

The Theoretical Minimum

Quantum Mechanics - Solutions

L06E06

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Exercise 1. Assume Charlie has prepared the two spins in the singlet state. This time, Bob measures τ_y and Alice measure σ_x . What is the expectation value of $\sigma_x \tau_y$?

What does this say about the correlation between the two measurements?

Let's recall the Pauli matrices involved:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

And the singlet state:

$$|\text{sing}\rangle = \frac{1}{\sqrt{2}} (|ud\rangle - |du\rangle)$$

Recall:

$$\begin{aligned} \sigma_x|u\rangle &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} =: |d\rangle; & \sigma_x|d\rangle &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} =: |u\rangle \\ \tau_y|u\rangle &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} =: i|d\rangle; & \tau_y|d\rangle &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} =: -i|u\rangle \end{aligned}$$

The expectation value of $\sigma_x \tau_y$ is then:

$$\begin{aligned} \langle \sigma_x \tau_y \rangle &:= \langle \text{sing} | \sigma_x \tau_y | \text{sing} \rangle \\ &= \langle \text{sing} | \sigma_x \tau_y \frac{1}{\sqrt{2}} (|ud\rangle - |du\rangle) \\ &= \frac{1}{\sqrt{2}} \langle \text{sing} | \left(\underbrace{(\sigma_x|u\rangle)}_{|d\rangle} \otimes \underbrace{(\tau_y|d\rangle)}_{-i|u\rangle} - \underbrace{(\sigma_x|d\rangle)}_{|u\rangle} \otimes \underbrace{(\tau_y|u\rangle)}_{i|d\rangle} \right) \\ &= \frac{-i}{\sqrt{2}} \langle \text{sing} | (|du\rangle + |ud\rangle) \\ &= \frac{-i}{2} (\langle ud| - \langle du|)(|du\rangle + |ud\rangle) \\ &= \frac{-i}{2} \left(\underbrace{\langle ud|du\rangle}_0 + \underbrace{\langle ud|ud\rangle}_1 - \underbrace{\langle du|du\rangle}_1 - \underbrace{\langle du|ud\rangle}_0 \right) \\ &= \boxed{0} \end{aligned}$$

Remember for the last step that $|du\rangle$ and $|ud\rangle$ are orthonormal basis vectors.

On to the correlation between the two measurements: recall from section 6.2 that the statistical correlation between an observable σ_A in Alice's space and an observable σ_B in Bob's space was defined as the quantity:

$$\langle \sigma_A \sigma_B \rangle - \langle \sigma_A \rangle \langle \sigma_B \rangle$$

Hence in our case, the correlation between the two measurements is (the authors previously computed $\langle \sigma_x \rangle = 0$ and $\langle \sigma_y \rangle = 0$: the computation of $\langle \tau_y \rangle$ would be identical as for the latter)

$$\langle \sigma_x \tau_y \rangle - \langle \sigma_x \rangle \langle \tau_y \rangle = 0$$

Hence, we can conclude that the two measurements aren't correlated at all.