بسم الله الرحمن الرحیم

نظریه زبان‌ها و ماشین‌ها

جلسه ۲۲

مجتبی خلیلی
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دانشگاه صنعتی اصفهان

مثال

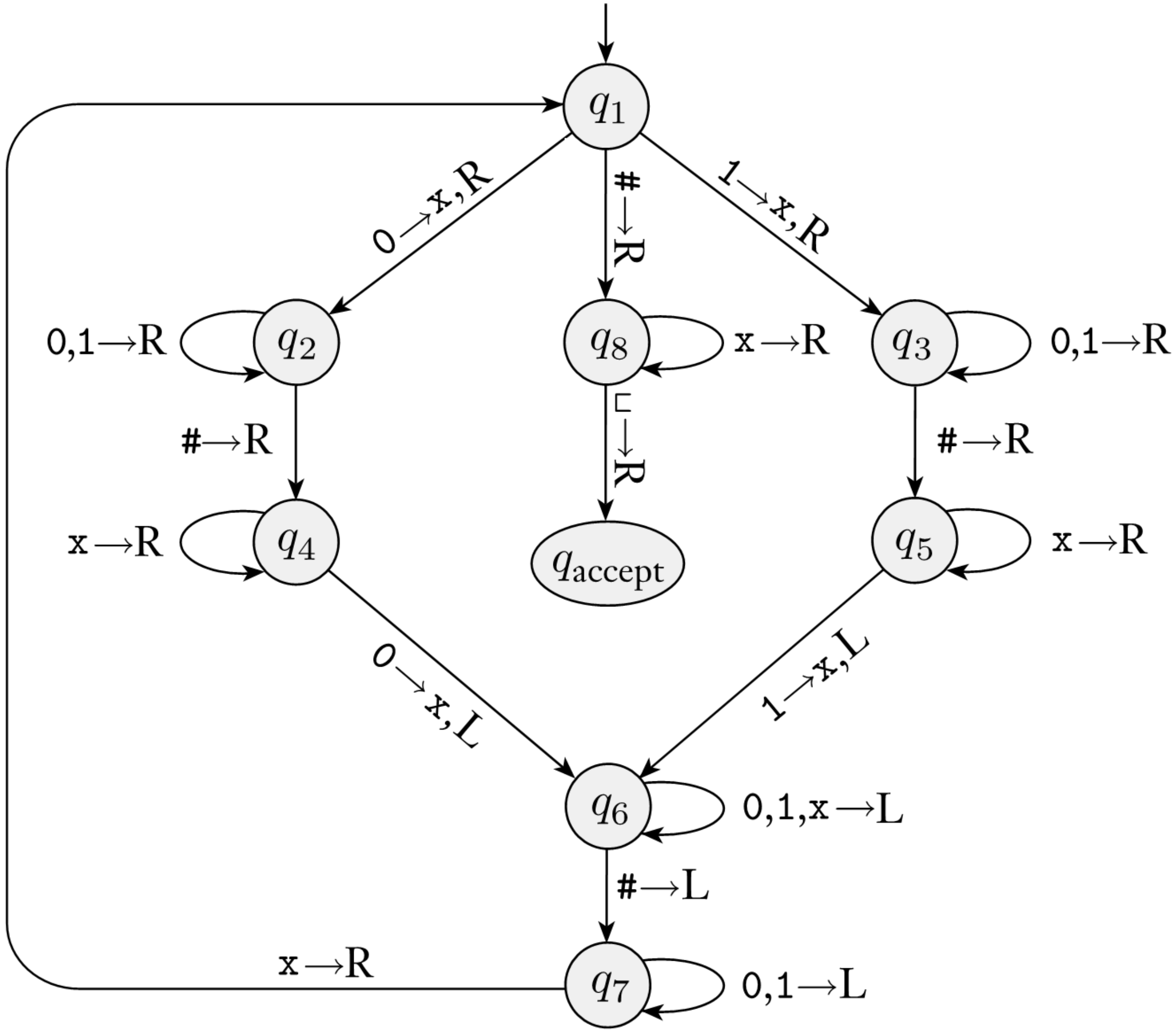
- یک TM که درباره زبان زیر تصمیم بگیرد.

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

مثال


```
0 1 1 0 0 0 # 0 1 1 0 0 0 ␣ ...
x 1 1 0 0 0 # 0 1 1 0 0 0 ␣ ...
x 1 1 0 0 0 # x 1 1 0 0 0 ␣ ...
x 1 1 0 0 0 # x 1 1 0 0 0 ␣ ...
x x 1 0 0 0 # x 1 1 0 0 0 ␣ ...
x x x x x x # x x x x x x ␣ ...
                          accept
```

مثال



To simplify the figure, we don't show the reject state or the transitions going to the reject state.

