P_Groudon Mock AMC 12

January 2020

Instructions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

1. In 2016, Atlantic Airlines were able to fill an aircraft fuel tank with 30000 liters of fuel for \$8000. In 2020, they were able to fill the tank of one of their larger airplanes with 50000 liters of fuel for \$24000. By what percentage has the cost of fuel increased over this time?

(A) 60% **(B)** $66\frac{2}{3}$% **(C)** 75% **(D)** 80% **(E)** 90%

2. Which of the following figures can be rotated about their center(s) by at least one of 90°, 180°, or 270° to give a figure that looks exactly the same?

I. Equilateral triangle

II. Parallelogram that is not a rectangle

III. Regular hexagon

(A) Only I **(B)** Only II **(C)** I and II **(D)** II and III **(E)** I, II, and III

3. How many 3-digit positive integers have exactly one odd digit?

(A) 325 **(B)** 350 **(C)** 360 **(D)** 375 **(E)** 400

4. A parabola whose equation is in the form $y = ax^2 + bx + c$ for some real numbers a, b, and c intersects the x-axis at $x = 3$ and $x = 7$. If the vertex of this parabola has y-coordinate 1, what is the y-intercept of the parabola?

(A) $-\frac{21}{2}$ **(B)** -7 **(C)** -6 **(D)** $-\frac{21}{4}$ **(E)** -5

5. Each vertex of a regular pentagon is colored red or blue. Two colorings are considered indistinguishable if one coloring can be rotated to obtain the other. How many distinguishable colorings are possible?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 11

6. Let A denote the set of all positive integer divisors of 2100, and let B denote the set of all positive integer divisors of 360. How many positive integers are in at least one of A or B?

(A) 42 **(B)** 48 **(C)** 53 **(D)** 54 **(E)** 66

7. Define $\lfloor x \rfloor$ to be the greatest integer less than or equal to x. Suppose that for some positive integer n, there are exactly 25 distinct integers m that satisfy $\lfloor \sqrt{m} \rfloor = n$. What is the sum of the digits of n?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 10

8. A polynomial $P(x) = 2x^3 + ax^2 + bx + 480$ has real coefficients and has $-3 + i$ as a root. What is the value of b?

(A) -154 **(B)** 77 **(C)** 154 **(D)** 188 **(E)** 308

9. Suppose $\{a_k\}$ is a sequence defined by $a_{k+2} = a_{k+1} - a_k$ for all nonnegative integer values of k. If $a_0 = m$ and $a_2 = n$ for some integers m and n with $|m|, |n| \leq 10$, for how many ordered pairs (m, n) do we have $a_{2020} = 10$?

(A) 11 **(B)** 20 **(C)** 21 **(D)** 30 **(E)** 44

10. Let x and y be positive real numbers satisfying $y = 5x$ and $\log_x y + \log_y x = \frac{10}{3}$. Let S be the sum of all possible values of x^2. Find the smallest integer greater than or equal to S

(A) 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

11. A paper trapezoid has side lengths $AB = 3, BC = 8, CD = 9$, and $DA = 9$ with AB parallel to DC. Let E be a point on line segment $\overline{BC}$ such that, when C is folded over the crease $\overline{DE}$, C coincides exactly with A. What is the length of CE?

(A) 3 **(B)** $\frac{15}{4}$ **(C)** 4 **(D)** $\frac{13}{3}$ **(E)** $\frac{24}{5}$

12. The curves with equations $x^2 + y^2 = a$ and $x + 2y = a$ meet at exactly one point, where a is a nonzero real number. The point of intersection can be written as (p, q). What is the value of $p + q$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

13. Let N be the smallest multiple of 63 whose base-6 representation contains exactly 6 digits, each of which is either a 1 or a 3. What is the sum of the digits of N when written in base-6?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

14. A regular tetrahedron $ABCD$ has 6 edges, each of which is painted red with probability $\frac{1}{2}$ or blue with probability $\frac{1}{2}$. An ant starts at vertex A and may walk along any red edge, but it cannot walk along a blue edge. What is the probability that it is possible for the ant to visit all the vertices of $ABCD$?

(A) $\frac{1}{2}$ **(B)** $\frac{17}{32}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{19}{32}$ **(E)** $\frac{5}{8}$

15. An ellipse has a major axis of length 10 and a minor axis of length 6. The major axis intersects the ellipse at two distinct points, A and D. Let B and C be the foci of the ellipse, with $AB < BD$ and $AC > CD$. A point E lies on the ellipse such that $CE = 3$. The length of DE can be expressed as $\sqrt{m}$ for some positive integer m. What is m?

