well-captured by the sensitive analysis. More accurate results could be achieved when applying more advanced analysis with expanding amount of calculation and optimization.

To improve the model, some suggestion is made as follow.

- (a) The water balance model used in this paper is relatively simple since the other processes are not taken into consideration. The model could be further improved by introducing the effect of temperature, biochemical reactions, soil mechanics and biological activities.
- (b) Field tests and laboratory tests can be adopted to get a better understanding of the range of the parameters. Moreover, a pre-processing of raw data may improve the performance of the model.
- (c) As mentioned above, more advanced methods (Markov Chain Monte Carlo) can be used for model calibration and will help the model to capture the water transition process during wetting and drying cycles.

## 4 | CONCLUSION

This report implements the model described by Benettin's study [3], and provides a robust simulation of the water balancing process in Wieringermeer landfill. Parameters are calibrated based on provided data, sensitivity analysis and characteristics of landfill storage dynamics.

The final modelling results show that the cover layer is affected by the environment, responded with precipitation and evapotranspiration in a time delay. On the contrary, the storage remain almost constant in waste body. Overall, the results indicate that the landfill has achieved a stable state.

The NSE of this model achieves 0.70, showing that the prediction of leachate production is rather accurate compared to observed data. It proves that the model is capable to predict the dynamic storage driven by the rainfall, temperature and evapotranspiration.

## ACKNOWLEDGMENTS

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