Christmas Mock AMC 8

Written by coolmath34 and RubixMaster21

RULES

1. This is a 40-minute, 25-question examination. Be sure to set a 40-minute timer before taking the test. This test runs along the honor system, which means we trust you to follow all rules including this one.

2. The score you get is a number between 0 and 25, the number of problems you got right. Guessing is not penalized, so it's to your benefit to answer every question.

3. The honor system means that all work is done by you. YOU ARE NOT ALLOWED TO USE THE INTERNET TO FIND THE ANSWERS TO PROBLEMS.

4. PM all final answers to coolmath34. DO NOT EDIT THE FIRST RESPONSE OF THE PRIVATE MESSAGE.

5. If you have any questions, PM coolmath34.

6. Most importantly, have fun!

QUESTIONS

1. Rubix has a cube with side length 2. What is the volume of his cube?

A) 2 B) 4 C) 8 D) 16 E) 6

2. Andy has 193 Christmas cookies that he baked. If he laid them out in rows of 8, making as many rows as possible, how many cookies would be left?

A) 0 B) 1 C) 3 D) 6 E) 7

3. Bob is practicing his present-wrapping skills. He has a $\frac{1}{4}$ chance at wrapping a present correctly. If he has two gifts to wrap, what is the probability that he incorrectly wraps both?

A) $\frac{1}{16}$ B) $\frac{1}{4}$ C) $\frac{9}{16}$ D) $\frac{1}{8}$ E) $\frac{1}{2}$

4. The number of ornaments on my Christmas tree is a number between 40 and 50 inclusive, leaves a remainder of 2 when divided by 3, and is a multiple of 4. How many ornaments are on my tree?

A) 41 B) 43 C) 44 D) 47 E) 48

5. Stan is randomly shuffling inside a box. He has a 50% chance at picking a green square. Suddenly, his brother accidentally drops five green squares into the box. Now, he has a $\frac{2}{3}$ probability to pick a green square. What is the number of green squares originally?

A) 5 B) 10 C) 15 D) 20 E) 25

6. Let the "square function" $s(n)$ denote $\frac{n(n+1)(n+2)}{6}$. What is the sum of $s(0) + s(1) + s(2) + s(3) + s(4) + s(5)$?

A) 55 B) 35 C) 20 D) 70 E) 15

7. Cubey likes to go skiing on the Cartesian plane. He starts skiing at (0, 20) and ends at (15, -30.) What is the distance of the path that he skiied?

A) 15 B) $\sqrt{545}$ C) $5\sqrt{109}$ D) 50 E) $\sqrt{109}$

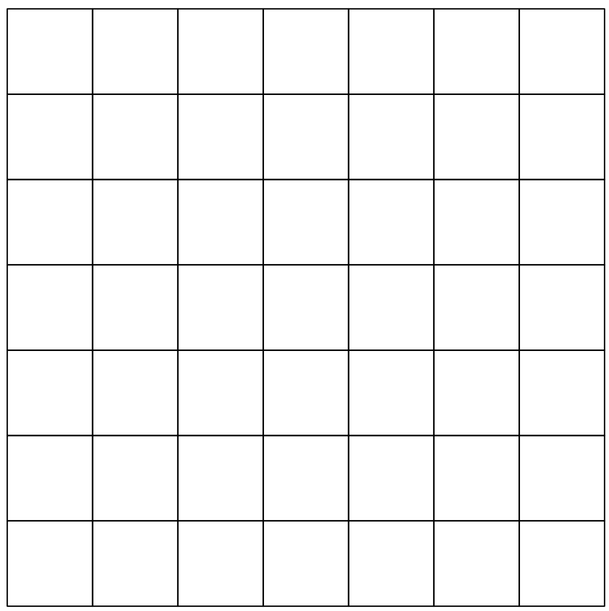
8. Cubey the Penguin has an unusual cube with side length 3. He manages to extract the central 1x1x1 cube. What is the surface area of the new solid?

A) 9   B) 27   E) 36   D) 54   E) 60

9. In a certain colony of penguins, the number of penguins doubles every day. The colony starts with 3 penguins, and has 6 at the end of day 1, 12 at the end of day 2, and so on. What is the number of the first day which ends with the colony having more than 100 penguins?

A) 4   B) 5   C) 6   D) 7   E) 8

10. Rubix finds a mesh on a shelf of his room. The side length of one small square of the mesh is 1. What is the sum of all line segments of the mesh?



A) 28   B) 70   C) 42   D) 84   E) 112

11. Jingle finds the sum of the first 47 odd numbers. Bells finds the sum of the first 21 odd numbers. What is the difference of their sums?

A) 2209   B) 781   C) 1809   D) 1768   E) 471

12. One of Cubey's friends does of the traditional Twelve Days of Christmas. As a rule, you get x more gifts than you got on day $x - 1$ on day x. By day 9, how many gifts will Cubey's friend have?

A) 35   B) 76   C) 91   D) 165   E) 140

13. Bells are ringing in Bob's Antique Shop for Items of Christmas. (Call it BASIC for now.) In BASIC, there are four bells that ring; one every ten seconds, one every fourteen seconds, one every 27 seconds, and one every 45 seconds. One day, all bells ring at the same time. How many seconds after that time will just three of the bells ring at the same time?

