

TTM - QM - L02E03

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Exercise 1.

For the moment, forget that Eqs. 2.10 give us working definitions for $|i\rangle$ and $|o\rangle$ in terms of $|u\rangle$ and $|d\rangle$, and assume that the components α, β, γ and δ are unknown:

$$|o\rangle = \alpha|u\rangle + \beta|d\rangle$$

$$|i\rangle = \gamma|u\rangle + \delta|d\rangle$$

a) Use Eqs. 2.8 to show that

$$\alpha^*\alpha = \beta^*\beta = \gamma^*\gamma = \delta^*\delta = \frac{1}{2}$$

b) Use the above results and Eqs. 2.9 to show that

$$\alpha^*\beta + \alpha\beta^* = \gamma^*\delta + \gamma\delta^* = 0$$

c) Show that $\alpha^*\beta$ and $\gamma^*\delta$ must each be pure imaginary.

If $\alpha^*\beta$ is pure imaginary, then α and β cannot both be real. The same reasoning applies to $\gamma^*\delta$.