به نام خدا

دانشکده برق و کامپیوتر

فصل دوم: آشنایی با سلف و خازن و تحلیل مدارهای مرتبه اول RL و RC

ارائه کننده:

روحانی

فروردین ۱۴۰۳

دانشکده برق و کامپیوتر

تعاریف اولیه

خازن الکتریکی

$$i = \frac{dq}{dt} \qquad \boxed{i = C\frac{dv}{dt}} \qquad [1]$$

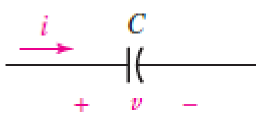

FIGURE 7.1 Electrical symbol and current-voltage conventions for a capacitor.

where v and i satisfy the conventions for a passive element, as shown in Fig. 7.1. We should bear in mind that v and i are functions of time; if needed, we can emphasize this fact by writing $v(t)$ and $i(t)$ instead. From Eq. [1], we may determine the unit of capacitance as an ampere-second per volt, or c$C = \varepsilon A/d,$olt. We will now define the ***farad***[1] (F) as one coulomb per volt, and use this as our unit of capacitance.

(a) (b) (c)

$$C = \varepsilon A/d,$$

FIGURE 7.2 Several examples of commercially available capacitors. (*a*) Left to right: 270 pF ceramic, 20 μF tantalum, 15 nF polyester, 150 nF polyester. (*b*) Left: 2000 μF 40 VDC rated electrolytic, 25,000 μF 35 VDC rated electrolytic. (*c*) Clockwise from smallest: 100 μF 63 VDC rated electrolytic, 2200 μF 50 VDC rated electrolytic, 55 F 2.5 VDC rated electrolytic, and 4800 μF 50 VDC rated electrolytic. Note that generally speaking larger capacitance values require larger packages, with one notable exception above. What was the tradeoff in that case?

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The capacitor voltage may be expressed in terms of the current by integrating Eq. [1]. We first obtain

$$dv = \frac{1}{C} i(t)\,dt$$

and then integrate[2] between the times t_0 and t and between the corresponding voltages $v(t_0)$ and $v(t)$:

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)} \qquad [2]$$

Equation [2] may also be written as an indefinite integral plus a constant of integration:

$$v(t) = \frac{1}{C}\int i\,dt + k$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt'$$

Since the integral of the current over any time interval is the corresponding charge accumulated on the capacitor plate into which the current is flowing, we may also define capacitance as

$$q(t) = Cv(t)$$

where $q(t)$ and $v(t)$ represent instantaneous values of the charge on either plate and the voltage between the plates, respectively.

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$$p = vi = Cv\frac{dv}{dt}$$

The change in energy stored in its electric field is simply

$$\int_{t_0}^{t} p\,dt' = C\int_{t_0}^{t} v\frac{dv}{dt'}\,dt' = C\int_{v(t_0)}^{v(t)} v'\,dv' = \frac{1}{2}C\left\{[v(t)]^2 - [v(t_0)]^2\right\}$$

and thus

$$w_C(t) - w_C(t_0) = \tfrac{1}{2}C\left\{[v(t)]^2 - [v(t_0)]^2\right\} \qquad [3]$$

where the stored energy is $w_C(t_0)$ in joules (J) and the voltage at t_0 is $v(t_0)$. If we select a zero-energy reference at t_0, implying that the capacitor voltage is also zero at that instant, then

$$\boxed{w_C(t) = \tfrac{1}{2}Cv^2} \qquad [4]$$

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تعاریف اولیه

سلف الکتریکی

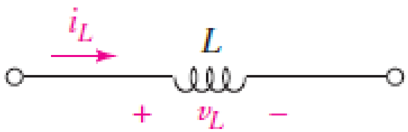
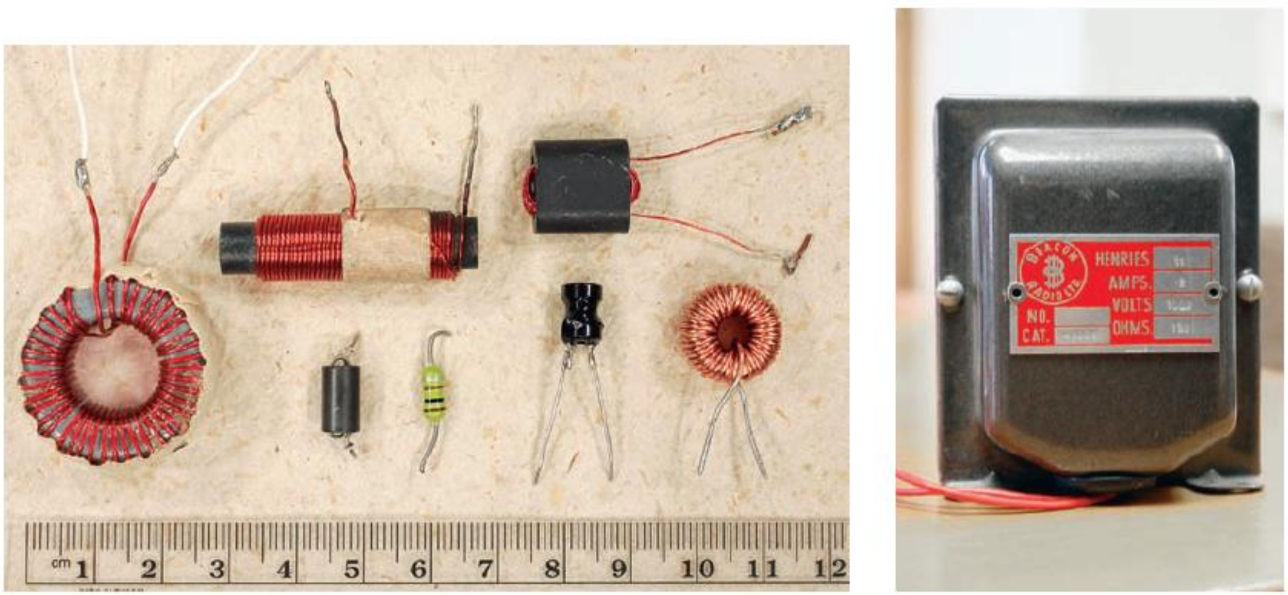


FIGURE 7.10 Electrical symbol and current-voltage conventions for an inductor.

$$\boxed{v = L\frac{di}{dt}}$$



(a) (b)

FIGURE 7.11 (*a*) Several different types of commercially available inductors, sometimes also referred to as "chokes." Clockwise, starting from far left: 287 μH ferrite core toroidal inductor, 266 μH ferrite core cylindrical inductor, 215 μH ferrite core inductor designed for VHF frequencies, 85 μH iron powder core toroidal inductor, 10 μH bobbin-style inductor, 100 μH axial lead inductor, and 7 μH lossy-core inductor used for RF suppression. (*b*) An 11 H inductor, measuring 10 cm (tall) $\times$ 8 cm (wide) $\times$ 8 cm (deep).

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We have defined inductance by a simple differential equation,

$$v = L\frac{di}{dt}$$

and we have been able to draw several conclusions about the characteristics of an inductor from this relationship. For example, we have found that we may consider an inductor to be a short circuit to direct current, and we have agreed that we cannot permit an inductor current to change abruptly from one value to another, because this would require that an infinite voltage and power be associated with the inductor. The simple defining equation for inductance contains still more information, however. Rewritten in a slightly different form,

$$di = \frac{1}{L}v\,dt$$

it invites integration. Let us first consider the limits to be placed on the two integrals. We desire the current i at time t, and this pair of quantities therefore provides the upper limits on the integrals appearing on the left and right sides of the equation, respectively; the lower limits may also be kept general by merely assuming that the current is $i(t_0)$ at time t_0. Thus,

$$\int_{i(t_0)}^{i(t)} di' = \frac{1}{L}\int_{t_0}^{t} v(t')\,dt'$$

which leads to the equation

$$i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v\,dt'$$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)} \qquad [6]$$

Equation [5] expresses the inductor voltage in terms of the current, whereas Eq. [6] gives the current in terms of the voltage. Other forms are

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Let us now turn our attention to power and energy. The absorbed power is given by the current-voltage product

$$p = vi = Li\,\frac{di}{dt}$$

The energy w_L accepted by the inductor is stored in the magnetic field around the coil. The change in this energy is expressed by the integral of the power over the desired time interval:

$$\int_{t_0}^{t} p\,dt' = L\int_{t_0}^{t} i\,\frac{di}{dt'}\,dt' = L\int_{i(t_0)}^{i(t)} i'\,di'$$
$$= \frac{1}{2}L\left\{[i(t)]^2 - [i(t_0)]^2\right\}$$

Thus,

$$w_L(t) - w_L(t_0) = \tfrac{1}{2}L\left\{[i(t)]^2 - [i(t_0)]^2\right\} \qquad [9]$$

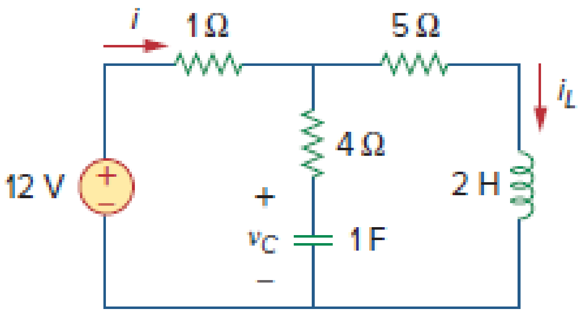
where we have again assumed that the current is $i(t_0)$ at time t_0. In using the energy expression, it is customary to assume that a value of t_0 is selected at which the current is zero; it is also customary to assume that the energy is zero at this time. We then have simply

$$\boxed{w_L(t) = \tfrac{1}{2}Li^2} \qquad [10]$$

تعاریف اولیه: سلف
انرژی ذخیره شده میان سیم پیچ های سلف: مثال

Example 6.10

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



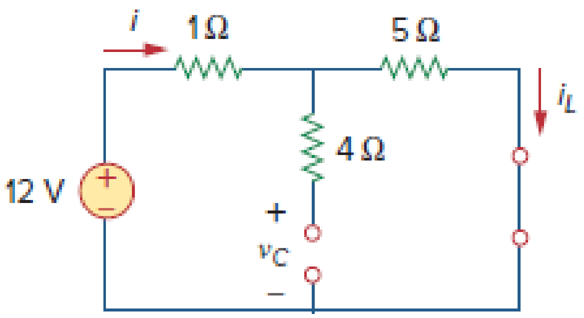


Figure 6.27
For Example 6.10.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1+5} = 2\text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10\text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50\text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4\text{ J}$$

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مدارهای مرتبه اول

پاسخ گذرای مدار RL (پاسخ ورودی صفر)

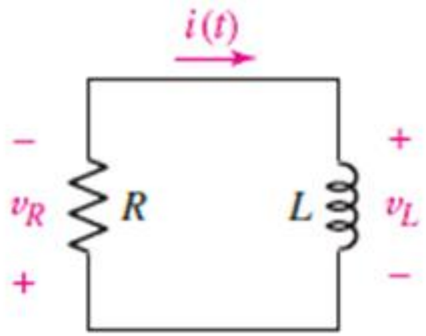

FIGURE 8.1 A series *RL* circuit for which $i(t)$ is to be determined, subject to the initial condition that $i(0) = I_0$.

$$Ri + L\frac{di}{dt} = 0$$

$$R + Lr = 0 \rightarrow r = -\frac{R}{L} \rightarrow i(t) = i_h(t) = Ke^{rt} = Ke^{-\frac{R}{L}t}$$

$$\xrightarrow{i_L(0)=I_0} I_0 = K \rightarrow i(t) = I_0 e^{-\frac{R}{L}t}$$

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مدارهای مرتبه اول
مدار RL : مثال

If the inductor of Fig. 8.2 has a current $i_L = 2$ A at $t = 0$, find an expression for $i_L(t)$ valid for $t > 0$, and its value at $t = 200\ \mu$s.

This is the identical type of circuit just considered, so we expect an inductor current of the form

$$i_L = I_0 e^{-Rt/L}$$

where $R = 200\ \Omega$, $L = 50$ mH and I_0 is the initial current flowing through the inductor at $t = 0$. Thus,

$$i_L(t) = 2e^{-4000t}$$

Substituting $t = 200 \times 10^{-6}$ s, we find that $i_L(t) = 898.7$ mA, less than half the initial value.

PRACTICE

8.1 Determine the current i_R through the resistor of Fig. 8.3 at $t = 1$ ns if $i_R(0) = 6$ A.

Ans: 812 mA.

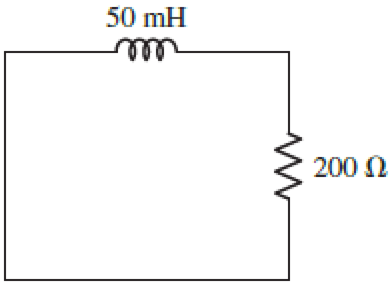

FIGURE 8.2 A simple *RL* circuit in which energy is stored in the inductor at $t = 0$.

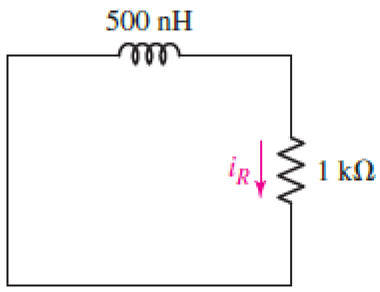

FIGURE 8.3 Circuit for Practice Problem 8.1.

