

The Theoretical Minimum

Quantum Mechanics - Solutions

L02E01

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Exercise 1. *Prove that the vector $|r\rangle$ in Eq. 2.5 is orthogonal to vector $|l\rangle$ in Eq. 2.6.*

Let us recall respectively Eq. 2.5 and Eq. 2.6:

$$|r\rangle = \frac{1}{\sqrt{2}}|u\rangle + \frac{1}{\sqrt{2}}|d\rangle \qquad |l\rangle = \frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle$$

Orthogonality can be detected with the inner-product: $|l\rangle$ and $|r\rangle$ are orthogonals $\Leftrightarrow \langle r|l\rangle = \langle l|r\rangle = 0$.

Remark 1.

The nullity of either inner-product is sufficient, because of the $\langle A|B\rangle = \langle B|A\rangle^$ axiom.*

For instance:

$$\begin{aligned} \langle l|r\rangle &= (\lambda_u^* \quad \lambda_d^*) \begin{pmatrix} \rho_u \\ \rho_d \end{pmatrix} \\ &= \left(\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}} \right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ &= 0 \quad \square \end{aligned}$$

Or, similarly:

$$\begin{aligned} \langle r|l\rangle &= (\rho_u^* \quad \rho_d^*) \begin{pmatrix} \lambda_u \\ \lambda_d \end{pmatrix} \\ &= \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ &= 0 \quad \square \end{aligned}$$