TTM - QM - L02E01

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Exercise 1.
Prove that the vector $|r\rangle$ in Eq. 2.5 is orthogonal to vector $|l\rangle$ in Eq. 2.6.

Let us recall respectively Eq. 2.5 and Eq. 2.6:

$$|r\rangle = \frac{1}{\sqrt{2}}|u\rangle + \frac{1}{\sqrt{2}}|d\rangle \qquad\qquad |l\rangle = \frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle$$

Orthogonality can be detected with the inner-product: $|l\rangle$ and $|r\rangle$ are orthogonals $\Leftrightarrow \langle r|l\rangle = \langle l|r\rangle = 0$.

Remark 1.
The nullity of either inner-product is sufficient, because of the $\langle A|B\rangle = \langle B|A\rangle^*$ axiom.

For instance:

$$\begin{aligned}
\langle l|r\rangle &= \begin{pmatrix}\lambda_u^* & \lambda_d^*\end{pmatrix}\begin{pmatrix}\rho_u \\ \rho_d\end{pmatrix} \\
&= \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \\
&= 0 \quad \square
\end{aligned}$$

Or, similarly:

$$\begin{aligned}
\langle r|l\rangle &= \begin{pmatrix}\rho_u^* & \rho_d^*\end{pmatrix}\begin{pmatrix}\lambda_u \\ \lambda_d\end{pmatrix} \\
&= \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \\
&= 0 \quad \square
\end{aligned}$$