Vfire's Mock AMC 12

1 Rules

1. DO NOT BEGIN THE TEST UNTIL YOU HAVE READ ALL RULES AND INSTRUCTIONS
2. This is a 25 - question multiple choice test. The answer to each questions will be one of the following: A, B, C, D or E. The problems generally in increasing order of difficulty. (If its not then blame the testsolvers)
3. This is a 75 minute test. No extra time should be allotted.
4. Remember to record your answers as you solve them in order to submit them.
5. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
6. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
7. Figures are not necessarily drawn to scale.
8. Once you are finished with your test, PM you answers to Vfire and I will respond with you score within 24 hours.
9. Have fun :)!

2 Problems

1. Given that $i = \sqrt{-1}$, evaluate the expression $\left(\frac{i+1}{i-1}\right)^2$.

   (A) $-i$ **(B)** i **(C)** $-1-i$ **(D)** $1+i$ **(E)** -1

2. Find the number of integers n such that $\sqrt{n^2-24}$ is an integer.

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 7 **(E)** 8

3. For how many natural number $n \leq 12$ is $(n-1)!$ not divisible by n?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

4. Bob walks a block east from his house. After each block he walks, he flips a fair coin and walks one block east if the coin is heads and one block west otherwise. He stops his walk the first time he returns home. What is the probability he stops his walk after walking exactly eight blocks?

   (A) $\frac{1}{64}$ **(B)** $\frac{5}{32}$ **(C)** $\frac{7}{32}$ **(D)** $\frac{13}{256}$ **(E)** $\frac{5}{128}$

5. If each letter in $\left(a+\frac{b}{c}\right)(d+e)$ is replaced by a different integer from 1 to 9 inclusive, what is the minimum integer value of the expression?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

6. Let x and y be positive real numbers such that $\frac{1}{x^2}+\frac{1}{xy}=\frac{1}{9}$ and $\frac{1}{y^2}+\frac{1}{xy}=\frac{1}{16}$. What is the value of $x+y$?

   (A) $\frac{97}{12}$ **(B)** 9 **(C)** $\frac{39}{4}$ **(D)** 10 **(E)** $\frac{125}{12}$

7. Which of the following could not be the discriminant of a quadratic in the form ax^2+bx+c where $a>0$ and b and c are integers? (The discriminant of ax^2+bx+c is b^2-4ac).

   (A)84 **(B)**85 **(C)**99 **(D)**121 **(E)**133

8. Alison, Beth, and Carol, are each driving on a trip.They are all trying to drive the same distance but they own different cars. Alison's car is refueled every 160 miles, Beth's car is refueled every 240 miles, and Carol's car is refueled every 400 miles. They all start with full gas tanks, but Alison finishes with an empty tank, Beth finishes with 80 miles worth of gas remaining, and Carol finishes with 160 miles worth of gas remaining. Given that the trip is between 5000 and 6000 miles long, what is the total length of the trip?
   (A) 5000 **(B)** 5160 **(C)** 5320 **(D)** 5400 **(E)** 5820

9. In quadrilateral $ABCD$ let E be the intersection of AC and BD. If $\angle ACD = \angle CAB = 90°$, $AE = a$, $EC = b$ and the area of AED is S, which of the following expresses the area of $ABCD$ in terms of S, a and b?

   (A)$S \cdot \frac{a^2}{b^2}$ **(B)** $S \cdot \frac{ab}{(a+b)^2}$ **(C)** $S \cdot \frac{ab}{a^2+b^2}$ **(D)** $S \cdot \frac{a^2+b^2}{ab}$ **(E)**$S \cdot \frac{(a+b)^2}{ab}$

10. Vfire and AIME12345 are having a picnic. In total, they know 6 other men and 6 other women. How many different sets of friends can they invite so that they invite the same number of men and women to the picnic? (Assume that they invite at least one person to the picnic.)

(A)720 **(B)**923 **(C)**1023 **(D)**1440 **(E)**1846

11. Holden2003 has 4 indistinguishable oranges, 13 indistinguishable grapes. He would like to hand out all 17 of his fruit to his 4 students such that each student has more grapes than oranges. How many ways can he do so?

    (A)840 **(B)**1960 **(C)**2250 **(D)**3024 **(E)**5040

12. Two ants are on the xy-plane, one positioned at $(-1, 1)$ and the other at $(2, 0)$. There is an ant-repellent puddle occupying the region $x^2 + y^2 < 1$. What is the length of the shortest path from one ant to the other which does not pass through the puddle?

    (A) $1 + \frac{\pi}{6} + \sqrt{3}$ **(B)** $\sqrt{10}$ **(C)** $5 - \sqrt{3}$ **(D)** $\frac{3\pi}{4} - \sqrt{3}$ **(E)** 4

13. Let $S(n)$ denote the sum of the digits of n. Find $S(S(k))$ if,

$$k = 1! + 2! + 3! + \cdots + 2016! + 2017!$$

    (A)24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

14. Let ABC be a triangle such that $\angle C = 90°$, $AB = 137$, and the distance from C to the incenter of the triangle is $28\sqrt{2}$. What is the area of $\triangle ABC$?

