Cavity-induced Eliashberg effect: SC vs CDW

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Md Mursalin Islam[1,2], Michele Pini[1,2], R. Flores-Calderón[2], & Francesco Piazza[1,2]

[1] *Theoretical Physics III, Institute of Physics, University of Augsburg, Augsburg, Germany*

[2] *Max Planck Institute for the Physics of Complex Systems, Dresden, Germany*

Universität Augsburg University

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Non-thermal control of material (1)

Controlling properties of materials via non-thermal redistribution of quasiparticles due to coupling to the light.

- **Classical EM pulse:** transient — A. de la Torre et. al., RMP 93, 041002 (2021)
  - Light-induced SC — D. Fausti et. al., Science. 331, 189 (2011)
  - Enhancement of SC — A. F. G. Wyatt et. al., PRL 16, 1166 (1966)
  - Eliashberg Effect — G. M. Eliashberg, JETP Lett. 11, 114 (1970)
- **Cavity photons:** steady state
  - Cavity Eliashberg effect — J. B. Curtis et. al., PRL 122, 167002 (2019)
  - Controlling CDW transition — G. Jarc et. al., Nature 622, 487-492 (2023)

Material inside a cavity (2)

System: Quasi-1D lattice

$$H_0 = \sum_k \left[c_+^\dagger(k) \quad c_-^\dagger(k)\right] \begin{bmatrix} \xi(k) & \Delta \\ \Delta^* & -\xi(k) \end{bmatrix} \begin{bmatrix} c_+(k) \\ c_-(k) \end{bmatrix} + \frac{\Delta^*\Delta}{\kappa}$$

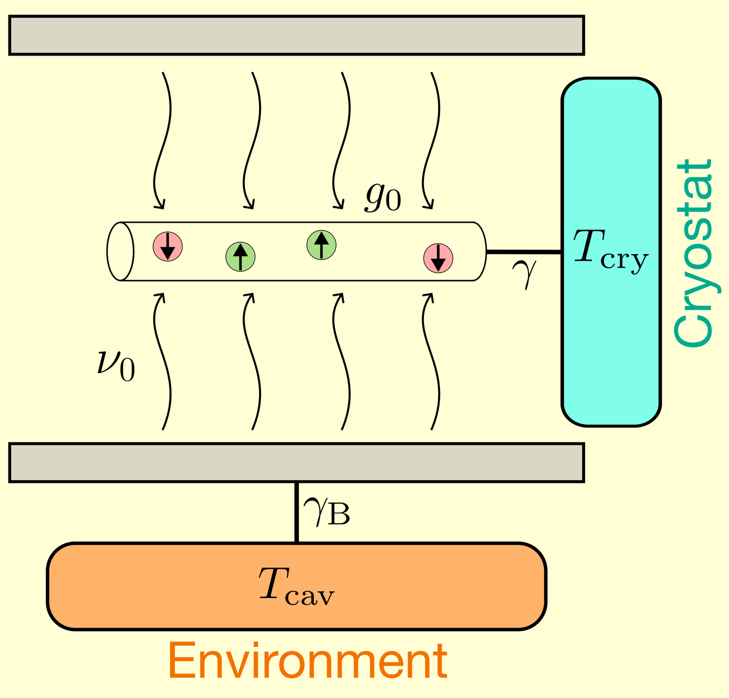
(κ is pairing coupling)

SC	CDW
$c_+(k) = c_\uparrow(k)$	$c_+(k) = c(k+k_F)$
$c_-(k) = c_\downarrow^\dagger(-k)$	$c_-(k) = c(k-k_F)$
$\xi(k) = -J\cos(ka)$	$\xi(k) = J\sin(ka)$

