بسم الله الرحمن الرحیم

نظریه زبان‌ها و ماشین‌ها

جلسه ۲۴

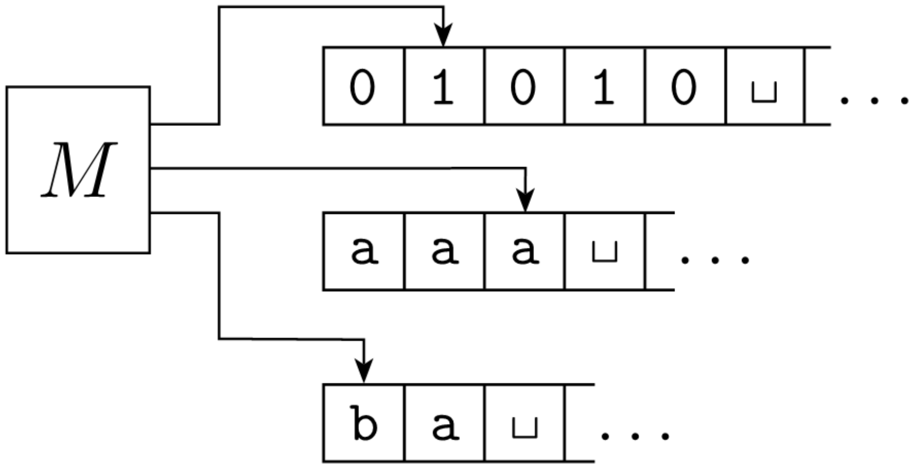
مجتبی خلیلی
دانشکده برق و کامپیوتر
دانشگاه صنعتی اصفهان

ماشین تورینگ

- تعریف ماشین تورینگ (معمولی یا استاندارد) را دیدیم (با یک نوار نامتناهی و حرکتهای قطعی یکی یکی به چپ یا راست).
- با LBA نیز آشنا شدیم.
- آیا میتوان ماشین تورینگ را قوی تر کرد؟

ماشین تورینگ چند نواره

A ***multitape Turing machine*** is like an ordinary Turing machine with several tapes. Each tape has its own head for reading and writing. Initially the input appears on tape 1, and the others start out blank. The transition function is changed to allow for reading, writing, and moving the heads on some or all of the tapes simultaneously.



ماشین تورینگ چند نواره

- به زبان فرمال:

$$\delta\colon Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{\mathrm{L}, \mathrm{R}, \mathrm{S}\}^k,$$

where k is the number of tapes. The expression

$$\delta(q_i, a_1, \ldots, a_k) = (q_j, b_1, \ldots, b_k, \mathrm{L}, \mathrm{R}, \ldots, \mathrm{L})$$

means that if the machine is in state q_i and heads 1 through k are reading symbols a_1 through a_k, the machine goes to state q_j, writes symbols b_1 through b_k, and directs each head to move left or right, or to stay put, as specified.

ماشین تورینگ چند نواره

- آیا از ماشین تورینگ تک نواره (معمولی یا استاندارد) قوی تر است؟
- معادل یا هم ارز ($L(TM_{single-tape}) = L(TM_{multitape})$) ؟

Equivalence of Classes of Automata

DEFINITION 10.1

Two automata are equivalent if they accept the same language. Consider two classes of automata C_1 and C_2. If for every automaton M_1 in C_1 there is an automaton M_2 in C_2 such that

$$L(M_1) = L(M_2),$$

we say that C_2 is at least as powerful as C_1. If the converse also holds and for every M_2 in C_2 there is an M_1 in C_1 such that $L(M_1) = L(M_2)$, we say that C_1 and C_2 are equivalent.

ماشین تورینگ چند نواره

THEOREM 3.13

Every multitape Turing machine has an equivalent single-tape Turing machine.

- اثبات:

$\boxed{\text{Multitape TM M}} \xleftarrow[\text{By simulation}]{} \boxed{\text{Single-tape TM S}}$

ماشین تورینگ چند نواره

THEOREM 3.13

Every multitape Turing machine has an equivalent single-tape Turing machine.

