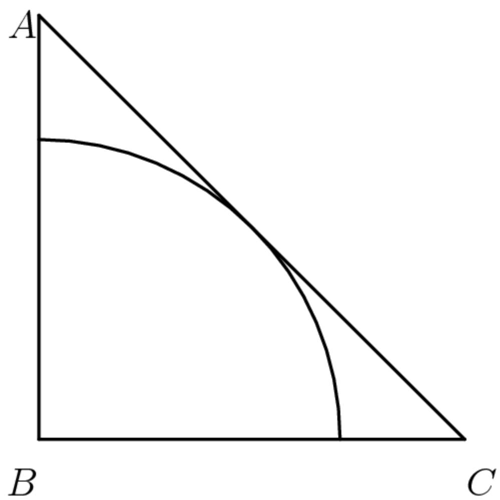
1. If x and y are positive integers such that their product is 64, what is the sum of all distinct values for $xy + x + y$?

   (A) 391 **(B)** 392 **(C)** 393 **(D)** 394 **(E)** 395

2. *(fidgetboss_4000)*
   In the diagram below, ABC is an isosceles right triangle with a right angle at B and with a hypotenuse of $40\sqrt{2}$ units. Find the greatest integer less than or equal to the value of the radius of the quarter circle inscribed inside ABC.



   (A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

3. There are $\binom{8}{4} = 70$ distinct quadrilaterals that can be formed from the vertices of a regular octagon. Which of these statements must hold true for all those quadrilaterals?

   (A) All of the 70 quadrilaterals are rectangles.
   (B) All of the 70 quadrilaterals are cyclic.
   (C) All of the 70 quadrilaterals are trapezoids.
   (D) All of the 70 quadrilaterals are kites.
   (E) None of the above statements are true.

4. *(fidgetboss_4000)* Mark rolled two standard dice. Given that he rolled two distinct values, find the probability that he rolled two primes.

$\textbf{(A)}\ \frac{1}{12} \qquad \textbf{(B)}\ \frac{1}{7} \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{2}{4} \qquad \textbf{(E)}\ \frac{2}{5}$

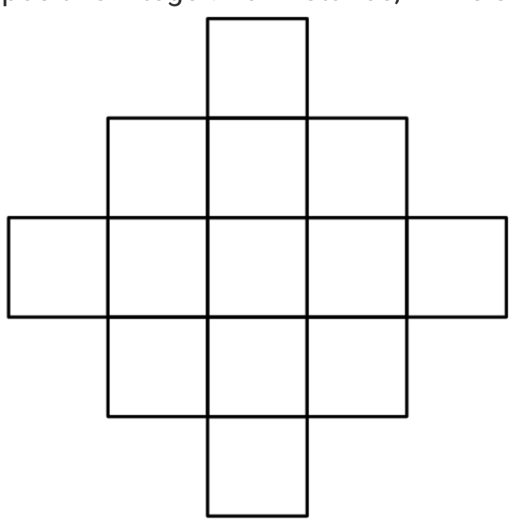
5. *(smartninja2000)* What is the sum of the solutions to $n^2 = x^2 - 8x + 96$?, where n is a positive integer?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

6. A certain period of time P starts at exactly 6:09PM on a Tuesday and ends at exactly 6:09AM on a Thursday. Which of these numbers listed in the choices here is a possible length in days for P?

$\text{(A)}\ 100.5 \qquad \text{(B)}\ 1000.5 \qquad \text{(C)}\ 10,000.5 \qquad \text{(D)}\ 100,000.5 \qquad \text{(E)}\ 1,000,000.5$

7. Let A_n be an array with n rows, n unit squares on the $\lceil \frac{n}{2} \rceil$th row, and $n-2k$ unit squares on the $\lceil \frac{n}{2} \rceil \pm k$th row, where n is an odd positive integer and k is a positive integer. For instance, A_5 is shown here:



Let m_n be the number of unit squares included in A_n.

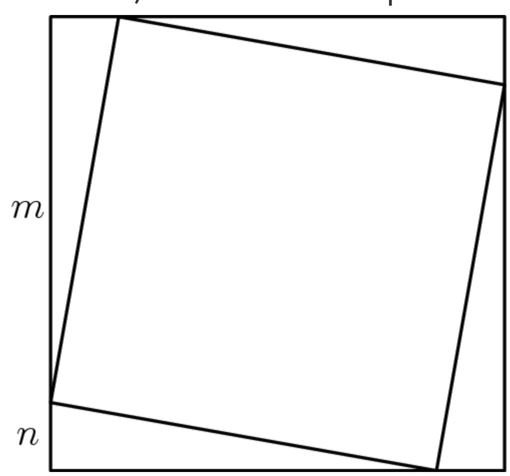
As n approaches ∞, what value does $\frac{m_n}{n^2}$ approach?

