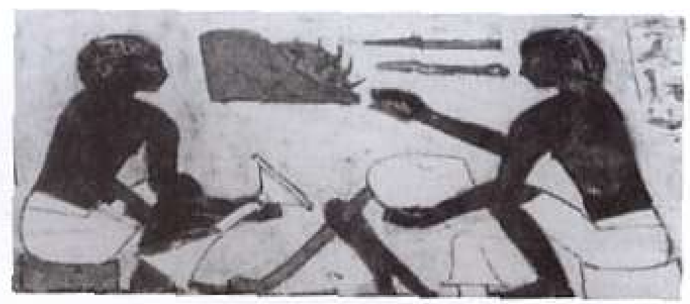
The Rise of Projective Geometry II

The Renaissance Artists

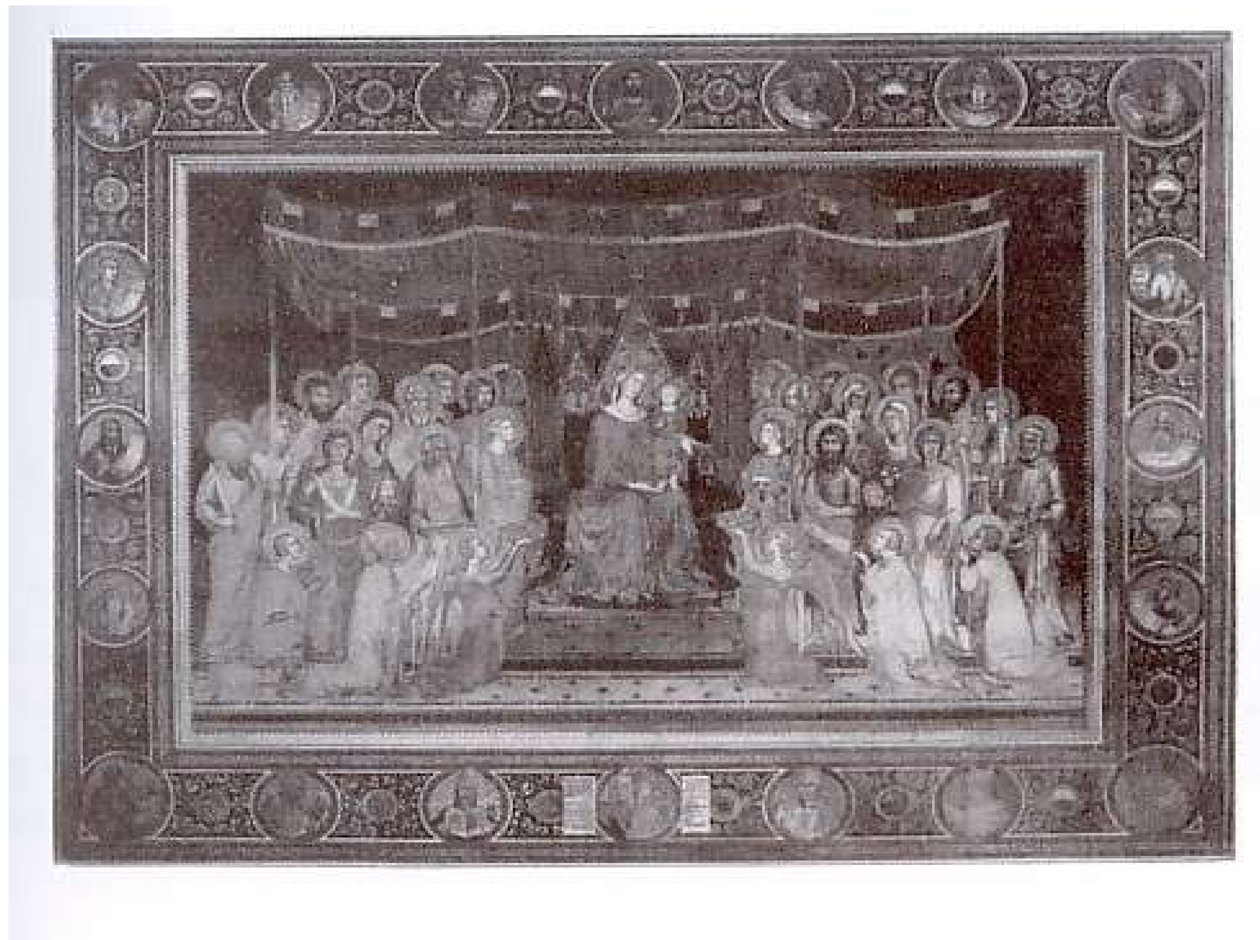
Although isolated results from earlier periods are now considered as belonging to the subject of projective geometry, the fundamental ideas that form the core of this area stem from the work of artists during the Renaissance.

Earlier art appears to us as being very stylized and flat.



The Renaissance Artists

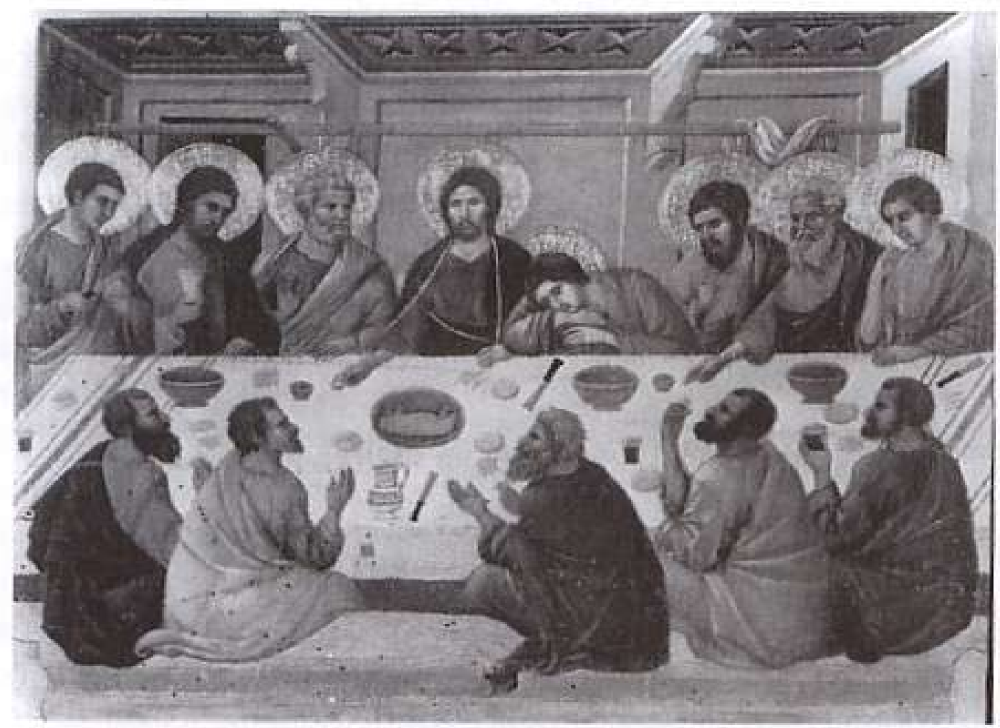
Towards the end of the 13th century, early Renaissance artists began to attempt to portray situations in a more realistic way. One early technique is known as terraced perspective, where people in a group scene that are further in the back are drawn higher up than those in the front.



Simone Martini: *Majesty*

The Renaissance Artists

As artists attempted to find better techniques to improve the realism of their work, the idea of ***vertical perspective*** was developed by the Italian school of artists (for example Duccio (1255-1318) and Giotto (1266-1337)). To create the sense of depth, parallel lines in the scene are represented by lines that meet in the centerline of the picture.



