Exercice 14.4-1

$T(n) = 1 + \sum_{i=o}^{n-1} T(i) = T(n-1) + 1 + \sum_{i=o}^{n-2} T(i) = T(n-1) + T(n-1) = 2 * T(n-1), \forall n > 0\ (1)$

$T(0) = 1 (2)$

Apply product from both sides:

$(1) \Rightarrow \prod_{i=1}^{n} T(i) = \prod_{i=1}^{n} [2 * T(i-1)] = 2^n \prod_{i=1}^{n} T(i-1), \forall n > 0\ (3)$

$(3) \Rightarrow T(n) = 2^n * T(0) = 2^n, \forall n > 0$

$(2) \Rightarrow T(n) = 2^n, \forall n \geq 0$

Exercice 14.4-1

a. For any given year i, let j_0 be the investment with max return.

→ Its obvious that placing all the money in that year on investment j_0 will exceed any other split.

b. Once the year i investment is made and maximized, the investment choice for next year doesnt depend on the previous because the transition and staying costs dont depend on the investment but are fixed (f_1 and f_2).

→ Therefore, the algorithm exhibits optimal substructure

c. Let:
   i. K_i be the capital realized at the end of year i
   ii. r_i be the maximum return for the year i

Then we have for the first year i=0: $K_0 = 10K\$ * r_0$

any consecutive year i: $K_{i+1} = K_i + max\{(K_i - f_1) * r_i, (K_i - f_2) * r_{i+1}\}$