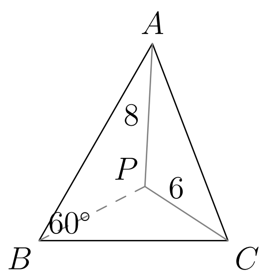
    (A)2870 **(B)**3276 **(C)**4400 **(D)**4620 **(E)**5280

15. What is the value of the following sum:

$$\sum_{k=0}^{89} \frac{1}{1 + \tan^3(k°)}$$

    (A) 45.5 **(B)** 50 **(C)** $30\sqrt{3}$ **(D)** $\frac{181}{4}$ **(E)** 270

16. In triangle ABC, $\angle ABC = 60°$. Let P be a point inside $\triangle ABC$ such that $\angle APB = \angle BPC = \angle CPA$, $PA = 8$, and $PC = 6$. What is the length of PB?



    (A)$3\sqrt{3}$ **(B)**5 **(C)**6 **(D)**$4\sqrt{3}$ **(E)**7

17. There exists a permutation $a_1, a_2, \cdots a_{15}, a_{16}$ of $\{1, 2, \cdots, 15, 16\}$ such $a_1 = 16$ and whenever $i < j < k$, we have that $a_k - a_j \neq a_j - a_i$. Which of these is a possible value of a_5.

    (A) 1 **(B)** 4 **(C)** 7 **(D)** 10 **(E)** 14

18. Let A and B be diametrically opposite points on the circular base of a cone sitting on a flat plane with radius $\frac{3}{4}$ and height $\frac{\sqrt{7}}{4}$. An ant on the point A wants to crawl along the surface of the cone to the point B. Find the minimum distance that ant has to travel.

(A) $\frac{3}{2}$ **(B)** $\frac{3\pi}{4}$ **(C)** $\sqrt{2+\sqrt{2}}$ **(D)** $1+\sqrt{2}$ **(E)** 2

19. Let $f(x) = 2x\sqrt{1-x^2}$, and let $f^{(k)}(x)$ denote the k^{th} iteration of $f(x)$, that is $f^{(0)}(x) = f(x)$ and $f^{(k+1)}(x) = f(f^{(k)}(x))$ for all positive integers k. For how many distinct real values n is $f^{(2017)}(n) = \frac{1}{2}$?

    (A) 1 **(B)** 2 **(C)** 2^{2017} **(D)** 2^{2018} **(E)** $2^{2018}+1$

20. A circle is in the coordinate plane such that the slope of the line from the origin to its center is 2 and the slope of one of its tangents through the origin is 3. Find the slope of the other tangent line through the origin.

    (A) 1 **(B)** $\frac{13}{9}$ **(C)** $\frac{17}{12}$ **(D)** $\frac{2\sqrt{6}}{3}$ **(E)** $\frac{4\sqrt{17}}{11}$

21. A 20 foot long line of army men are marching in a straight line at a constant speed. A messenger at the back of the line sprints to the front of the line at a constant speed to deliver a message to the captain. Once the message is delivered, the messenger sprints back to the back of the line at the same speed. Once the messenger reached the back of the line, the army men had traveled a total of 20 feet. How many feet did the messenger sprint?

    (A) 20 **(B)** $20+20\sqrt{2}$ **(C)** 200 **(D)** $200+100\sqrt{2}$ **(E)** $40\sqrt{2}$

22. Claserken places 3 identical tennis balls in each of 3 different bowls. He then chooses one ball from a random bowl (that has at least one ball in it) and moves it into another random bowl. He repeats this process 4 more times. After he has finished, what is the probability that there is still 3 balls in each bowl?

    (A) $\frac{1}{9}$ **(B)** $\frac{5}{32}$ **(C)** $\frac{3}{18}$ **(D)** $\frac{17}{324}$ **(E)** $\frac{5}{108}$

23. Vfire and algebrastar1234 live on the opposite vertices of a planet that is the shape of a cube. One day they decide to meet and set of from their homes at the same time. They both walk along the edges of the cube at a rate of one edge per minute. Each time they reach a vertex, they choose one of the 3 neighboring vertices to travel to with equal probability. What is the expected number of minutes it will take for them to meet? (Vfire and algebrastar1234 can meet at the midpoint of an edge)

    (A) 3 **(B)** 5.5 **(C)** 8.5 **(D)** 9 **(E)** 13

24. Vfire is standing at the edge of a cliff. If he moves one unit towards the edge he will fall off the cliff. Every minute Vifre moves 1 unit away from the edge of the cliff with probability $\frac{3}{4}$ and moves one unit towards the edge of the cliff with probability $\frac{1}{4}$. What is the probability that Vfire falls of the cliff?

    (A) $\frac{1}{5}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{12}$ **(E)** 1

25. Vfire likes to play with powers of 2. After listing all the powers of 2 from 2^0 to 2^{2017}, he makes three observations: 2^{2017} has a leading digit of 1, 250 numbers have a leading digit of 3 and 161 numbers have a leading digit of 5. Find the amount of numbers in his list that have a leading digit of 1 or 2.

    (A) 943 **(B)** 956 **(C)** 965 **(D)** 971 **(E)** 982