Duccio's *Last Supper*

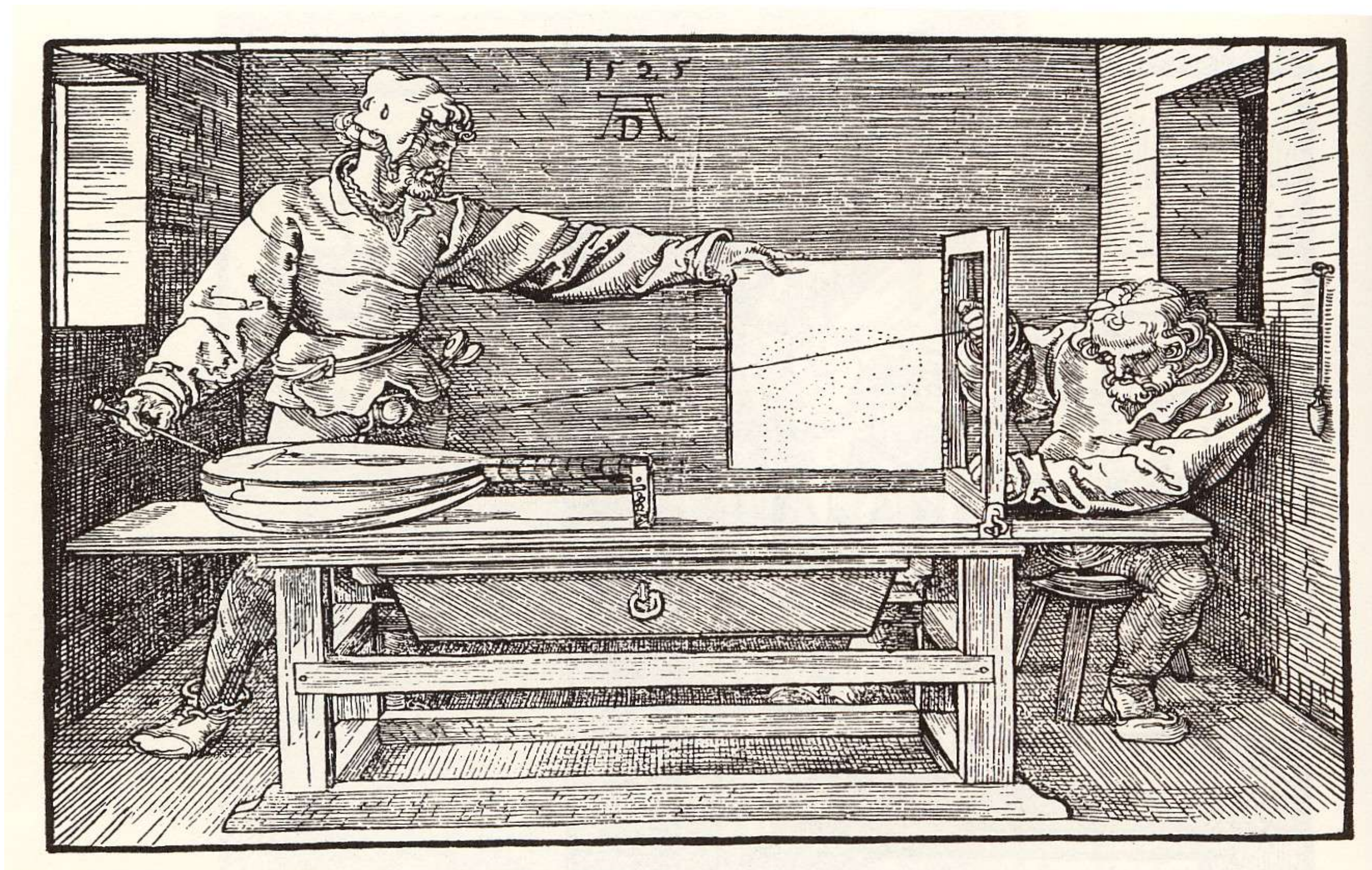
The Renaissance Artists

The modern system of ***focused perspective*** was discovered around 1425 by the sculptor and architect Brunelleschi (1377-1446), and formulated in a treatise a few years later by the painter and architect Leone Battista Alberti (1404-1472). The method was perfected by Leonardo da Vinci (1452 – 1519).

The German artist Albrecht Dürer (1471 – 1528) introduced the term ***perspective*** (from the Latin verb meaning "to see through") to describe this technique and illustrated it by a series of well-known woodcuts in his book *Underweysung der Messung mit dem Zyrkel und Rychtsscheyed* [Instruction on measuring with compass and straight edge] in 1525.