(A) 0.4 **(B)** 0.45 **(C)** 0.5 **(D)** 0.55 **(E)** 0.6

8. The sum of the squares of two numbers is 11, and the sum of their fourth powers is 102. The sum of their reciprocals can be expressed in the form $\sqrt{\frac{a+\sqrt{b}}{19}}$, where a and b are positive integers. Find the value of $a+b$.

(A) 171 **(B)** 172 **(C)** 173 **(D)** 174 **(E)** 175

9. In the figure below, a square of area 108 is inscribed inside a square of area 144. There are two segments, labeled m and n. The value of m can be expressed as $a+b\sqrt{c}$, where a, b, c are positive integers and c is square-free. Find $a+b+c$.

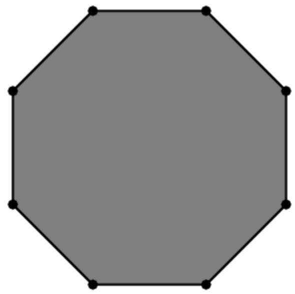


(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

10. *(fidgetboss_4000)* For a particular positive integer n, the number of ordered sextuples of positive integers (a, b, c, d, e, f) that satisfy $a+b+c+d+e+f \leq n$ is exactly 3003. Find n.

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

11. *(fidgetboss_4000)* Let S be a regular octagon. How many distinct quadrilaterals can be formed from the vertices of S given that two quadrilaterals are not distinct if the latter can be obtained by a rotation of the former?

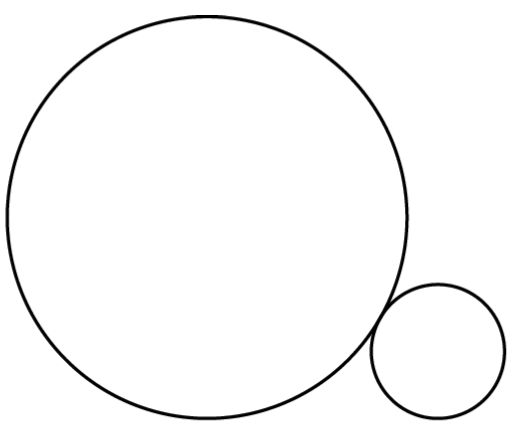


$\textbf{(A) } 9 \qquad \textbf{(B) } 10 \qquad \textbf{(C) } 16 \qquad \textbf{(D) } 35 \qquad \textbf{(E) } 70$

12. Let the number of distinct 8 letter combinations of the word $\textbf{CONNECTICUT}$, where order does not matter, be N. For example, $\textbf{CONNECTI}$ and $\textbf{ECNTINOC}$ would be the same 8 letter combination, because they contain the same letters, but are ordered differently. Find the sum of the digits of N.

$\textbf{(A) } 5 \qquad \textbf{(B) } 6 \qquad \textbf{(C) } 7 \qquad \textbf{(D) } 8 \qquad \textbf{(E) } 9$

13. *(smartninja2000)* Two logs of length 10 are laying on the ground touching each other. Their radii are 3 and 1, and the smaller log is fastened to the ground. The bigger log rolls over the smaller log without slipping, and stops as soon as it touches the ground again. What is the volume of the set of points swept out by the larger log as it rolls over the smaller one?



$\textbf{(A) } 250\pi \qquad \textbf{(B) } 260\pi \qquad \textbf{(C) } 270\pi \qquad \textbf{(D) } 280\pi \qquad \textbf{(E) } 290\pi$

14. Let N be the number of ordered octuples $(a_1, a_2, a_3, ..., a_8)$ of positive integers such that $1 \leq a_1, a_2, a_3, ..., a_8 \leq 4$ and $i^{a_1} + i^{a_2} + i^{a_3} + ... + i^{a_8}$ is a real number. Find the sum of the digits of N.

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 20$

15. The sum of the base 10 logarithms of the divisors of 10^{111} is A. Compute the remainder when A is divided by 20.

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 14 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 18$

16. *(fidgetboss_4000)* Define a permutation $a_1a_2a_3a_4a_5a_6$ of the set $1, 2, 3, 4, 5, 6$ to be factor-hating if $\gcd(a_k, a_{k+1}) = 1$ for all $1 \leq k \leq 5$.Find the number of factor-hating permutations.

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 56 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 72$

17. *(fidgetboss_4000)* Consider Triangle $A_0B_0C_0$, with $A_0B_0 = A_0C_0 = 13$ and $B_0C_0 = 10$. Points M and N are on B_0C_0 and A_0C_0 such that $MC_0 = \frac{1}{10}B_0C_0$ and $NC_0 = \frac{1}{5}A_0C_0$. Cevian B_0P_0 is drawn, with P_0 be a randomly chosen point on A_0C_0. Then, Cevian P_0P_1 is drawn, with P_1 being a randomly chosen point on B_0C_0. Generally $P_{n-1}P_n$ is drawn and P_n a randomly chosen point on $P_{n-2}C_0$, for all $n \geq 2$. The probability that the cevian $P_{12}P_{13}$ is completely outside of Triangle MNC_0 can be expressed in lowest terms as $\frac{m^\gamma}{n^\gamma}$, where m, n, γ are positive integers. Find $m + n + \gamma$.

