2018 Memorial Day Mock AMC 10

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Instructions:

1. Set a timer for **75 minutes** before starting the test.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
4. No aids are permitted other than pencils, pens, scratch paper, graph paper, rulers, compasses, protractors and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. Submit your answers by sending a private message (PM) to QIDb602.
7. A leaderboard with scores will be posted. If you want to remain anonymous in the leaderboard, please indicate in the private message (PM) that you want to remain anonymous.
8. If there are errors, please indicate the errors in the private message (PM).

1. What is $\sqrt{6^4 + 2 \cdot 6^2 \cdot 8^2 + 8^4}$?

   (A) 10 **(B)** $10\sqrt{3}$ **(C)** $2\sqrt{1398}$ **(D)** $4\sqrt{481}$ **(E)** 100

2. Joe wants to buy a shirt. He notices that the original price is \$25, but it is on sale for 20% off. The tax is 8% of the sale price. How much money does Joe need to pay for the shirt?

   (A) \$16.00 **(B)** \$18.40 **(C)** \$21.60 **(D)** \$22.00 **(E)** \$36.00

3. Two fair six-sided dice numbered with integers from 1 through 6 are tossed. What is the probability that the sum and the product of the two values that show up on top of the dice are both odd?

   (A) 0 **(B)** $\frac{1}{16}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

4. A picture frame is made by cutting out a $7 \times 10 \times 1$ rectangular prism from the center of a $9 \times 12 \times 1$ rectangular prism. What is the surface area of the picture frame?

   (A) 128 **(B)** 136 **(C)** 144 **(D)** 152 **(E)** 160

5. Manuel chooses a random real number from the interval $[0, 2018]$. Nicole, Olivia, and Peter try to guess the number that Manuel chose. Their guesses are also random real numbers from the interval $[0, 2018]$. What is the probability that Nicole's guess is too low, Olivia's guess is too high, and Peter's guess is also too high?

   (A) $\frac{1}{16}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

6. n is a positive integer. The decimal representation of $n!$ ends in r zeros. The decimal representation of $n! + (n-1)!$ ends in s zeros. Which of the following values of n causes s to be greater than r?

   (A) 2014 **(B)** 2015 **(C)** 2016 **(D)** 2017 **(E)** 2018

7. a and b are positive integers in the equation $a^2 + 4b = b^2 + a$, and $a > 1$. What is the only possible value of $a + b$?

   (A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 24

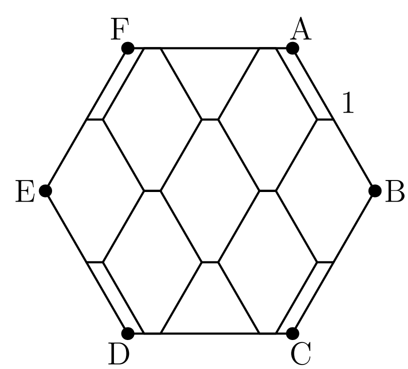
8. Each element of the set $\{1, 2, 3, \ldots, 9\}$ is placed into either set X or set Y but not both. All of the elements in set X make up a single arithmetic progression with at least three elements. All of the elements in set Y also make up a single arithmetic progression with at least three elements. For example, a possibility is $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6, 7, 8, 9\}$. How many distinct sets can X be equal to?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

9. Ashley and Ben are playing a game. At the beginning of the game, $n = 1$. Starting with Ashley, the players then take turns multiplying n by an integer between 2 and 9, inclusive. The first player to get a product of at least 100 wins. If neither player makes any mistakes, how many different integers can Ashley multiply n by on her first turn in order to win?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. Seven congruent equilangular hexagons are drawn inside regular hexagon $ABCDEF$ as shown. If $\overline{AB} = 1$, the perimeter of one of the smaller hexagons is equal to $\frac{a}{b}$ where a and b are relatively prime positive integers. What is $a + b$?