The Renaissance Artists

The Renaissance Artists



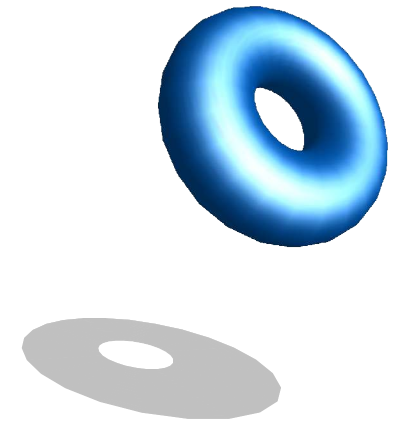
The Renaissance Artists

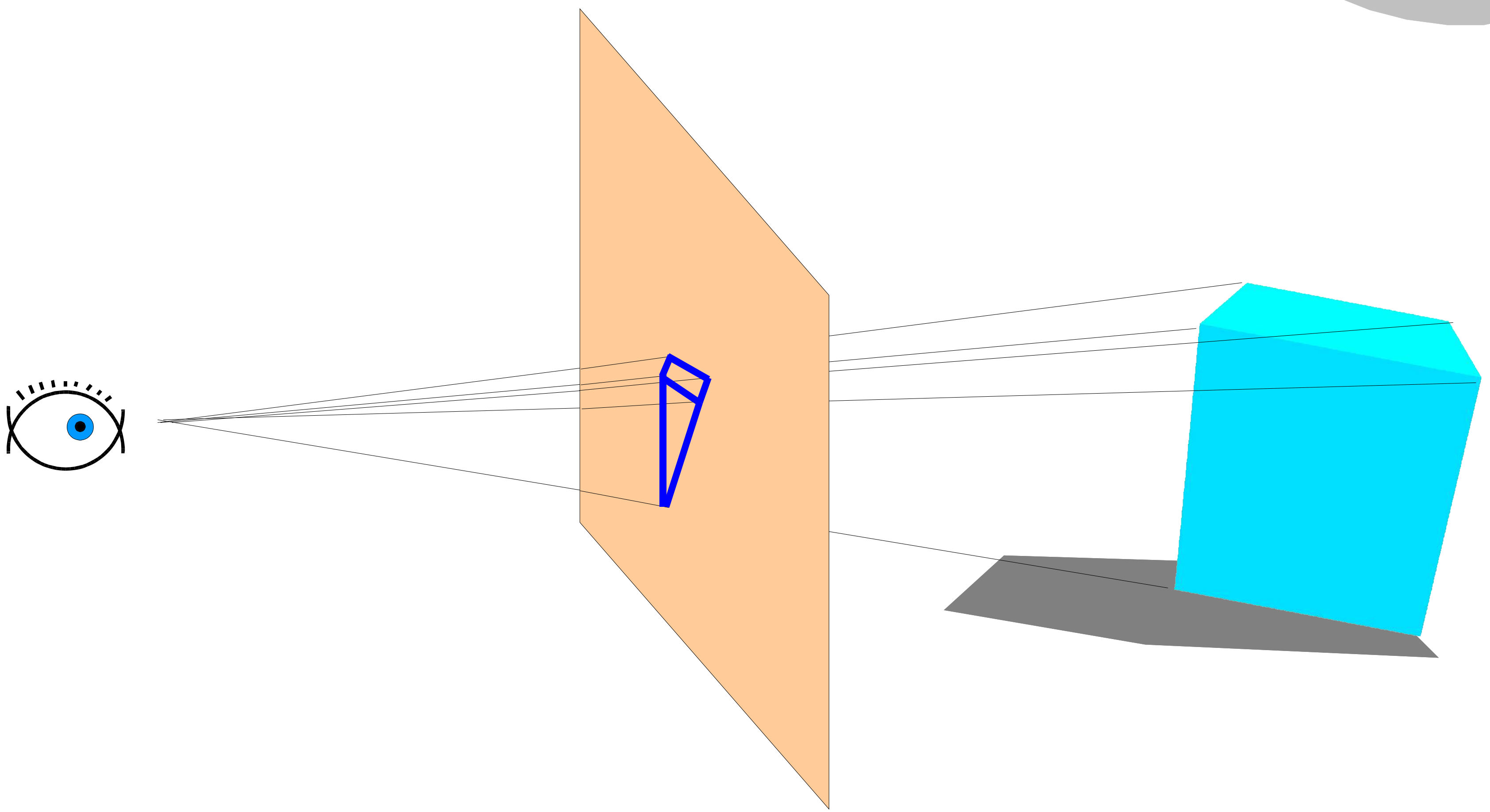
Dürer's
St. Jerome in his cell

The Renaissance Artists

16. Hogarth's comment on the importance of correct perspective, *False Perspectives*.

Perspective





Desargues

Little is known about **Girard Desargues**' (1593-1662) personal life. His family had been very rich for several generations and had supplied lawyers and judges to the Parlement in Paris as well as to that in Lyon.

Desargues seems to have made several extended visits to Paris in connection with a lawsuit for the recovery of a huge debt. Despite this loss, the family still owned several large houses in Lyon, a manor house (and its estate) at the nearby village of Vourles, and a small chateau surrounded by the best vineyards in the vicinity. Desargues had every opportunity of acquiring a good education and had leisure to indulge in whatever pursuits he might enjoy. In his later years, these seem to have included designing an elaborate spiral staircase, and an ingenious new form of pump, but the most important of Desargues' interests was Geometry. He invented a new, non-Greek way of doing geometry, now called 'projective' or 'modern' geometry. As a mathematician he was very good indeed: highly original and completely rigorous. He is, however, far from lucid in his mathematical style.

Desargues

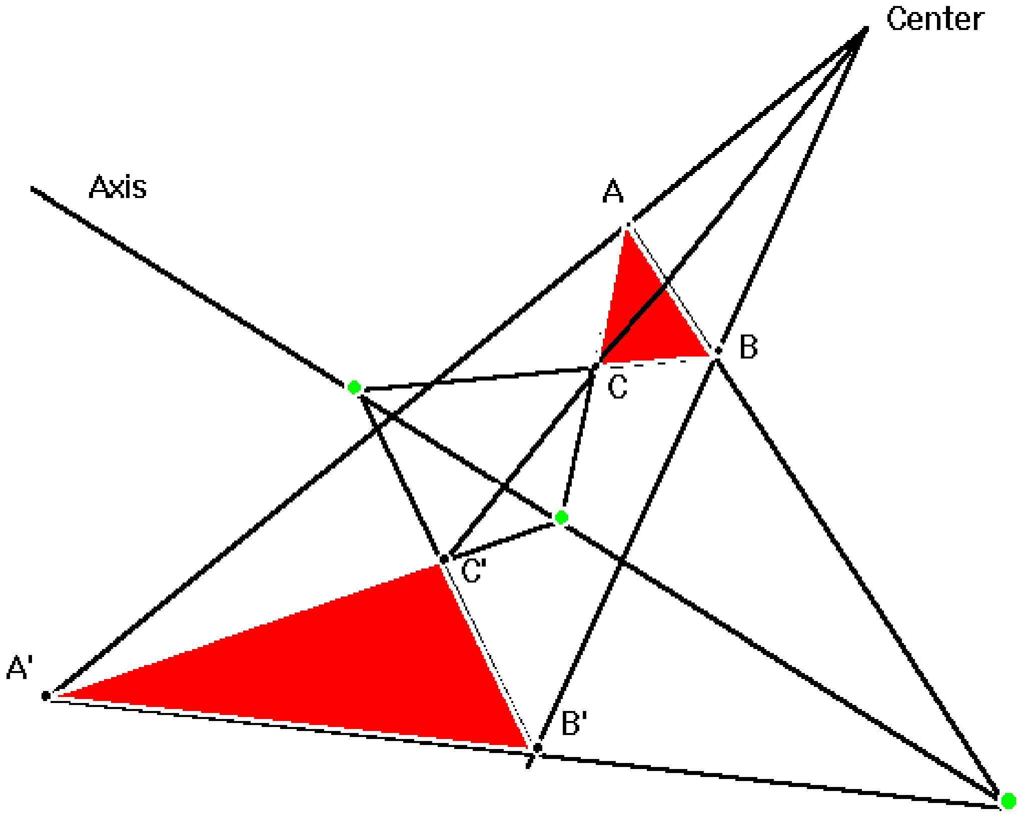
When in Paris, Desargues became part of the mathematical circle surrounding Marin Mersenne (1588 - 1648). This circle included Rene Descartes (1597 -1650), Étienne Pascal (1588 -1651) and his son Blaise Pascal (1623 - 1662). It was probably essentially for this limited readership of friends that Desargues prepared his mathematical works, and had them printed. Some of them were later expanded into more publishable form by Abraham Bosse (1602 -1676), who is now best remembered as an engraver, but was also a teacher of perspective.

Desargues wrote on 'practical' subjects such as perspective (1636), the cutting of stones for use in building (1640) and sundials (1640). His writings are, however, dense in content and theoretical in their approach to the subjects concerned. There is none of the wordy and elementary step-by-step explanation which one finds in texts that are truly addressed to artisans.

Desargues

Desargues' most important work, the one in which he invented his new form of geometry, has the title *Rough draft for an essay on the results of taking plane sections of a cone* (Brouillon project d'une atteinte aux evenemens des rencontres du Cone avec un Plan). A small number of copies were printed in Paris in 1639. Only one is now known to survive, and until this was rediscovered, in 1951, Desargues' work was known only through a manuscript copy made by Philippe de la Hire (1640 - 1718). The book is short, but very dense. It begins with pencils of lines and ranges of points on a line, considers involutions of six points (Desargues does not use or define a cross ratio), gives a rigorous treatment of cases involving 'infinite' distances, and then moves on to conics, showing that they can be discussed in terms of properties that are invariant under projection. We are given a unified theory of conics.

Desargues' Theorem



Desargues' famous 'perspective theorem' - that when two triangles are in perspective the meets of corresponding sides are collinear - was first published in 1648, as an appendix in a work on perspective by his friend Abraham Bosse.

Desargues

It is clear that, despite his determination to explain matters in the vernacular, and without direct reference to the theorems or the vocabulary of Ancient mathematicians, Desargues is well aware of the work of ancient geometers, for instance Apollonius and Pappus. His choosing to explain himself differently may perhaps be due to his recognition that his own work was also deeply indebted to the practical tradition, specifically to the study of perspective. It seems highly likely that it was in fact from his work on perspective and related matters that Desargues' new ideas arose. When projective geometry was reinvented, by the pupils of Gaspard Monge (1746 -1818), the reinvention was from descriptive geometry, a technique that has much in common with perspective.