Light-matter coupling:

$$H_{LM} = \sum_{\alpha=\{+,-\}} \sum_{k,k'} g^\alpha_{k,k'}\, \underbrace{c_\alpha^\dagger(k)c_\alpha(k')}_{\text{electron}}\, \overbrace{\phi(k-k')}^{\text{photon}}$$

$$g^\pm_{k,k'} = \begin{cases} g_0 \sin\left(\frac{ka+k'a}{2}\right) & \text{for SC} \\ \pm g_0 \cos\left(\frac{ka+k'a}{2}\right) & \text{for CDW} \end{cases}$$



G. Jarc et. al., Nature 622, 487-492 (2023)
R. Flores-Calderón, **MMI**, M. Pini & F. Piazza PRR 7, 013073 (2025)
F. Schlawin, A. Cavalleri, and D. Jaksch, PRL 122, 133602 (2019)

Non-thermal gap equation (3)

$$\boxed{\int dE \Lambda(E) \frac{1 - 2[n_0(E) + \delta n(E)]}{E} = \frac{1}{\kappa}}$$

where $n_0(\omega) = \dfrac{1}{\exp(\omega/T_{cry}) + 1}$ (Fermi function)

$\Lambda(E)$ is DOS on a 1D lattice

The non-thermal part

$$\delta n(E) = \frac{2g_0^2}{\gamma} \int_\Delta^\infty dE'\, \Lambda(E') \left[\overbrace{\Gamma_\Delta^\eta(E,E')}^{\text{intra-band scattering}} + \underbrace{\Gamma_\Delta^\eta(E,-E')}_{\text{inter-band scattering}}\right],$$

Scattering amplitude: $\Gamma_\Delta^\eta(E,E') = \left(1 + \eta\frac{\Delta^2}{EE'}\right) \mathrm{Im} D_0^R(E-E') H_0(E,E')$

Photon spectrum: $\mathrm{Im} D_0^R(\omega) = -\dfrac{1}{4\pi}\left[\dfrac{\gamma_B}{(\omega-\nu_0)^2 + \gamma_B^2} - \dfrac{\gamma_B}{(\omega+\nu_0)^2 + \gamma_B^2}\right]$

Distribution factor: $H_0(\omega,\omega') = \left[n_0(\omega) - n_0(\omega')\right]\left[N_{T_{cav}}(\omega-\omega') - N_{T_{cry}}(\omega-\omega')\right]$

Where, $N_T(\omega) = \dfrac{1}{\exp(\omega/T) - 1}$ (Bose function)

$g_0 = 0$:

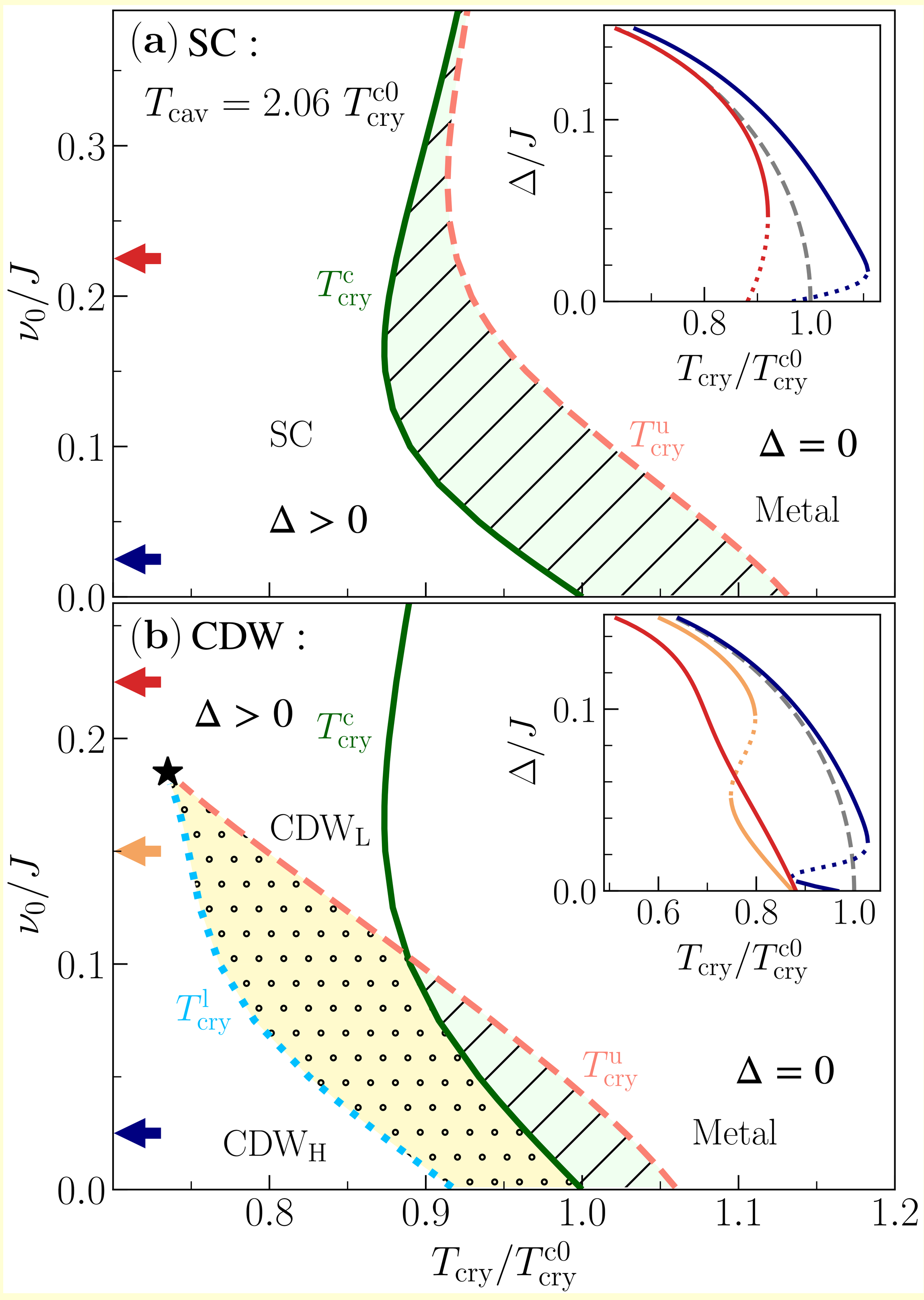
Thermal equilibrium → SC & CDW are equivalent → continuous transition

$g_0 \neq 0$:

Out of equilibrium SC & CDW differ as $\eta = \begin{cases} 1 & \text{for SC} \\ -1 & \text{for CDW} \end{cases}$ Particle and hole currents couple to photons differently

Non-thermal steady-state phase diagram (4)

Parameters: $g_0 = 0.1J$, $\gamma = 0.01\,J$, $\gamma_B = 0.5\,\nu_0$, $T_{cav} = 2.06\,T^{c0}_{cry}$



- Phase coexistence for $T^c_{cry} < T_{cry} < T^u_{cry}$
  ↓
  First-order transition
- Enhancement of gap and of transition temperature