مثال

The following is a formal description of $M_1 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$,

- $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$,
- $\Sigma = \{\texttt{0},\texttt{1},\texttt{\#}\}$, and $\Gamma = \{\texttt{0},\texttt{1},\texttt{\#},\texttt{x},\sqcup\}$.
- We describe δ with a state diagram (see the following figure).
- The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively.

مثال

EXAMPLE 9.7

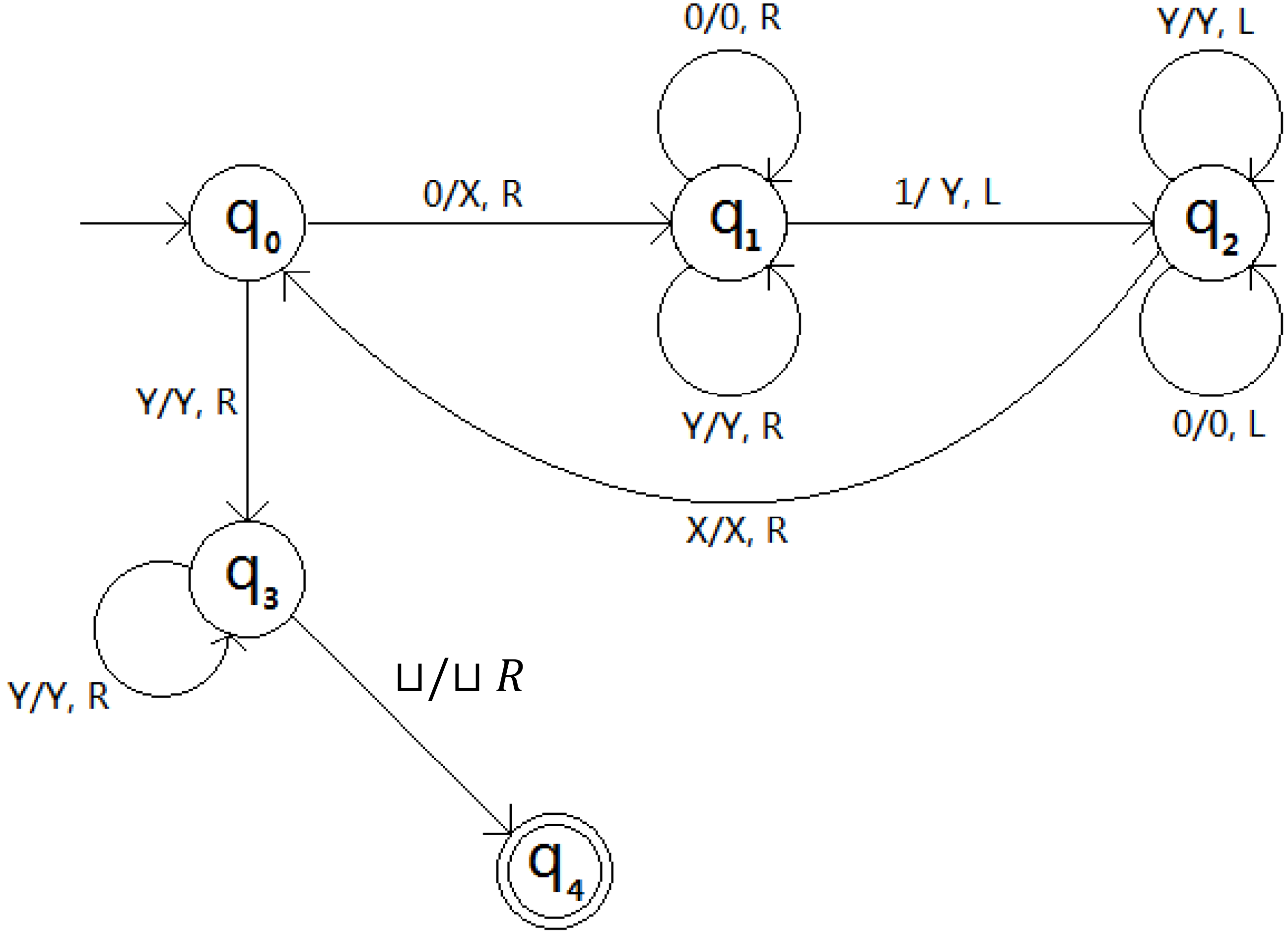
For $\Sigma = \{a, b\}$, design a Turing machine that accepts

