The Rise of Projective Geometry

Part III

Introduction

We will look at how the emergence of projective geometry effected the educational system in England in the 19th century. This material is largely based on Joan L. Richards, *Mathematical Visions: The Pursuit of Geometry in Victorian England* (Academic Press, 1988).

One of the ways we can trace this influence is by tracking the changes in the "infamous" Mathematical Tripos exams at Cambridge University. A side benefit of this approach is that we may learn something which is relevant to the educational issues that we face today.

The Mathematical Tripos

The 19th century intellectual emphasis in education was accompanied by an increased focus on written examinations as a way to determine a student's rank.

The unique aspect of the education at Cambridge, which set it off from Oxford, was that the major emphasis of its curriculum was upon mathematics. Until the 1850's, no matter what subject formed a student's primary interest, he had to study mathematics to obtain an honors degree. At Oxford, the primary focus was on the classics. But at Cambridge, even if Greek and Latin were his major interest, a student could not take the classics examination without first passing the Mathematical Tripos.

The Mathematical Tripos

The Tripos, officially known as the Senate House Examination, was named after a three-legged stool upon which a student sat while he was being verbally grilled by professors and others at the university in its early days. In those days a student was evaluated on how well he could “wrangle” out of the theological and ethical conundrums that were put to him by the examiners.

The results of the Tripos, which could be critically important to a student's future, were grouped into three major categories – Wrangler, First Class and Second Class (the last two categories were renamed as Senior Optime and Junior Optime in 1907) – and students were rank listed within each category. In order to achieve this ranking the exam was full of picky details, trick questions and memorized proofs.

The Mathematical Tripos

Furthermore, the examination contained more problems than anyone could possibly finish in the alloted time. This design was intended to give students choice. However, in practice, it meant that for the good students, the speed with which they wrote became a major factor determining how well they placed.

Before 1848, the Tripos was an undifferentiated six-day exam. It was then lengthened to eight days and divided into two parts. The first three days (later, four days) were designed to cover the material essential for anyone to receive an ordinary degree. Only after he had completed the first part of the exam could a student sit for the more advanced, second part to determine whether or not he would receive honors.

The Mathematical Tripos

In 1848, the first part of the Tripos was to test

the portions of Euclid usually read [first four books]; Arithmetic; parts of Algebra, embracing the Binomial Theorem and the Principles of Logarithms; Plane Trigonometry, so far as to include the solution of triangles; Conic Sections, treated geometrically; the elementary parts of Statics and Dynamics, treated without the Differential Calculus; the first three sections of Newton, the Propositions to be proved in Newton's manner; the elementary parts of Hydrostatics, without the Differential Calculus; the simpler propositions of Optics, treated geometrically; the parts of Astronomy required for the explanation of the more simple phenomena, without calculation.

The Mathematical Tripos

This list of subjects clearly shows a strong emphasis on geometry and applied mathematics. The geometric emphasis existed before the 1848 reform.

The requirement of taking the mathematical tripos was not based on a belief of the applicability of mathematics for solving problems, but rather on the belief that studying mathematics was the best way to increase ones ability to reason. Geometry, due to its perceived clarity of its axioms, was considered the ideal mathematical subject for this purpose.

The Mathematical Tripos

There were three major reforms of the Tripos that we wish to consider. These occurred in 1819, 1848 and 1907.

In 1819, Newton's fluxional notation was abandoned on the Senate House Examination. In the ensuing years this innovation took hold, and analytical notations were universally adopted. Abandoning the geometrical, Newtonian notation of the previous century was advocated as a significant step towards bringing English scientists in line with their continental counterparts. It was an attempt to include all of the most up-to-date developments in mathematics, both pure and applied, on the Tripos. This development is mentioned as a central feature of 19th century British mathematics by virtually everyone who has considered it.

The Mathematical Tripos

In the wake of the liberating reforms the purview of the examination expanded unmanageably. Since any aspect of mathematics might be included on the Tripos, it became virtually impossible to prepare for it. By the middle of the century, it became clear that an attempt had to be made to define and delimit the subjects appropriate to the examination. As part of this effort the place of mathematics in the Cambridge curriculum was searchingly reexamined and reevaluated. This led to the 1848 reform which divided the exam into two parts and provided the listing of topics that we have seen.

The Textbook Wars

In the 1860's a new educational battle broke out in England. This involved the teaching of elementary geometry out of the time honored Euclid. J. J. Sylvester, in his inaugural presidential address to the British Association for the Advancement of Science of 1869, said

... there are some who rank Euclid as second in sacredness to the Bible alone, and as one of the advanced outposts of the British Constitution.

His request that mathematics be taught with "life and animation" was directly linked to abandoning "our traditional and mediaeval modes of teaching."