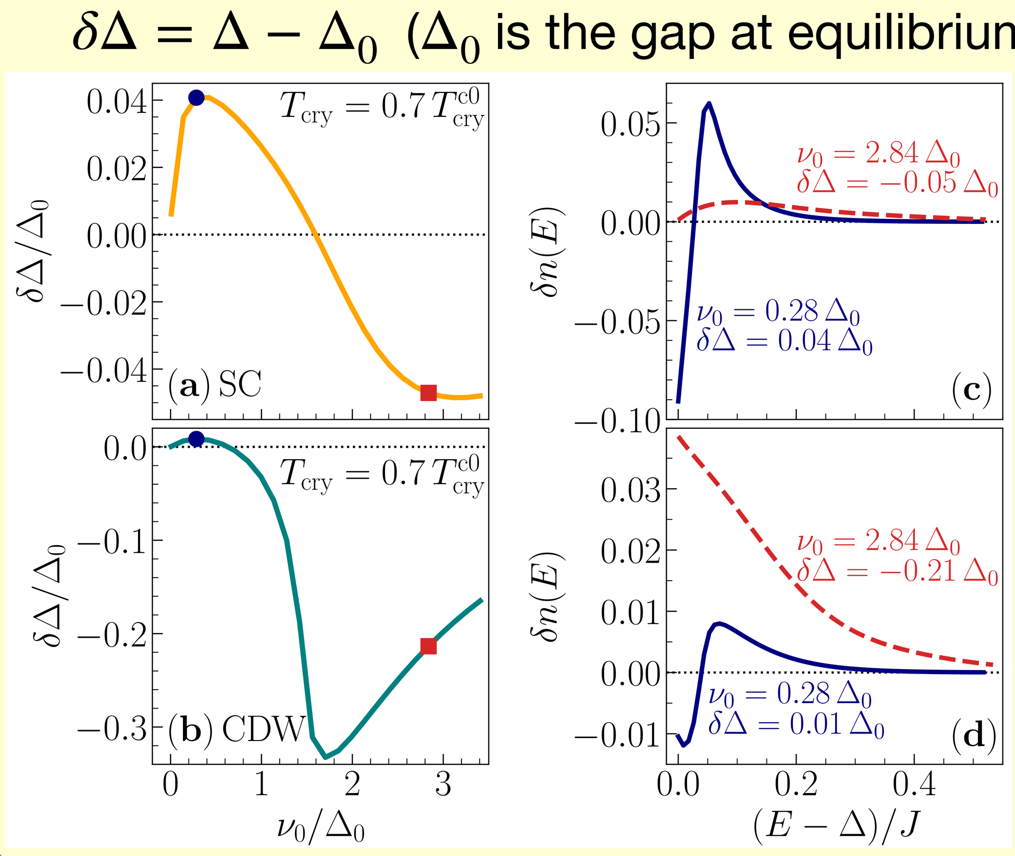
  Eliashberg effect!
- 2 kind of ordered phases: CDW_L (smaller Δ) CDW_H (larger Δ)
- Phase-coexistences: Metal & CDW_H (striped) CDW_L & CDW_H (dotted)
- Critical point (black star)
- Multi-staged transitions

Out of equilibrium, the CDW phase diagram shows more features than SC!

Gap enhancement and distribution function (5)

Enhancement of gap Δ is a necessary (but not sufficient) condition for the increment of transition temperature.

$\delta\Delta = \Delta - \Delta_0$ (Δ_0 is the gap at equilibrium)



- Gap is enhanced for $\nu_0 \lesssim \Delta_0$.
- For the blue curves ($\nu_0 < \Delta_0$), occupation is depleted ($\delta n < 0$) enough near $E = \Delta$ to enhance the gap.
- For the red curves ($\nu_0 > \Delta_0$), the non-thermal redistribution of particles is not favorable to enhance the gap.

Non-equilibrium Ginzburg-Landau theory (6)

For small Δ and $T_{cry} \approx T^c_{cry}$ (where $\Delta \to 0$),

$$\mathcal{F}_{neq}[\Delta] = \frac{T_{cry} - T^c_{cry}}{2T^c_{cry}}\Delta^2 + \eta\frac{g_0^2[N_{T_{cav}}(\nu_0) - N_{T_{cry}}(\nu_0)]}{3\pi\gamma J T^c_{cry}}\Delta^3 + \frac{7\zeta(3)}{32\pi^2}\frac{1}{(T^c_{cry})^2}\Delta^4 + \cdots,$$

- Cubic term is present only out-of-equilibrium
- Differentiates between SC and CDW through η

↓

$$\Delta[T_{cry}] \approx \frac{\pi\gamma J}{g_0^2}\frac{\eta}{N_{T_{cav}}(\nu_0) - N_{T^c_{cry}}(\nu_0)}(T_{cry} - T^c_{cry})$$

Linear growth of the order parameter!

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Queries? Suggestions?

contact: md.islam@uni-a.de

arXiv link