$$L = \{a^n b^n : n \geq 1\}.$$

Intuitively, we solve the problem in the following fashion. Starting at the leftmost a, we check it off by replacing it with some symbol, say x. We then let the read-write head travel right to find the leftmost b, which in turn is checked off by replacing it with another symbol, say y. After that, we go left again to the leftmost a, replace it with an x, then move to the leftmost b and replace it with y, and so on. Traveling back and forth this way, we match each a with a corresponding b. If after some time no a's or b's remain, then the string must be in L.

مثال

$\{0^n 1^n : n \geq 1\}$



مثال

EXAMPLE 9.8

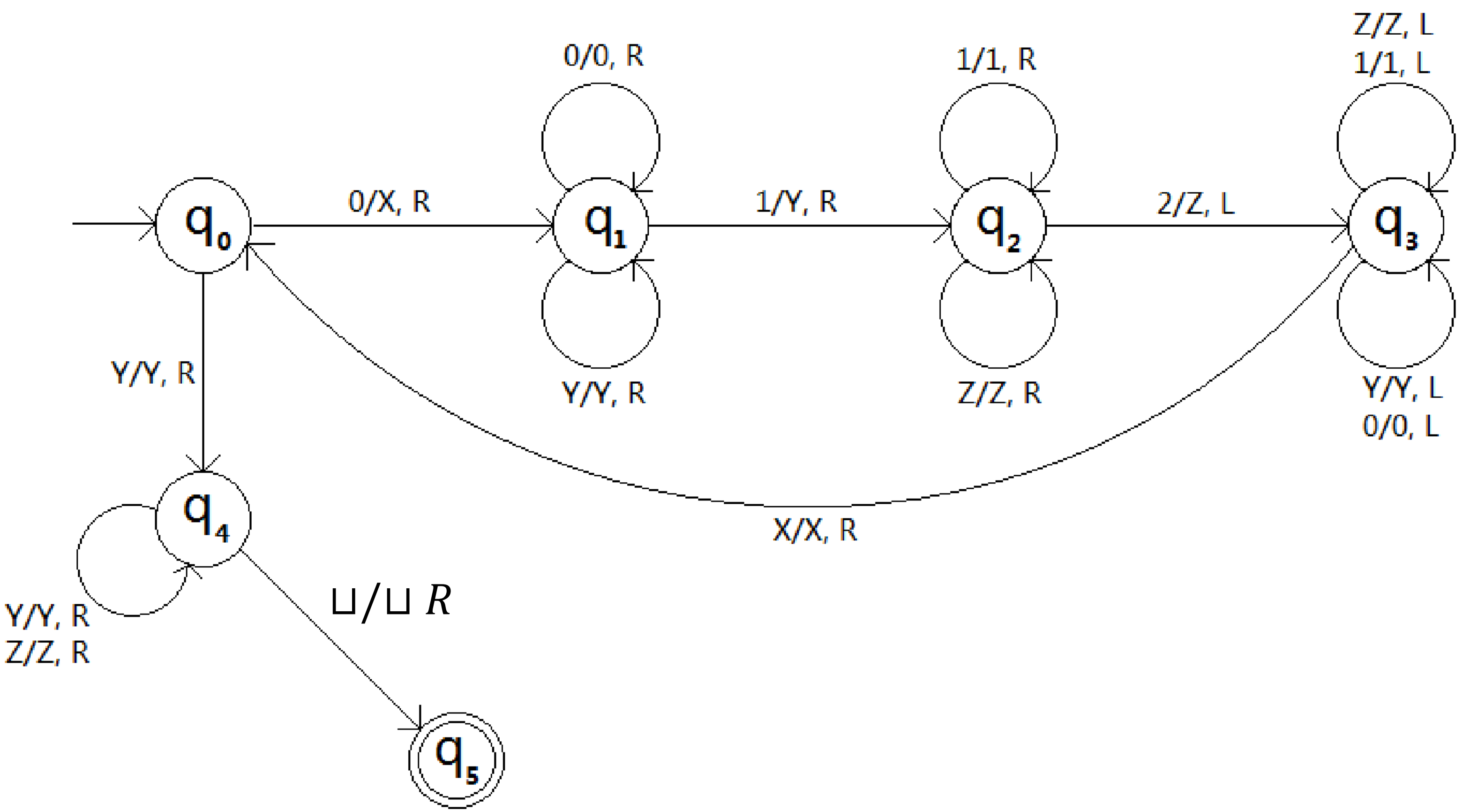
Design a Turing machine that accepts

$$L = \{a^n b^n c^n : n \geq 1\}.$$

