The Theoretical Minimum

Classical Mechanics - Solutions

L03E02

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Exercise 1. *Integrate this equation. Hint: Use definite integrals.*

The equation in question resulting from Newton's second law in the case of a constant force F_z being applied to an object of mass m following the z-axis:

$$\dot{v_z} = \dot{v_z(t)} = \frac{F_z}{m}$$

By integrating both sides, thanks to the fundamental theorem of calculus, assuming the mass is constant over time, we obtain:

$$\begin{aligned} v_z(t) &= \int \frac{F_z}{m}\, dt \\ &= \frac{F_z}{m} \int dt \\ &= \frac{F_z}{m} t + c,\ c \in \mathbb{R} \end{aligned}$$

Generally, c would be determine from an initial condition $v_z(0)$, which is our case, would precisely be c, hence:

$$\boxed{v_z(t) = v_z(0) + \frac{F_z}{m} t} \quad \square$$

Which is exactly the solution proposed in the book.