

The Theoretical Minimum

Classical Mechanics - Solutions

I02E03

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Exercise 1. *Treat the expressions from Exercise 1 as expressions for the acceleration of a particle. Integrate them once, with respect to time, and determine the velocities, and a second time to determine the trajectories. Because we will use t as one of the limits of integration we will adopt the dummy integration variable t' . Integrate them from $t' = 0$ to $t' = t$.*

$$\begin{aligned}v(t) &= \int_0^t t'^4 dt' \\v(t) &= \int_0^t \cos t' dt' \\v(t) &= \int_0^t (t'^2 - 2) dt'\end{aligned}$$

Conceptually, the exercise seems to be about creating functions from integrals, by having a variable of the integration limit being a function parameter, which forces to use a different name for the integration variable; multiple choices are obviously possible, as long as the result is consistent:

$$v(t) = \int_0^t a(T) dT$$

$$v(t) = \int_0^t t'^4 dt'$$

Let's recall our results from either I02E01 or I02E02. If we work from the former, we first would need to change the variable name from the primitive, say to t' , and evaluate the indefinite integral between t and 0. Working from the later, we would need to replace T by t .

$$\begin{aligned}v(t) &= \int_0^t t'^4 dt' \\&= \left[\frac{1}{5} t'^5 + c \right]_0^t, \quad c \in \mathbb{R} \\&= \boxed{\frac{1}{5} t^5}\end{aligned}$$

Now we can repeat the same process to compute the position $x(t)$:

$$\begin{aligned}x(t) &= \int_0^t v(t') dt' \\&= \int_0^t \frac{1}{5} t'^5 dt' \\&= \left[\frac{1}{30} t'^6 + c \right]_0^t, \quad c \in \mathbb{R} \\&= \boxed{\frac{1}{30} t^6}\end{aligned}$$

$v(t) = \int_0^t \cos t' dt'$
 Same exact process:

$$\begin{aligned}
 v(t) &= \int_0^t \cos t' dt' \\
 &= [\sin t' + c]_0^t, \quad c \in \mathbb{R} \\
 &= \boxed{\sin t} \\
 x(t) &= \int_0^t v(t') dt' \\
 &= \int_0^t \sin t' dt' \\
 &= [-\cos t' + c]_0^t, \quad c \in \mathbb{R} \\
 &= \boxed{-\cos t}
 \end{aligned}$$

$v(t) = \int_0^t (t'^2 - 2) dt'$
 No surprises here either:

$$\begin{aligned}
 v(t) &= \int_0^t (t'^2 - 2) dt' \\
 &= \left[\frac{1}{3} t'^3 - 2t' + c \right]_0^t, \quad c \in \mathbb{R} \\
 &= \boxed{\frac{1}{3} t^3 - 2t} \\
 x(t) &= \int_0^t v(t') dt' \\
 &= \int_0^t \frac{1}{3} t'^3 - 2t' dt' \\
 &= \left[\frac{1}{12} t'^4 - t'^2 + c \right]_0^t, \quad c \in \mathbb{R} \\
 &= \boxed{\left(\frac{1}{12} t^2 - 1 \right) t^2}
 \end{aligned}$$