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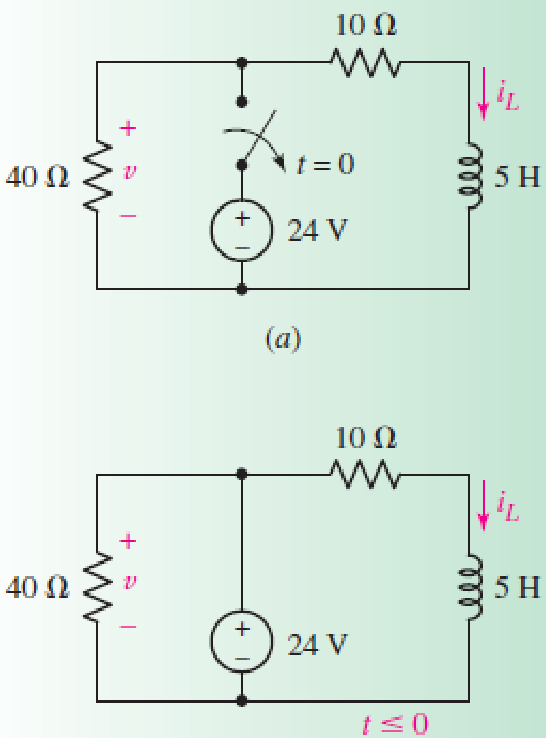
مدارهای مرتبه اول

مدار RL : مثال (پاسخ ورودی صفر)



(*a*)

(*b*)

For the circuit of Fig. 8.5*a*, find the voltage labeled v at $t = 200$ ms.

- ***Identify the goal of the problem.***

The schematic of Fig. 8.5*a* actually represents *two different* circuits: one with the switch closed (Fig. 8.5*b*) and one with the switch open (Fig. 8.5*c*). We are asked to find $v(0.2)$ for the circuit shown in Fig. 8.5*c*.

- ***Collect the known information.***

Both new circuits are drawn and labeled correctly. We next make the assumption that the circuit in Fig. 8.5*b* has been connected for a long time, so that any transients have dissipated. We may make such an assumption as a general rule unless instructed otherwise. This circuit determines $i_L(0)$.

- ***Devise a plan.***

The circuit of Fig. 8.5*c* may be analyzed by writing a KVL equation. Ultimately we want a differential equation with only v and t as variables; we will then solve the differential equation for $v(t)$.

- ***Construct an appropriate set of equations.***

Referring to Fig. 8.5*c*, we write

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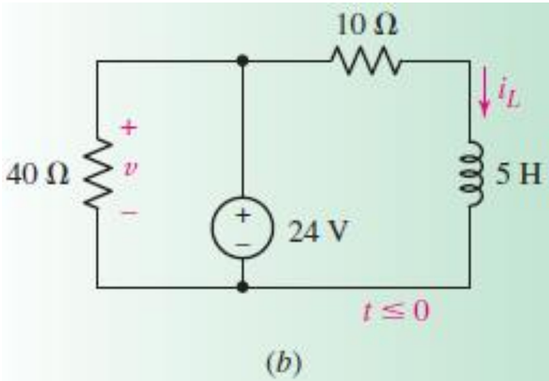
مدار RL: (پاسخ ورودی صفر) ادامه مثال



$$t < 0$$

$$KVL: 24 = 10 i_L \rightarrow i_L(0^-) = 2.4A, v(0^-) = 24v$$

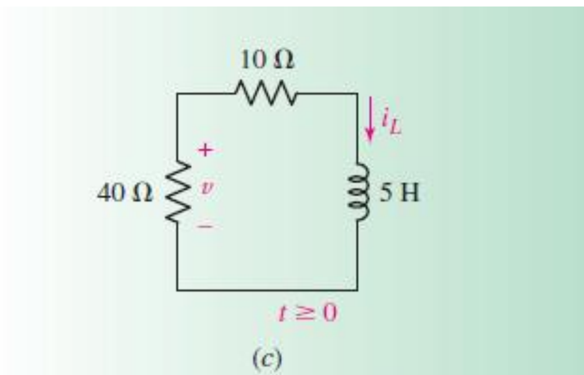


FIGURE 8.5 (*a*) A simple *RL* circuit with a switch thrown at time $t = 0$. (*b*) The circuit as it exists prior to $t = 0$. (*c*) The circuit after the switch is thrown, and the 24 V source is removed.

$$t > 0$$

$$KVL: 40 i_L + 10 i_L + 5\frac{di_L}{dt} = 0 \rightarrow 5\frac{di_L}{dt} + 50 i_L = 0$$

$$5r + 50 = 0 \rightarrow r = -10 \rightarrow i_L = Ke^{-10t} \xrightarrow{i_L(0^-)=i_L(0^+)=2.4A} K = 2.4$$

$$i_L(t) = 2.4e^{-10t} \rightarrow v(t) = -40 i_L(t) = -96e^{-10t}$$

$$i_L(t) = \begin{cases} 2.4 & t \le 0 \\ 2.4e^{-10t} & t > 0 \end{cases}, v(t) = \begin{cases} 24 & t \le 0 \\ -96e^{-10t} & t > 0 \end{cases}$$

$$v(0.2s) = -12.99v$$

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مدارهای مرتبه اول

مدار RL : (پاسخ ورودی صفر) انرژی **اولیه** ذخیره شده در سلف

Before we turn our attention to the interpretation of the response, let us return to the circuit of Fig. 8.1, and check the power and energy relationships. The power being dissipated in the resistor is

$$p_R = i^2 R = I_0^2 R e^{-2Rt/L}$$

and the total energy turned into heat in the resistor is found by integrating the instantaneous power from zero time to infinite time:

$$w_R = \int_0^\infty p_R \, dt = I_0^2 R \int_0^\infty e^{-2Rt/L} \, dt$$
$$= I_0^2 R \left(\frac{-L}{2R} \right) e^{-2Rt/L} \Big|_0^\infty = \frac{1}{2} L I_0^2$$

This is the result we expect, because the total energy stored initially in the inductor is $\frac{1}{2} L I_0^2$, and there is no longer any energy stored in the inductor at infinite time since its current eventually drops to zero. All the initial energy therefore is accounted for by dissipation in the resistor.

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مدارهای مرتبه اول

مدار RL : (پاسخ ورودی صفر) خواص پاسخ نمایی مدار RL

$$i(t) = I_0 e^{-Rt/L}$$

At $t = 0$, the current has value I_0, but as time increases, the current decreases and approaches zero. The shape of this decaying exponential is seen by the plot of $i(t)/I_0$ versus t shown in Fig. 8.7. Since the function we are plotting is $e^{-Rt/L}$, the curve will not change if R/L remains unchanged. Thus, the same curve must be obtained for every series RL circuit having the same L/R ratio. Let us see how this ratio affects the shape of the curve.

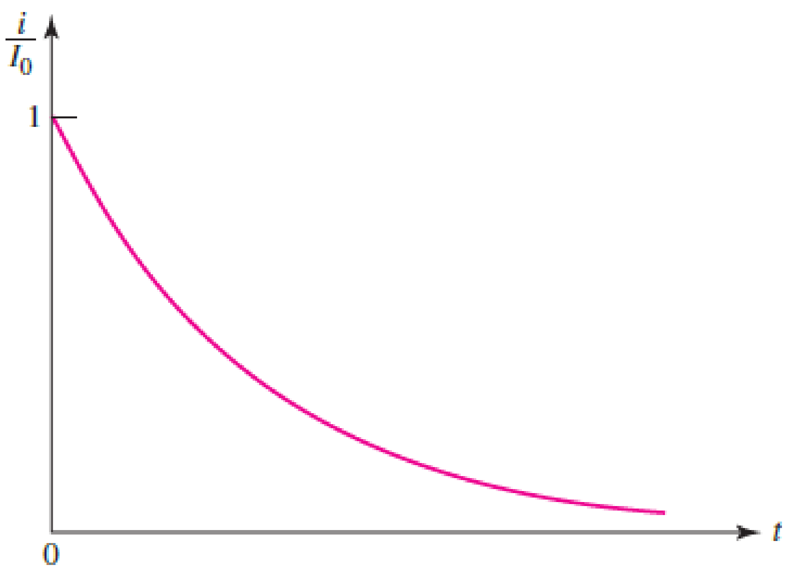

FIGURE 8.7 A plot of $e^{-Rt/L}$ versus t.

$$\boxed{\tau = \frac{L}{R}}$$

مدارهای مرتبه اول

مدار RL : (پاسخ ورودی صفر) خواص پاسخ نمایی مدار RL

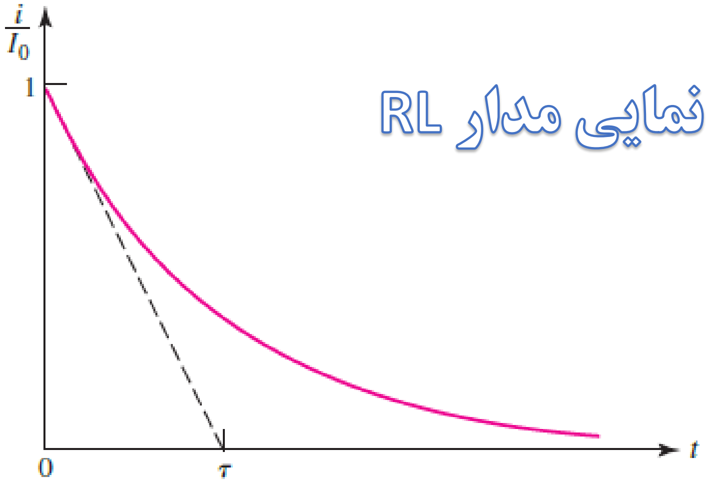

FIGURE 8.8 The time constant τ is L/R for a series RL circuit. It is the time required for the response curve to drop to zero if it decays at a constant rate equal to its initial rate of decay.

An equally important interpretation of the time constant τ is obtained by determining the value of $i(t)/I_0$ at $t = \tau$. We have

$$\frac{i(\tau)}{I_0} = e^{-1} = 0.3679 \qquad \text{or} \qquad i(\tau) = 0.3679 I_0$$

Thus, in one time constant the response has dropped to 36.8 percent of its initial value; the value of τ may also be determined graphically from this fact, as indicated by Fig. 8.9. It is convenient to measure the decay of the current at intervals of one time constant, and recourse to a hand calculator shows that $i(t)/I_0$ is 0.3679 at $t = \tau$, 0.1353 at $t = 2\tau$, 0.04979 at $t = 3\tau$, 0.01832 at $t = 4\tau$, and 0.006738 at $t = 5\tau$. At some point three to five time constants after zero time, most of us would agree that the current is a negligible fraction of its former self. Thus, if we are asked, "*How long does it take for the current to decay to zero?*" our answer might be, "*About five time constants.*" At that point, the current is less than 1 percent of its original value!

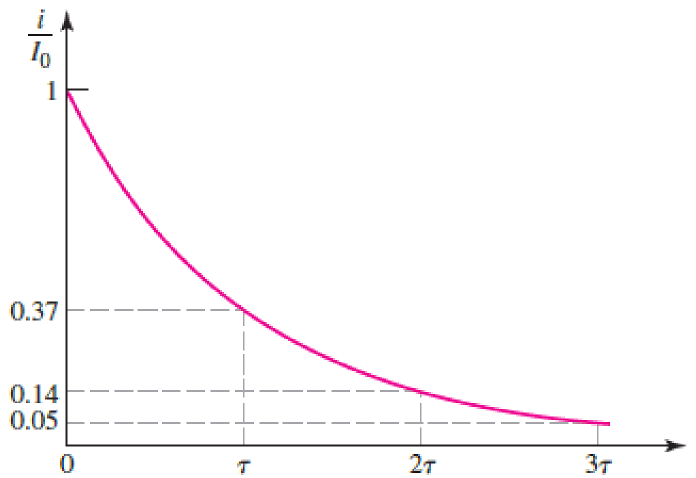

FIGURE 8.9 The current in a series RL circuit is reduced to 37 percent of its initial value at $t = \tau$, 14 percent at $t = 2\tau$, and 5 percent at $t = 3\tau$.

مدارهای مرتبه اول

مدار RC (پاسخ ورودی صفر)

Let us see how closely the analysis of the parallel (or is it series?) *RC* circuit shown in Fig. 8.15 corresponds to that of the *RL* circuit. We will assume an initial stored energy in the capacitor by selecting

$$v(0) = V_0$$

$$KVL: Ri_R(t) - v = 0 \rightarrow -Ri_C(t) - v = 0 \rightarrow -RC\frac{dv}{dt} - v = 0$$

$$KCL\ A: i_C(t) + i_R(t) = 0 \rightarrow C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$Cr + \frac{1}{R} = 0 \rightarrow r = -\frac{1}{RC} \Rightarrow v(t) = Ke^{-\frac{t}{RC}}$$

$$\xrightarrow{v(0)=V_0} V_0 = K$$

enables us to immediately write

$$v(t) = v(0)e^{-t/RC} = V_0 e^{-t/RC} \qquad [14]$$

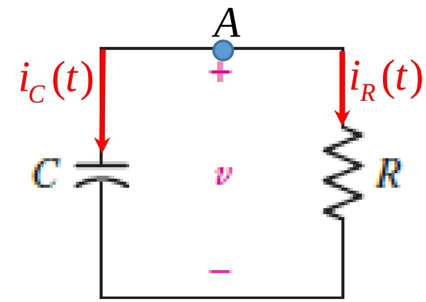

FIGURE 8.15 A parallel *RC* circuit for which $v(t)$ is to be determined, subject to the initial condition that $v(0) = V_0$.

