

## Definitions

Let:

$A$ : be a positive definite matrix

$n$ : size of the square matrix  $A$

$i, j$ : be the rows and columns index respectively

$x$ : be a column vector of size  $n$

## Exercise 28.3-1

Let:

$x^k$ : be the column vector where all values are zero except index  $k$

Then

$$x^{k^T} A x^k = a_{kk} > 0 \text{ by definition of } A$$

## Exercise 28.1-3

Lets assume that the maximum isnt on the diagonal.

Let:  $i_0, j_0$  be the indexes of the maximum cell where  $i_0 \neq j_0$

Additionally,

$$x^0 \in \mathbb{R}^{n \times 1} / x_i^0 = 0 \text{ if } i \neq i_0, j_0 \text{ and } x_{i_0}^0 = 1 \text{ and } x_{j_0}^0 = -1$$

It follows that:

$$(x^0)^T A x^0 = a_{i_0 i_0} + a_{j_0 j_0} - (a_{i_0 j_0} + a_{j_0 i_0}) = a_{i_0 i_0} + a_{j_0 j_0} - 2a_{i_0 j_0}$$

$$\text{Since } a_{i_0 j_0} \text{ is the max cell then } a_{i_0 i_0} + a_{j_0 j_0} - 2a_{i_0 j_0} \leq 0$$

This contradicts the definition  $(x^0)^T A x^0 > 0$

Hence the maximum lays on the diagonal