Part I

**Preface** 

#### **Purpose and Scope of This Book**

- 1.1 What This Book Covers
- 1.2 What This Book Does Not Cover
- 1.3 Target Audience: Students, Researchers, and Practitioners

## Why "Precise Analysis" Matters — From Theory to Engineering

- 2.1 The Gap Between Theoretical Complexity and Real-World Performance
- 2.2 Case Studies: When Big-O Isn't Enough
- 2.3 The Role of Constants, Lower-Order Terms, and Hardware

### Mathematical and Algorithmic Prerequisites

- 3.1 Discrete Mathematics
- 3.1.1 Sets, Functions, and Relations
- 3.1.2 Combinatorics: Permutations, Combinations, and Binomial Coefficients
- 3.1.3 Graph Theory Basics
- 3.1.4 Proof Techniques: Induction, Contradiction, and Contrapositive
- 3.2 Elementary Probability Theory
- 3.2.1 Sample Spaces, Events, and Probability Measures
- 3.2.2 Random Variables and Expectations
- 3.2.3 Basic Distributions: Uniform, Bernoulli, Geometric, Binomial
- 3.2.4 Linearity of Expectation
- 3.2.5 Conditional Probability and Independence
- 3.2.6 Variance and Standard Deviation
- 3.2.7 Moment Generating Functions (Brief Introduction)
- 3:3:00-20 Mathematical Analysis

### Structure of the Book: Theorems, Proofs, Examples, and Exercises

- 4.1 How to Read This Book
- 4.2 Notation and Conventions
- 4.3 Types of Exercises: Conceptual, Computational, and Proof-Based
- 4.4 Using Examples Effectively
- 4.5 The Role of Rigor vs. Intuition

## Primary References and Parallel Reading Guide

- 5.1 Classic Textbooks (CLRS, Sedgewick, Kleinberg-Tardos)
- 5.2 Research Papers and Monographs
- 5.3 Online Resources and Lecture Notes
- 5.4 Recommended Reading Order and Study Plans

#### Part II

Foundations of Algorithmic Analysis

### **Introduction to Algorithm Analysis**

6.1	What Is Algorithm Analysis?
6.1.1	Correctness vs. Efficiency
6.1.2	Resource Measures: Time, Space, Energy, I/O
6.1.3	The Need for Mathematical Models
6.2	The RAM Model of Computation
6.2.1	Basic Operations and Unit-Cost Assumption
6.2.2	Memory Access Model
6.2.3	Limitations and Extensions of the RAM Model
6.3	Measuring Algorithm Performance
6.3.1	Input Size and Problem Instances
6.3.2	Counting Basic Operations
6.3.3	Exact vs. Asymptotic Analysis
6.4	Overview of Complexity Classes
6.4.1	P, NP, NP-Complete, and NP-Hard (Brief Introduction)
6.4.2	Why We Focus on Polynomial-Time Algorithms

#### **Asymptotic Notation**

<b>7.1</b>	The Need	for Asym	ptotic Analysis

- 7.1.1 Why Exact Counts Are Often Impractical
- 7.1.2 Growth Rates and Scalability
- 7.2 Big-O Notation (O)
- 7.2.1 Formal Definition
- 7.2.2 Intuition: Upper Bounds
- 7.2.3 Common Functions and Their Growth Rates
- 7.2.4 Examples and Non-Examples
- 7.2.5 Properties of Big-O

**Transitivity** 

Addition and Multiplication Rules

Reflexivity and Asymmetry

#### 7.3 Big-Omega Notation ( $\Omega$ )

- 7.3.1 Formal Definition
- 7.3.2 Intuition: Lower Bounds
- 7.3.3 Examples and Applications
- 7.3.4 Relationship Between O and  $\Omega$

#### 7.4 Big-Theta Notation (⊕)

### **Recurrence Relations and Their Solutions**

8.1 Introduction to Recurrence Relation
---

- 8.1.1 What Are Recurrences?
- 8.1.2 Why They Arise in Algorithm Analysis
- 8.1.3 Examples from Divide-and-Conquer Algorithms

#### 8.2 The Substitution Method

- 8.2.1 Guessing the Solution
- 8.2.2 Proving by Induction
- 8.2.3 Examples: Mergesort, Binary Search
- 8.2.4 Strengthening the Inductive Hypothesis