Desargues

Desargues' work was not well received, partly because he invented and used so many new technical terms that few could follow it, and partly because mathematicians were just beginning to appreciate Descartes' analytic unification of geometry and were not ready to consider a new synthetic version. At any rate the work was soon forgotten and remained almost unknown until Chasles happened to find a copy in 1845, since which time it has been looked upon as one of the classics in the early development of modern pure geometry. He introduced the notions of the point at infinity, the line at infinity, the straight line as a circle of infinite radius, geometric involution, the tangent as a limiting case of a secant, the asymptote as a tangent at infinity, poles and polars, homology and perspective, thus laying a substantial basis for the modern theory of projective geometry.

Projective Geometry

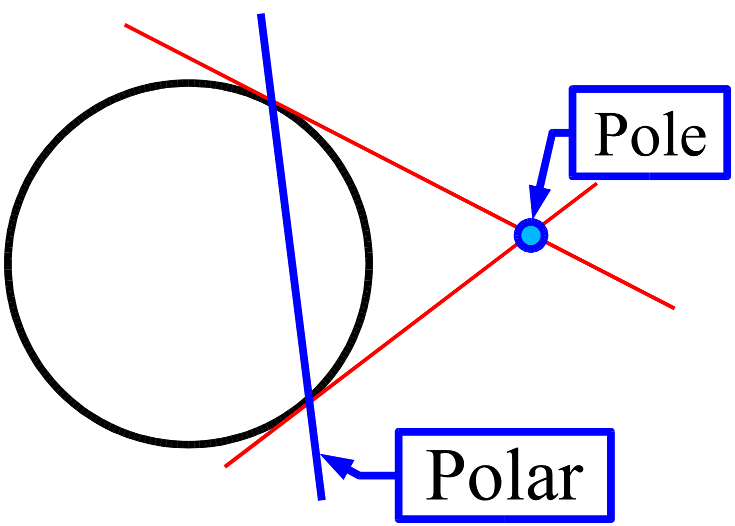
In projective geometry there are no parallel lines. Any two lines in a common plane **must** intersect!

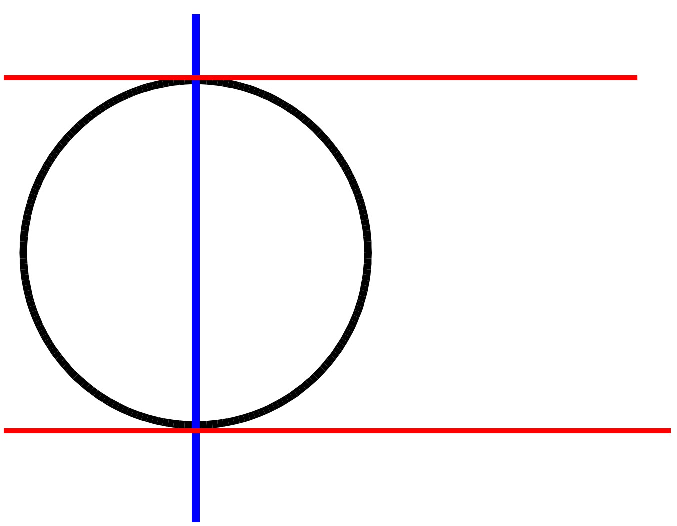
The usual Euclidean plane is contained in what we call the ***real projective plane***. To construct the real projective plane we need to introduce several new points and one new line which contains them all to the Euclidean plane. As these points can not be seen in the Euclidean plane, they are called ***points at infinity*** and the line they are on is the ***line at infinity***. The construction is simply to add one point at infinity to every line of a parallel class of lines (thus there will be one point at infinity for each parallel class). The formerly parallel lines will now meet – at the point of infinity of their parallel class.

Projective Geometry

While it appears that the line at infinity and its points are "special", this is really not the case – all lines and points are created equal!

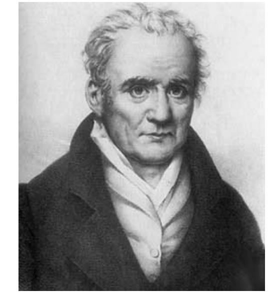
Geometrical statements are simpler in projective geometry than in Euclidean geometry since many special cases due to parallel lines will not arise in projective geometry.





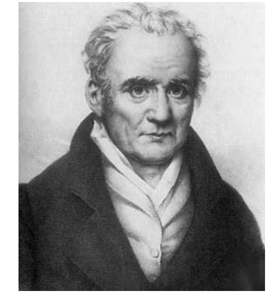
Pole ?

Monge



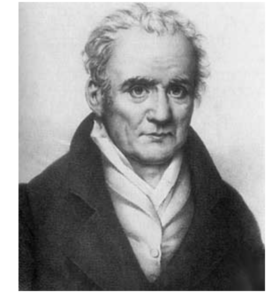
Gaspard Monge (1746 -1818) attended the Oratorian College in Beaune. This school was intended for young nobles and was run by priests. The school offered a more liberal education than other religious schools, providing instruction not only in the humanities but also in history, mathematics, and the natural sciences. It was at this school that Monge first showed his brilliance. In 1762, at the age of 16, Monge went to Lyons where he continued his education at the Collège de la Trinité. Despite being only 17 years of age at the time, Monge was put in charge of teaching a course in physics. Completing his education there in 1764, Monge returned to Beaune where he drew up a plan of the city.

Monge



The plan of Beaune that Monge constructed was to have a major influence in the direction that his career took, for the plan was seen by a member of staff at the École Royale du Génie at Mézières. He was very impressed by Monge's work and, in 1765, Monge was appointed to the École Royale du Génie as a draftsman. Of course, in this post Monge was undertaking tasks that were not entirely to his liking, for he aspired to a position in life which made far more use of his mathematical talents. However the École Royale du Génie brought Monge into contact with Charles Bossut who was the professor of mathematics there. At first Monge's post did not require him to use his mathematical talents, but Monge worked in his own time developing his own ideas of geometry.

Monge



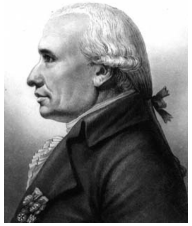
About a year after becoming a draftsman, Monge was given a task which allowed him to use his mathematical skill to attack the task he was given. Asked to draw up a fortification plan which prevented an enemy from either seeing or firing at a military position no matter what the position of the enemy, Monge devised his own graphical method to construct such a fortification rather than use the complicated methods then available. This method made full use of the geometrical techniques which Monge was developing in his own time. His mathematical abilities were now recognised at the École Royale du Génie and it was realised that Monge was someone with exceptional abilities in both theoretical and practical subjects.

Monge

When Bossut left the École Royale du Génie at Mézières, Monge was appointed to succeed him in January 1769. In 1770 he received an additional post at the École Royale du Génie when he was appointed as instructor in experimental physics. Although this was a large step forward for Monge's career, he was more interested in making his name as a mathematician in the highest circles. Condorcet recommended that he present memoirs to the Académie des Sciences in each of the four areas of mathematics in which he was undertaking research.

The four memoirs that Monge submitted to the Académie were on a generalisation of the calculus of variations, infinitesimal geometry, the theory of PDEs, and combinatorics. Over the next few years he submitted a series of important papers to the Académie on PDEs which he studied from a geometrical point of view. His interest in subjects other than mathematics began to grow and he became interested in problems in both physics and chemistry.