The ideas used in Example 9.7 are easily carried over to this case. We match each a, b, and c by replacing them in order by x, y, and z, respectively. At the end, we check that all original symbols have been rewritten. Although conceptually a simple extension of the previous example, writing the actual program is tedious. We leave it as a somewhat lengthy, but straightforward exercise. Notice that even though $\{a^n b^n\}$ is a context-free language and $\{a^n b^n c^n\}$ is not, they can be accepted by Turing machines with very similar structures.

مثال

$$\{0^n 1^n 2^n : n \geq 1\}$$



مثال

- یک TM که درباره زبان زیر تصمیم بگیرد.

$$A = \{0^{2^n} \mid n \geq 0\}$$

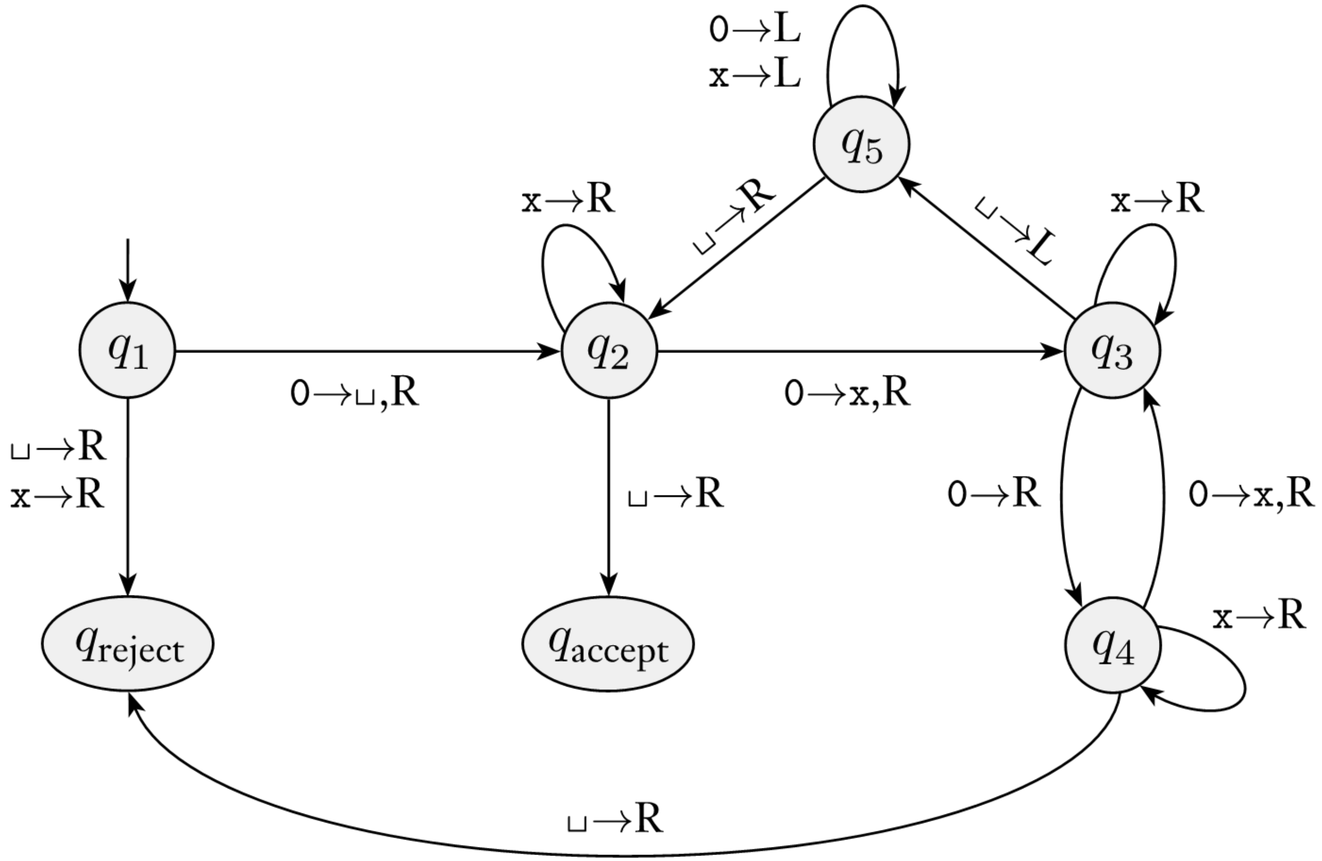
$M_2 =$ "On input string w:

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0, *accept*.
3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, *reject*.
4. Return the head to the left-hand end of the tape.
5. Go to stage 1."

