The Theoretical Minimum

Quantum Mechanics - Solutions

L07E07

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Exercise 1. *Use Eq. 7.24 to calculate ρ^2. How does this result confirm that ρ represents an entangled state? We'll soon discover that there are other ways to check for entanglement.*

Here's Eq. 7.24:

$$\rho = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

From there it's trivial to see that:

$$\rho^2 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}^2 = \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix}$$

The authors demonstrated earlier a criteria to determine whether a density matrix corresponds to an entangled state or not, at the end of section 7.5: for a pure state, and a density matrix ρ, we *must* have:

$$\rho^2 = \rho \text{ and } \mathrm{Tr}(\rho)^2 = 1$$

While for a mixed or entangled state, we *must* have:

$$\rho^2 \neq \rho \text{ and } \mathrm{Tr}(\rho)^2 < 1$$

Hence, $\boxed{\rho \text{ represents an entangled state}}$.