ثابت زمانی مدار RC

$$\boxed{\tau = RC}$$

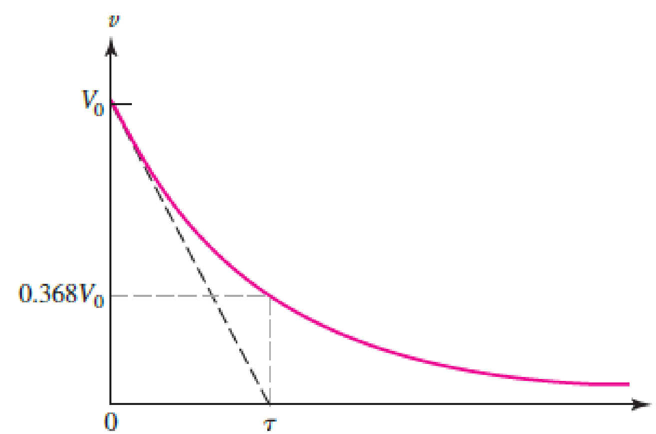

FIGURE 8.16 The capacitor voltage $v(t)$ in the parallel *RC* circuit is plotted as a function of time. The initial value of $v(t)$ is V_0.

Let us discuss the physical nature of the voltage response of the *RC* circuit as expressed by Eq. [14]. At $t = 0$ we obtain the correct initial condition, and as t becomes infinite, the voltage approaches zero. This latter result agrees with our thinking that if there were any voltage remaining across the capacitor, then energy would continue to flow into the resistor and be dissipated as heat. *Thus, a final voltage of zero is necessary*. The time constant of the *RC*

مدارهای مرتبه اول
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$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}} \tag{7.53}$$

where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value. Thus, to find the step response of an *RC* circuit requires three things:

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ.

We obtain item 1 from the given circuit for $t < 0$ and items 2 and 3 from the circuit for $t > 0$. Once these items are determined, we obtain the response using Eq. (7.53). This technique equally applies to *RL* circuits, as we shall see in the next section.

Note that if the switch changes position at time $t = t_0$ instead of at $t = 0$, there is a time delay in the response so that Eq. (7.53) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau} \tag{7.54}$$

where $v(t_0)$ is the initial value at $t = t_0^+$. Keep in mind that Eq. (7.53) or (7.54) applies only to step responses, that is, when the input e xcitation is constant.

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مدارهای مرتبه اول

روش ثابت زمانی در حالت پاسخ ورودی صفر

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}} \qquad (7.53)$$

$$v(\infty) = 0v \rightarrow v(t) = 0 + \left[v(0) - 0\right]e^{-\frac{t}{\tau}}$$

$$\boxed{v(t) = v(0)e^{\frac{-t}{\tau}}}$$

مدارهای مرتبه اول

مدار RC : مثال (پاسخ ورودی صفر)

For the circuit of Fig. 8.17*a*, find the voltage labeled v at $t = 200\ \mu$s.

To find the requested voltage, we will need to draw and analyze two separate circuits: one corresponding to before the switch is thrown (Fig. 8.17*b*), and one corresponding to after the switch is thrown (Fig. 8.17*c*).

The sole purpose of analyzing the circuit of Fig. 8.17*b* is to obtain an initial capacitor voltage; we assume any transients in that circuit died out long ago, leaving a purely dc circuit. With no current through either the capacitor or the 4 Ω resistor, then,

$$v(0) = 9 \text{ V} \qquad [17]$$

روش اول: تشکیل معادله دیفرانسیل

$$KVL: v + 2i_C(t) + 4i_C(t) = 0 \rightarrow v + 6C\frac{dv}{dt} = 0$$

$$1 + 6Cr = 0 \rightarrow r = -\frac{1}{6C} \Rightarrow v(t) = Ke^{\frac{-t}{6C}}$$

$$\xrightarrow{v(0^+)=V(0^-)=9} K = 9 \rightarrow v(t) = 9e^{\frac{-t}{60\times10^{-6}}}$$

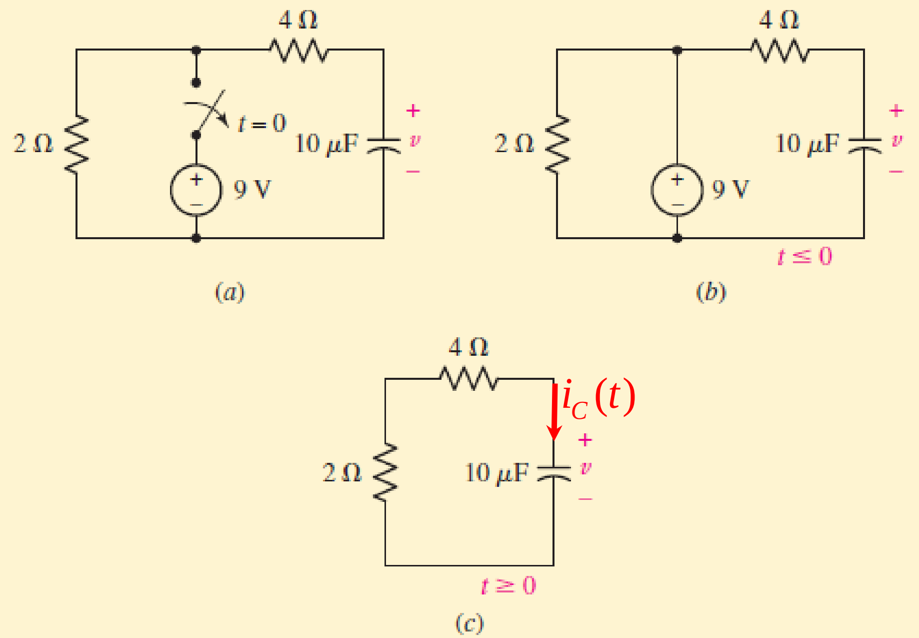
روش دوم: ثابت زمانی کل مدار

We next turn our attention to the circuit of Fig. 8.17*c*, recognizing that

$$\tau = RC = (2+4)(10 \times 10^{-6}) = 60 \times 10^{-6} \text{ s}$$

Thus, from Eq. [14],

$$v(t) = v(0)e^{-t/RC} = v(0)e^{-t/60\times10^{-6}} \qquad [18]$$



■ **FIGURE 8.17** (*a*) A simple *RC* circuit with a switch thrown at time $t = 0$. (*b*) The circuit as it exists prior to $t = 0$. (*c*) The circuit after the switch is thrown, and the 9 V source is removed.

The capacitor voltage must be the same in both circuits at $t = 0$; no such restriction is placed on any other voltage or current. Substituting Eq. [17] into Eq. [18],

$$v(t) = 9e^{-t/60\times10^{-6}} \text{ V}$$

so that $v(200 \times 10^{-6}) = 321.1$ mV (less than 4 percent of its maximum value).

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ ورودی صفر مدار RL حالت کلی‌تر: تمرین (به عهده دانشجو)

Example 7.5

In the circuit shown in Fig. 7.19, find i_o, v_o, and i for all time, assuming that the switch was open for a long time.

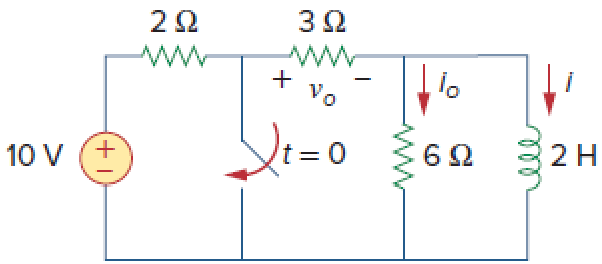


Figure 7.19
For Example 7.5.

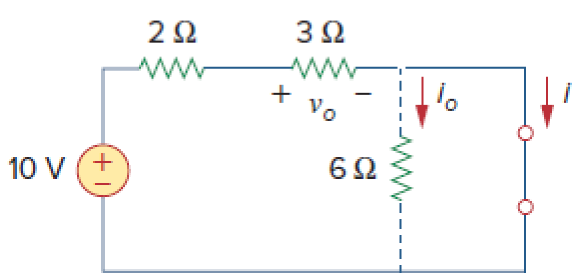
Solution:
It is better to first find the inductor current i and then obtain other quantities from it.

For $t < 0$, the switch is open. Since the inductor acts like a short circuit to dc, the 6-Ω resistor is short-circuited, so that we ha ve the circuit shown in Fig. 7.20(a). Hence, $i_o = 0$, and

$$i(t) = \frac{10}{2+3} = 2 \text{ A}, \qquad t < 0$$

$$v_o(t) = 3i(t) = 6 \text{ V}, \qquad t < 0$$

Thus, $i(0) = 2$.



(a)

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ ورودی صفر مدار RL حالت کلی‌تر: ادامه تمرین (به عهده دانشجو)

روش اول: تشکیل معادله دیفرانسیل

$$v_O + v_L = 0 \rightarrow 3\left(i_O + i_L\right) + L\frac{di_L}{dt} = 0 \xrightarrow{i_O = \frac{v_L}{6} = \frac{L}{6}\frac{di_L}{dt}} 3i_L + \frac{3}{2}L\frac{di_L}{dt} = 0$$

$$3 + 3r = 0 \rightarrow r = -1 \rightarrow i_L = Ke^{-t} \xrightarrow{i_L(0^-) = i_L(0^+) = 2} i_L(t) = 2e^{-t}$$

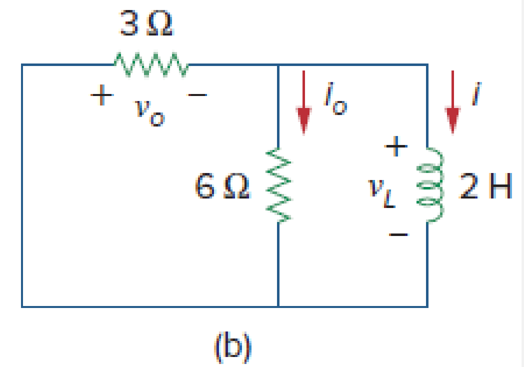


Figure 7.20
The circuit in Fig. 7.19 for: (a) $t < 0$, (b) $t > 0$.

روش دوم: ثابت زمانی کل مدار

For $t > 0$, the switch is closed, so that the v oltage source is short-circuited. We now have a source-free RL circuit as shown in Fig. 7.20(b). At the inductor terminals,

$$R_{\text{Th}} = 3 \parallel 6 = 2\ \Omega$$

$$\tau = \frac{L}{R_{\text{Th}}} = 1\ \text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t}\text{A}, \qquad t > 0$$

محاسبه سایر پارامترهای مدار:

$$i_O = \frac{v_L}{6} = \frac{L}{6}\frac{di_L}{dt} = \frac{1}{3}\left(-2e^{-t}\right) = \frac{-2}{3}e^{-t}$$

$$v_O = 3\left(i_O + i_L\right) = 3\left(\frac{-2}{3}e^{-t} + 2e^{-t}\right) = 4e^{-t}$$

$$i_O(0^+) = -i(0^+) \times \frac{3}{9} = -\frac{1}{3}i(0^+) = -\frac{2}{3} \rightarrow i_o(t) = -\frac{2}{3}e^{-t}$$

$$v_O(0^+) = 3 \times i(0^+) \times \frac{6}{9} = 2i(0^+) = 4 \rightarrow v_o(t) = 4e^{-t}$$

مدارهای مرتبه اول

پاسخ ورودی صفر مدار RL حالت کلی‌تر: ادامه **تمرین** **(به عهده دانشجو)**

Thus, for all time,

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\dfrac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \qquad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

We notice that the inductor current is continuous at $t = 0$, while the current through the 6-Ω resistor drops from 0 to $-2/3$ at $t = 0$, and the voltage across the 3-Ω resistor drops from 6 to 4 at $t = 0$. We also notice that the time constant is the same regardless of what the output is defined to be. Figure 7.21 plots i and i_o.

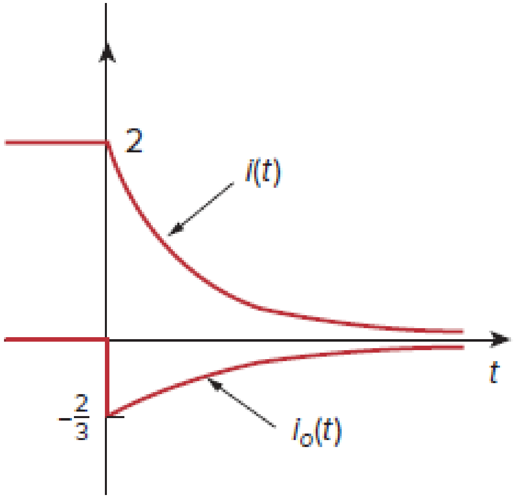


Figure 7.21
A plot of i and i_o.

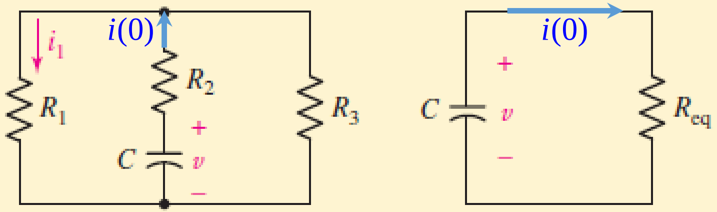
دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: مثال

(پاسخ ورودی صفر): روش ثابت زمانی کل مدار

Find $v(0^+)$ and $i_1(0^+)$ for the circuit shown in Fig. 8.22*a* if $v(0^-) = V_0$.



