

Exercise 5.1-2

$n = b - a + 1$: The width of the selection interval

$p = E[\ln_2(n)] + 1$: The nearest power of 2 bigger than n

Example:

$Random(3, 7)$

$n = 5$

$p = 3: 2^3$

There are 8 different combinations for a 3 run sequence of $Random(0,1)$ each with equal probability.

Pseudocode:

$Random(a, b)$:

- $n = b - a + 1$
- $p = E[\ln_2(n)] + 1$
- Generate a list L of the first n permutations in $\{0, 1\}$ of size p
- Do:
 - Generate a random permutation P of size p
- While (P not in L)
- Getindex i of the permutation in L
- **Return** Integer with index i in the $\{a, \dots, b\}$

Exercise 5.1-3

Runtime $\approx 1/p \times \text{BIASED-RANDOM time}$

Explanation:

- We wish to have a balanced dice from an unbalanced one.
- We need to run the dice $1/p$ times to record a fair measurement of the minority class.
- For each call of $UNBIASED-RANDOM$ we run $BIASED-RANDOM$ $1/p$ times

Exercise 5.2-5

N total number of customers

X_n : Customer n gets his correct hat

$$E[X_n] = 1/n$$

S : sum of customer receiving their correct hats

$$S = E[\sum X_n] = \sum E[X_n] = \sum 1/n \approx \ln(N)$$