

1. C
2. A
3. A, C, A
4. B, D, A
5. C
6. C
7. C

1. A simple divide-and-conquer algorithm with parallel recursive calls working on  $A[i+1..n+1]$ .
2. Running time is  $n^2$ . A simple counterexample is [1,4,2,3]. When constructing a subsequence starting at 1, the algorithm greedily picks 4, which precludes it from discovering 1,2,3 - the longest increasing subsequence.
3. Hints:

One simple possible solution is to consider a binary choice of including the first element of a subproblem in the subsequence or not. Then the subproblem is defined by two parameters: the starting index  $i$  and the index of the previously picked element  $j$  ( $j < i$ ). The subproblem is to find the length of the longest increasing subsequence in  $A[i..n]$ , such that its elements are larger than  $A[j]$ . The whole problem is the subproblem ( $i=1, j=0$ ), assuming that  $A[0] = \text{minus infinity}$ . The resulting dynamic-programming algorithm will have  $n^2$  running time and will use  $n^2$  space.

Another possible solution:

Assume that the subproblem is to construct in  $A[i..n]$  an increasing subsequence that starts with  $A[i]$ . Then the choice is  $k$ -nary: what is the index of the next element to include after  $A[i]$ . We consider all elements from  $A[i+1..n]$  that are larger than  $A[i]$ . The solution to the whole problem is then the maximum of the solutions to all the possible subproblems  $i = 1, 2, \dots, n$ .

The resulting algorithm will still have  $n^2$  running time, but will use  $n$  space.