(*a*) (*b*)

FIGURE 8.22 (*a*) A given circuit containing one capacitor and several resistors. (*b*) The resistors have been replaced by a single equivalent resistor; the time constant is simply $\tau = R_{eq}C$.

We first simplify the circuit of Fig. 8.22*a* to that of Fig. 8.22*b*, enabling us to write

$$v = V_0 e^{-t/R_{eq}C}$$

where

$$v(0^+) = v(0^-) = V_0 \qquad \text{and} \qquad R_{eq} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Every current and voltage in the resistive portion of the network must have the form $Ae^{-t/R_{eq}C}$, where A is the initial value of that current or voltage. Thus, the current in R_1, for example, may be expressed as

$$i_1 = i_1(0^+)e^{-t/\tau}$$

where

$$\tau = \left(R_2 + \frac{R_1 R_3}{R_1 + R_3}\right) C$$

$$-V_0 + R_{eq} i(0) = 0 \to i(0) = \frac{V_0}{R_{eq}}$$

$$i_1(0) = i(0) \times \frac{R_3}{R_1 + R_3} = \frac{V_0 R_3}{R_{eq}\left(R_1 + R_3\right)}$$

$$i_1(t) = \frac{V_0 R_3}{R_{eq}\left(R_1 + R_3\right)} e^{\frac{-t}{R_{eq}C}}$$

$$i_1(t) = \frac{R_3 V_0}{R_2\left(R_1 + R_3\right) + R_1 R_3} e^{\frac{-t}{R_{eq}C}}$$

$$i_1(0) = \frac{R_3 V_0}{R_2\left(R_1 + R_3\right) + R_1 R_3}$$

مدارهای مرتبه اول
مدار RC: حالت کلی تر: تمرین (به عهده دانشجو)

PRACTICE

8.6 Find values of v_C and v_o in the circuit of Fig. 8.23 at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) 1.3 ms.

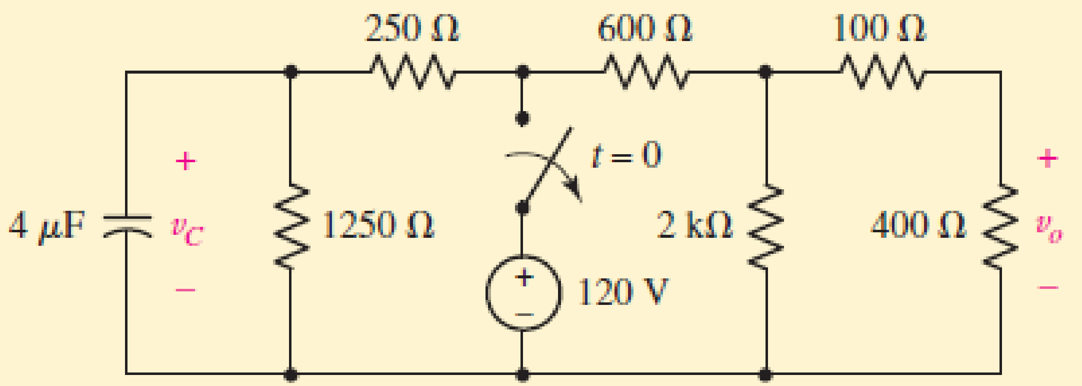


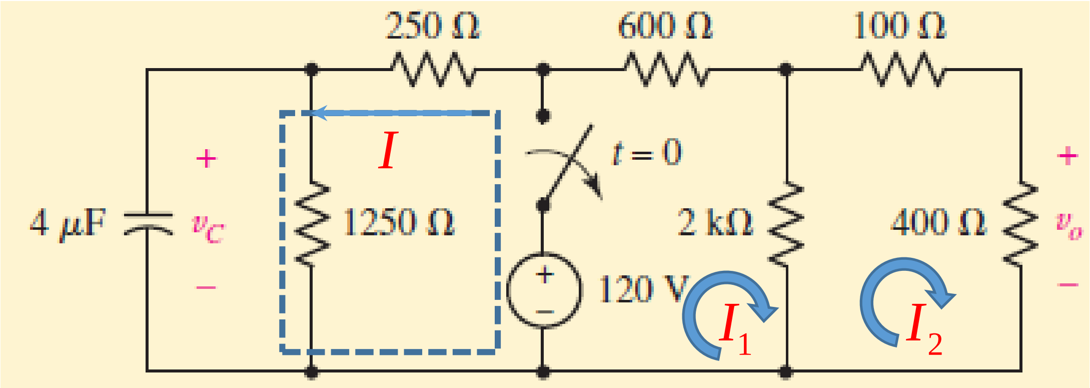
FIGURE 8.23

Ans: 100 V, 38.4 V; 100 V, 25.6 V; 59.5 V, 15.22 V.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه تمرین (به عهده دانشجو)



$t < 0:$

$$KVL: -120 + 250I + 1250I = 0 \to I = 0.08A$$

$$v_C(0^-) = 1250I = 100v$$

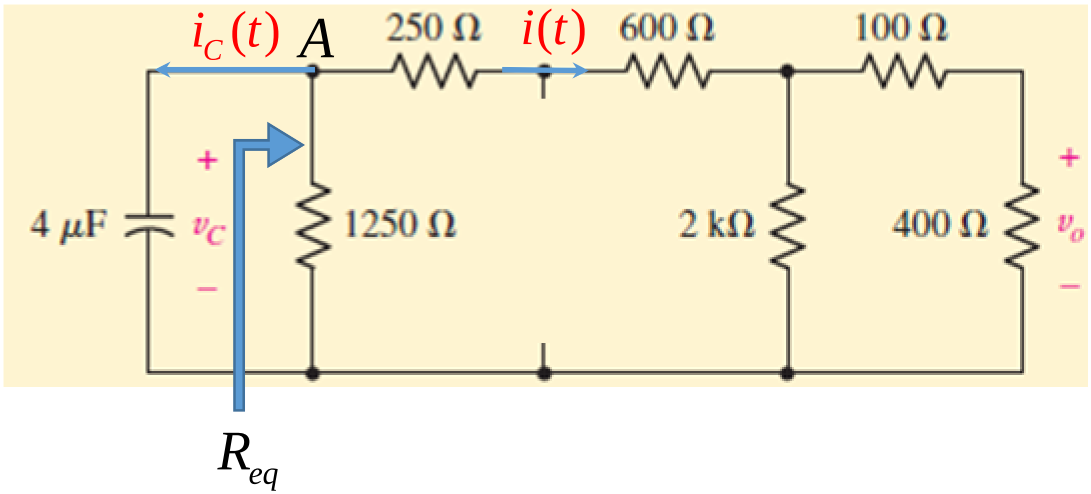
$$\begin{cases} KVL\ I: 120 = 600I_1 + 2000(I_1 - I_2) \\ KVL\ II: 0 = 500I_2 + 2000(I_2 - I_1) \end{cases} \to I_2 = 0.096A \to v_o(0^-) = 400I_2 = 38.4v$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر:(پاسخ ورودی صفر) ادامه تمرین (به عهده دانشجو)

روش اول:

محاسبه ثابت زمانی کل مدار



$$t>0: R_{eq}=\left(2000\Vert(400+100)+600+250\right)\Vert 1250=625\Omega \rightarrow \tau=R_{eq}C=2.5ms$$

$$v_C(t)=Ke^{-\frac{t}{\tau}}=Ke^{-400t}\xrightarrow{v_C(0^-)=v_C(0^+)=100}K=100\rightarrow v_C(t)=100e^{-400t},i_C(t)=C\frac{dv_C}{dt}=-0.16e^{-400t}$$

$$KCL\ A: i(t)=-i_C(t)-\frac{v_C(t)}{1250}=0.16e^{-400t}-0.08e^{-400t}=0.08e^{-400t}\rightarrow v_o(t)=400\left(i(t)\frac{2000}{2000+500}\right)=25.6e^{-400t}$$

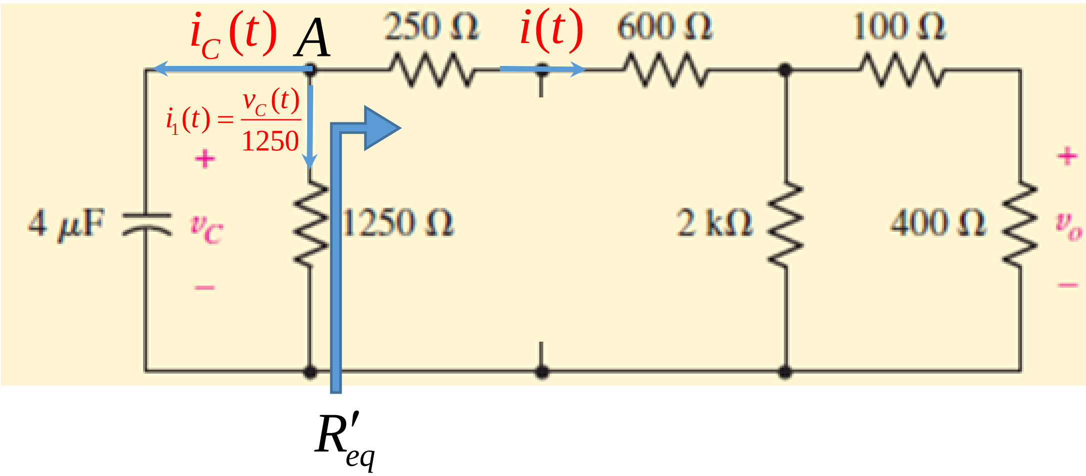
$$v_C(t)=\begin{cases}100 & t\le 0\\ 100e^{-400t} & t>0\end{cases},v_o(t)=\begin{cases}38.4 & t<0\\ 25.6e^{-400t} & t>0\end{cases}\rightarrow\begin{cases}v_C(0^-)=v_C(0^+)=100,v_C(1.3ms)=59.45v\\ v_o(0^-)=38.4v,v_o(0^+)=25.6,v_o(1.3ms)=15.21v\end{cases}$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر: (پاسخ ورودی صفر) ادامه **تمرین** (به عهده دانشجو)

روش دوم:

تشکیل معادله دیفرانسیل



$$t>0: R'_{eq}=500\|2000+850=1250\Omega \rightarrow Node\ A: i_1(t)=\frac{v_C}{1250}=-i_C(t)\frac{R'_{eq}}{1250+R'_{eq}}$$

$$\frac{v_C}{1250}=-C\frac{dv_C}{dt}\frac{1}{2}\rightarrow -\frac{C}{2}\frac{dv_C}{dt}-\frac{v_C}{1250}=0\rightarrow -r\frac{C}{2}-\frac{1}{1250}=0\rightarrow r=\frac{-2}{1250C}=-400$$

$$v_C(t)=Ke^{-400t}\xrightarrow{v_C(0^-)=v_C(0^+)=100}K=100\rightarrow v_C(t)=100e^{-400t}, i_C(t)=C\frac{dv_C}{dt}=-0.16e^{-400t}$$

$$KCL\ A: i(t)=-i_C(t)-\frac{v_C(t)}{1250}=0.16e^{-400t}-0.08e^{-400t}=0.08e^{-400t}\rightarrow v_o(t)=400\left(i(t)\frac{2000}{2000+500}\right)=25.6e^{-400t}$$

$$v_C(t)=\begin{cases}100 & t\le 0\\ 100e^{-400t} & t>0\end{cases}, v_o(t)=\begin{cases}38.4 & t<0\\ 25.6e^{-400t} & t>0\end{cases}\rightarrow\begin{cases}v_C(0^-)=v_C(0^+)=100, v_C(1.3ms)=59.45v\\ v_o(0^-)=38.4v, v_o(0^+)=25.6, v_o(1.3ms)=15.21v\end{cases}$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر: مثال **منبع وابسته**: راه حل اول **(پاسخ ورودی صفر)**

For the circuit of Fig. 8.24a, find the voltage labeled v_C for $t > 0$ if $v_C(0^-) = 2$ V.

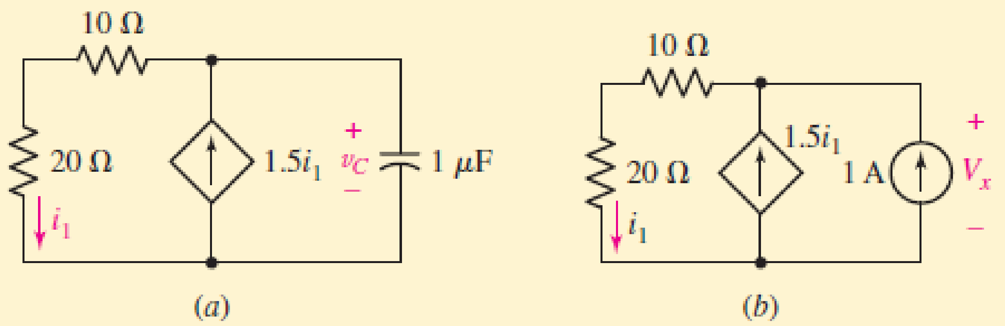


FIGURE 8.24 (a) A simple RC circuit containing a dependent source not controlled by a capacitor voltage or current. (b) Circuit for finding the Thévenin equivalent of the network connected to the capacitor.