#### 8.3 The Recursion-Tree Method

- 8.3.1 Visualizing the Recurrence
- 8.3.2 Summing Over Levels
- 8.3.3 Examples and Illustrations
- 8.3.4 Limitations and When to Use

#### 8.4 The Master Theorem

8.4.1 Statement of the Master Theorem (Standard Form)

#### Best-Case, Worst-Case, and Average-Case Analysis

9.1	<b>Defining</b>	Input	Classes
-----	-----------------	-------	---------

- 9.1.1 What Constitutes an "Input"?
- 9.1.2 Problem Instances and Instance Distributions
- 9.2 Best-Case Analysis
- 9.2.1 Definition and Purpose
- 9.2.2 Examples: Insertion Sort, Linear Search
- 9.2.3 When Best-Case Matters (and When It Doesn't)
- 9.3 Worst-Case Analysis
- 9.3.1 Definition and Motivation
- 9.3.2 Guarantees and Robustness
- 9.3.3 Examples: Quicksort, Searching in Unsorted Arrays
- 9.3.4 Lower Bounds and Optimality
- 9.4 Average-Case Analysis
- 9.4.1 Definition: Expected Running Time
- 9.4.2 Assumptions About Input Distributions
- 9.4.3 Probabilistic Models: Uniform, Gaussian, etc.
- 7.4.5 I Tobabilistic Widacis. Offitolini, Gaussian, etc.

**Examples: Quicksort, Hashing, Skip Lists** 

#### **Probabilistic Analysis of Algorithms**

10.1	Foundations of Probabilistic Analysis	
10.1.1	Random Variables in Algorithm Analysis	
10.1.2	Indicator Random Variables	
10.1.3	Linearity of Expectation	
10.2	<b>Expected Running Time</b>	
10.2.1	Formal Definition	
10.2.2	Computing Expectations via Indicator Variables	
10.2.3	Examples: Hiring Problem, Randomized Quicksort	
10.3	Probabilistic Bounds	
10.3.1	Markov's Inequality	
10.3.2	Chebyshev's Inequality	
10.3.3	Chernoff Bounds	
10.3.4	Applications to Load Balancing and Hashing	
10.4	Randomized Algorithms	
10.4.1	Randomized Quicksort (Detailed Analysis)	
10.4.2	Randomized Selection (Quickselect)	
<b>1.0</b> :4:3:0	Hashing and Universal Hash Functions	<b>17</b>  45

10.4.4 Bloom Filters and Probabilistic Data Structures

# Part III Advanced Analysis Techniques

#### **Amortized Analysis**

11.1	Introduction to Amortized Analysis
11.1.1	Motivation: Why Average Per-Operation Cost?
11.1.2	Amortized vs. Average-Case Analysis
11.1.3	When to Use Amortized Analysis
11.2	Aggregate Analysis
11.2.1	Definition and Methodology
11.2.2	Example: Dynamic Array (Vector) Resizing
11.2.3	<b>Example: Binary Counter Increment</b>
11.2.4	Example: Stack with Multipop
11.3	The Accounting Method
11.3.1	Conceptual Framework: Credits and Debits
11.3.2	<b>Defining Amortized Costs</b>
11.3.3	Example: Dynamic Array via Accounting
11.3.4	<b>Example: Splay Trees (Introduction)</b>
11.3.5	<b>Ensuring Non-Negative Credit Balance</b>
11.4	The Potential Method

12.5

#### **Space Complexity Analysis**

12.1	Introduction to Space Complexity
12.1.1	Why Space Matters
12.1.2	Types of Memory: Stack, Heap, Static
12.1.3	In-Place vs. Out-of-Place Algorithms
12.2	Measuring Space Usage
12.2.1	Auxiliary Space vs. Total Space
12.2.2	Recursive Call Stack Depth
12.2.3	Implicit vs. Explicit Data Structures
12.3	<b>Examples of Space Complexity Analysis</b>
12.3.1	Iterative Algorithms: Loops and Arrays
12.3.2	Recursive Algorithms: Mergesort, Quicksort
12.3.3	Dynamic Programming: Memoization vs. Tabulation
12.3.4	Graph Algorithms: BFS, DFS, Shortest Paths
12.4	Space-Time Tradeoffs
12.4.1	Caching and Memoization
12.4.2	Lookup Tables and Precomputation
FrsPEdAn 3202	Compression and Succinct Data Structures