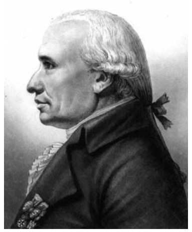
Monge



In 1777 Monge married Cathérine Huart and, since his wife had a forge, he became interested in metallurgy. From 1780, however, he devoted less time to his work at the École at Mézières since in that year he was elected as adjoint géomètre at the Académie des Sciences in Paris. From that time on he spent long periods in Paris, teaching a course in hydrodynamics as a substitute for Bossut as well as participating in projects undertaken by the Académie in mathematics, physics and chemistry. It was not possible to do all this and to teach all his courses at Mézières but he kept his posts there and received his full salary out of which he paid others to teach some courses in his place.

After three years of dividing his time between Paris and Mézières, Monge was offered yet another post, namely to replace Bézout as examiner of naval cadets. Monge would have liked to keep all these positions, but after attempting to organise an impossible schedule for about a year, he decided that he would have to resign his posts in Mézières, which he did in December 1784.

Monge



1789 was an eventful year in French history with the storming of the Bastille on 14 July 1789 marking the start of the French Revolution. This was to completely change the course of Monge's life. At the onset of the Revolution he was one of the leading scientists in Paris with an outstanding research record in a wide variety of sciences, experience as an examiner and experience in school reforms which he had undertaken in 1786 as part of his duties as an examiner. Politically Monge was a strong supporter of the Revolution, and his first actions were to show his support by joining various societies supporting the Revolution, but he continued his normal duties as an examiner of naval cadets, and as a major figure in the work of the Académie. By this time he was on the major Académie Commission on Weights and Measures.

Monge

Through his political connections, Monge was appointed Minister of the Navy. Monge's period as Minister of the Navy cannot be viewed as a success. Although he tried hard in difficult circumstances, he survived only eight months in the post before he gave up the incessant battle with those around him, and he submitted his resignation on 10 April 1793. For a few months Monge returned to his work with the Académie des Sciences but this did not last long for, on 8 August 1793, the Académie des Sciences was abolished by the National Convention.

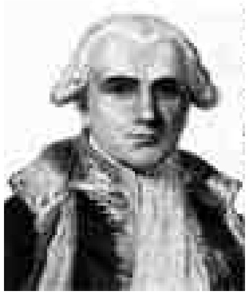
Still a strong republican and supporter of the Revolution, Monge worked on various military projects relating to arms and explosives. He wrote papers and also gave courses on these military topics. He continued to serve on the Commission on Weights and Measures which survived despite the ending of the Académie des Sciences. He also proposed educational reforms to the National Convention but, despite being accepted on 15 September 1793, it was rejected on the following day. Such was the volatile nature of decisions at this unstable time.

Monge

Monge was appointed by the National Convention on 11 March 1794 to the body that was put in place to establish the École Centrale des Travaux Publics (soon to become the École Polytechnique). Not only was he a major influence in setting up the École using his experience at Mézières to good effect, but he was appointed as an instructor in descriptive geometry on 9 November 1794. His first task as instructor was to train future teachers of the school which began to operate from June 1795. Monge's lectures on infinitesimal geometry were to form the basis of his book *Application de l'analyse à la géométrie*.

Another educational establishment, the École Normale, was set up to train secondary school teachers and Monge gave a course on descriptive geometry. He was also a strong believer in the Académie des Sciences and worked hard to see it reinstated as the Institut National. The National Convention approved the new body on 26 October 1795.

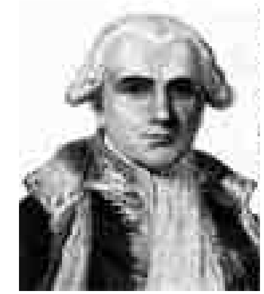
Monge



However from May 1796 to October 1797, Monge was in Italy on a commission to select the best art treasures for the conquerors and bring them to France. Of particular significance was the fact that he became friendly with Napoleon Bonaparte during his time in Italy. Napoleon had defeated Austria and signed the Treaty of Campo Formio on 17 October 1797. Monge returned to Paris bringing the text of the Treaty of Campo Formio with him.

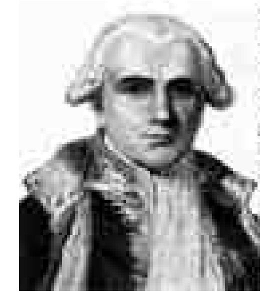
Back in Paris Monge slotted back into his previous roles and was appointed to the prestigious new one of Director of the École Polytechnique. By February 1798 Monge was back in Rome, involved with the setting up of the Republic of Rome. In particular Monge proposed a project for advanced schools in the Republic of Rome. Napoleon Bonaparte now asked Monge to join him on his Egyptian expedition and, somewhat reluctantly, Monge agreed.

Monge



Monge left Italy on 26 May 1798 and joined Napoleon's expeditionary force. The expedition, which included the mathematicians Fourier and Malus as well as Monge, was at first a great success. Malta was occupied on 10 June 1798, Alexandria taken by storm on 1 July, and the delta of the Nile quickly taken. However, on 1 August 1798 the French fleet was completely destroyed by Nelson's fleet in the Battle of the Nile, so that Napoleon found himself confined to the land that he was occupying. Monge was appointed president of the Institut d'Egypte in Cairo on 21 August. The Institut had twelve members of the mathematics division, including Fourier, Monge, Malus and Napoleon Bonaparte. During difficult times with Napoleon in Egypt and Syria, Monge continued to work on perfecting his treatise *Application de l'analyse à la géométrie.*

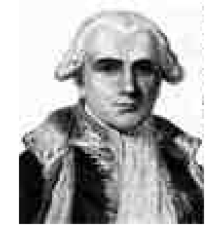
Monge



Napoleon abandoned his army and returned to Paris in 1799, he soon held absolute power in France. Monge was back in Paris on 16 October 1799 and took up his role as director of the École Polytechnique. He discovered that his memoir *Géométrie descriptive* had been published earlier in 1799. This had been done at his wife's request from Monge's lectures at the École Normale. On 9 November 1799 Napoleon and two others seized power in a coup and a new government, the Consulate, was set up. Napoleon named Monge a senator on the Consulate for life. Monge accepted with pleasure, although his republican views should have meant that he was opposed to the military dictatorship imposed by Napoleon on France. Monge was

... dazzled by Napoleon ... and accepted all the honours and gifts the emperor bestowed upon him: grand officer of the Legion of Honour in 1804, *president of the Senate in* 1806, *Count of Péluse in* 1808, *among others.*

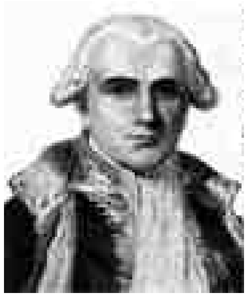
Monge



Over the next few years Monge continued a whole range of activities, undertaking his role as a senator while maintaining an interest in research in mathematics but mostly his mathematical work involved teaching and writing texts for the students at the École Polytechnique. Slowly he became less involved in mathematical research, then from 1809 he gave up his teaching at the École Polytechnique as his health began to fail.

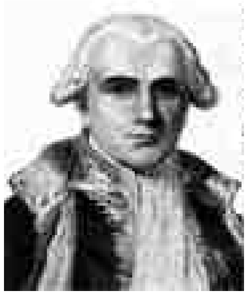
In June 1812 Napoleon assembled his Grande Armée of about 453,000 men, including men from Prussia and from Austria who were forced to serve, and marched on Russia. The campaign was a disaster but by September Napoleon's army had entered a deserted Moscow. Napoleon withdrew, the Prussians and Austrians deserted the Grande Armée and there were attempts at a coup against Napoleon in Paris. Monge was dismayed at the situation and his health suddenly collapsed. Slowly his health returned after Napoleon left the remains of his army and returned to Paris to assert his authority. After Napoleon had some military success in 1813, the allied armies against him strengthened. Monge was sent to Liège to organize the defense of the town against an attack.

