# Exercice 27.1-1

$h(m) = m + 1,\ m < p$
$h(m) = k + p,\ m \geq p$
We wish to find p that minimises the competitive ratio C
If p < k:
$C(m)_p = \{\frac{1}{1} ... \frac{p}{p}, \frac{p+k}{p+1} ... \frac{p+k}{k}\} \Rightarrow max_m\{C(m)_p\} = \frac{p+k}{p+1}$
If p>=k:
$C(m)_p = \{\frac{1}{1} ... \frac{k}{k} ... \frac{p}{k}, \frac{p+k}{k} ... \frac{p+k}{k}\} \Rightarrow max_m\{C(m)_p\} = \frac{p+k}{k}$
The minimum competitive ratio is achieved for **p=k-1**
Where we get: $max_m\{C(m)_p\} = \frac{2k-1}{k} = 2 - \frac{1}{k}$

# Exercice 27.1-2

Lets denote by:
$d = 1...D$: the possible number of days of skiing
$R$: rent value per day
$B$: buy price
$d_0 = E[B/R] + 1$, day starting from which it becomes more profitable to buy than to rent
$F(d)$: The optimal algorithm
$A(d)$: proposed online algorithm
$C$: set of ratio yields
The optimal algorithm is the one that minimises total spending after d days of skiing given a priori knowledge of total skiing duration:
$F(d) = d * R,\ d < d_0$
$F(d) = B,\ d \geq d_0$
We propose the following online algorithm:
- So long as the number of days of skiing is below $d_0$, **rent**
- Otherwise, **buy**

Then, the algorithm looks as below
$A(d) = d * R,\ d < d_0$
$A(d) = B + (d_0 - 1) * R,\ d \geq d_0$
The ratio yields for all values of d for this algorithm is:
$C = \{\frac{R}{R}, \frac{2R}{2R} ... \frac{(d_0-1)R}{(d_0-1)R}, \frac{(d_0-1)R+B}{B} ... \frac{(d_0-1)R+B}{B}\}$
Following computed max ratio:
$max_d(C) = \frac{(d_0-1)R+B}{B}$
On the other side:
$d_0 - 1 = E[B/R] \leq B/R \rightarrow (d_0 - 1)R + B \leq 2B \rightarrow max_d(C) \leq 2$