Ex 1.2.4. Coord sys is orthogonal means $g_{ij} \overset{\text{defn}}{=} e_i \cdot e_j = K_{ij}\delta^i_j$. $\therefore \hat{G} = \begin{pmatrix} K_{11} & & 0 \\ & K_{22} & \\ 0 & & K_{33} \end{pmatrix}$

Ex 1.2.6 In Example 1.2.1, $\mu_i = v\delta^1_i - u\delta^2_i + \delta^3_i$. Find μ^i.

$$M^* = (\mu_i) = \begin{pmatrix} v \\ -u \\ 1 \end{pmatrix}$$

$$(\mu^i) = M = \hat{G} M^* = \begin{pmatrix} \frac{1}{2} & 0 & -v \\ 0 & \frac{1}{2} & -u \\ -v & -u & 2u^2+2v^2+1 \end{pmatrix}\begin{pmatrix} v \\ -u \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}v \\ -\frac{3}{2}u \\ v^2+3u^2+1 \end{pmatrix}$$

$$\Rightarrow \mu^i = -\frac{1}{2}v\delta^1_i - \frac{3}{2}u\delta^2_i + (v^2+3u^2+1)\delta^3_i$$

Ex 1.2.7 (a) $\delta^i_i = \delta^1_1 + \delta^2_2 + \delta^3_3 = 3$ (b) $\delta^A_A = \delta^1_1 + \delta^2_2 = 2$

(c) $\delta^a_a = \delta^{a_1}_{a_1} + \cdots + \delta^{a_n}_{a_n} = n$ (d) $\delta^\mu_\mu = \delta^1_1 + \cdots + \delta^4_4 = 4$

Ex 1.4.1 Show $U^k_{i'} U^{i'}_j = \delta^k_j$ and $U^{k'}_i U^i_{j'} = \delta^{k'}_{j'}$

$$\delta^k_j = \frac{\partial u^k}{\partial u^j} \underset{\text{Rule}}{\overset{\text{ch.}}{=}} \frac{\partial u^k}{\partial u^{i'}}\frac{\partial u^{i'}}{\partial u^j} = U^k_{i'} U^{i'}_j$$