(A) 10 **(B)** 12 **(C)** 13 **(D)** 15 **(E)** 18

16. Triangle $\triangle ABC$ with centroid G has $AB = 8$. If BG and CG are perpendicular, what is the maximum possible area of $\triangle ABC$?

(A) 20 **(B)** $\frac{45}{2}$ **(C)** $\frac{95}{4}$ **(D)** 24 **(E)** $\frac{74}{3}$

17. How many ways are there to place 3 Os and 3 Xs in a 3×4 rectangular grid, given that each row and column may contain at most one O and at most one X? Each cell of the grid may not contain more than one letter.

(A) 216 **(B)** 264 **(C)** 288 **(D)** 336 **(E)** 360

18. A non-right cone C with volume 50π has a circular base of radius 5. The shortest distance from the apex of C to a point on the circumference of the circular base of C is $3\sqrt{5}$. A sphere S is circumscribed around C such that the apex of C and every point on the circumference of the circular base of C lies on S. If the apex of C lies directly above some point on the circular base of C, the radius of S can be expressed in the form $\frac{m\sqrt{n}}{p}$, where m and p are relatively prime positive integers and n is not divisible by the square of any prime. What is $m + n + p$?

(A) 18 **(B)** 21 **(C)** 23 **(D)** 24 **(E)** 26

19. Suppose 10 distinct points $P_1, P_2, \ldots, P_9, P_{10}$ are placed in a plane such that no three points are collinear and no three points form a right triangle. Among the angles that can be formed with some set of 3 of these 10 points, let N be the minimum number of acute angles that can be formed. What is the sum of the digits of N?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

20. Define sequences p_k, q_k, and r_k for all positive integers k by $p_k = \sqrt[3]{(k+1)^2(k-1)}$, $q_k = \sqrt[3]{(k+1)(k-1)^2}$, and $r_k = \sqrt[3]{2-3(p_k-q_k)}$. If the sum $r_1 + r_2 + ... + r_{214} + r_{215}$ can be expressed in the form $\sqrt[3]{m} + n$ for some positive integers m and n, what is $m+n$?

(A) 220 **(B)** 221 **(C)** 222 **(D)** 223 **(E)** 224

21. For each positive integer n, define $\phi(n)$ to be the number of positive integers less than or equal to n which are relatively prime to n. For example, $\phi(10) = 4$ and $\phi(23) = 22$. For how many positive integers $a < 2020$ does there exist a positive integer b such that $4\phi(ab) = 7\phi(a)\phi(b)$?

(A) 72 **(B)** 73 **(C)** 95 **(D)** 96 **(E)** 288

22. Bertie Bee is placed at the point $(0,0)$ in the x-y plane. Every minute, if she is at the point (x,y), she moves to one of the four points $(x+1,y)$, $(x-1,y)$, $(x,y+1)$, or $(x,y-1)$. After 2020 minutes, she is back at the point $(0,0)$. If N is the number of possible sequences of moves she could have made, what is the highest power of 10 that divides N?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 8

23. A triangle $\triangle ABC$ has a right angle at B. A point D lies on segment AC such that $CD = 6$. The circle ω with diameter CD intersects AB at two distinct points, E and F, such that $AE < AF$. If $AE = 2$ and $DE = EF$, the length BF can be written in the form $\frac{m}{n}$, where m and b are relatively prime positive integers. What is $m+n$?

(A) 33 **(B)** 34 **(C)** 35 **(D)** 36 **(E)** 37

24. A hexagon $ABCDEF$ is convex and all of its sides except AF have length 2020. In the hexagon, $\angle ABC = 108°$, $\angle BCD = 126°$, $\angle CDE = 162°$, $\angle DEF = 108°$, and $\angle FAB > 108°$. There exists a unique point P such that P is on the same side of line AE as F, $\angle APE = 153°$, and $AP \cdot \sqrt{2} = PE$. Which of the following is the closest to the degree measure of $\angle AFP$?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

25. For how many ordered pairs of complex numbers (x,y) does there exist some positive real number k for which all of the following equations are true?

- $|x^3 + y^3 - k^3 i - 3ki \cdot xy| = 32$
- $x^2 + y^2 = k^2$
- $|x| = |y| = k$

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8