مثال

- یک TM که درباره زبان زیر تصمیم بگیرد.

$$A = \{0^{2^n} \mid n \geq 0\}$$



مثال

- یک TM که درباره زبان زیر تصمیم بگیرد.

$$A = \{\texttt{0}^{2^n} \mid n \geq 0\}$$

Now we give the formal description of $M_2 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$:

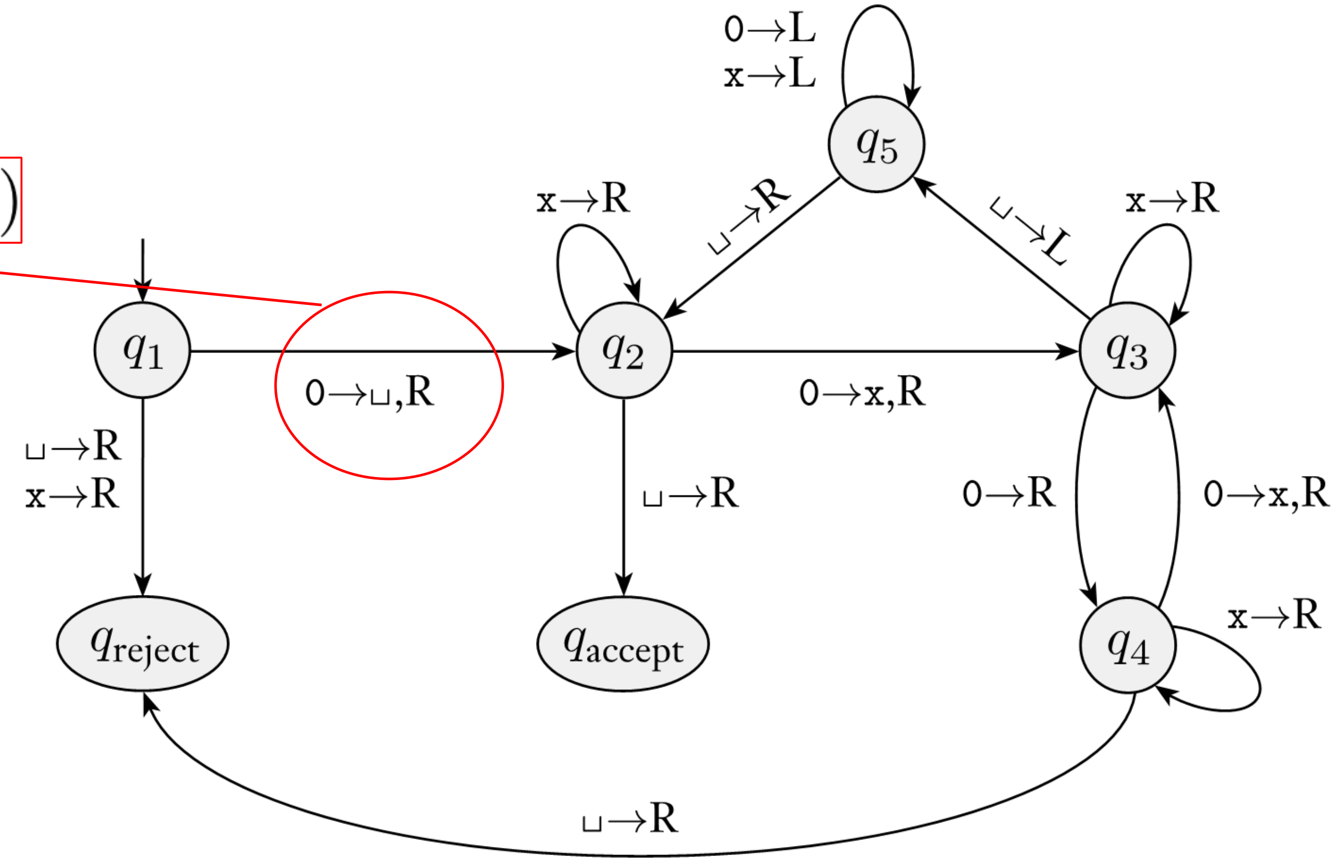
- $Q = \{q_1, q_2, q_3, q_4, q_5, q_{\text{accept}}, q_{\text{reject}}\}$,
- $\Sigma = \{\texttt{0}\}$, and
- $\Gamma = \{\texttt{0}, \texttt{x}, \sqcup\}$.
- We describe δ with a state diagram (see Figure 3.8).
- The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively.