The dependent source is not controlled by a capacitor voltage or current, so we can start by finding the Thévenin equivalent of the network to the left of the capacitor. Connecting a 1 A test source as in Fig. 8.24b,

$$V_x = (1 + 1.5i_1)(30)$$

where

$$i_1 = \left(\frac{1}{20}\right)\frac{20}{10+20}V_x = \frac{V_x}{30}$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه مثال منبع وابسته (راه حل اول: ثابت زمانی کل مدار)

Performing a little algebra, we find that $V_x = -60$ V, so the network has a Thévenin equivalent resistance of $-60\ \Omega$ (unusual, but not impossible when dealing with a dependent source). Our circuit therefore has a *negative* time constant

$$\tau = -60(1 \times 10^{-6}) = -60\ \mu s$$

The capacitor voltage is therefore

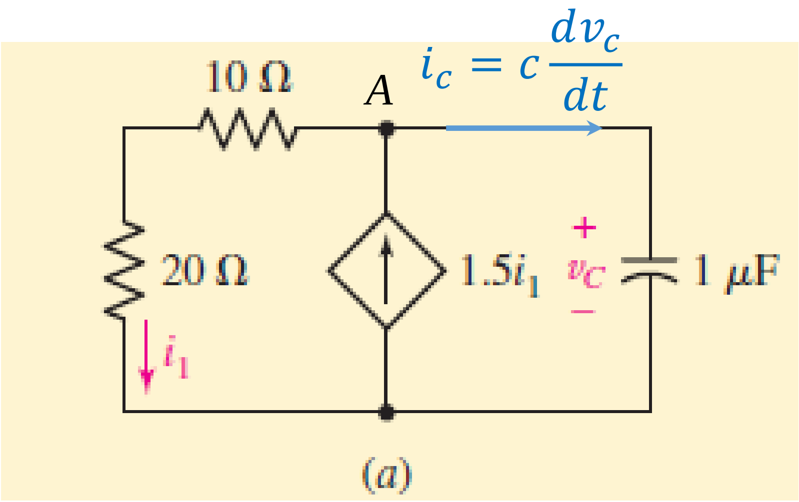
$$v_C(t) = Ae^{t/60\times10^{-6}} \qquad \text{V}$$

where $A = v_C(0^+) = v_C(0^-) = 2$ V. Thus,

$$v_C(t) = 2e^{t/60\times10^{-6}} \qquad \text{V} \qquad [21]$$

which, interestingly enough is unstable: it grows exponentially with time. This cannot continue indefinitely; one or more elements in the circuit will eventually fail.

مدار RC: حالت کلی تر: ادامه مثال منبع وابسته: راه حل دوم (تشکیل معادله دیفرانسیل)



$$KCL\ A: 1.5i_1 = i_1 + i_c \rightarrow 0.5i_1 = C\frac{dv_C}{dt}$$

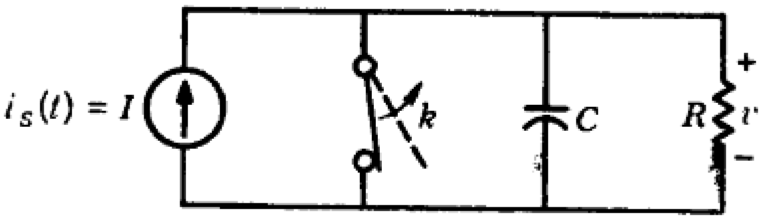
$$i_1 = \frac{v_C}{30} \rightarrow C\frac{dv_C}{dt} - \frac{v_C}{60} = 0, Cr - \frac{1}{60} = 0 \rightarrow r = \frac{1}{60C}$$

$$v_C(t) = Ke^{\frac{t}{60C}} \xrightarrow{v_C(0)=2v} K = 2, v_C(t) = 2e^{\frac{t}{60C}} = 2e^{\frac{t}{60\times10^{-6}}}$$

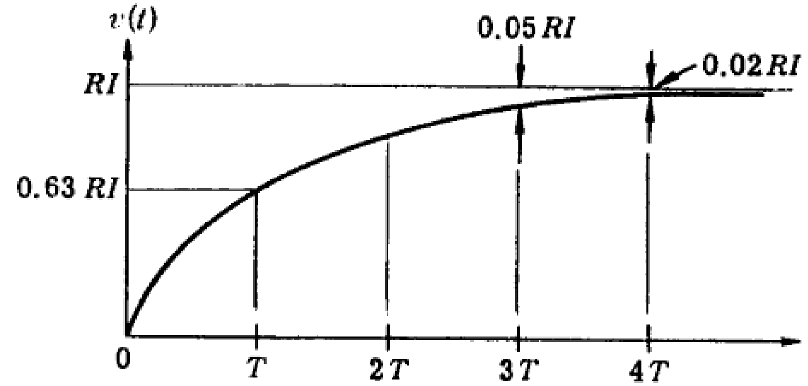
دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: مثال: (پاسخ حالت صفر)



شکل ۱-۲- مدار RC با ورودی منبع جریان . درلحظه $t=0$ کلید باز میشود .



شکل ۳-۲- پاسخ ولتاژ مدار RC ناشی ازمنبع ثابت I چنانکه درشکل (۲-۱) با $v(0)=0$ نشان داده شده است .

$$C\frac{dv}{dt}+\frac{v}{R}=I \rightarrow \begin{cases} v_h : Cr+\frac{1}{R}=0 \rightarrow r=-\frac{1}{RC} \rightarrow v_h = Ke^{-\frac{t}{RC}} \\ v_p : v_p = A \rightarrow C\times 0+\frac{A}{R}=I \rightarrow A=RI \rightarrow v_p = RI \end{cases}$$

$$v(t)=RI+Ke^{-\frac{t}{RC}} \xrightarrow{v(0^-)=v(0^+)=0} K=-RI \rightarrow v(t)=RI-RIe^{-\frac{t}{RC}}=RI\left(1-e^{-\frac{t}{RC}}\right)$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

تعریف **پاسخ حالت صفر**

در دوحالتی که دراین بخش دیدیم ولتاژ v را پاسخ و منبع جریان i_s را ورودی درنظر گرفتیم . شرط اولیه درمدار صفر بوده یعنی پیش از وارد آوردن ورودی، ولتاژ دوسر خازن برابر با صفر بود . درحالت کلی اگر همه شرطهای اولیه درمدار صفر باشند گوئیم مدار در **حالت صفر** (۱) است+ . پاسخ مداری که ازحالت صفر شروع میکند منحصراً معلول ورودی آنست. بموجب تعریف ، **پاسخ حالت صفر** یک مدار پاسخ آن به یک ورودی است که در زمان دلخواه t_0 بمدار وارد شود بشرط آنکه مدار درست پیش از وارد آوردن این ورودی (یعنی در زمان t_{0-}) درحالت صفر باشد . درمحاسبهٔ پاسخ حالت صفر هدف اصلی، رفتار پاسخ برای $t \geqslant t_0$ است. بدین منظورچنین «قرار میگذاریم» : برای $t < t_0$ ورودی و پاسخ حالت صفر را متحد باصفر میگیریم .

مدارهای مرتبه اول

روش ثابت زمانی در پاسخ حالت صفر

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}} \qquad (7.53)$$

سوئیچ زنی در زمان صفر

$$v(0) = 0v \rightarrow v(t) = v(\infty) + \left[0 - v(\infty)\right]e^{\frac{-t}{\tau}}$$

$$v(t) = v(\infty)\left[1 - e^{\frac{-t}{\tau}}\right]$$

سوئیچ زنی در زمان $t = t_0$

$$v(t_0) = 0v \rightarrow v(t) = v(\infty) + \left[0 - v(\infty)\right]e^{\frac{-(t-t_0)}{\tau}}$$

$$v(t) = v(\infty)\left[1 - e^{\frac{-(t-t_0)}{\tau}}\right]$$

مدارهای مرتبه اول

مدار RL: مثال: (پاسخ حالت صفر) – روش اول: حل معادله دیفرانسیل

PRACTICE

8.11 The circuit shown in Fig. 8.41 has been in the form shown for a very long time. The switch opens at $t = 0$. Find i_R at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) ∞; (*d*) 1.5 ms.

Ans: 0; 10 mA; 4 mA; 5.34 mA.

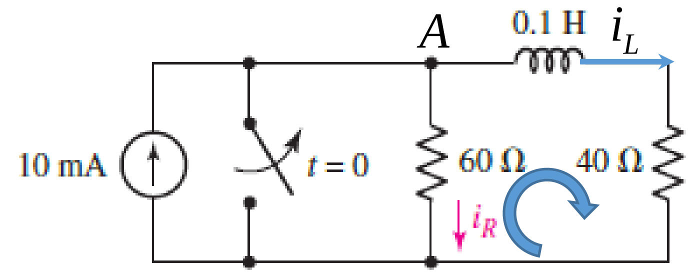


FIGURE 8.41

$$t>0: KCL\ A: 0.01 = i_R + i_L \rightarrow i_R = 0.01 - i_L$$

$$KVL: -60i_R + L\frac{di_L}{dt} + 40i_L = 0 \rightarrow 0.1\frac{di_L}{dt} + 40i_L - 60(0.01 - i_L) = 0$$

$$0.1\frac{di_L}{dt} + 100i_L = 0.6 \rightarrow \begin{cases} i_{Lh}: 0.1r + 100 = 0 \rightarrow r = -1000 \rightarrow i_{Lh} = Ke^{-1000t} \\ i_{Lp}: i_{Lp} = 0.006 \end{cases}$$

$$i_L(t) = i_{Lh} + i_{Lp} = Ke^{-1000t} + 0.006 \xrightarrow{i_L(0^-)=i_L(0^+)=0} i_L(t) = 0.006\left(1 - e^{-1000t}\right)$$

$$i_R(t) = 0.01 - i_L = 0.01 - 0.006\left(1 - e^{-1000t}\right) = 0.004 + 0.006e^{-1000t}$$

$$i_R(t) = \begin{cases} 0 & t<0 \\ 0.004 + 0.006e^{-1000t} & t>0 \end{cases}$$

$$\rightarrow i_R(0^-) = 0A, i_R(0^+) = 0.01A, i_R(\infty) = 0.01\frac{40}{60+40} = 0.004A, i_R(0.0015) = 0.00533A$$

مدارهای مرتبه اول

مدار RL: ادامه مثال: (پاسخ حالت صفر) – روش دوم: ثابت زمانی

PRACTICE

8.11 The circuit shown in Fig. 8.41 has been in the form shown for a very long time. The switch opens at $t = 0$. Find i_R at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) ∞; (*d*) 1.5 ms.

Ans: 0; 10 mA; 4 mA; 5.34 mA.

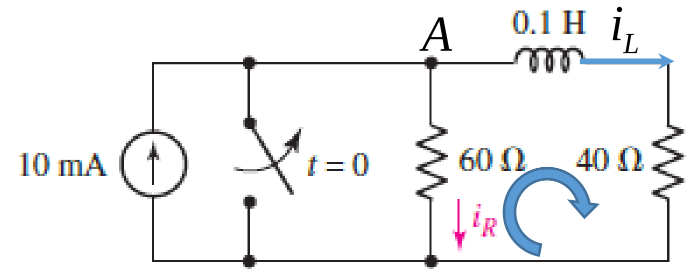


FIGURE 8.41

$$i_L(0^-) = 0A, i_L(\infty) = 0.01 \times \frac{60}{100} = 0.006A$$

$$R_{th} = 100\Omega \rightarrow \tau = \frac{L}{R_{th}} = \frac{0.1}{100} = 0.001s$$

$$i_L(t) = i_L(\infty) + \left[i_L(0^+) - i_L(\infty)\right]e^{\frac{-t}{\tau}} = 0.006 - 0.006e^{-1000t} = 0.006\left(1 - e^{-1000t}\right)$$

$$i_R(t) = 0.01 - i_L = 0.01 - 0.006\left(1 - e^{-1000t}\right) = 0.004 + 0.006e^{-1000t}$$

$$i_R(t) = \begin{cases} 0 & t < 0 \\ 0.004 + 0.006e^{-1000t} & t > 0 \end{cases}$$

$$\rightarrow i_R(0^-) = 0A, i_R(0^+) = 0.01A, i_R(\infty) = 0.01\frac{40}{60+40} = 0.004A, i_R(0.0015) = 0.00533A$$

مدارهای مرتبه اول
تعریف **پاسخ کامل**

۳- پاسخ کامل : حالت گذرا و حالت دائمی

۳-۱- پاسخ کامل

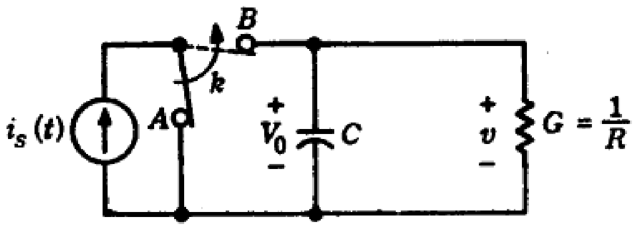
پاسخ یک مدار به تحریک ورودی وشرطهای اولیه رویهم، پاسخ کامل[۱] نام دارد. بنابراین پاسخ ورودی صفر و پاسخ حالت صفر حالتهای خاص پاسخ کامل هستند. دراین بخش نشان خواهیم داد که :

« برای مدار ساده خطی تغییرناپذیر با زمان RC پاسخ کامل برابر است با مجموع پاسخ ورودی صفر و پاسخ حالت صفر آن مدار+. »

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ کامل : مثال



شکل ۱-۳-۴ مدار RC با $v(o)=V_0$ با یک منبع جریان $i_s(t)$ تحریک میشود. درلحظه $t=0$ کلید k ازنقطه A به نقطه B چرخانیده میشود.

$$KCL\ B: i_s(t) = I = C\frac{dv}{dt} + \frac{v}{R}, v(0) = V_0 \rightarrow \begin{cases} v_h: cr + \frac{1}{R} = 0 \rightarrow r = -\frac{1}{RC} \rightarrow v_h(t) = Ke^{-\frac{t}{RC}} \\ v_p: v_p = RI \end{cases}$$

$$v(t) = Ke^{-\frac{t}{RC}} + RI \xrightarrow{v(0^-)=v(0^+)=V_0} V_0 = K + RI \rightarrow K = V_0 - RI$$

$$v(t) = \left(V_0 - RI\right)e^{-\frac{t}{RC}} + RI = V_0 e^{-\frac{t}{RC}} + RI\left(1 - e^{-\frac{t}{RC}}\right)$$

روش ثابت زمانی در پاسخ کامل مدارهای مرتبه اول

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}} \tag{7.53}$$

where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value. Thus, to find the step response of an *RC* circuit requires three things:

Note that if the switch changes position at time $t = t_0$ instead of at $t = 0$, there is a time delay in the response so that Eq. (7.53) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t - t_0)/\tau} \tag{7.54}$$

where $v(t_0)$ is the initial value at $t = t_0^+$. Keep in mind that Eq. (7.53) or (7.54) applies only to step responses, that is, when the input e xcitation is constant.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ کامل: مثال:

Example 7.10

The switch in Fig. 7.43 has been in position A for a long time. At $t = 0$, the switch moves to B. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ and 4 s.