$\textbf{(A)}\ 47 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 49 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 51$

18. *(fidgetboss_4000)* There are N distinct 4×4 arrays of integers that satisfy:
    1. Each integer in the array is a $1, 2, 3$ or 4.
    2. Every row and column contains all the integers $1, 2, 3$ and 4.
    3. No row or column contains two of the same number.

    Find N.

    $\textbf{(A)}\ 432 \qquad \textbf{(B)}\ 576 \qquad \textbf{(C)}\ 864 \qquad \textbf{(D)}\ 1,152 \qquad \textbf{(E)}\ 1,296$

19. *(fidgetboss_4000)* Let P be a regular 12-gon with sides $A_1A_2, A_2A_3, A_3A_4, A_4A_5, ..., A_{12}A_1$. The diameters of semicircles $S_1, S_2, S_3, ..., S_{12}$coincide with $A_1A_2, A_2A_3, A_3A_4, ..., A_{12}A_1$, with one vertex of each of the semicircles $S_1, S_2, S_3, ..., S_{12}$ coinciding with Points $A_1, A_2, A_3, ..., A_{12}$. Given that semicircles with diameters coinciding on adjacent sides of the 12-gon are tangent to each other with the point of tangency being inside P, the ratio of the diameter of S_1 to the side length of P can be expressed as $a\sqrt{b}+c\sqrt{d}-p\sqrt{q}-r$,where a, b, c, d, p, q, r are positive integers and b, d, q are square-free. Find $a+b+c+d+p+q+r$.

    $\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 37 \qquad \textbf{(C)}\ 38 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 40$

20. How many ordered pairs of positive integers (m, n) are there such that $0 \leq m, n \leq 1000$ and $2^m + 3^n \equiv 1 \bmod 1000$?

    $\textbf{(A)}\ 2480 \qquad \textbf{(B)}\ 2490 \qquad \textbf{(C)}\ 2500 \qquad \textbf{(D)}\ 2510 \qquad \textbf{(E)}\ 2520$

21. Let $S = \{r_1, r_2, r_3, ..., r_\mu\}$ be the set of all possible remainders when $15^n - 7^n$ is divided by 256, where n is a positive integer and μ is the number of elements in S. The sum $r_1 + r_2 + r_3 + ... + r_\mu$ can be expressed as $p^q r$,where p, q, r are positive integers and p and r are as small as possible. Find $p+q+r$.

    $\textbf{(A)}\ 40 \qquad \textbf{(B)}\ 41 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 43 \qquad \textbf{(E)}\ 44$

22. Let $r_0 = r_1 = r_2 = 1$ and let $r_n = 3^{r_{n-0}+r_{n-1}+r_{n-2}}$ for $n \geq 3$. Find the least value of n such that $r_n - 1$ is a multiple of 169.

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

23. *(fidgetboss_4000)* Let $$S_{n,k} = \sum_{a=0}^{n} \binom{a}{k}\binom{n-a}{k}$$. Find the remainder when $\sum_{n=0}^{200}\sum_{k=0}^{200} S_{n,k}$ is divided by 1000.

$\textbf{(A)}\ 374 \qquad \textbf{(B)}\ 375 \qquad \textbf{(C)}\ 503 \qquad \textbf{(D)}\ 750 \qquad \textbf{(E)}\ 751$

24. How many distinct sequences of integers a_1, a_2, a_3, a_4, a_5 satisfy:

- $-3 \leq a_k \leq 3$ for $1 \leq k \leq 5$
- $\sum_{n=1}^{\mu} a_k$ is positive for all $1 \leq \mu \leq 5$

$\textbf{(A)}\ 661 \qquad \textbf{(B)}\ 662 \qquad \textbf{(C)}\ 663 \qquad \textbf{(D)}\ 664 \qquad \textbf{(E)}\ 665$

25. Define a k-array as an k-row array of congruent equilateral triangles of side length $2\sqrt{3}$ that contains $2k-1$ equilateral triangles on row k. For example, an 10-array is shown here. Let S_k be the set of all distinct triangles in a k-array. Let N_k equal the sum of the areas of all the incircles inside all of the triangles in S_k. Find the remainder obtained when $\frac{N_{2019}}{\pi}$ is divided by 1000.

$\text{(A) } 074 \qquad \text{(B) } 266 \qquad \text{(C) } 336 \qquad \text{(D) } 374 \qquad \text{(E) } 724$