مثال

○ یک TM که درباره زبان زیر تصمیم بگیرد.

$$A = \{0^{2^n} \mid n \geq 0\}$$

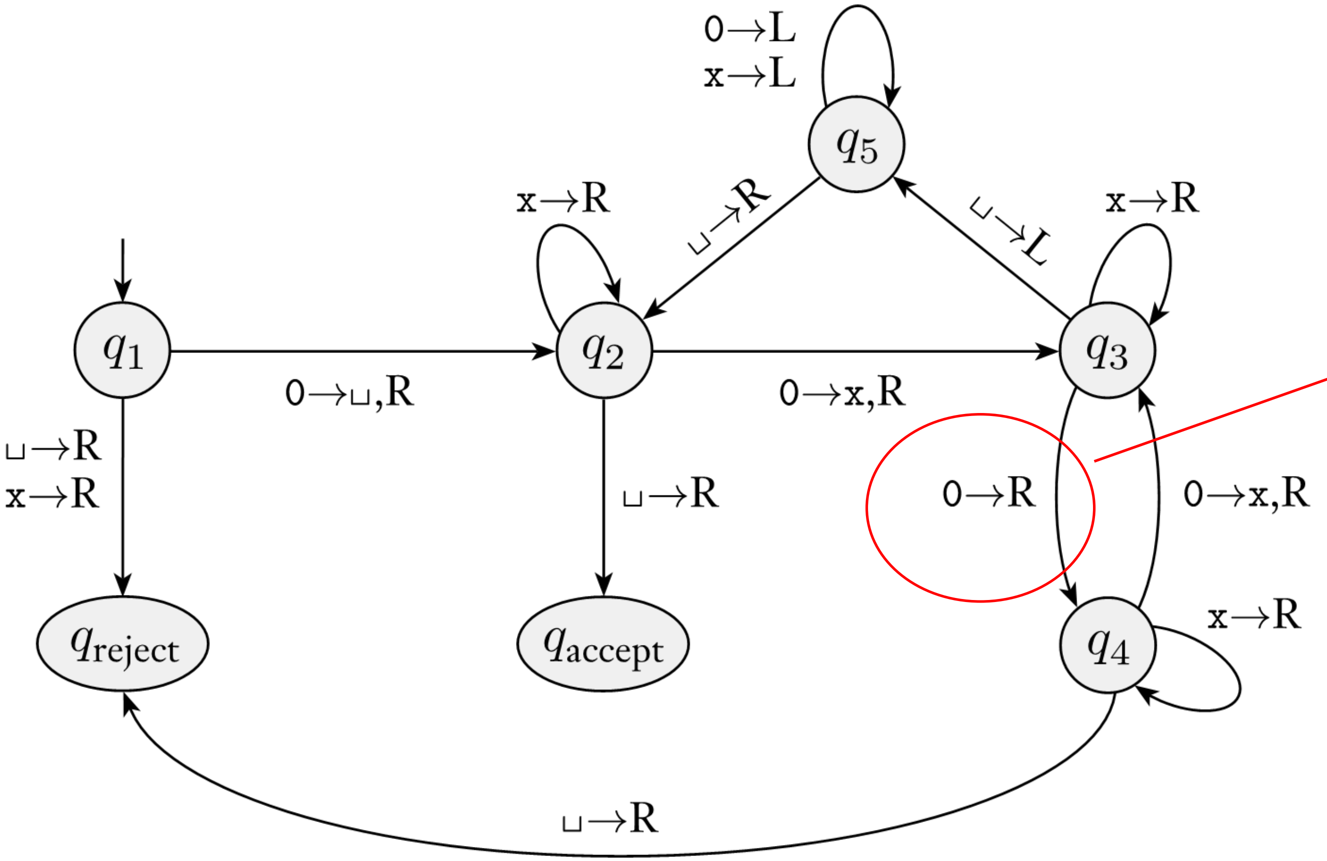
$$\boxed{\delta(q_1,0) = (q_2,\sqcup,\mathrm{R})}$$



مثال

- یک TM که درباره زبان زیر تصمیم بگیرد.