Monge



The allied armies began to move against France and Monge fled. When Napoleon abdicated on 6 April 1814, Monge was not in Paris, but soon after he did return and tried to pick up his life again. Napoleon escaped from Elba, where he had been banished, and by 20 March 1815 he was back in Paris. Monge immediately rallied to Napoleon and gave him his full support. After Napoleon was defeated at Waterloo, Monge continued to see him until he was put on board a ship on 15 July. By October Monge feared for his life and fled from France.

Monge



Monge returned to Paris in March 1816. Two days after his return he was expelled from the Institut de France and from then on his life was desperately difficult as he was harassed politically and his life was continually threatened. On his death the students of the École Polytechnique paid tribute to him despite the insistence of the French Government that no tributes should be paid.

Of his political career, it has been said :-

[*Monge's*] *tenure at the Ministry of the Navy was a complete failure and he presided over the cultural pillage of Italy and Egypt. If Napoleon actually said that Monge loved him like a mistress, it proves that the utmost mathematical clarity can go hand in hand with political blindness.*

Monge

The basic philosophy behind Monge's approach to mathematics is the

... geometrisation of mathematics based on:

(a) *the analogy or correspondence of operations in analysis with geometric transformations;*

(b) *the genetic classification and parametrisation of surfaces through analysis of the movement of generating lines.*

Monge regarded analysis as being :-

... not a self-contained language but merely the 'script' of the 'moving geometrical spectacle' that constitutes reality.

His:-

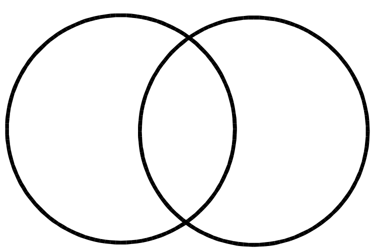
... new approach addressed itself to the most profound, intimate and universal relations in space and their transformations, putting him in a position to interconnect geometry and analysis in a fertile, previously unheard-of fashion. Practical concerns induced Monge to perceive the object and function of mathematics in a new way, in violation of the formalistic (linguistic) standards set by the approved patrons of mathematics ...

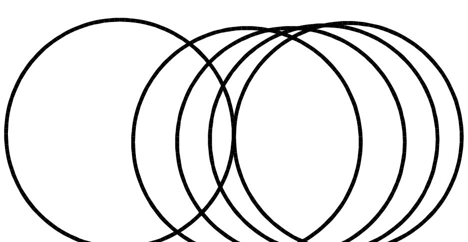
Principle of Continuity

Underlying Monge's technique of descriptive geometry was the ***principle of continuity*** (explicitly stated by Poncelet in 1822):

... theorems proved for one figure are equally true for corresponding figures formed from the original by a continuous transformation.

For example, consider two circles:

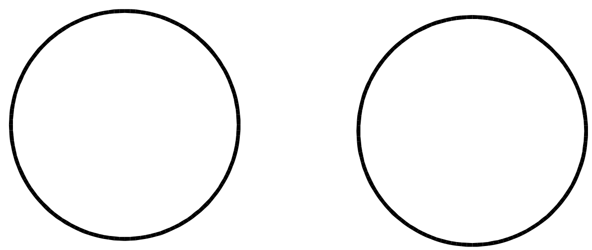




Euclid deals with these two situations in seperate propositions (III:10 and III:13), but by the principle of continuity, since one can be obtained from the other by continuously moving one of the circles, these are really the same proposition: the circles meet at two points, and in the second case the two points are identical.

Principle of Continuity

Now, continue to move the second circle to get:



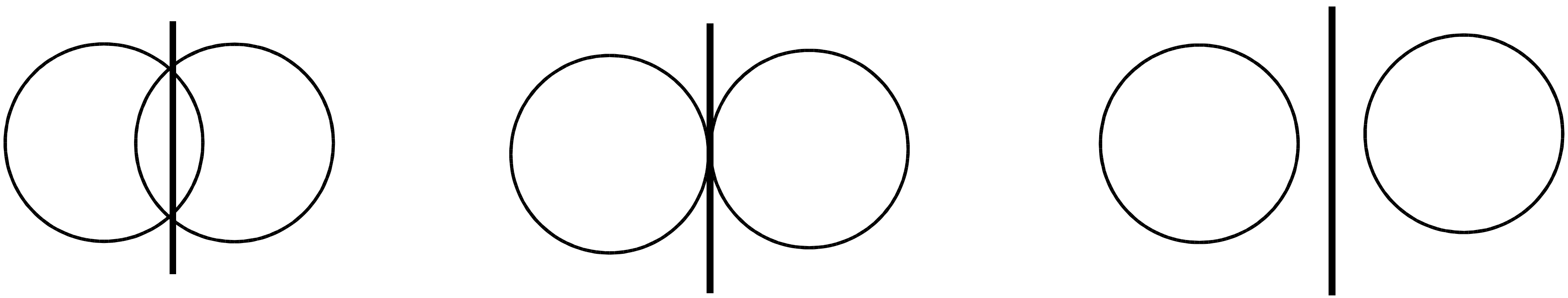
The principle of continuity says that these circles also meet in two points!

That this must be so can be "seen" analytically. The general intersection of the two circles is given by a quadratic equation. The first instance corresponds to two real solutions, the second to two identical solutions, and this last case to two imaginary solutions.

Monge would have argued that these circles met in two imaginary points which were *real* points, just not in the Euclidean plane. He would treat these as other points and determine their properties.

Principle of Continuity

For instance, the ***radical axis*** of the pair of circles is defined as the line determined by these points of intersection. The tangents drawn from any point on the radical axis to the two circles have the same length.



In this last case, the two imaginary points of the circle must lie on the radical axis but not in the Euclidean plane (the line can be drawn because it is simply the perpendicular bisector of the line segment joining the centers of the two circles.)

Poncelet

Jean-Victor Poncelet (1788-1867) was a pupil of Monge. His development of the pole and polar lines associated with conics led to the principle of duality.

Poncelet took part in Napoleon's 1812 Russian campaign as an engineer. He was left for dead at Krasnoy and imprisoned until 1814 when he returned to France. During his imprisonment he worked on projective geometry. He also wrote a treatise on analytic geometry *Applications d'analyse et de géométrie* based on what he had learnt at the École Polytechnique but it was only published 50 years later.

From 1815 to 1825 he was a military engineer at Metz and from 1825 to 1835 professor of mechanics there. He applied mechanics to improve turbines and waterwheels more than doubling the efficiency of the waterwheel.

Poncelet

Poncelet was one of the founders of modern projective geometry simultaneously discovered by Joseph Gergonne and Poncelet. His development of the pole and polar lines associated with conics led to the principle of duality. He also discovered circular points at infinity.

He published *Traité des propriétés projectives des figures* in 1822 which is a study of those properties which remain invariant under projection. This work contains fundamental ideas of projective geometry such as the cross-ratio, perspective, involution and the circular points at infinity.

Poncelet published *Applications d'analyse et de géométrie* in two volumes: 1862 and 1864.

