Exercice 6.1-1

For a binary tree of height h:
Minimum and maximum number of nodes are respectively: 2^h, $2^{h+1} - 1$

Exercice 6.1-2

For a given height h, the number of nodes is $n = 2^h$, this means the depth $h = log_2(n)$

Exercice 6.1-3

By definition of the data structure

Exercice 6.1-4

As corollary, the smallest nodes for each subtree reside on the leafs

Exercice 6.2-2

Notations:
lg designates log base 2
$E[x]$ designates floor value i.e. E[2.3] = 2
We want to show that: $E[n/2^{h+1}] + 1 \geq 1/2, \forall\, 0 \leq h \leq E[lg(n)]$
Equivalent to showing: $E[n/2^{h+1}] \geq - 1/2, \forall\, 0 \leq h \leq E[lg(n)]$
Proof:
$lg(n/2^{h+1}) = lg(n) - lg(2^{h+1}) = lg(n) - h - 1$
Given: $0 \leq h \leq E[lg(n)] \Rightarrow - 1 + lg(n) - E[lg(n)] \leq lg(n) - h - 1$
However: $lg(n) - E[lg(n)] \geq 0$
This means that: $- 1 \leq lg(n) - h - 1 \Rightarrow - 1 \leq E[lg(n) - h - 1]$
On the other side: $- 1 = lg(- 1/2)$
So: $lg(- 1/2) \leq E[lg(n/2^{h+1})]$
Since log is monotonous we have the result

Exercice 6.2-3

Starting from 1 means that all subtress below 1 are ordered which is not the case.