**22**|45

#### Cache-Aware and I/O Complexity

13.1	Introduction to the Memory Hierarchy
13.1.1	Registers, Cache (L1, L2, L3), RAM, Disk
13.1.2	Latency and Bandwidth Characteristics
13.1.3	Why Algorithm Design Must Consider Memory
13.2	The External Memory Model (I/O Model)
13.2.1	Parameters: $N$ (data size), $M$ (memory size), $B$ (block size)
13.2.2	I/O Complexity: Counting Block Transfers
13.2.3	Comparison with RAM Model
13.3	I/O-Efficient Algorithms
13.3.1	Scanning and Sorting
Externa	l Merge Sort

13.3.2 Matrix Operations

I/O Complexity:  $O((N/B)\log_{M/B}(N/B))$ 

**Matrix Transposition** 

Matrix Multiplication

13.3.3 Graph Algorithms

I/O-Efficient BFS and DFS

**Minimum Spanning Tree** 

**24**|45

### Cache-Aware Scheduling and Analysis for Multicores

14.1	Introduction to Multicore and Parallel Computing
14.1.1	Shared vs. Distributed Memory
14.1.2	Parallel Models: PRAM, Fork-Join, Work-Stealing
14.1.3	Performance Metrics: Work, Span, Parallelism
14.2	Cache Coherence and Consistency
14.2.1	MESI and MOESI Protocols
14.2.2	False Sharing in Multicore Systems
14.2.3	Impact on Algorithm Design
14.3	Cache-Aware Parallel Algorithms
14.3.1	Parallel Sorting with Cache Awareness
14.3.2	Parallel Matrix Multiplication (Strassen, Coppersmith-Winograd)
14.3.3	Load Balancing and Task Granularity

**Real-Time and Embedded Systems** 

WCET Analysis in Cache-Aware Contexts

# Part IV Lower Bounds and Optimality

15.5 Exercises

## Lower Bounds for Comparison-Based Algorithms

15.1	Decision Trees
15.1.1	Modeling Algorithms as Decision Trees
15.1.2	Height of Decision Trees and Worst-Case Complexity
15.2	<b>Sorting Lower Bound</b>
15.2.1	Information-Theoretic Argument
15.2.2	$\Omega(n \log n)$ Lower Bound for Comparison Sorting
15.2.3	Implications and Optimal Algorithms
15.3	<b>Selection and Searching Lower Bounds</b>
	Selection and Searching Lower Bounds Finding the Minimum: $\Omega(n)$
15.3.1	
15.3.1 15.3.2	Finding the Minimum: $\Omega(n)$
15.3.1 15.3.2 15.3.3	Finding the Minimum: $\Omega(n)$ Finding Median: Adversary Arguments
15.3.1 15.3.2 15.3.3 15.4	Finding the Minimum: $\Omega(n)$ Finding Median: Adversary Arguments Searching in Sorted Arrays: $\Omega(\log n)$

First Edition • 2025

### Algebraic and Non-Comparison Lower Bounds

16.1	Algebraic Decision Trees
16.1.1	<b>Extending Beyond Comparisons</b>
16.1.2	<b>Element Distinctness Lower Bound</b>
16.2	<b>Communication Complexity</b>
16.2.1	Models and Definitions
16.2.2	Applications to Data Structures
16.3	Cell-Probe Model
16.3.1	<b>Lower Bounds for Data Structures</b>
16.3.2	Dynamic vs. Static Data Structures
16.4	Exercises

#### Part V

**Specialized Topics and Applications** 

## Analysis of Specific Algorithm Paradigms

<b>17.1</b>	Divide-and-Conquer Algorithms
17.1.1	General Framework and Recurrence Relations
17.1.2	Examples: Mergesort, Quicksort, Strassen's Algorithm
17.1.3	Optimality and Lower Bounds
17.2	Greedy Algorithms
17.2.1	Correctness via Exchange Arguments
17.2.2	Matroid Theory (Brief Introduction)
17.2.3	Examples: Huffman Coding, Kruskal's MST
17.3	Dynamic Programming
17.3.1	Optimal Substructure and Overlapping Subproblems
17.3.2	Memoization vs. Tabulation
17.3.3	Time and Space Complexity Analysis