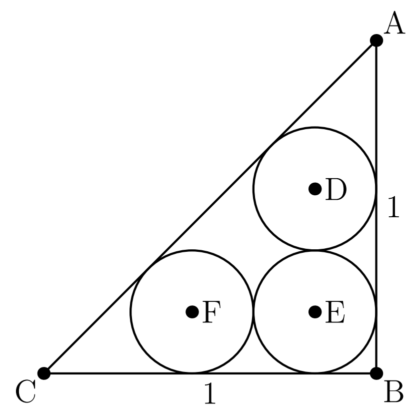


(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

11. How many numbers from the set $\{8^2, 18^2, 28^2, \ldots, 2018^2\}$ have a remainder of 64 when divided by 100?

(A) 40 **(B)** 41 **(C)** 403 **(D)** 404 **(E)** 405

12. In triangle ABC, $AB = BC = 1$ and $m\angle ABC = 90°$. Non-overlapping congruent circles D, E, and F are placed inside triangle ABC so that each circle is internally tangent to triangle ABC at two different points. If circle E is externally tangent to circles D and F, what is the radius of circle D?



(A) $\frac{2-\sqrt{2}}{4}$ **(B)** $\frac{5-\sqrt{2}}{23}$ **(C)** $3-2\sqrt{2}$ **(D)** $\frac{4-\sqrt{2}}{14}$ **(E)** $\frac{3-\sqrt{2}}{7}$

13. A three-digit positive integer n is called *triskaidekaphilic* if it satisfies at least one of the following conditions:

- n is evenly divisible by 13.
- n contains 13 in its digits. For example, 135 and 513 both satisfy this condition, but 153 and 531 don't.

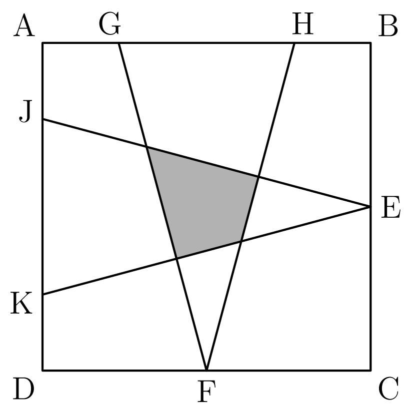
How many three-digit positive integers are *triskaidekaphilic*?

(A) 78 **(B)** 86 **(C)** 87 **(D)** 94 **(E)** 95

14. 6 students each took a 10-point science quiz. Each student received a score within the set $\{7, 8, 9, 10\}$, and each score in the set $\{7, 8, 9, 10\}$ was received by at least one student. In how many ways could the students have scored?

(A) 1320 **(B)** 1560 **(C)** 2400 **(D)** 2640 **(E)** 5760

15. In the diagram shown below, square $ABCD$ has a side length of 10. Point E is the midpoint of $\overline{BC}$ and point F is the midpoint of $\overline{CD}$. If $m\angle EJD = m\angle EKA = m\angle FGB = m\angle FHA = 75°$, the area of the shaded quadrilateral is equal to $\frac{r\sqrt{t}}{s}$ where r, s, and t are positive integers, r and s are relatively prime, and t is not evenly divisible by the square of any integer greater than 1. What is $r + s + t$?



(A) 31 **(B)** 34 **(C)** 37 **(D)** 43 **(E)** 49

16. Anna and Beth are 10 kilometers apart from each other on a road. They both start walking toward each other. Anna walks at 4 kilometers per hour, and Beth walks at 5 kilometers per hour. After a while, Anna runs at 10 kilometers per hour, while Beth still walks at 5 kilometers per hour. Anna and Beth both stopped when they met each other. If the distance that Anna walked is equal to the distance that Anna ran, the total distance that Anna traveled in kilometers is equal to $\frac{m}{n}$ where m and n are relatively prime positive integers. What is $m + n$?

(A) 11 **(B)** 17 **(C)** 19 **(D)** 29 **(E)** 41

17. The diagonals of quadrilateral $ABCD$ are perpendicular to each other. Let point E be the intersection of the diagonals. If $AE = 3$, $BE = 4$, $CE = 5$, and $DE = 6$, the side length of a square inscribed in quadrilateral $ABCD$ with two sides parallel to $\overline{AC}$ and the other two sides parallel to $\overline{BD}$ is equal to $\frac{a}{b}$ where a and b are relatively prime positive integers. What is $a + b$?