- اثبات:

Multitape TM M → Single-tape TM S

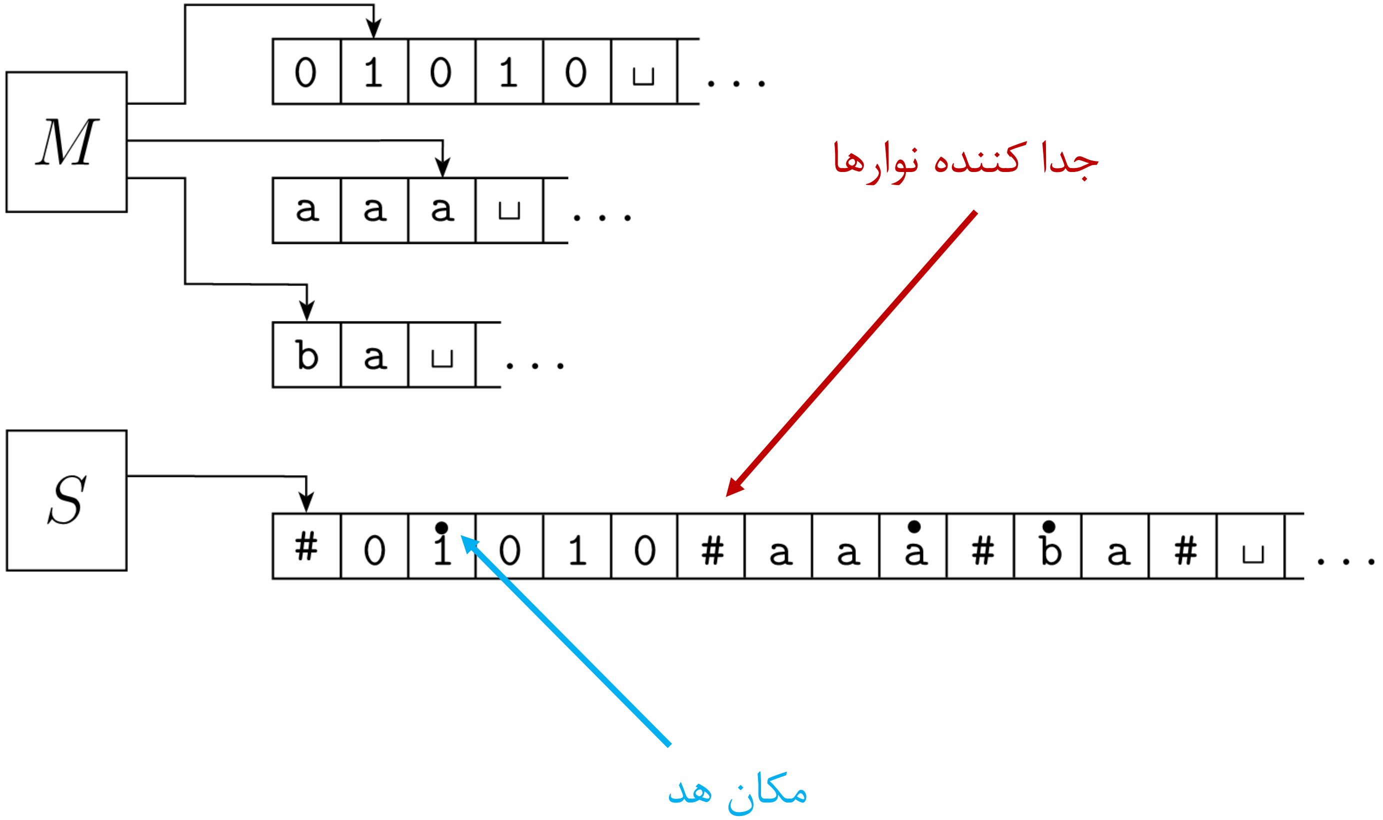
By simulation

S simulates the effect of k tapes by storing their information on its single tape.

ماشین تورینگ چند نواره

IUT-ECE

- اثبات (ادامه):



ماشین تورینگ چند نواره

$S =$ "On input $w = w_1 \cdots w_n$:

1. First S puts its tape into the format that represents all k tapes of M. The formatted tape contains
$$\#\dot{w}_1 w_2 \cdots w_n \#\dot{\sqcup}\#\dot{\sqcup}\# \cdots \#.$$
2. To simulate a single move, S scans its tape from the first #, which marks the left-hand end, to the $(k+1)$st #, which marks the right-hand end, in order to determine the symbols under the virtual heads. Then S makes a second pass to update the tapes according to the way that M's transition function dictates.
3. If at any point S moves one of the virtual heads to the right onto a #, this action signifies that M has moved the corresponding head onto the previously unread blank portion of that tape. So S writes a blank symbol on this tape cell and shifts the tape contents, from this cell until the rightmost #, one unit to the right. Then it continues the simulation as before."

