2021 March MIMC 10

Michael595 and Interstigation

March 2021

Instructions:
This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct. You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. No aids are permitted other than scratch paper, graph paper, ruler, compass, and erasers (no calculators are accepted after 2006, and no protractors are accepted after 2020). Figures are not necessarily drawn to scale. You will have 75 minutes working time to complete the test.

1. What is the value of $\frac{1}{2} + 2^{-8}$?
(A)$\frac{127}{256}$ **(B)**$\frac{1}{2}$ **(C)**$\frac{129}{256}$ **(D)**$\frac{65}{128}$ **(E)**$\frac{131}{256}$

2. How many integers x **are there such that** $-2\pi < x \leq 3\pi$?
(A)14 **(B)**15 **(C)**16 **(D)**17 **(E)**18

3. There exist an integer x **such that the twice of the reciprocal of** x **is** 8 **times the square of the reciprocal of** x**. Find** $\frac{2x}{3}$
(A)2 **(B)**$\frac{8}{3}$ **(C)**3 **(D)**$\frac{10}{3}$ **(E)**4

4. Find the area between the circumcircle and incircle of an equilateral triangle with area of $4\sqrt{3}$.
(A)2π **(B)**4π **(C)**6π **(D)**8π **(E)**10π

5. Mr. James has 8 **books on his shelf, which includes** 3 **math books,** 4 **English books and a Spanish book. He wants to rearrange them so that the math books are all adjacent and English books are all adjacent as well. Find the number of ways he can do that.**
(A)280 **(B)**840 **(C)**864 **(D)**5040 **(D)**40320

6. What is the probability that a subset with more than 1 **element randomly chosen in the set** $\{1, 2, 3, 4, 5, 6, 7\}$ **have no two elements that has** $gcd > 1$**, and that subset does not contain** 1**? The answer can be expressed in the form of** $\frac{x}{y}$**. What is** $2x + y$**?**

(A) 13 **(B)** 22 **(C)** 33 **(D)** 54 **(E)** 120

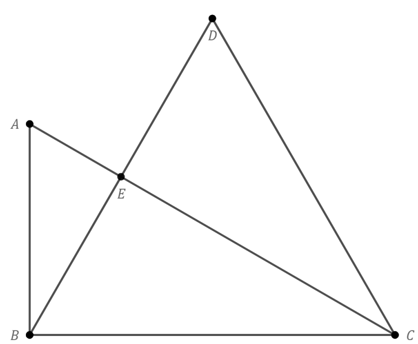
7. If $xy + 2x + 3y = 2$**, What is the sum of all possible values of** x **if they are both integers?**
(A) -36 **(B)** -24 **(C)** -5 **(D)** 5 **(E)** 36

8. Let x **be the sum of the last four digits and** y **be the sum of all digits of the expression** $99 + 9999 + 999999 + 99999999 + 9999999999 + ... + 999...99$**(two hundred** $9s$**). Find** $x + y$**.**
(A) 97 **(B)** 98 **(C)** 99 **(D)** 100 **(E)** 101

9. Define a function $f(x)$ **such that** $f(x) = \lfloor\sqrt{x}\rfloor$**. What is** $f(201600)$**?**
(A) 145 **(B)** 146 **(C)** 447 **(D)** 448 **(E)** 449

10. $(100!)^9 + 506$ **is a multiple of what number?**
(A) 97 **(B)** 101 **(C)** 103 **(D)** 107 **(E)** 503

11. A $30 - 60 - 90$ **ABC triangle with the longer leg** BC **length 8 is overlapping with an equilateral triangle** DBC **with area** $16\sqrt{3}$**, and they shared a common base. The hypotenuse,** AC**, intersects the side** DB**, at** E**. The length of** CE **can be expressed as** $x\sqrt{y}$**. Find** $xy + x + y$**.**



(A) 11 **(B)** 12 **(C)** 19 **(D)** 23 **(E)** 24

12. If there are x **ordered six-ples** (a, b, c, d, e, f) **of positive integers such that all of them are multiples of** 4 **and** $a + b + c + d + e + f = 1956$**. Find the remainder when** x **is divided by 11.**
(A) 0 **(B)** 2 **(C)** 7 **(D)** 8 **(E)** 9

13. There is an equilateral triangle with area $16\sqrt{3}$**, Let point** X, Y, Z **be on side** AB, BC, AC**, respectively, such that** $AX : XB = 3 : 1$**,** $BY : YC = 3 : 1$**,** $AZ : ZC = 1 : 3$**. The perimeter of triangle** XYZ

can be expressed as $m\sqrt{n}$ such that n does not have perfect square factors. Find mn.

