

The Theoretical Minimum

Classical Mechanics - Solutions

L06E06

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Exercise 1. *Explain how we derived this.*

Let us recall that "this" refers to the following expression for the kinetic energy:

$$T = m(\dot{x}_+^2 + \dot{x}_-^2)$$

Starting from the following Lagrangian, involving two particles moving on a line with respective position and velocity x_i, \dot{x}_i :

$$L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - V(x_1 - x_2) \quad (1)$$

After having performed the following change of coordinates:

$$x_+ = \frac{x_1 + x_2}{2} \qquad x_- = \frac{x_1 - x_2}{2} \quad (2)$$

From the Lagrangian, (1) we have the kinetic energy:

$$T = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) \quad (3)$$

By first both summing and subtracting the two equations of (2), and then by linearity of the derivation, we get:

$$\begin{aligned} x_+ + x_- &= x_1 & x_+ - x_- &= x_2 \\ \dot{x}_+ + \dot{x}_- &= \dot{x}_1 & \dot{x}_+ - \dot{x}_- &= \dot{x}_2 \end{aligned} \quad (4)$$

It's now simply a matter of injecting (4) into (3):

$$\begin{aligned} T &= \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) \\ &= \frac{m}{2}((\dot{x}_+ + \dot{x}_-)^2 + (\dot{x}_+ - \dot{x}_-)^2) \\ &= \frac{m}{2}(2\dot{x}_+^2 + 2\dot{x}_-^2 + 2\dot{x}_+\dot{x}_- - 2\dot{x}_+\dot{x}_-) \\ &= m(\dot{x}_+^2 + \dot{x}_-^2) \quad \square \end{aligned}$$