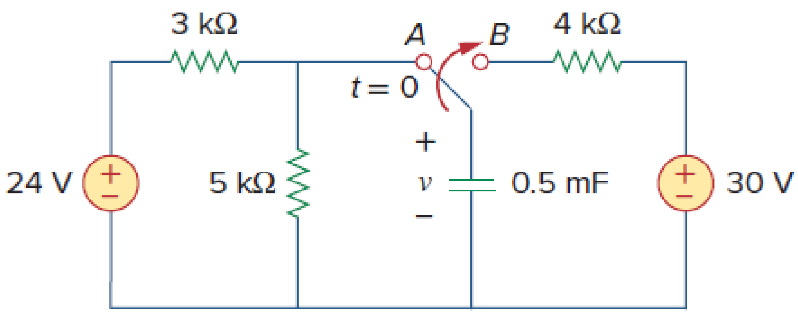
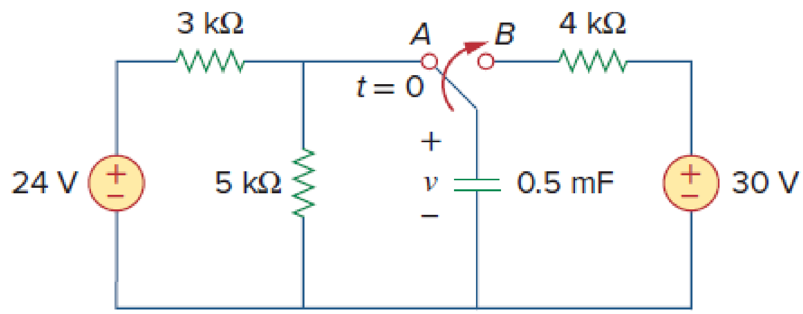


Figure 7.43
For Example 7.10.

مدارهای مرتبه اول

پاسخ کامل: ادامه مثال: (روش تشکیل معادله دیفرانسیل)

$$t<0: v(0^-)=15v$$



$$t>0: KVL: -v-4K\times 0.5m\frac{dv}{dt}+30=0$$

$$v+2\frac{dv}{dt}=30 \rightarrow \begin{cases} v_h : 1+2r=0 \rightarrow r=-0.5 \\ v_p=30 \end{cases}$$

$$v(t)=Ke^{-0.5t}+30 \xrightarrow{v(0^+)=v(0^-)=15v} K=-15$$

$$v(t)=\begin{cases} 15 & t\le 0 \\ 30-15e^{-0.5t} & t>0 \end{cases}$$

At $t = 1$,

$$v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$

At $t = 4$,

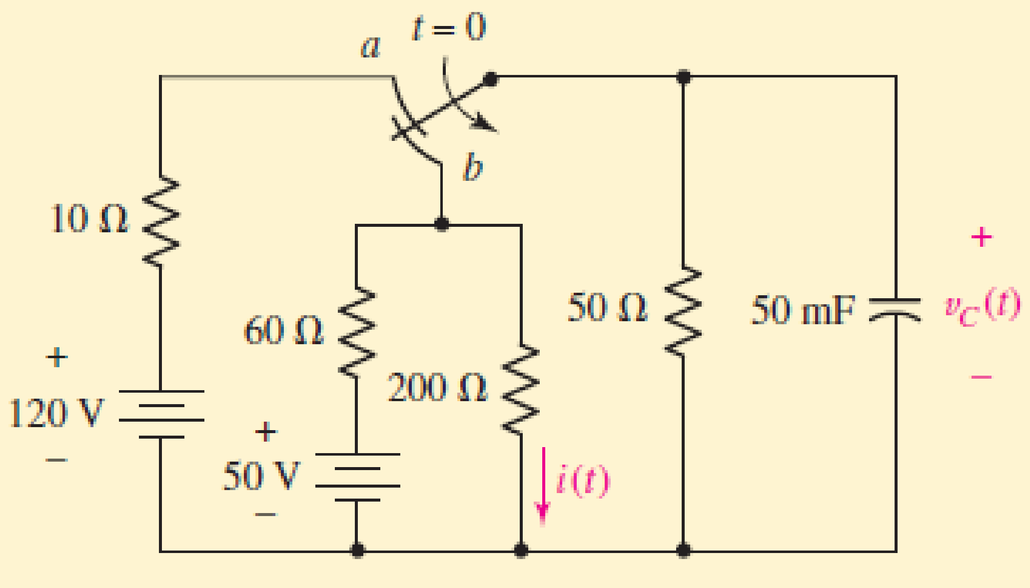
$$v(4) = 30 - 15e^{-2} = 27.97 \text{ V}$$

مدارهای مرتبه اول

مدار RC: مثال (پاسخ کامل)

دانشکده برق و کامپیوتر

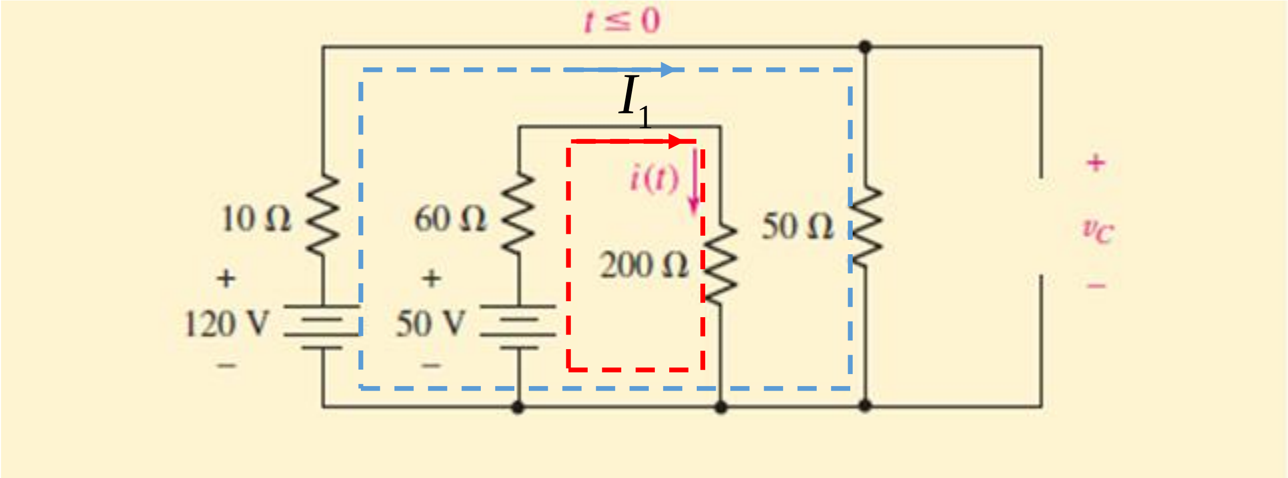
Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200 Ω resistor of Fig. 8.42 for all time.



(a)

مدارهای مرتبه اول: مدار RC: ادامه مثال

$t < 0$



(b)

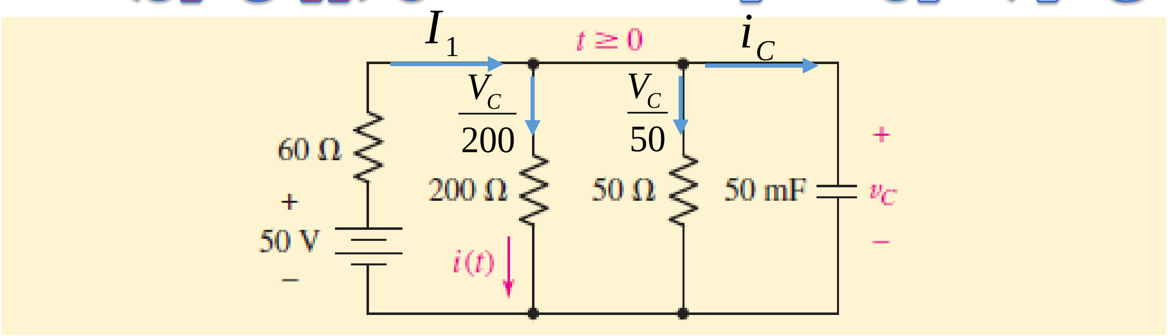
$$KVL: -120 + 10I_1 + 50I_1 = 0 \rightarrow I_1 = 2A$$

$$V_C(0^-) = 50 \times 2A = 100v = V_C(0^+)$$

$$KVL: -50 + 60i(t) + 200i(t) = 0 \rightarrow i(t) = 0.1923A$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول: مدار RC: ادامه مثال (روش اول)



$$t > 0$$

$$KCL: I_1 = \frac{V_C}{200} + \frac{V_C}{50} + C\frac{dV_C}{dt}$$

$$KVL: -50 + 60I_1 + V_C = 0 \rightarrow -50 + 60\left(\frac{V_C}{200} + \frac{V_C}{50} + C\frac{dV_C}{dt}\right) + V_C = 0$$

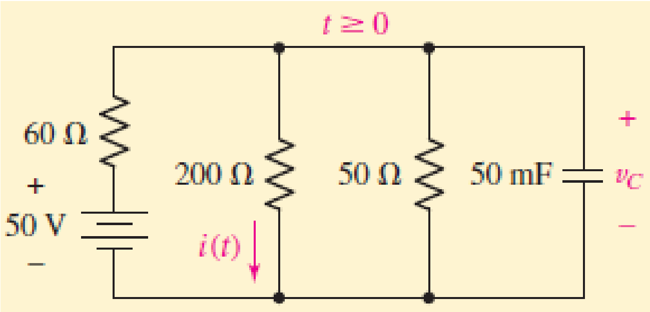
$$60C\frac{dV_C}{dt} + 2.5V_C = 50 \rightarrow V_C(t) = V_{Ch}(t) + V_{CP}(t) \rightarrow \begin{cases} 60Cr + 2.5 = 0 \rightarrow r = -0.833 \\ V_{CP}(t) = \dfrac{50}{2.5} \end{cases}$$

$$V_C(t) = Ke^{-0.833t} + \frac{50}{2.5} \xrightarrow{V_C(0^-)=100v=V_C(0^+)} 100 = K + \frac{50}{2.5} \rightarrow K = 80$$

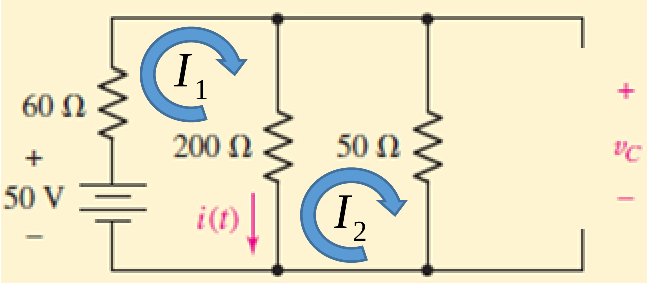
$$V_C(t) = 80e^{-0.833t} + 20 \xrightarrow{i(t)=\frac{V_C(t)}{200}} i(t) = 0.4e^{-0.833t} + 0.1$$

$$V_C(t) = \begin{cases} 100 & t < 0 \\ 80e^{-0.833t} + 20 & t \geq 0 \end{cases}, i(t) = \begin{cases} 0.1923 & t < 0 \\ 0.4e^{-0.833t} + 0.1 & t > 0 \end{cases}$$

مدارهای مرتبه اول: مدار RC: ادامه مثال (روش دوم)