Examples: Knapsack, Edit Distance, Matrix Chain Multipli-

cation

17.3.4

# Online Algorithms and Competitive Analysis

18.1	Introduction to Online Algorithms
18.1.1	Online vs. Offline Problems
18.1.2	Competitive Ratio
18.2	<b>Examples of Online Problems</b>
18.2.1	Paging and Caching (LRU, FIFO, LFU)
18.2.2	Load Balancing
18.2.3	Online Scheduling
18.3	<b>Competitive Analysis Techniques</b>
18.3.1	Deterministic vs. Randomized Algorithms
18.3.2	Lower Bounds via Adversary Arguments
18.4	Exercises

### **Approximation Algorithms**

19.1	Introduction to Approximation
19.1.1	NP-Hardness and Intractability
19.1.2	Approximation Ratios
19.2	Examples of Approximation Algorithms
19.2.1	Vertex Cover (2-Approximation)
19.2.2	<b>Set Cover (Greedy,</b> $\log n$ <b>-Approximation)</b>
19.2.3	Traveling Salesman Problem (Metric TSP)
19.3	Analysis Techniques
19.3.1	<b>Bounding Optimal Solutions</b>
19.3.2	Linear Programming Relaxations
19.4	Exercises

### **Parameterized Complexity**

20.1	<b>Introduction to Parameterized Algorithms</b>
20.1.1	Fixed-Parameter Tractability (FPT)
20.1.2	Kernelization
20.2	Examples and Analysis
20.2.1	Vertex Cover Parameterized by Solution Size
20.2.2	Treewidth and Graph Algorithms
20.3	W-Hierarchy and Hardness
20.3.1	W[1], W[2], and Beyond
20.4	Exercises

#### Part VI

### **Practical Considerations and Case Studies**

### **From Theory to Practice**

21.1	Hidden Constants and Lower-Order Terms
21.1.1	When $O(n \log n)$ Beats $O(n)$ in Practice
21.1.2	<b>Empirical Performance Measurements</b>
21.2	Algorithm Engineering
21.2.1	Profiling and Benchmarking
21.2.2	Tuning for Specific Hardware
21.2.3	Libraries and Implementations (STL, Boost, etc.)
21.3	Parallel and Distributed Algorithm Analysis
	Parallel and Distributed Algorithm Analysis Scalability and Speedup
21.3.1	
21.3.1 21.3.2	Scalability and Speedup
21.3.1 21.3.2 21.4	Scalability and Speedup  Amdahl's Law and Gustafson's Law
21.3.1 21.3.2 21.4 21.4.1	Scalability and Speedup  Amdahl's Law and Gustafson's Law  Energy Efficiency

#### **Case Studies**

22.1	Sorting Algorithms in Practice
22.1.1	Timsort, Introsort, Radix Sort
22.1.2	Comparison of Theoretical vs. Empirical Performance
22.2	<b>Graph Algorithms in Large-Scale Systems</b>
22.2.1	Web Graphs and PageRank
22.2.2	Social Network Analysis
22.3	Machine Learning and Data Science
22.3.1	Complexity of Training Algorithms
22.3.2	SGD, AdaGrad, Adam: Time and Space Analysis
22.4	Database Systems
22.4.1	Query Optimization
22.4.2	Indexing Structures (B-Trees, LSM-Trees)
22.5	Exercises

#### Appendix A

#### **Mathematical Background**

- A.1 Summation Formulas
- A.2 Logarithms and Exponentials
- A.3 Recurrence Relations (Quick Reference)
- A.4 Probability Distributions
- A.5 Matrix Operations

#### Appendix B

#### **Pseudocode Conventions**

- **B.1** Notation and Style
- **B.2** Common Data Structures

#### Appendix C

#### **Solutions to Selected Exercises**

#### Appendix D

### **Glossary of Terms**

# Appendix E Index of Algorithms

#### Appendix F

#### **Annotated Bibliography**

- F.1 Foundational Texts
- F.2 Research Papers by Topic
- F.3 Online Courses and Resources