SUMMER MOCK AMC 10
INSTRUCTIONS

1. DO NOT OPEN THIS TEST UNTIL YOU SET YOUR TIMER.

2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.

3. You are only given one option at filling out the Google Form, so make sure you select all of your answers correctly.

4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. When you start your timer, begin working on the problems. You will have 75 minutes to complete the test.

8. If you have any issues, message one of the following people on the AoPS forums: Rowechen, ItzVineeth, KenV, Kvedula2004, OmicronGamma, or FedeX333X. You can also post on the forum thread with any questions.

1. Find $\frac{x^2 \cdot 5x^4}{12x^3}$ given that $x = 6$.

(A) 30 **(B)** 90 **(C)** 150 **(D)** 216 **(E)** 250

2. Brianna picks three integers from a hat and adds them two at a time. Their sums are 10, 24, and 36. What is the number of positive divisors of $|a \cdot b \cdot c|$ including 1 and itself?

(A) 4 **(B)** 6 **(C)** 10 **(D)** 16 **(E)** 20

3. Two fair six-sided dices are rolled. The probability that the positive difference between the two rolls is at least 3 can be written as $\frac{m}{n}$ in the simplest form. Compute $m + n$.

(A) 4 **(B)** 7 **(C)** 9 **(D)** 13 **(E)**37

4. Mark is thinking of a two-digit number. The tens digit of the number exceeds the units digit by 6. The number is 2 less than three times the number formed when the digits of the original number are reversed. What is the sum of the digits of the original number?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 10 **(E)**11

5. A special binary operator, $\diamondsuit$, has a property that $a\diamondsuit b = b((a-1)\diamondsuit(b-1))$. Also, $0\diamondsuit 1 = 1$. What is the remainder when $5\diamondsuit 6$ is divided by 100?

(A) 00 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

6. At a restaurant, chips cost 91 cents a bag and salsa costs 39 cents per small container. While going there, Joe spent all his money on chips and salsa. Which of the following could have been the amount he started with?

(A) 1.43 **(B)** 3.73 **(C)** 5.13 **(D)** 7.29 **(E)**10.01

7. What is the sum of all the positive integer values of x such that the elements of the set $A = 11, 26, 40, x$ have an integer arithmetic mean and a non-integer median?

(A) 132 **(B)** 156 **(C)** 189 **(D)** 200 **(E)** 210

8. Right triangle $\triangle ABC$ with right angle at B has an equilateral triangle $\triangle ABD$ inscribed inside it, such that D lies on side AC. The angle bisector of $\angle BDC$ intersects side BC at G. AG intersects BD at a point P. Find the ratio of the area of $\triangle APD$ to the area of $\triangle ABD$.

(A)$1:4$ **(B)**$3:11$ **(C)**$2:7$ **(D)**$1:3$ **(E)**$1:2$

9. A cube is composed of 343 unit cubes (all of which are white). All of the faces of the cube are colored red. The outer "shell" of unit cubes (just painted red), is removed, leaving a cube cube composed of 125 unit cubes. This process is repeated until you are left with a single unit cube. How many unit cubes now have paint on an odd number of faces?

(A) 124 **(B)** 158 **(C)** 199 **(D)** 216 **(E)** 234

10. As per every standard magic square, the sums of all the diagonals, columns, and rows are equal. Find the sum of the missing terms in the following magic square.

18	X	30
33	B	D
A	27	E

(A) 66 **(B)** 75 **(C)** 81 **(D)** 87 **(E)** 96

11. The probability that the product of two distinct positive integers $1 \leq m, n \leq 100$ is divisible by 3 can be expressed in the form $\frac{a}{b}$, where a, b are relatively prime positive integers. What is the value of $a + b$?

(A) 183 **(B)** 217 **(C)** 233 **(D)** 256 **(E)** 313

12. Given that the area of a rectangle is 1, and its perimeter is 8, find the length of its diagonal.

(A) 2 **(B)** $\sqrt{6}$ **(C)** $\sqrt{10}$ **(D)** $\sqrt{14}$ **(E)** 4

13. A positive integer is called *n-wacky* if, when its leftmost digit is deleted, the remaining number is equal to $\frac{1}{n}$ of the original number. For example, 15 is *3-wacky* because 5 is $\frac{1}{3}$ of 15. How many positive integers are *35-wacky*?