$$A = \{0^{2^n} \mid n \geq 0\}$$

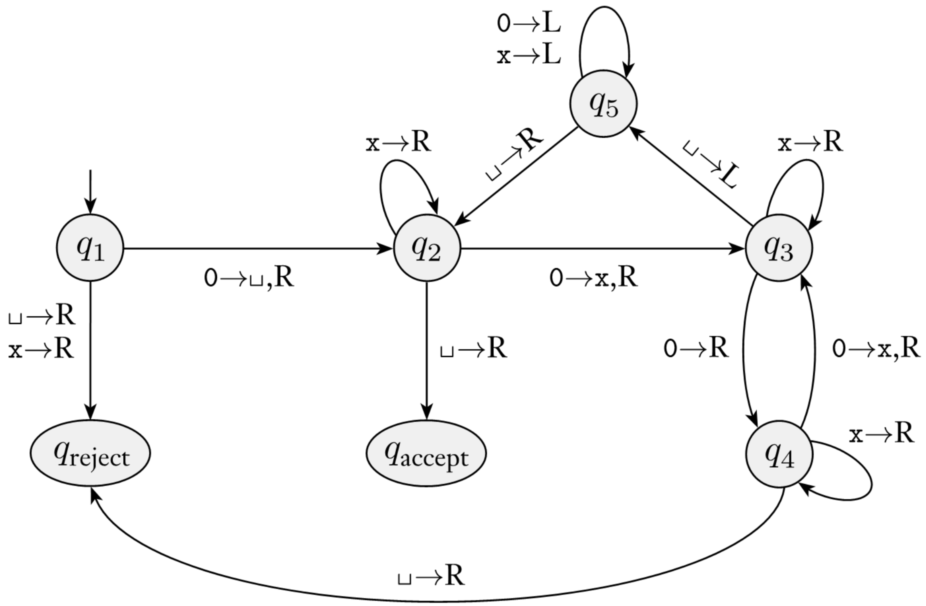


$$\boxed{\delta(q_3, 0) = (q_4, 0, R)}$$

مثال

IUT-ECE

Next we give a sample run of this machine on input 0000.



$q_1 0000$	$\sqcup q_5 x0x\sqcup$	$\sqcup x q_5 xx\sqcup$
$\sqcup q_2 000$	$q_5 \sqcup x0x\sqcup$	$\sqcup q_5 xxx\sqcup$
$\sqcup x q_3 00$	$\sqcup q_2 x0x\sqcup$	$q_5 \sqcup xxx\sqcup$
$\sqcup x0 q_4 0$	$\sqcup x q_2 0x\sqcup$	$\sqcup q_2 xxx\sqcup$
$\sqcup x0x q_3 \sqcup$	$\sqcup xx q_3 x\sqcup$	$\sqcup x q_2 xx\sqcup$
$\sqcup x0 q_5 x\sqcup$	$\sqcup xxx q_3 \sqcup$	$\sqcup xx q_2 x\sqcup$
$\sqcup x q_5 0x\sqcup$	$\sqcup xx q_5 x\sqcup$	$\sqcup xxx q_2 \sqcup$
		$\sqcup xxx\sqcup q_{\text{accept}}$