Principle of Duality

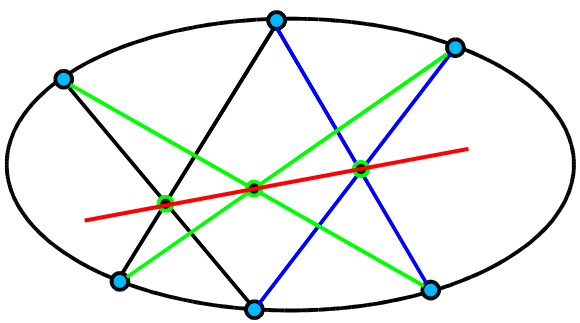
One of the advantages of working in projective geometry is the validity of the Principle of Duality. This principle, stated for projective planes, says that any true statement about the relationship between points and lines in a projective plane is transformed into a true statement when the words "point" and "line" are interchanged.

This "meta" theorem was proved by Poncelet from a consideration of the pole/polar relationship of conics. Today it is more commonly derived from an examination of the axioms of the system.

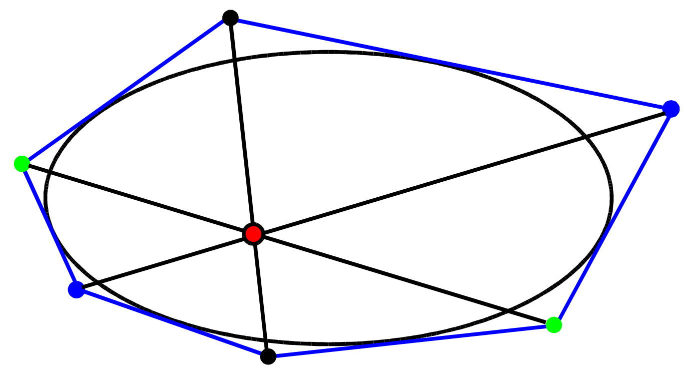
The theorems of projective geometry thus come in (dual) pairs, and only one of each pair requires a proof.

Principle of Duality

As an example of this principle we have the dual pair of Pascal's Theorem (proved in 1639) and Brianchon's Theorem (proved in the 19^{th} century).



Pascal's Theorem



Brianchon's Theorem

Steiner

Jakob Steiner (1796-1863) did not learn to read and write until he was 14 and only went to school at the age of 18, against the wishes of his parents. He then studied at the Universities of Heidelberg and Berlin, supporting himself with a very modest income from tutoring.

He was an early contributor to Crelle's Journal, the first journal devoted entirely to mathematics founded in 1826. He was appointed to a chair at the University of Berlin in 1834, a post he held until his death.

He was one of the greatest contributors to projective geometry. He discovered the Steiner surface which has a double infinity of conic sections on it. The Steiner theorem states that the two pencils by which a conic is projected from two of its points are projectively related.

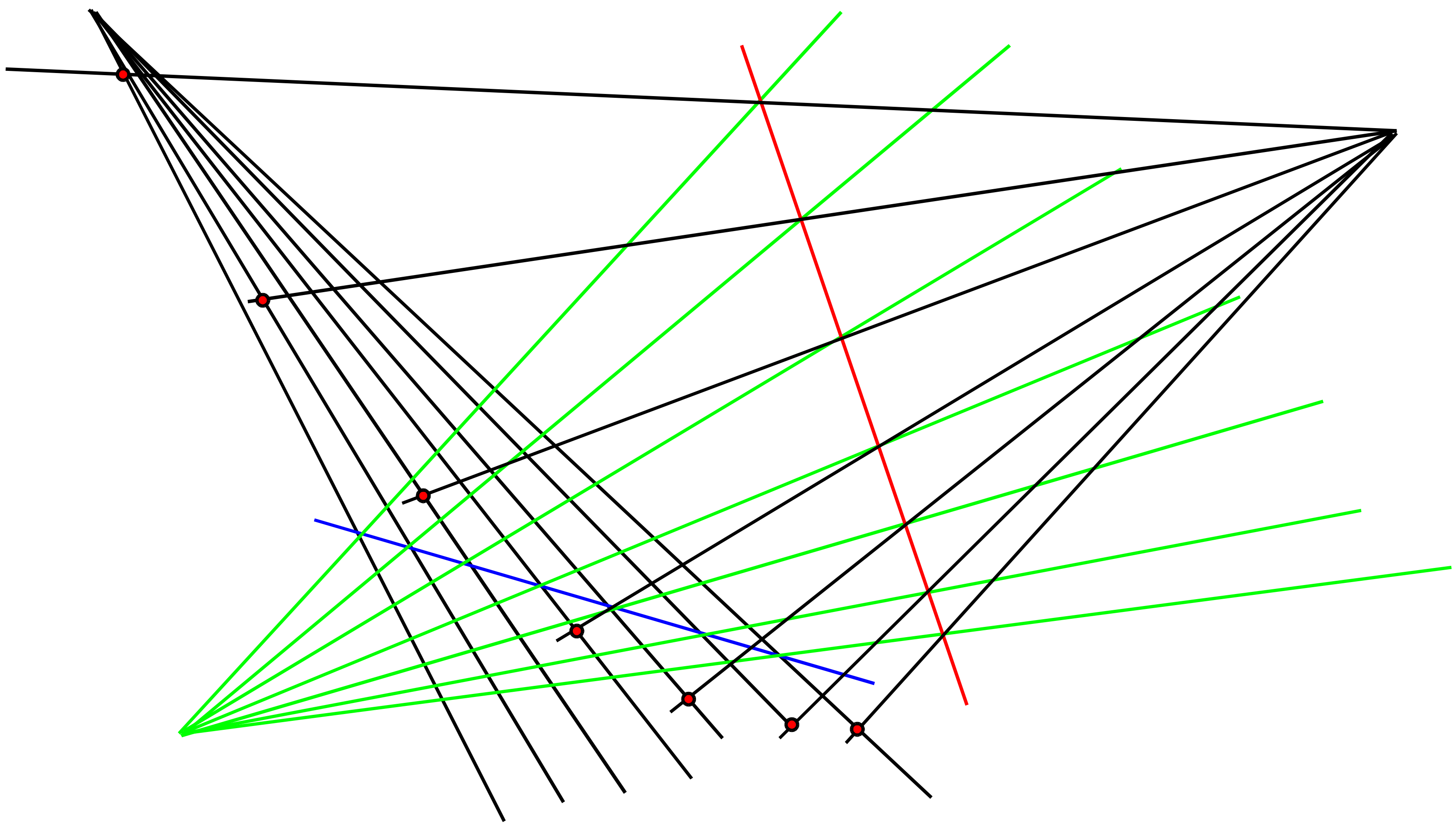
Steiner

Another famous result is the Poncelet-Steiner theorem which shows that only one given circle and a straight edge are required for Euclidean constructions.

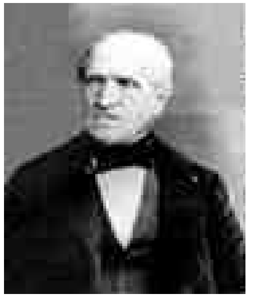
He disliked algebra and analysis and believed that calculation replaces thinking while geometry stimulates thinking. He was described as follows:-

He is a middle-aged man, of pretty stout proportions, has a long intellectual face, with beard and moustache and a fine prominant forehead, hair dark rather inclining to turn grey. The first thing that strikes you on his face is a dash of care and anxiety, almost pain, as if arising from physical suffering - he has rheumatism. He never prepares his lectures beforehand. He thus often stumbles or fails to prove what he wishes at the moment, and at every such failure he is sure to make some characteristic remark.

