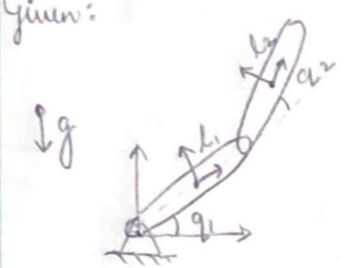
Given:



$l = 1,\ m_l = 1,\ I_l = 1/12,\ \tau_1, \tau_2, g$

$q(t_0) = \left[-\frac{\pi}{2}, 0\right]^T,\quad \dot{q}(t_0) = [0,0]^T$

$q(t_f) = \left[\frac{\pi}{2}, 0\right]^T,\quad \dot{q}(t_0) = [0,0]^T$

$\Delta t = h = 20\times10^{-3}s,\quad t_f - t_0 = 1.5s$

Objective function: $\int \tau^T \tau \, dt$

1.1 $M\ddot{q} + C\dot{q} + N = \Upsilon \Rightarrow \ddot{q} = M^{-1}(-C\dot{q} - N + \Upsilon)$ — ③

Linear Interpolation: $\ddot{q}(t) \approx \ddot{q}(t_k) + \left(\frac{t - t_k}{t_{k+1} - t_k}\right)\left(\ddot{q}(t_{k+1}) + \ddot{q}(t_k)\right)$

For $t = t_{k+1} \Rightarrow$ Similarly,

$\tau(t) \approx \tau(t_k) + \left(\frac{t - t_k}{t_{k+1} - t_k}\right)\left(\tau(t_{k+1}) + \tau(t_k)\right)$

Quadratic Interpolation:

$\dot{q}(t_{k+1}) = \dot{q}(t_k) + \frac{\overbrace{(t_{k+1} - t_k)}^{h}}{2}\left[\ddot{q}(t_{k+1}) - \ddot{q}(t_k)\right]$ — ①

$q(t_{k+1}) = q(t_k) + \frac{h}{2}\left[\dot{q}(t_{k+1}) - \dot{q}(t_k)\right]$ — ②

Decision Variables : $\boxed{q, \dot{q}, \tau}$ → 3 decision variables

Other constraints: $q(t_0),\ \dot{q}(t_0),\ q(t_f),\ \dot{q}(t_f)$

Cost function to minimize: $\int \tau^T \tau \, dt \approx \sum_{k=0}^{N-1} \frac{h}{2}\left[\tau^T\tau(t_k) + \tau^T\tau(t_{k+1})\right]$

Total constraints: $\boxed{14}$ → 2 for each $\ddot{q}$, $q(t_0)$, $\dot{q}$, q, $q(t_f)$, $\dot{q}(t_0)$, $\dot{q}(t_f)$

1.3 The trajectory for the unbounded case is such that the second link almost folds onto link 1 but for the bounded case, second link traces a wider path. The wider path also requires more torque input as a result. Thus, the cost is higher for the bounded case.

Optimized Trajectory for Unbounded case (1.2)

