

# The Theoretical Minimum

## Classical Mechanics - Solutions

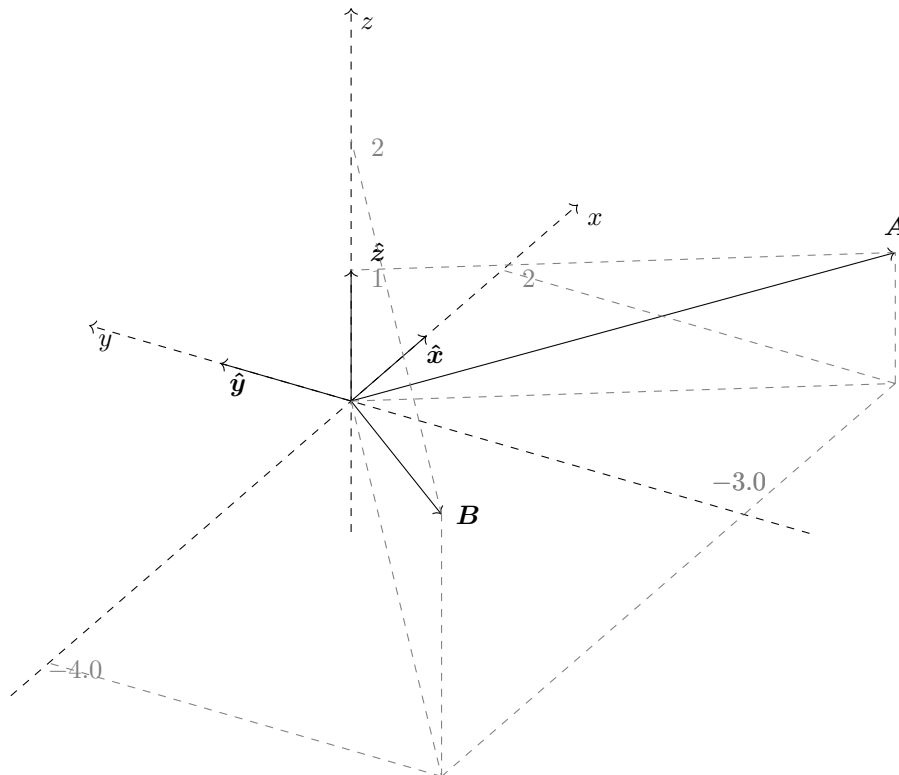
I01E04

M. Bivert

December 26, 2022

**Exercise 1.** Let  $(A_x = 2, A_y = -3, A_z = 1)$  and  $(B_x = -4, B_y = -3, B_z = 2)$ . Compute the magnitude of  $\vec{A}$  and  $\vec{B}$ , their dot product, and the angle between them.

This is an immediate application of the formulas. Let's start by plotting those vectors:



Using slightly different notations, let us then recall first how the magnitude of a vector  $\mathbf{u}$  is defined:

$$\|\mathbf{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2} = u \quad (1)$$

There are two formulas for the dot product: one involving the magnitudes and the angle between the vectors, and the other one involving the components:

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= u_x v_x + u_y v_y + u_z v_z \\ &= \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta_{uv} \end{aligned} \quad (2)$$

By applying (1), we have:

$$\begin{aligned} A &= \sqrt{2^2 + (-3)^2 + 1^2} & B &= \sqrt{(-4)^2 + (-3)^2 + 2^2} \\ &= \sqrt{4 + 9 + 1} & &= \sqrt{16 + 9 + 4} \\ &= \boxed{\sqrt{14}} & &= \boxed{\sqrt{29}} \end{aligned}$$

We can also compute the dot product from the vectors' components, using the first form of (2):

$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= 2(-4) + (-3)(-3) + 1 \times 2 \\ &= \boxed{3}\end{aligned}$$

From the second form of (2), we can deduce a formula for the angle between  $\mathbf{A}$  and  $\mathbf{B}$ ,  $\theta_{AB}$ :

$$\begin{aligned}\cos \theta_{AB} &= \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} \\ \Leftrightarrow \theta_{AB} &= \cos^{-1} \left( \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} \right) \\ \Leftrightarrow &= \cos^{-1} \left( \frac{3}{\sqrt{14 \times 29}} \right) \\ \Leftrightarrow &= \boxed{81.43753893^\circ}\end{aligned}$$

**Remark 1.** Looking at our plots, the angle feels to be something a little less than  $90^\circ$ , which is coherent with what we've found.