

# The Theoretical Minimum

## Classical Mechanics - Solutions

L11E02

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**Exercise 1.** *Prove Eq. (4).*

Where Eq. (4) is the following, for  $V$  a scalar field:

$$\vec{\nabla} \times (\vec{\nabla} V(x)) = 0$$

If think we can agree that  $V(x)$  is actually a  $V(x, y, z)$ .

And  $\vec{\nabla}$  is the differentiation vector operator:

$$\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

By this definition,

$$\vec{\nabla} V(x, y, z) = \begin{pmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{pmatrix}$$

We also have previously established that for a field  $F = (F_x, F_y, F_z)$ :

$$\vec{\nabla} \times F = \begin{pmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{pmatrix}$$

And so,

$$\vec{\nabla} \times (\vec{\nabla} V(x, y, z)) = \begin{pmatrix} \frac{\partial}{\partial y} \frac{\partial V}{\partial z} - \frac{\partial}{\partial z} \frac{\partial V}{\partial y} \\ \frac{\partial}{\partial z} \frac{\partial V}{\partial x} - \frac{\partial}{\partial x} \frac{\partial V}{\partial z} \\ \frac{\partial}{\partial x} \frac{\partial V}{\partial y} - \frac{\partial}{\partial y} \frac{\partial V}{\partial x} \end{pmatrix} = \vec{0}$$

Where we can conclude because of Schwarz/Clairaut's theorem<sup>1</sup>. This means we consider  $V$  to have continuous second partial derivatives on its domain (or, at least in a neighborhood of a point  $x$  of its domain), which is often a reasonable assumption in Physics.

<sup>1</sup>[https://en.wikipedia.org/wiki/Symmetry\\_of\\_second\\_derivatives](https://en.wikipedia.org/wiki/Symmetry_of_second_derivatives)