$t = \infty$



$$V_C(0^+) = 100v \rightarrow i(0^+) = \frac{100}{200} = 0.5A$$

$$KVL: \begin{cases} 50 = 60I_1 + 200(I_1 - I_2) \\ 0 = 50I_2 + 200(I_2 - I_1) \end{cases} \rightarrow I_1 = 0.5A; I_2 = 0.4A$$

$$v_C(\infty) = 50I_2(\infty) = 20v, i(\infty) = I_1(\infty) - I_2(\infty) = 0.1A$$

$$R_{eq} = 50 \| 200 \| 60 = 23.98\Omega \rightarrow \tau = R_{eq}C = 1.199s$$

$$v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right]e^{\frac{-t}{\tau}}$$

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{\frac{-t}{\tau}}$$

$$v_C(t) = 20 + 80e^{\frac{-t}{1.199}}$$

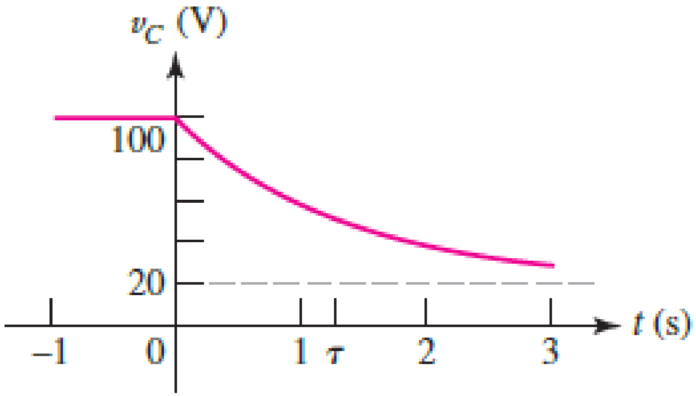
$$i(t) = 0.1 + 0.4e^{\frac{-t}{1.199}}$$

$$v_C(t) = \begin{cases} 100 & t < 0 \\ 80e^{-0.833t} + 20 & t \ge 0 \end{cases}$$

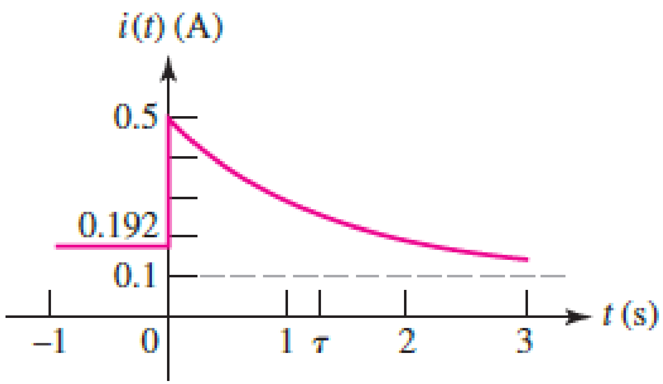
$$i(t) = \begin{cases} 0.1923 & t < 0 \\ 0.4e^{-0.833t} + 0.1 & t > 0 \end{cases}$$

مدارهای مرتبه اول
مدار RC: ادامه مثال

$$\begin{cases} v_C(t \le 0) = 100v \\ v_C(t > 0) = 80e^{-0.833t} + 20 \end{cases}$$



$$\begin{cases} i(t < 0) = 0.1923A \\ i(t > 0) = 0.4e^{-0.833t} + 0.1 \end{cases}$$



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مدارهای مرتبه اول
معرفی تابع پله واحد برای جایگزین سوئیچ در مدار

We define the unit-step forcing function as a function of time which is zero for all values of its argument less than zero and which is unity for all positive values of its argument. If we let $(t - t_0)$ be the argument and represent the unit-step function by u, then $u(t - t_0)$ must be zero for all values of t less than t_0, and it must be unity for all values of t greater than t_0. At $t = t_0$, $u(t - t_0)$ changes *abruptly* from 0 to 1. Its value at $t = t_0$ is not defined, but its value is known for all instants of time that are arbitrarily close to $t = t_0$. We often indicate this by writing $u(t_0^-) = 0$ and $u(t_0^+) = 1$. The concise mathematical definition of the unit-step forcing function is

$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

and the function is shown graphically in Fig. 8.26. Note that a vertical line of unit length is shown at $t = t_0$. Although this "riser" is not strictly a part of the definition of the unit step, it is usually shown in each drawing.

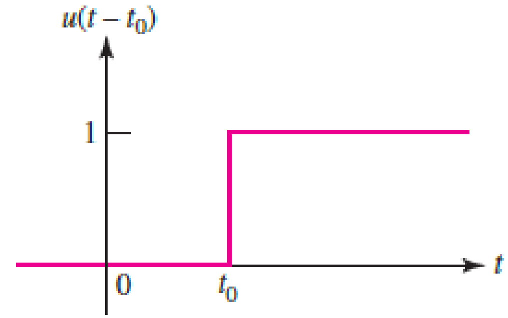


FIGURE 8.26 The unit-step forcing function, $u(t - t_0)$.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول
معرفی تابع پله واحد: ادامه

We also note that the unit step need not be a time function. For example, $u(x-x_0)$ could be used to denote a unit-step function where x might be a distance in meters, for example, or a frequency.

Very often in circuit analysis a discontinuity or a switching action takes place at an instant that is defined as $t=0$. In that case $t_0=0$, and we then represent the corresponding unit-step forcing function by $u(t-0)$, or more simply $u(t)$. This is shown in Fig. 8.27. Thus

$$u(t)=\begin{cases}0 & t<0\\ 1 & t>0\end{cases}$$

The unit-step forcing function is in itself dimensionless. If we wish it to represent a voltage, it is necessary to multiply $u(t-t_0)$ by some constant voltage, such as 5 V. Thus, $v(t)=5u(t-0.2)$ V is an ideal voltage source which is zero before $t=0.2$ s and a constant 5 V after $t=0.2$ s. This forcing function is shown connected to a general network in Fig. 8.28*a*.

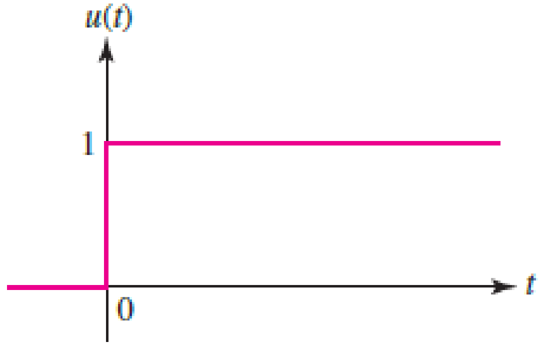


FIGURE 8.27 The unit-step forcing function $u(t)$ is shown as a function of t.

$$u(-t)=\begin{cases}1 & t<0\\ 0 & t>0\end{cases}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

راه انداز مدار RL با استفاده از تابع پله واحد

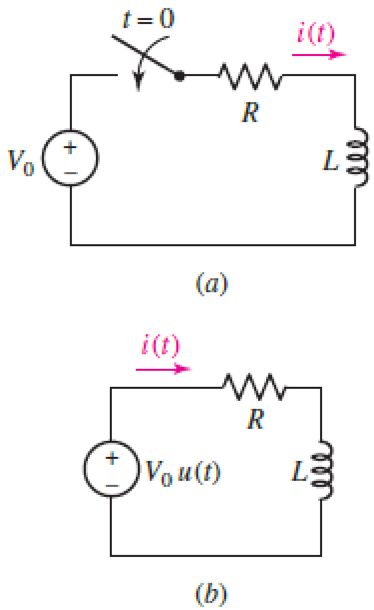


FIGURE 8.33 (*a*) The given circuit. (*b*) An equivalent circuit, possessing the same response *i*(*t*) for all time.

$$Ri + L\frac{di}{dt} = V_0 u(t)$$

Since the unit-step forcing function is discontinuous at $t = 0$, we will first consider the solution for $t < 0$ and then for $t > 0$. The application of zero voltage since $t = -\infty$ forces a zero response, so that

$$i(t) = 0 \qquad t < 0$$

For positive time, however, $u(t)$ is unity and we must solve the equation

$$Ri + L\frac{di}{dt} = V_0 \qquad t > 0$$

$$\begin{cases} i_h(t): R + Lr = 0 \rightarrow r = -R/L \rightarrow i_h(t) = Ke^{-R/Lt} \\ i_P(t) = V_0/R \end{cases} \rightarrow i(t) = Ke^{-R/Lt} + V_0/R$$

$$\xrightarrow{i(0^-)=i(0^+)=0} i(t) = -V_0/Re^{-R/Lt} + V_0/R \Rightarrow i(t) = V_0/R\left(1 - e^{-R/Lt}\right)u(t)$$

مدارهای مرتبه اول

مدار RL: تمرین (به عهده دانشجو)

The circuit depicted in Fig. 8.79 contains two independent sources, one of which is only active for $t > 0$. (*a*) Obtain an expression for $i_L(t)$ valid for all t; (*b*) calculate $i_L(t)$ at $t = 10\ \mu s$, $20\ \mu s$, and $50\ \mu s$.

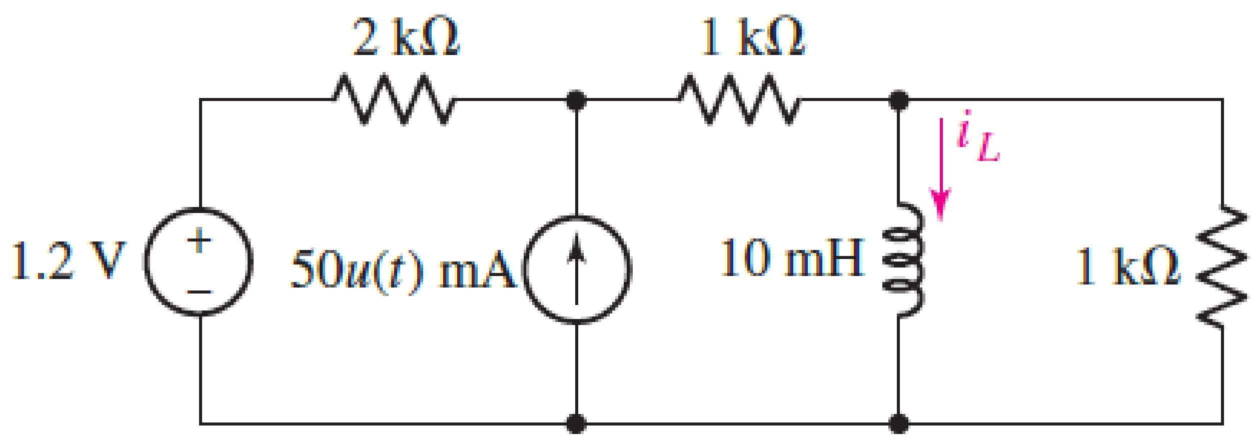
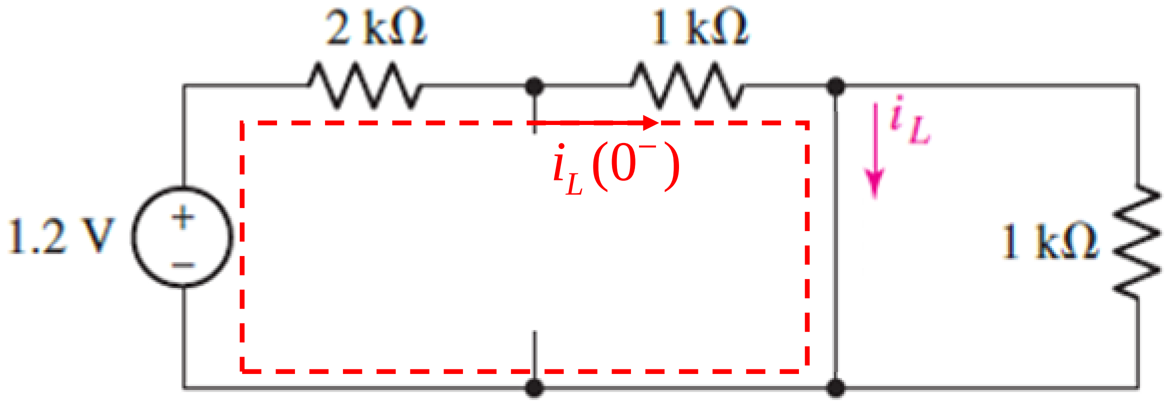


FIGURE 8.79

مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو)

$t < 0$



$$KVL: -1.2 + 2000 i_L + 1000 i_L = 0 \rightarrow i_L = \frac{1.2}{3000} = 0.4mA$$

$$i_L(0^-) = i_L(0^+) = 0.4mA$$

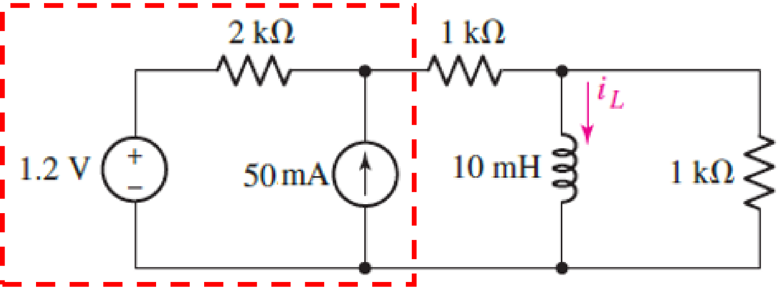
دانشکده برق و کامپیوتر

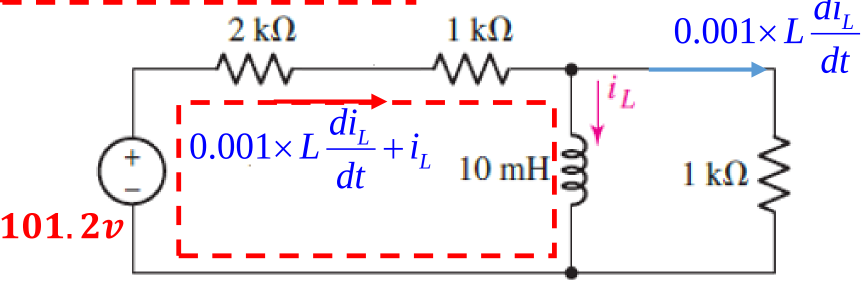
مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو)