- اثبات (ادامه): روند شبیه سازی که S انجام میدهد:

ماشین تورینگ چند نواره

COROLLARY 3.15

A language is Turing-recognizable if and only if some multitape Turing machine recognizes it.

PROOF A Turing-recognizable language is recognized by an ordinary (single-tape) Turing machine, which is a special case of a multitape Turing machine. That proves one direction of this corollary. The other direction follows from Theorem 3.13.

خواص بستاری

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- زبان‌های تصمیم‌پذیر تحت عملگرهای زیر بسته اند:
  - اجتماع
  - الحاق
  - استار
  - اشتراک
  - معکوس
  - مکمل

- زبان‌های RE تحت عملگرهای زیر بسته اند:
  - اجتماع
  - الحاق
  - استار
  - اشتراک
  - معکوس

ماشین تورینگ نامعین

- همانی که انتظار داریم. در هر گام ماشین میتواند چند امکان برای حرکت داشته باشد. این یعنی:

$$\delta: Q \times \Gamma \longrightarrow \mathcal{P}(Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}\}).$$

- مانند قبل چند شاخه دنبال میشود و هر کدام که وارد accept شدند برای پذیرش رشته ورودی کافی است.

ماشین تورینگ نامعین

EXAMPLE 10.2

If a Turing machine has transitions specified by

$$\delta(q_0, a) = \{(q_1, b, R), (q_2, c, L)\},$$

it is nondeterministic. The moves

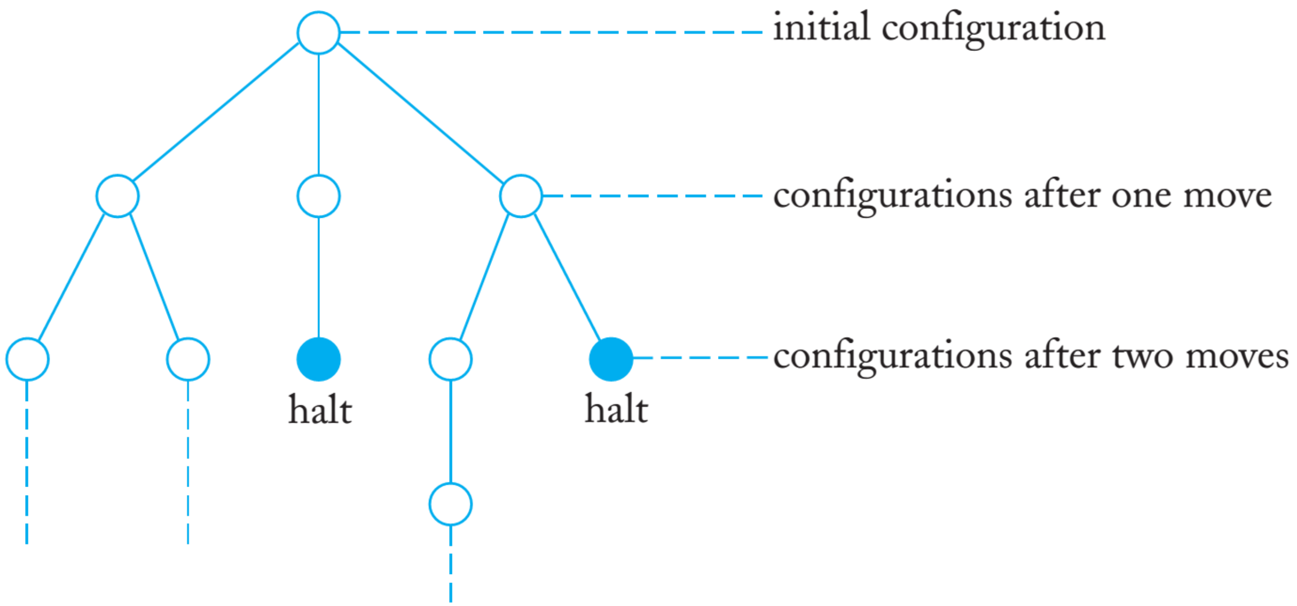
$$bq_0aaa \vdash bbq_1aa$$

and

$$bq_0aaa \vdash q_2bcaa$$

are both possible.

