

The Theoretical Minimum

Classical Mechanics - Solutions

I01E03

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October 7, 2022

Exercise 1. *Show that the magnitude of a vector satisfies $|\vec{A}|^2 = \vec{A} \cdot \vec{A}$.*

Remark 1. *We'll again use a **bold** font to denote vectors instead of arrows and use a slightly different symbol for the magnitude; the change can be summed up by stating: $\|\mathbf{u}\| = |\vec{u}|$ ($= u$).*

Let's recall that the magnitude of a vector was *defined* as:

$$\|\mathbf{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

And the dot product between two vectors as:

$$\mathbf{u} \cdot \mathbf{v} = u_x v_x + u_y v_y + u_z v_z$$

From there, we quickly reach the expected result:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{u} &= u_x u_x + u_y u_y + u_z u_z \\ &= u_x^2 + u_y^2 + u_z^2 \\ &= \left(\sqrt{u_x^2 + u_y^2 + u_z^2} \right)^2 \\ &= \|\mathbf{u}\|^2 \quad \square\end{aligned}$$