Steiner's Conic



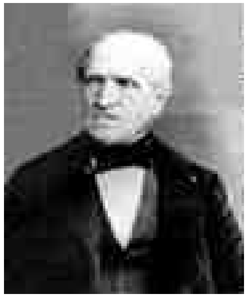
Chasles



Michel Chasles' (1793-1880) father, Charles-Henri Chasles, was a wood merchant who became president of the chamber of commerce in Chartres. Chasles was born into a fairly well off Catholic family. A rather strange fact is that Chasles was christened Floréal Chasles by his parents. A court order was obtained to change his name from Floréal to Michel a few days after his sixteenth birthday.

Chasles attended the Lycée Impérial for his secondary education. Then, in 1812, he entered the École Polytechnique in Paris. This was the period when Napoleon was desperately trying to call up conscripts for his armies as he attempted to replenish the troops lost in the fighting. In January 1813, after the disaster of the Russian campaign, Napoleon called up more men to fill the dwindling numbers in his armies. Chasles was called up to take part in the defence of Paris in early 1814. Shortly after Paris fell, Napoleon abdicated on 6 April 1814 and the war was over. Chasles was able to return to his studies at the École Polytechnique.

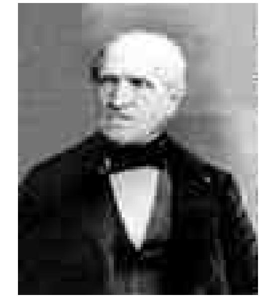
Chasles



Having obtained a place in the engineering corps, Chasles decided not to accept it but to give his place to one of his fellow students who was in financial difficulties. At this point Chasles returned to living at home but his father insisted that he join a firm of stockbrokers in Paris. This was not the occupation for Chasles but he obeyed his father's wishes and went to join the firm in Paris to learn the trade of a stockbroker. However, Chasles was interested in history and in mathematics and he was not successful as a trainee in the firm. He returned again to his home where he could pursue his historical and mathematical interests.

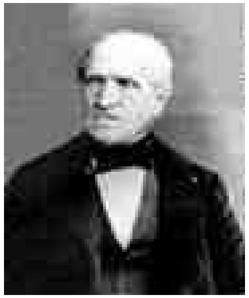
In 1837 Chasles published his first major work *Aperçu historique sur l'origine et le développement des méthodes en géométrie* (Historical view of the origin and development of methods in geometry) which quickly made his reputation as both a mathematician and as an historian of mathematics and is still an important historical reference.

Chasles



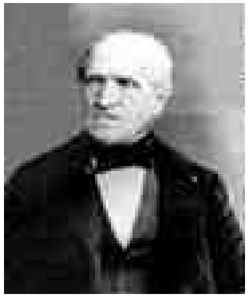
In *Aperçu historique* Chasles studied the method of reciprocal polars as an application of the principle of duality in projective geometry; in the same way the principle of homography leads to a great number of properties of quadric surfaces. The Académie des Sciences wanted to publish the work which Chasles submitted to them but he asked to be able to add to the historical introduction as well as to add further historical notes to the text and include some new material and notes. This work in many ways is the crucial one for Chasles's future research since almost all of the many works he produced throughout the rest of his career elaborate on points discussed in these notes he added to the *Aperçu historique*. The extended version is the one which the Academy published in 1830. Koppelman notes that the work had one weakness. This was that Chasles could not read German so he was not so familiar with the recent results published in that language.

Chasles



On the strength of his fine work Chasles became professor at the École Polytechnique in Paris in 1841, at the age of nearly 48. Topics he taught were geodesy, mechanics, and astronomy. In 1846 he was appointed to a chair of higher geometry at the Sorbonne which had been specially created for him. He continued to teach at the École Polytechnique after this appointment at the Sorbonne but he resigned his post at the École Polytechnique in 1851, retaining his chair at the Sorbonne until his death.

Chasles



He also wrote an extremely important text on geometry showing the power of synthetic geometry. In his text *Traité de géométrie* in 1852 Chasles discusses cross ratio, pencils and involutions, all notions which he introduced, although Möbius independently introduced the cross ratio. A second text, *Traité des sections coniques* (1865), applied these techniques to conic sections. The principle of duality occurs throughout his work which was carried further by Steiner.

One of the results for which Chasles is well known is his enumeration of conics. Questions of this type go back to Apollonius, but such questions had arisen while Chasles was working on geometry, in particular the Steiner "problem of five conics" was posed in 1848. This problem, namely to determine the number of conics tangent to five given conics, was solved incorrectly by Steiner who gave the answer 7776. Chasles solved this problem correctly in 1864 when he gave the answer of 3264. Chasles' developed a theory of characteristics to solve this problem.

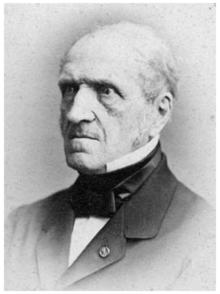
Chasles

Koppelman writes :-

Chasles published highly original work until his very last years. He never married, and his few interests outside his research, teaching, and the Academy, which he served on many commissions, seem to have been in charitable organisations.

Chasles received many honours for this highly original work. He was elected a corresponding member of the Académie des Sciences in 1839 and a full member in 1851. He was elected a Fellow of the Royal Sciety of London in 1854 and won its Copley Medal in 1865. The London Mathematical Society was founded in 1865 and it elected Chasles in 1867 as its first foreign member. He was also a member of academies in Brussels, Copenhagen, Naples, Stockholm, St Petersburg, and the United States

Chasles



There is one aspect of Chasles's life which seems so out of character with the brilliant man that he was that it caused him great distress. He was the victim of a celebrated fraud paying the equivalent of 20,000 pounds for various letters from famous men of science and others which turned out to be forged. Chasles collected autographs and manuscripts but appears to have displayed a naiveté which is almost unbelievable. Chasles bought thousands of manuscripts from Denis Vrain-Lucas between 1861 and 1869. Vrain-Lucas sold Chasles documents which purported to be part of correspondence between Newton, Pascal, and Boyle. Chasles presented the letters to the Académie des Sciences in 1867 for they "proved" that Pascal was the first to propose the universal law of gravitation, and not Newton.

Chasles

As one might expect this created a great controversy. Chasles argued strongly that the letters were genuine. However Vrain-Lucas was tried in 1869-70 for forging the documents and Chasles had to appear at the trial. It was an extremely uncomfortable experience for Chasles since he had to admit in court that he had purchased documents supposedly written by Galileo, Cleopatra and Lazarus and how someone of Chasles's intelligence and a deep interest in history would have believed that these all these wrote in French is beyond belief! Vrain-Lucas was found guilty and Chasles, although 77 by this time, must have looked extremely silly.

Cross Ratio

Since distance is not preserved under projections it was difficult to talk about metric concepts in projective geometry. What was needed was a numerical invariant of projection which could then be used as a basis for metrical ideas. Chasles provided this with the idea of a cross ratio, a numerical value associated to four points on a line.

The cross ratio of the four points

is:

$$(AB, CD) = \frac{(AC)(BD)}{(AD)(BC)}.$$

Von Staudt

Karl von Staudt (1798 - 1867) attended the University of Göttingen from 1818 to 1822. His early work was on determining the orbit of a comet and, based on this work, he received a doctorate from Erlangen in 1822 . Von Staudt was appointed professor of mathematics at the Polytechnic School at Nürnberg in 1827 and he was appointed to the University of Erlangen in 1835.

Von Staudt showed how to construct a regular inscribed polygon of 17 sides using only compasses. He turned to projective geometry and Bernoulli numbers. An important work on projective geometry, *Geometrie der Lage* was published in 1847. It was the first work to completely free projective geometry from any metrical basis. Another of his publications on projective geometry was *Beiträge zur Geometrie der Lage* (1856-60). He also gave a nice geometric solution to quadratic equations.