(روش اول): تشکیل و حل معادله دیفرانسیل

$t \geq 0$





$$KVL: -101.2+3000\left(i_L+0.001\times L\frac{di_L}{dt}\right)+L\frac{di_L}{dt}=0 \rightarrow 4L\frac{di_L}{dt}+3000i_L=101.2$$

$$\begin{cases} i_{Lh}(t): 4Lr+3000=0 \rightarrow r=-\dfrac{3000}{4L}=\dfrac{-750}{L} \\ i_{LP}(t)=\dfrac{101.2}{3000}=0.0337A \end{cases} \rightarrow i_L(t)=Ke^{\frac{-750}{L}t}+0.0337$$

$$\xrightarrow{i_L(0^-)=i_L(0^+)=0.4mA} K=-0.0333 \rightarrow i_L(t)=-0.0333e^{\frac{-750}{L}t}+0.0337$$

$$\begin{cases} i_L(t)=-0.0333e^{\frac{-750}{L}t}+0.0337 & t>0 \\ i_L(t)=0.4mA & t\leq 0 \end{cases}$$

$$i_L(10^{\mu s})=18mA, i_L(20^{\mu s})=26.3mA, i_L(50^{\mu s})=32.9mA$$

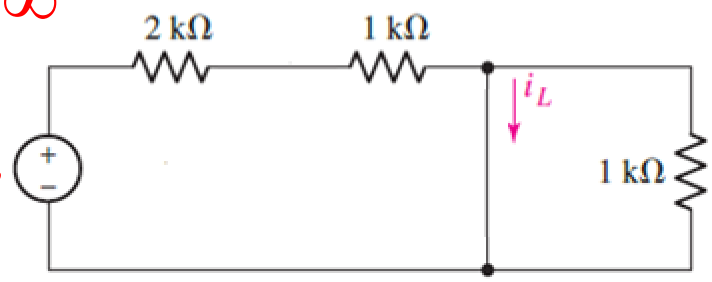
دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو)

(روش دوم): روش ثابت زمانی

$t = \infty$



$$i_L(0^+) = 0.4mA$$

$$i_L(\infty) = \frac{101.2}{3000} = 0.0337A$$

$$R_{eq} = 3000 \| 1000 = 750\Omega \rightarrow \tau = \frac{L}{R_{eq}} = \frac{L}{750}$$

$$i_L(t) = i_L(\infty) + \left[i_L(0^+) - i_L(\infty) \right] e^{\frac{-t}{\tau}} \rightarrow i_L(t) = 0.0337 + \left[0.0004 - 0.0337 \right] e^{\frac{-750t}{L}}$$

$$i_L(t) = 0.0337 - 0.0333e^{\frac{-750t}{L}}$$

$$\begin{cases} i_L(t) = -0.0333e^{\frac{-750}{L}t} + 0.0337 & t > 0 \\ i_L(t) = 0.4mA & t \le 0 \end{cases}$$

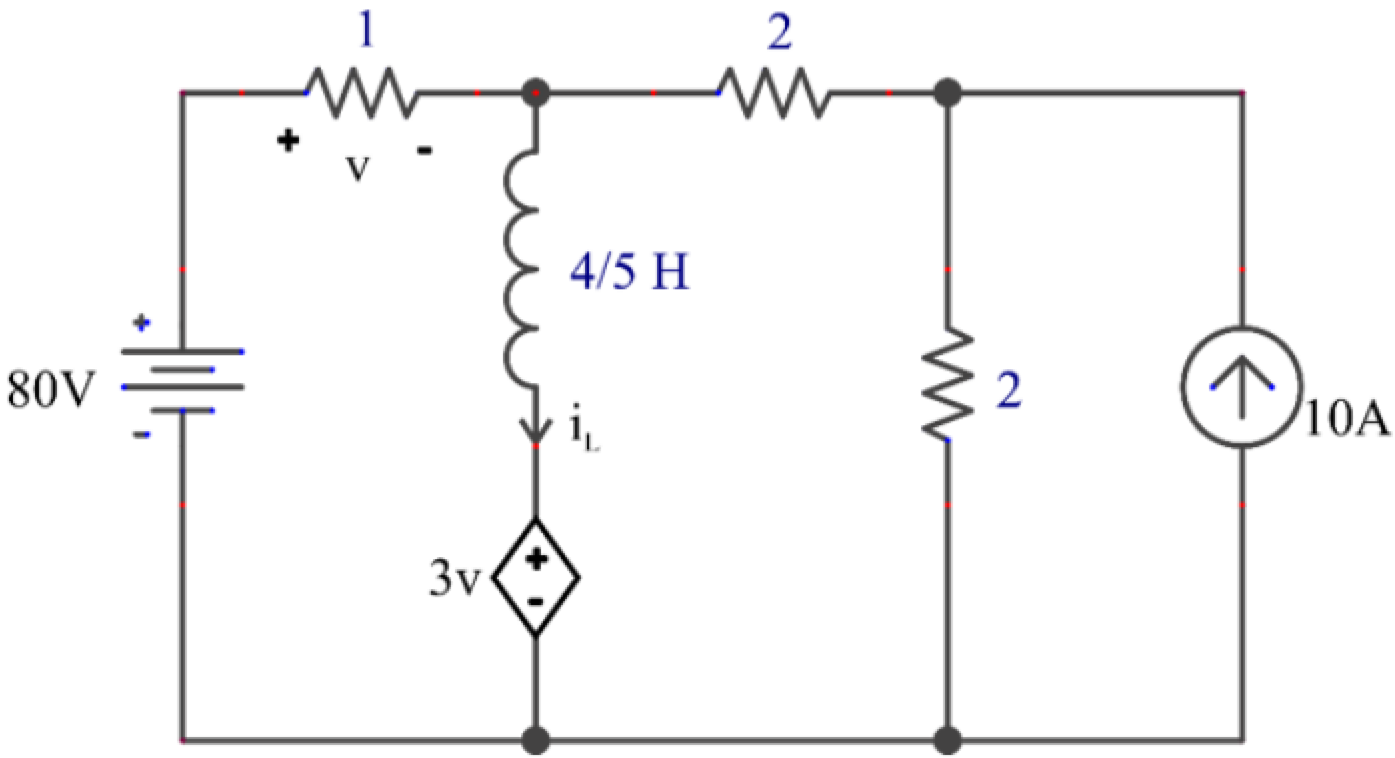
$$i_L(10^{\mu s}) = 18mA, i_L(20^{\mu s}) = 26.3mA, i_L(50^{\mu s}) = 32.9mA$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: تمرین (به عهده دانشجو)

در مدار شکل زیر اگر بدانیم $i_L(0^-) = 2$ آمپر است، جریان $i_L(t)$ را برای $t > 0$ بیابید. (مقاومت ها بر حسب اهم هستند)

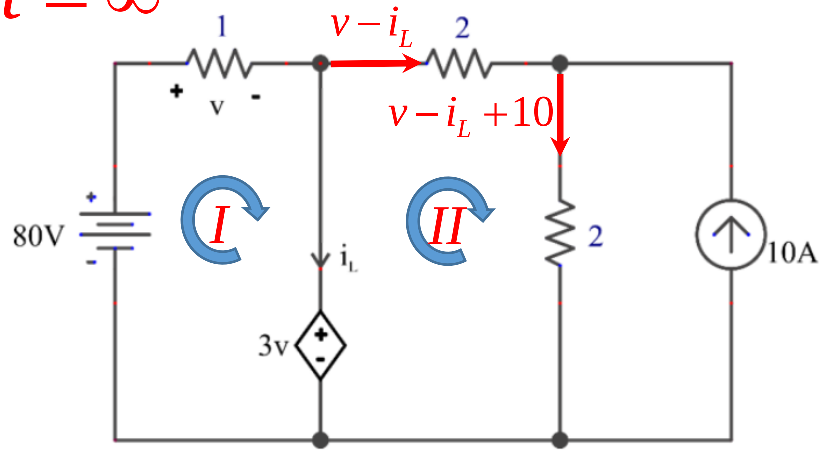


دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو) : استفاده از روش حل ثابت زمانی

$t=\infty$

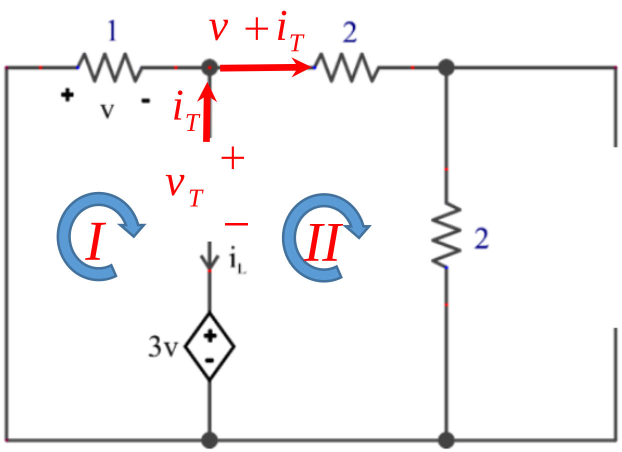


$$KVL\ I: 80 = v + 3v \rightarrow v = 20$$

$$KVL\ II: 2\left[v - i_L(\infty)\right] + 2\left[v - i_L(\infty) + 10\right] - 3v = 0$$

$$v + 20 = 4i_L(\infty) \rightarrow \boxed{i_L(\infty) = \frac{40}{4} = 10A}$$

محاسبه مقاومت دیده شده از دو سر سلف



$$KVL\ I: 0 = v + v_T + 3v \rightarrow \boxed{4v = -v_T}$$

$$KVL\ II: 4\left[v + i_T\right] - 3v - v_T = 0$$

$$v + 4i_T - v_T = 0 \rightarrow \frac{-v_T}{4} + 4i_T - v_T = 0$$

$$\frac{5}{4}v_T = 4i_T \rightarrow \boxed{R_{th} = \frac{v_T}{i_T} = \frac{16}{5}\Omega}$$

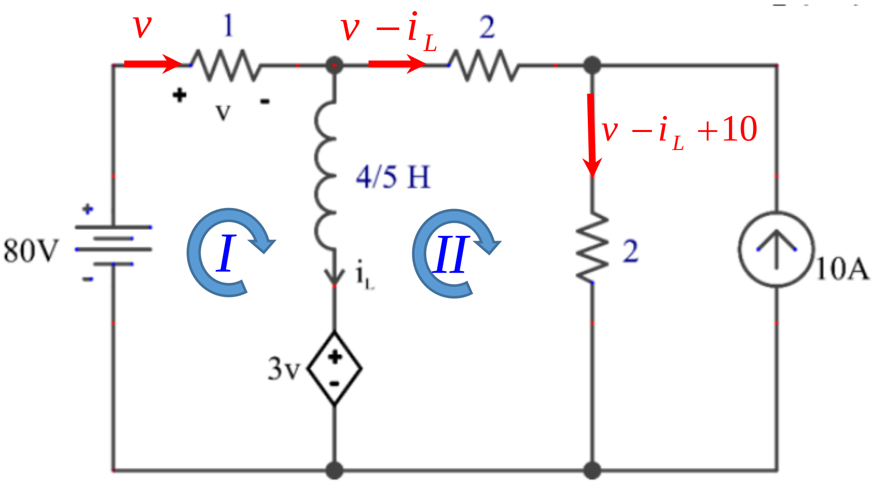
$$\boxed{\tau = \frac{L}{R_{th}} = \frac{4/5}{16/5} = 0.25s}$$

مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو) : استفاده از روش حل ثابت زمانی

$$\begin{cases} i_L(0) = 2A \\ i_L(\infty) = 10A \rightarrow i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{\frac{-t}{\tau}} \rightarrow \boxed{i_L(t) = 10 - 8e^{-4t}} \\ \tau = 0.25s \end{cases}$$

مدار RL: ادامه تمرین (به عهده دانشجو) : استفاده از روش حل معادله دیفرانسیل



$$KVL\ I: 80 = v + v_L + 3v \rightarrow \boxed{80 = 4v + L\frac{di_L}{dt}}$$

$$KVL\ II: -3v - L\frac{di_L}{dt} + 2(v - i_L) + 2(v - i_L + 10) = 0$$

$$\boxed{v - L\frac{di_L}{dt} - 4i_L = -20}$$

$$\left(\frac{80 - L\frac{di_L}{dt}}{4}\right) - L\frac{di_L}{dt} - 4i_L = -20 \rightarrow \frac{-5L}{4}\frac{di_L}{dt} - 4i_L = -40 \xrightarrow{L=\frac{4}{5}H} \boxed{i_L(t) = 10 - 8e^{-4t}}$$