A) 1890   B) 270   C) 630   D) 1050   E) 580

14. A penguin purchases an igloo. His friend is content with his Grade 6 igloo, having spent \$1000 on it. As a general rule, any Grade x igloo where $x > 6$ costs $\frac{x}{2}$ as much as a Grade $x - 1$ igloo. Also, any Grade x igloo where $x < 6$ costs $\frac{1}{x}$ as much as a Grade $x + 1$ igloo. Knowing that the penguin spent at least \$5000, what is the least number grade of igloo that he bought?

A) 7 B) 8 C) 9 D) 10 E) 11

15. During a celebration at Santa's house, there are $12^{12^{12}}$ relatives. Everyone files into groups of 13. Some way or another, there's always a group of leftovers. How many people are in the leftover group?

A) 1 B) 4 C) 7 D) 12 E) 10

16. Joy has been brought to the world. If it starts with three people, and the people affected double every day, when will the whole world population be joyous? Assume the world population is exactly 7 billion.

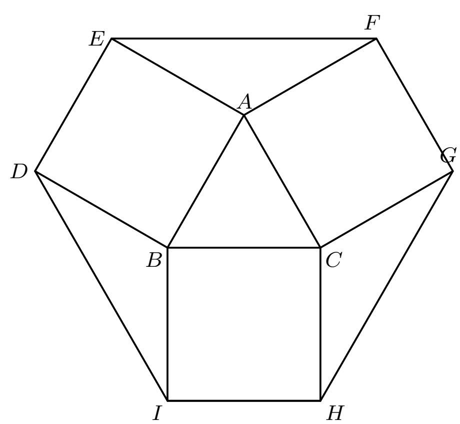
A) 30 B) 31 C) 32 D) 33 E) 34

17. Due to a strange infection, Vanilla Gorilla the elf has to eat two types of cookies. Blue cookies must be eaten every three days and prevent five days of infection. Red cookies must be taken every week and prevent twelve days of infection. Eating lots of cookies at the same time won't overlap infection protection. Over a one-year period of recovery, and Vanilla Gorilla takes at least five cookies of each type, what is the least amount of cookies she needs to take?

Note that doesn't matter: In my Algebra class, we have an elf called Vanilla Gorilla. Don't ask how it was named.

A) 31 B) 30 C) 56 D) 72 E) 60

18. The star of Bethlehem was finally available to theological scientists for research. This is the shape they have come up with:



All number units are supposed to be "thousands of kilometers." If $\triangle ABC$ is equilateral and $AB = BC = AC = 1$, then what is the area of the star?

A) $3+\sqrt{3}$ B) $3\sqrt{3}$ C) $\frac{3\sqrt{3}}{2}$ D) $\sqrt{3}$ E) 3

19. Rubix has decided to join the festivities. His friend gives him a house number from 100-999 inclusive. He gives Rubix the following clues:

1. My house number is divisible by four.

2. My house number is divisible by the first prime in the range 20 to 29.

3. My house number is the product of two consecutive integers.

Rubix now knows what his friend's house number is. What is that number?

A) 472 B) 552 C) 156 D) 572 E) 928

20. The bell tower has started to ring again. In the first layer, there are four bells. In the second, there are four more, and in the third, there are four more than the second layer, and so on. If the bell tower has 8 layers, then how many bells are on the tower?

A) 32 B) 42 C) 106 D) 112 E) 144

21. One of Santa's elves, Holly Ivy, has taken a premium helper application test. There are 625 problems on the test, and she scores 80% the first time. Not satisfied, she asks Santa for a retake. Fortunately, Santa is feeling generous, and gives her two retakes. Every time she retakes the test, she gets two-fifths of the problems she initially had wrong correct. At the end of the two retakes, what is the number of problems that she originally got wrong that are still wrong?

A) 500 B) 45 C) 20 D) 50 E) 125

22. Cubey notes that the ice is very slippery. On day one, he has a 50% chance of slipping. For each following day, the chance he slips is reduced by half. After three days, what is the probability that Cubey doesn't slip at all?

A) $\frac{1}{8}$ B) $\frac{17}{64}$ C) $\frac{21}{64}$ D) $\frac{9}{16}$ E) $\frac{7}{8}$

23. Santa has lots of penguin toys to give to the children of the world. He has seven bags of penguins. Each bag has the same number of penguins. One day, Santa finds a bag of 53 penguins. He decides to redistribute the number of penguins he has so that all eight bags he holds have the same number of penguins. Santa successfully manages to redistribute all the penguins, and also notes that he has more than 200 penguins. What is the smallest number of penguins Santa could have had before finding the bag of 53 penguins?

A) 189 B) 264 C) 203 D) 247 E) 217

24. What is the difference in the number of zeros of $100(1! + 2! + 3! + 4! + 5!)$ and $100! + 200! + 300! + 400! + 500!$?

A) 100 B) 64 C) 122 D) 370 E) 368

25. The infinite sequence $T = \{t_0, t_1, t_2, \ldots\}$ is defined as $t_0 = 0$, $t_1 = 1$, and $t_n = t_{n-2} + t_{n-1}$ for all integers $n > 1$. If a, b, c are fixed non-negative integers such that

$$\begin{aligned} a &\equiv 5 \pmod{16} \\ b &\equiv 10 \pmod{16} \\ c &\equiv 15 \pmod{16}, \end{aligned}$$

then what is the remainder when $t_a + t_b + t_c$ is divided by 7?

A) 4 B) 5 C) 6 D) 0 E) 1

ADJUDICATING TIEBREAKERS

Tiebreakers will be judged by day of submission, because the last tiebreaker format didn't work well.

FINAL NOTE

Thank you, and have a wonderful Christmas!