The Theoretical Minimum

Classical Mechanics - Solutions

L06E02

M. Bivert

September 15, 2022

Exercise 1. *Show that Eq.* (6) *is just another form of Newton's equation of motion* $F_i = m_i \ddot{x}_i$.

Where Eq. (6) are the following set of equation, defined for all $i \in [\![1, n]\!]$:

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x_i}} L\right) = \frac{\partial}{\partial x_i} L \tag{1}$$

Remark 1. *This exercice is simply a generalization of the previous exercice (L06E01) to a configuration space of size* $n \in \mathbb{N}$.

Then again, let us recall the Lagrangian defined slightly earlier in the related section of the book:

$$L = \sum_{i=1}^{n} \left(\frac{1}{2} m_i \dot{x_i}^2\right) - V(\{x\}) \tag{2}$$

Hence, $(\forall i \in [\![1, n]\!])$:

$$\begin{aligned} \frac{\partial}{\partial \dot{x_i}} L &= \frac{\partial}{\partial \dot{x_i}} \sum_{j=1}^{n} \frac{1}{2} m_j \dot{x_j}^2 & \qquad \frac{\partial}{\partial x_i} L = -\frac{\partial}{\partial x_i} V(\{x\}) \\ &= \sum_{j=1}^{n} m_j \dot{x_j} \delta_{ij} \\ &= m_i \dot{x_i} \end{aligned} \tag{3}$$

Again, we need the *potential energy principle*, stated as Eq. (5) of the previous chapter *Lecture 5: Energy*, for abstract configuration space $\{x\} = \{x_i\}$, as:

$$F_i(\{x\}) = -\frac{\partial}{\partial x_i} V(\{x\}) \tag{4}$$

From which we can conclude, by injecting (4) in the second half of (3), and connecting each side with Euler-Lagrange's equations (1), $(\forall i \in [\![1, n]\!])$:

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x_i}} L\right) = \frac{\partial}{\partial x_i} L \\ \Leftrightarrow &\frac{d}{dt} m_i \dot{x_i} = F_i(\{x\}) \\ \Leftrightarrow &\boxed{F_i = m_i \ddot{x_i}} \quad \square \end{aligned}$$