مثال

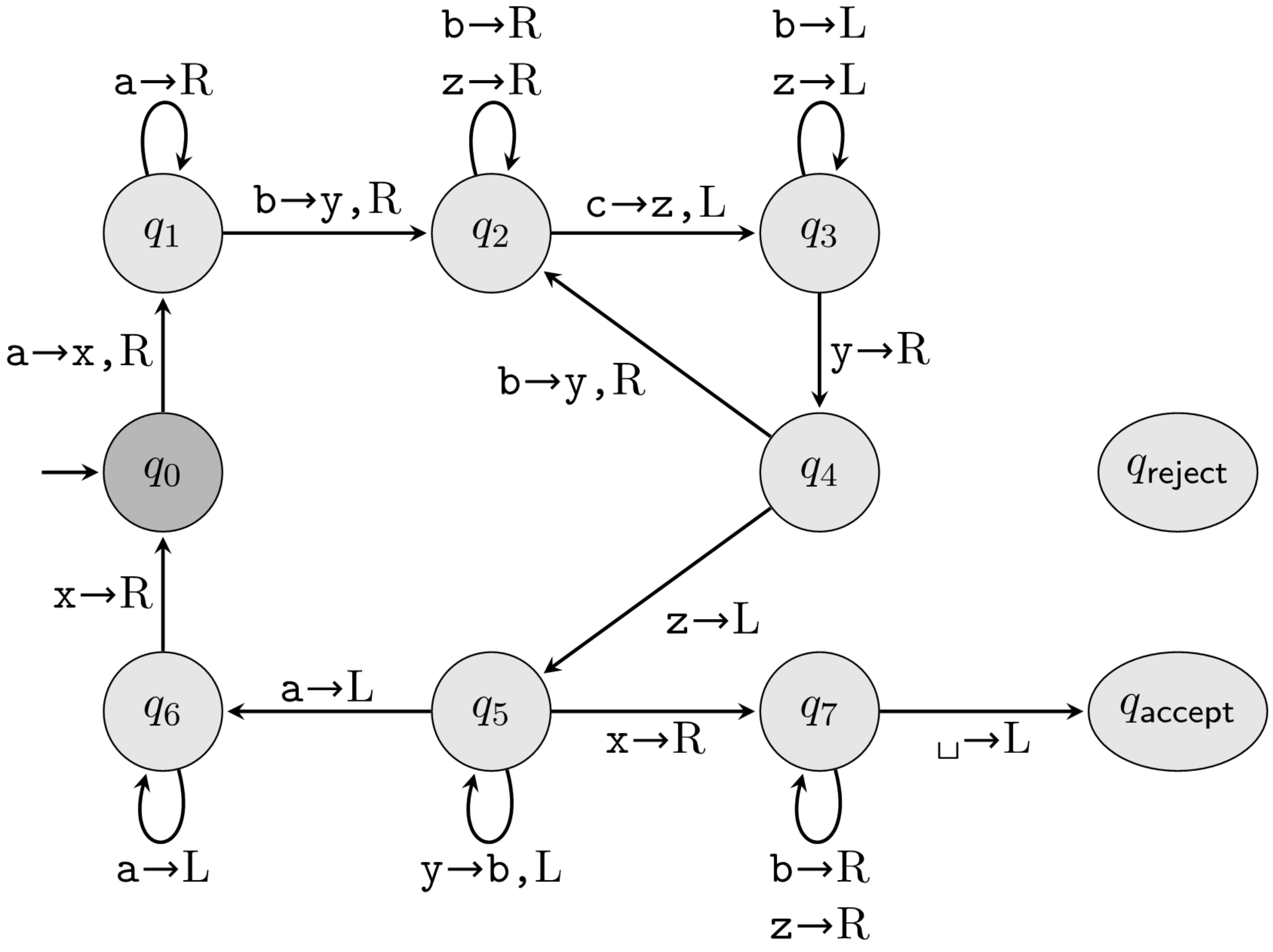
EXAMPLE 3.11

Here, a TM M_3 is doing some elementary arithmetic. It decides the language $C = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$.

M_3 = "On input string w:

1. Scan the input from left to right to determine whether it is a member of $\mathtt{a}^+\mathtt{b}^+\mathtt{c}^+$ and *reject* if it isn't.
2. Return the head to the left-hand end of the tape.
3. Cross off an a and scan to the right until a b occurs. Shuttle between the b's and the c's, crossing off one of each until all b's are gone. If all c's have been crossed off and some b's remain, *reject*.
4. Restore the crossed off b's and repeat stage 3 if there is another a to cross off. If all a's have been crossed off, determine whether all c's also have been crossed off. If yes, *accept*; otherwise, *reject*."

مثال


a→R
b→R
z→R
b→L
z→L
b→y,R
c→z,L
a→x,R
b→y,R
y→R
q_1
q_2
q_3
q_0
q_4
q_{reject}
x→R
z→L
a→L
q_6
q_5
q_7
q_{accept}
x→R
␣→L
a→L
y→b,L
b→R
z→R

ماشین تورینگ

IUT-ECE

- تاکنون:
  - ماشین تورینگ با تابع گذار و جزئیات
  - توصیف سطح پیاده‌سازی ماشین تورینگ
  - توصیف سطح بالای ماشین تورینگ (مانند شبه کد یا بلوک دیاگرام- الگوریتم)
- توصیف سطح بالا برای ترکیب ماشین‌های تورینگ (مثلا شبه کد و زیربرنامه)

ماشین تورینگ

- ترکیب ماشینهای تورینگ برای حل مسائل پیچیده تر

EXAMPLE 9.12

Design a Turing machine that computes the function

$$f(x, y) = x + y \quad \text{if } x \geq y,$$
$$= 0 \quad \text{if } x < y.$$

For the sake of discussion, assume that x and y are positive integers in unary representation. The value zero will be represented by 0, with the rest of the tape blank.