

The Theoretical Minimum

Quantum Mechanics - Solutions

L04E02

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Exercise 1. *Prove that if M and L are both Hermitian, $i[M, L]$ is also Hermitian. Note that the i is important. The commutator is, by itself, not Hermitian.*

$$\begin{aligned}
 (i[M, L])^\dagger &= (i(ML - LM))^\dagger \\
 &= (iML - iLM)^\dagger \\
 &= ((iML - iLM)^T)^* && (\dagger\text{'s definition}) \\
 &= ((iML)^T - (iLM)^T)^* && ((A + B)^T = A^T + B^T) \\
 &= (iL^T M^T - iM^T L^T)^* && ((AB)^T = B^T A^T) \\
 &= (-i(L^T M^T)^* + i(M^T L^T)^*) && ((zt)^* = z^* t^*; (z + t)^* = z^* + t^*) \\
 &= i(M^\dagger L^\dagger - L^\dagger M^\dagger) && (\dagger\text{'s definition}) \\
 &= i(ML - LM) && (L = L^\dagger; M = M^\dagger) \\
 &= i[M, L] && \square
 \end{aligned}$$