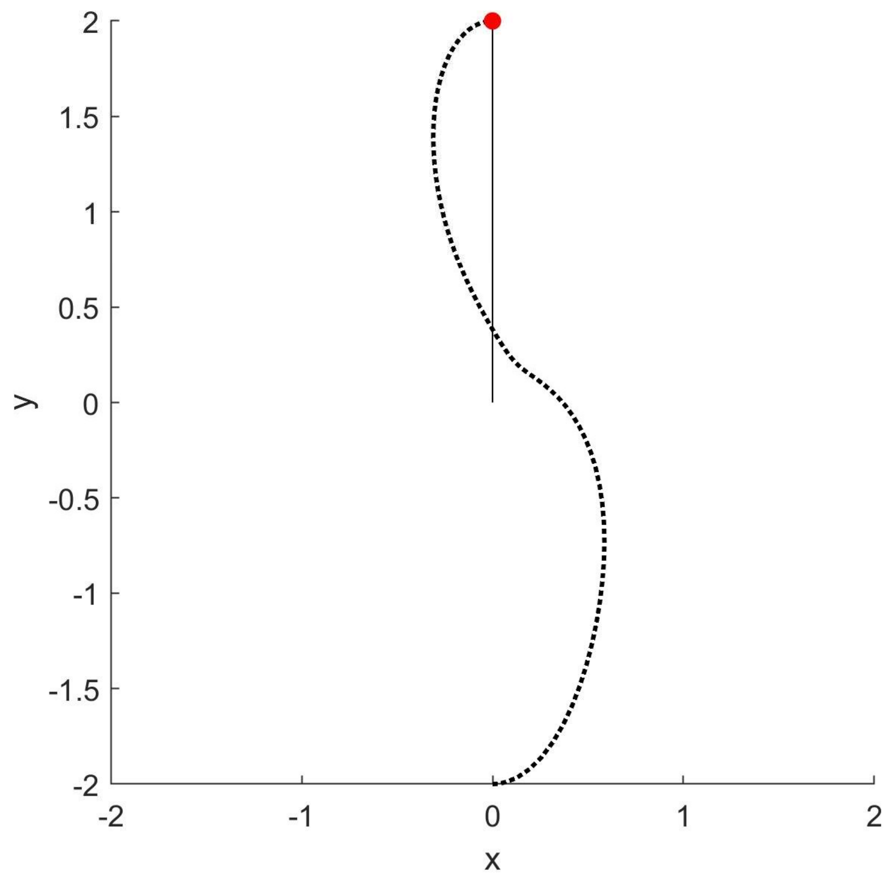


Figure 1: Optimized Trajectory for Unbounded case (1.2)

Optimized Trajectory for bounded case (1.3)

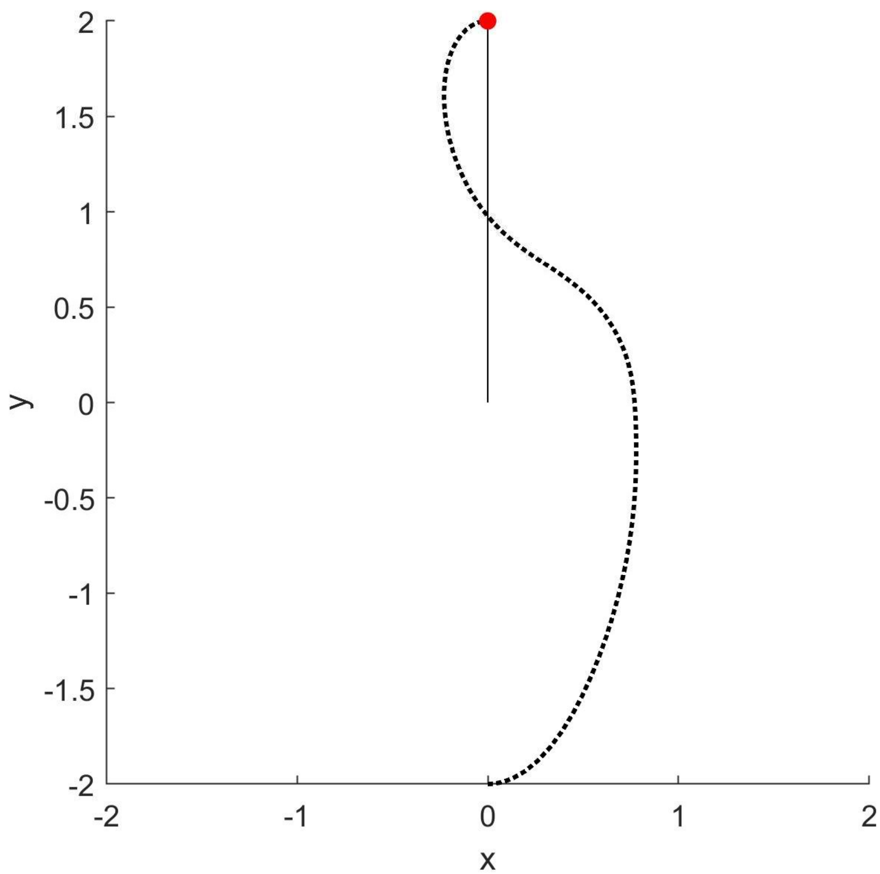


Figure 2: Optimized Trajectory for bounded case (1.3)