

The Theoretical Minimum

Classical Mechanics - Solutions

L06E02

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Exercise 1. Show that Eq. (6) is just another form of Newton's equation of motion $F_i = m_i \ddot{x}_i$.

Where Eq. (6) are the following set of equation, defined for all $i \in \llbracket 1, n \rrbracket$:

$$\frac{d}{dt} \left(\frac{\partial}{\partial \dot{x}_i} L \right) = \frac{\partial}{\partial x_i} L \quad (1)$$

Remark 1. This exercise is simply a generalization of the previous exercise (L06E01) to a configuration space of size $n \in \mathbb{N}$.

Then again, let us recall the Lagrangian defined slightly earlier in the related section of the book:

$$L = \sum_{i=1}^n \left(\frac{1}{2} m_i \dot{x}_i^2 \right) - V(\{x\}) \quad (2)$$

Hence, $(\forall i \in \llbracket 1, n \rrbracket)$:

$$\begin{aligned} \frac{\partial}{\partial \dot{x}_i} L &= \frac{\partial}{\partial \dot{x}_i} \sum_{j=1}^n \frac{1}{2} m_j \dot{x}_j^2 & \frac{\partial}{\partial x_i} L &= -\frac{\partial}{\partial x_i} V(\{x\}) \\ &= \sum_{j=1}^n m_j \dot{x}_j \delta_{ij} \\ &= m_i \dot{x}_i \end{aligned} \quad (3)$$

Again, we need the *potential energy principle*, stated as Eq. (5) of the previous chapter *Lecture 5: Energy*, for abstract configuration space $\{x\} = \{x_i\}$, as:

$$F_i(\{x\}) = -\frac{\partial}{\partial x_i} V(\{x\}) \quad (4)$$

From which we can conclude, by injecting (4) in the second half of (3), and connecting each side with Euler-Lagrange's equations (1), $(\forall i \in \llbracket 1, n \rrbracket)$:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial}{\partial \dot{x}_i} L \right) &= \frac{\partial}{\partial x_i} L \\ \Leftrightarrow \frac{d}{dt} m_i \dot{x}_i &= F_i(\{x\}) \\ \Leftrightarrow \boxed{F_i = m_i \ddot{x}_i} &\quad \square \end{aligned}$$