I should rejoice to see ... Euclid honourably shelved or buried 'deeper than did ever plummet sound' out of the schoolboy's reach.

The Textbook Wars

The other camp, upholding the virtue of Euclid, was led by Augustus de Morgan until he died in 1871.

Geometry is intended, in education, ... to [teach] the tricks which reason plays on all but the cautious, plus the dangers arising out of caution itself. Let him that thinketh he standeth take heed lest he fall, is the motto on the door; augmented by, Let him that thinketh he falleth be not quite sure he doth not stand both until after very close examination ... Such a pause [or examination] is never made by a young student: he is quick to see everything except that he sees too much and too fast. Of all his studies, geometry is the one in which experience may cure him of this nasty habit, if he be properly exercised in the very field of danger.

The Textbook Wars

Others in this camp, such as the Reverend Charles Dodgson (an Oxford don, but better known by his pen name Lewis Carroll), added a more humanistic element to De Morgan's argument that Euclid was valuable largely *because* of the difficulties of its reasoning. As Dodgson noted while defending Euclid's ordering of theorems:

> The Propositions have been known by those numbers for two thousand years; they have been referred to, probably, by hundreds of writers ... and some of them, I.5 and I.47 for instance – 'the Asses Bridge' and 'The Windmill' – are now historical characters, and their nicknames are 'familiar as household words'.

The Textbook Wars

The reformers were faced with a difficult problem. In England, not only educational but often professional position and promotion in the army, navy, civil service and so on were routinely judged by standardized examinations. In geometry, these examinations assumed a thorough knowledge of specific theorems, proofs, orderings and so forth from Euclid's *Elements*. Rejecting Euclid as the basic text required drastically revising the examinations and developing a new equally clear-cut and universal standard by which students could be judged.

The reformers formed the Association for the Improvement of Geometrical Teaching (AIGT) in 1871 to create this new standard. This group met until 1893, but never succeeded in creating revolutionary changes in geometrical education. Part of the failure is no doubt due to the Tripos which set the standards for mathematical preparation in the schools.

Changing the Tripos

In patriotic duty bound, the Cambridge of Newton adhered to Newton's fluxions, to Newton's geometry, to the very text of Newton's *Principia*: in my own Tripos of 1881 we were expected to know any lemma in that great work by its number alone, as if it were one of the commandments or the 100th Psalm. Thus English mathematics were isolated: Cambridge became a school that was self-satisfied, self-supporting, self-content, almost marooned in its limitations.

A. R. Forsyth (Senior Wrangler)

Changing the Tripos

The mathematical teaching at Cambridge when I was an undergraduate was definitely bad ... The necessity for nice discrimination between the abilities of different examinees led to an emphasis on 'problems' [memorized] as opposed to 'bookwork' [freeform]. The 'proofs' that were offered of mathematical theorems were an insult to the logical intelligence. Indeed, the whole subject of mathematics was presented as a series of clever tricks by which to pile up marks in the Tripos.

- Bertrand Russell

Changing the Tripos

At the end of the century, a number of reforms were proposed to deal with the educationally destructive idiosyncrasies of the Tripos.

The first, instituted in 1892, allowed students to take the Tripos during their second rather than third year of their career. This was intended to free them to pursue their own interests in the final two years. This reform was highly ineffective ... hardly anyone took advantage of it.

An additional reform, first suggested in 1899 but not implemented until 1907, designed to mitigate the intense pressure to do well on the examination, was to abolish the rank ordering on the exam. The results would now be listed alphabetically in the three categories.

Consequences

Although these reforms might look like minor changes, they both had a radical effect on the Tripos and signified a major change in the way mathematics was viewed as a component of a liberal education.

Without the need to differentiate the students so finely, the exam was loosened up and became more reasonable. The institution of tripos "coaches" virtually vanished overnight. The bookies no longer took bets on the senior wrangler. Students took the exam in their second year as preparation for further studies, rather than the culmination of them.

In the wider view, this de-emphasis of the Tripos had a freeing effect on students. Mathematics students, who bitterly ranted against the Tripos, were now able to study mathematics that would be useful for them. Others were freed from what they considered a complete waste of their time at college.

High Stake Exams

Today, the question of high stake exams is being hotly debated in educational circles. We should look at the experience of the mathematical tripos, the ultimate high stake exam, to discern what pitfalls may present themselves in this arena.

1. The Tripos was grounded in the educational theory of its time, but that theory moved on while the Tripos did not.
2. The Tripos proved to be very resistant to change.
3. The Tripos indirectly controlled the curriculum of the grade schools.
4. Teaching to the test became the standard, lecture courses were poorly attended and poorly taught.
5. The Tripos had a dampening effect on originality and creativity – there was only one first rate mathematician in England during the heyday of the exam.