The Theoretical Minimum
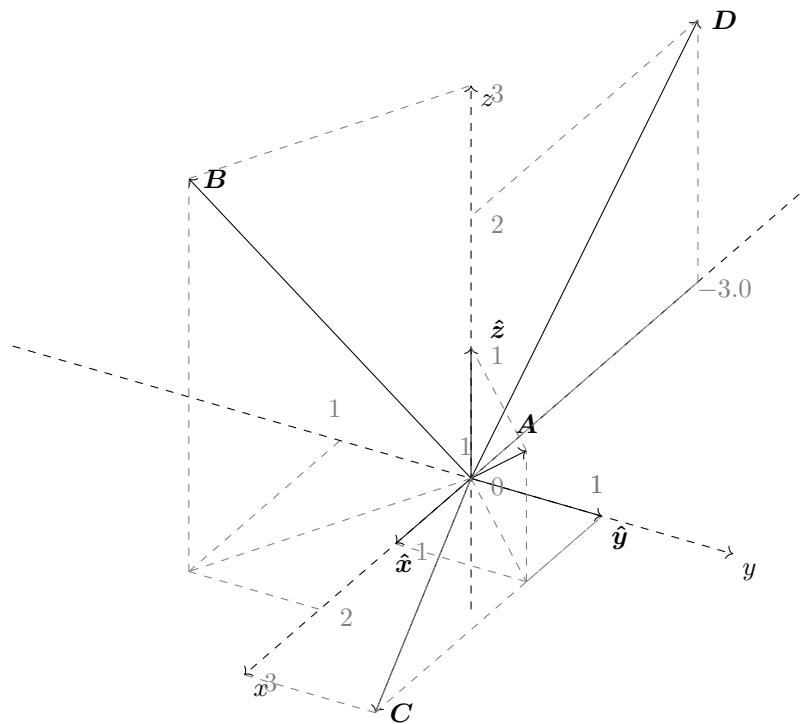## Classical Mechanics - Solutions
I01E05

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Exercise 1. *Determine which pair of vectors are orthogonal.* $(1, 1, 1), (2, -1, 3), (3, 1, 0), (-3, 0, 2)$.

This is again an immediate application of dot product formula. We'll respectively name the vectors $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$; let's start by plotting them:

Right before this exercise, the authors wrote:

An important property of the dot product is that it is zero if the vectors are *orthogonal*.

Let us recall the "components-based" dot product formula we'll be using:

$$\boldsymbol{u} \cdot \boldsymbol{v} \quad = \quad u_x v_x + u_y v_y + u_z v_z$$

Then, it's just a matter of crunching numbers:

$$
\begin{aligned}
\boldsymbol{A} \cdot \boldsymbol{B} &= A_x B_x + A_y B_y + A_z B_z \\
&= (1 \times 2) + (1 \times (-1)) + (1 \times 3) \\
&= \boxed{4}
\end{aligned}
\qquad
\begin{aligned}
\boldsymbol{A} \cdot \boldsymbol{C} &= A_x C_x + A_y C_y + A_z C_z \\
&= (1 \times 3) + (1 \times 1) + (1 \times 0) \\
&= \boxed{4}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{A} \cdot \boldsymbol{D} &= A_x D_x + A_y D_y + A_z D_z \\
&= (1 \times (-3)) + (1 \times 0) + (1 \times 2) \\
&= \boxed{-1}
\end{aligned}
\qquad
\begin{aligned}
\boldsymbol{B} \cdot \boldsymbol{C} &= B_x C_x + B_y C_y + B_z C_z \\
&= (2 \times 3) + (-1 \times 1) + (3 \times 0) \\
&= \boxed{5}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{B} \cdot \boldsymbol{D} &= B_x D_x + B_y D_y + B_z D_z \\
&= (2 \times (-3)) + (-1 \times 0) + (3 \times 2) \\
&= \boxed{0}
\end{aligned}
\qquad
\begin{aligned}
\boldsymbol{D} \cdot \boldsymbol{C} &= D_x C_x + D_y C_y + D_z C_z \\
&= (-3 \times 3) + (0 \times 1) + (2 \times 0) \\
&= \boxed{-9}
\end{aligned}
$$

Hence, the only two orthogonal vectors are $\boldsymbol{B}$ and $\boldsymbol{D}$, or $(2, -1, 3)$ and $(-3, 0, 2)$.