(A) 37 **(B)** 39 **(C)** 49 **(D)** 61 **(E)** 155

18. How many integer values of n between 1 and 100, inclusive, cause the value of the following expression to be an integer?

$$\frac{\sqrt[2]{n^{3n}}}{\sqrt[3]{n^{2n}}}$$

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

19. In the xy-coordinate plane, three distinct points with integer x- and y- coordinates are randomly chosen within the area bounded by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. What is the probability that the three points, when connected, form a triangle with an area of 1?

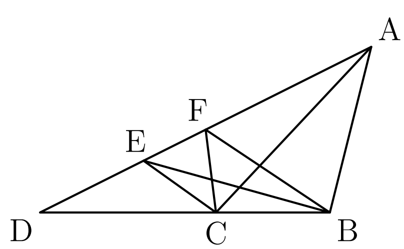
(A) $\frac{4}{21}$ **(B)** $\frac{5}{21}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{8}{21}$

20. q and r are positive integers in the following equation. There is only one possible value of $q+r$. Let $S(n)$ represent the sum of the digits of positive integer n. What is $S(q+r)$?

$$\frac{q}{r} + \frac{r}{q} = \frac{qr}{144}$$

(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 11

21. In the following diagram, $m\angle BAC = m\angle BFC = 40°$, $m\angle ABF = 80°$, and $m\angle FEB = 2m\angle DBE = 2m\angle FBE$. What is $m\angle ADB$?



(A) 12° **(B)** 15° **(C)** 24° **(D)** 30° **(E)** 48°

22. The quadratic equations $y = ax^2 + 20x + 32$ and $y = x^2 + bx + 16$ share the same vertex in the xy-coordinate plane. If a and b are integers, what is $a + b$?

    (A) 9 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 21

23. In triangle ABC, $AB = AC = 5$ and $BC = 6$. Point D is placed on $\overline{AB}$ and point E is placed on $\overline{AC}$ so that $BD = DE$ and $m\angle BDE = 90°$. BD is equal to $\frac{u}{v}$ where u and v are relatively prime positive integers. What is $u + v$?

    (A) 151 **(B)** 157 **(C)** 163 **(D)** 229 **(E)** 239

24. n is a nonnegative integer with up to four digits. If n has less than four digits, leading zeros are added to n until n has four digits. Let $R(n)$ represent n with its digits reversed after adding leading zeros to n if necessary. For example, $R(476) = R(0476) = 6740$ and $R(1453) = 3541$. The function f is defined by $f(x) = |x - R(x)|$. Which integer is closest to the average value of $f(x)$ for all nonnegative integer values of x less than 10000?

    (A) 3267 **(B)** 3297 **(C)** 3300 **(D)** 3326 **(E)** 3594

25. A particle starts at the origin in the xy-coordinate plane. A *move* is defined as going from (x, y) to any one of $(x, y+1)$, $(x+1, y+1)$, $(x+1, y)$, $(x+1, y-1)$, $(x, y-1)$, $(x-1, y-1)$, $(x-1, y)$, or $(x-1, y+1)$ with one additional rule: the x- and y-coordinates of the particle must always stay between 0 and 5, inclusive. The particle stops forever when it gets to $(5, 5)$. In how many ways can the particle can get to $(5, 5)$ using no more than 7 moves?

    (A) 471 **(B)** 473 **(C)** 475 **(D)** 477 **(E)** 479

Here are some optional but encouraged questions about your experience of taking this test.

1. On a scale of 0 to 10, with 0 being very low quality and 10 being perfect, how would you rate the quality of the test?
2. On a scale of 0 to 10, with 0 being much easier and 10 being much more difficult, how would you rate the difficulty of the test in comparison with real AMC 10 tests during the last three years?
3. What additional comments or suggestions do you have about the test?