ماشین تورینگ نامعین



The width of such a configuration tree depends on the branching factor, that is, the number of options available on each move. If k denotes the maximum branching, then

$$M = k^n \tag{10.1}$$

is the maximum number of configurations that can exist after n moves.

ماشین تورینگ نامعین

DEFINITION 10.3

A nondeterministic Turing machine M is said to *accept* a language L if, for all $w \in L$, at least one of the possible configurations accepts w. There may be branches that lead to nonaccepting configurations, while some may put the machine into an infinite loop. But these are irrelevant for acceptance.

A nondeterministic Turing machine M is said to *decide* a language L if, for all $w \in \Sigma^*$, there is a path that leads either to acceptance or rejection.

ماشین تورینگ نامعین

- آیا از ماشین تورینگ (معمولی) قوی تر است؟
- معادل یا هم ارز؟

ماشین تورینگ نامعین

THEOREM 3.16

Every nondeterministic Turing machine has an equivalent deterministic Turing machine.

- اثبات:

$$\boxed{\text{nondeterministic TM N}} \xleftarrow[\text{By simulation}]{\hspace{4em}} \boxed{\text{deterministic TM D}}$$

ماشین تورینگ نامعین

THEOREM 3.16

Every nondeterministic Turing machine has an equivalent deterministic Turing machine.

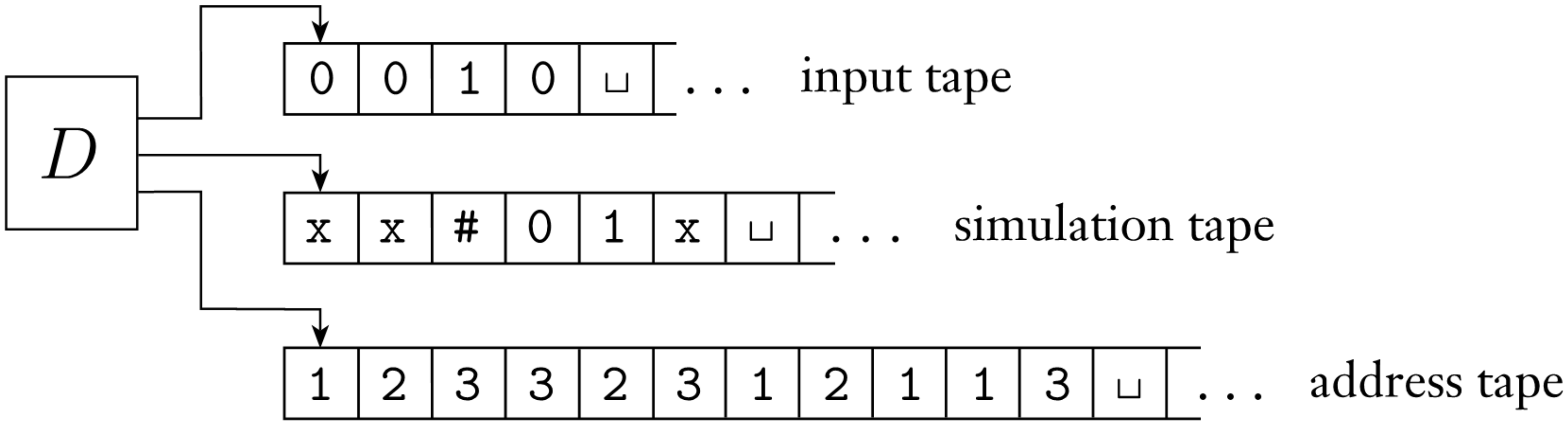
- اثبات:

nondeterministic TM N $\xrightarrow{\text{By simulation}}$ deterministic TM D

ماشین تورینگ نامعین

IUT-ECE

- اثبات (ادامه):



ماشین تورینگ نامعین

IUT-ECE

COROLLARY 3.18

A language is Turing-recognizable if and only if some nondeterministic Turing machine recognizes it.

PROOF Any deterministic TM is automatically a nondeterministic TM, and so one direction of this corollary follows immediately. The other direction follows from Theorem 3.16.

ماشین تورینگ نامعین

COROLLARY 3.19

A language is decidable if and only if some nondeterministic Turing machine decides it.