

# The Theoretical Minimum

## Classical Mechanics - Solutions

L02E03

M. Bivert

October 9, 2022

**Exercise 1.** Use the chain rule to find the derivatives on each of the following functions:

$$\begin{aligned}g_0(t) &= \sin(t^2) - \cos(t^2) \\ \theta_0(\alpha) &= e^{3\alpha} + 3\alpha \ln(3\alpha) \\ x_0(t) &= \sin^2(t^2) - \cos(t^2)\end{aligned}$$

**Remark 1.** We've slightly altered the functions names compared to what they have in the book. The reason will be apparent very soon.

**Remark 2.** As in previous exercises (e.g. L02E01), we'll be using some additional mathematical notation to avoid us the need to define functions. The meaning should be obvious from the context.

We will also use the prime notation to denote differentiation, as we also did in earlier exercises.

There are two main ways of proceeding: either applying the *chain rule* to each individual term of each function, for instance to compute  $g'$ , we would first compute  $(\sin(t^2))' = (\sin((u \mapsto u^2)(t)))'$ , using the chain rule, then  $(\cos(t^2))'$  in a similar fashion, etc.

Or, and we're going to use this approach as it's likely to have been expected one, we can consider each function "globally" as a composition of two smaller functions. While doing so, we will find that  $g_0$ ,  $\theta_0$  and  $x_0$  are respectively created by composing functions  $f$ ,  $\theta$  and  $x$  of an earlier exercise, with two simple functions  $(v \mapsto v^2)$  and  $(v \mapsto 3v)$ :

$$\begin{aligned}g_0(t) &= \underbrace{((u \mapsto \sin(u) - \cos(u)))}_g \circ (v \mapsto v^2))(t) \\ \theta_0(\alpha) &= \underbrace{((u \mapsto e^u + u \ln(u)))}_\theta \circ (v \mapsto 3v)(t) \\ x_0(t) &= \underbrace{((u \mapsto \sin^2(u) - \cos(u)))}_x \circ (v \mapsto v^2)(t)\end{aligned}$$

Let's remember the derivative of  $g$ ,  $\theta$  and  $x$ , that we've computed in L02E01:

$$\begin{aligned}g'(x) &= \cos(x) + \sin(x) \\ \theta'(\alpha) &= e^\alpha + \ln(\alpha) + 1 \\ x'(t) &= (1 + 2 \cos t) \sin t\end{aligned}$$

Finally, let's recall the *chain rule*:

$$\frac{d}{dt}(\psi \circ \varphi) = \frac{d}{dt}(\psi(\varphi(t))) = \varphi'(t)\psi'(\varphi(t))$$

Then, the derivative of  $(v \mapsto v^2)$  and  $(v \mapsto 3v)$  being respectively  $(v \mapsto 2v)$  and  $(v \mapsto 3)$  (constant function), we have the following derivatives for our functions:

$$\begin{aligned}g'_0(t) &= \boxed{2t(\cos t^2 + \sin t^2)} \\ \theta'_0(\alpha) &= \boxed{3(e^{3\alpha} + \ln(3\alpha) + 1)} \\ x'_0(t) &= \boxed{2t((1 + 2 \cos t^2) \sin t^2)}\end{aligned}$$