(A)40 **(B)**42 **(C)**44 **(D)**46 **(E)**68

14. When 2579 is expressed in base b, the resulting number is $5n5$ which both b and n are integers. Find $b+n$.

(A)14 **(B)**20 **(C)**29 **(D)**31 **(E)**36

15. Castoy, the frog, is at $(-3,4)$. He want to reach $(0,10)$ as his final destination. Given that he can only move 1 unit in the positive x or y direction. Find the number of ways he can accomplish this.

(A)8 **(B)**35 **(C)**36 **(D)**70 **(E)**84

16. What is the area inside of $\frac{x^2}{4}+\frac{y^2}{9}=1$ but outside of $x^2+y^2=4$.

(A)0π **(B)**2π **(C)**3π **(D)**4π **(E)**5π

17. The numerical representation of $21!$ in base 10 has over 60000 positive integer divisors. Find the probability that a random chosen positive integer divisor is a power of 6. The probability can be written as $\frac{x}{y}$, find $x+y$.

(A)176 **(B)**6001 **(C)**6081 **(D)**6091 **(E)**12162

18. Find the largest real value of x that is a root of the polynomial $f(x)=x^4-14x^3+47x^2+14x-15$. The largest real root can be expressed as $\frac{a+\sqrt{b}}{c}$. Find $2a+b+c$.

(A)81 **(B)**82 **(C)**83 **(D)**84 **(E)**85

19.Given $x+y=2$ and $xy=-1$, what is the value of $\frac{(x^2+y^2)(x^4+y^4)(x^6+y^6)}{x^3y^3-3xy}$

(A)198 **(B)**20196 **(C)**20198 **(D)**40392 **(E)**40396

20. Find the remainder when $1+11+11^2+11^3+11^4+...+11^{1009}$ is divided by 1000.

(A)60 **(B)**260 **(C)**460 **(D)**660 **(E)**860

21. Given that r,s,t are the three roots of polynomial x^3-2x^2+4x-8, find $|sr^2+rt^2+ts^2+st^2+rs^2+tr^2|$.

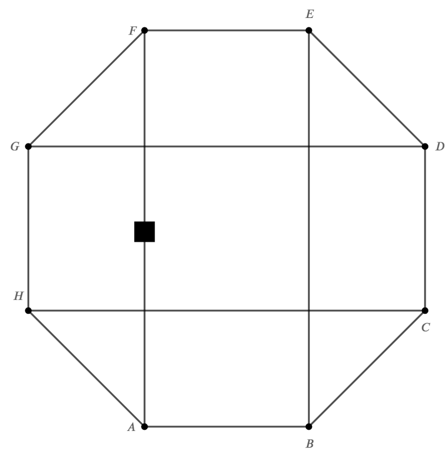
(A)4 **(B)**8 **(C)**12 **(D)**16 **(E)**20

22. What is the remainder when $10001^{111}+10001^{222}+10001^{333}+10001^{444}+10001^{555}+10001^{666}+10001^{777}+10001^{888}+10001^{999}+10001^{1100}+9990$ is divided by 16?

(A)0 **(B)**1 **(C)**10 **(D)**11 **(E)**12

23. In the regular octagon $ABCDEFGH$ with side length 8, diagonals BG, CF, AD, EH are drew to form 4 isoceles right triangle at the

four corners, 4 rectangles on sides, and a giant square in center. A piece of unit square landed in the region of octagon. The sides of the unit square must be parallel to the diagonals. Find the area of region that the center of the unit square can locate in the octagon given that the landed unit square must have at least half of the unit square in the middle giant square and the unit square must intersect or touch the side of the giant square.



(A)11 **(B)**14 **(C)**31 **(D)**64 **(E)**80

24. Given that $(x-2\sqrt{2})^2+(y+2\sqrt{2})^2=8$ and the two axises x and y on the standard Cartesian plane intersects the equation at A and B, respectively. Also, a line with slope -1 both intersects the equation at A and intersects y axis at C, and another line with slope -1 both intersects the equation at B and intersect the x axis at point D. Also given that $(-2\sqrt{2},2\sqrt{2})$ is point E, the enclosed area of the curve AB and line segments $BD, DE, EC,$ and AC can be expressed as $x\cdot(y-z\pi)$, where x,y,z are distinct integers such that $gcd(y,z)=1$. Find $2x+y-z$.

(A)11 **(B)**12 **(C)**13 **(D)**14 **(E)**15

25. How many six-digit integers $abcdef$ has the property that the three four-digit integers with $abcd>bcde>cdef$ where $abcd,bcde,cdef\neq 0$ and forms a decreasing geometric sequence?

(A)0 **(B)**1566 **(C)**1567 **(D)**1568 **(E)**14112