(A) 0 **(B)** 9 **(C)** 36 **(D)** 81 **(E)**256

14. The decimal number $20ab$, where a and b are digits of a number, has a special property. When it is written in the binary system, it has as many 0s in its representation as the number of digits it has in its decimal representation. Find the maximum value of $a + b$.

(A) 8 **(B)** 9 **(C)** 12 **(D)** 14 **(E)**16

15. If x and y are real numbers such that $x^4y + 2x^2y^3 = 90$ and $\frac{xy^4}{2} + x^3y^2 = 45$ and $x^5 + y^5 = 60$, find $(x + y)^{10}$.

(A) $460,800$ **(B)** $614,400$ **(C)** $768,000$ **(D)** $921,600$ **(E)** $1,075,200$

16. Let n be the smallest positive integer such that the first digit of 2^n is 7. What is the first digit of 5^n?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 7

17. Let $\triangle ABC$ be a triangle with sides $AB = 17, AC = 25, BC = 26$, and let X, Y be intersections of the line parallel to AB passing through C with the bisectors of $\angle CAB$ and $\angle ABC$ respectively. What is the area of $ABXY$?

(A) 782 **(B)** 794 **(C)** 816 **(D)** 844 **(E)**870

18. In how many 0s does 2018! end in when expressed in base 48?

(A) 502 **(B)** 520 **(C)** 562 **(D)** 612 **(E)** 670

19. The Tribonacci sequence is defined as $T_1 = 1, T_2 = 1, T_3 = 2$, and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for all $n > 3$. What is the remainder when T_{2017} is divided by 12?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 7 **(E)**11

20. Given that $\frac{5}{x+y} - \frac{1}{xy} = 1$, for positive reals x, y, the maximum value of x can be expressed in the form $\frac{a+\sqrt{b}}{c}$. What is the value of $a + b + c$?

(A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

21. Let $ABCD$ be a convex quadrilateral with $\angle ABD = 18°, \angle ACB = 54°, \angle ACD = 36°, \angle ADB = 27°$, and let P be the intersection of the two diagonals AC and BD. What is the degree value of $\angle APB$?

(A) 75° **(B)** 91° **(C)** 93° **(D)** 99° **(E)**105°

22. Let f be a function such that for every positive integer n, $f(n)$ is equal to the number obtained by writing consecutively 2018 times the number n. For example, $f(1234) = 123412341234...1234$ where the number 1234 is repeated 2018 times. For how many integers $1 \leq n \leq 2018$, $f(n)$ is divisible by 11?

(A) 909 **(B)** 918 **(C)** 993 **(D)** 1011 **(E)** 1020

23. Let A denote the set of all positive integers that can be expressed as the sum of some not necessarily distinct perfect squares $\neq 1$. For example, 4 and $102 = 49 + 49 + 4$ belong to A, while 3 doesn't. The probability that a randomly chosen positive integer $1 \leq n \leq 2018$ belongs to A can be express as $\frac{a}{b}$ where a, b are relatively prime positive integers. What is the remainder when $a + b$ is divided by 100?

(A) 12 **(B)** 19 **(C)** 27 **(D)** 48 **(E)** 69

24. How many distinct planes passing through at least 3 distinct vertices of a regular icosahedron (a polyhedron with 20 sides) cut the solid in 2 not necessarily equal parts?

(A) 54 **(B)** 60 **(C)** 67 **(D)** 72 **(E)**75

25. Let C be a cube with side length 12. Rotate the cube 60 degrees counterclockwise along its diagonal to get a new cube C'. What is the volume of the solid of intersection of the two cubes C and C'?

(A) 972 **(B)** 1120 **(C)** 1296 **(D)** 1444 **(E)** 1592

END OF TEST

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Collaborators:

Pointy-Hair Boss:
Rowechen

Problem Czars:
ItzVineeth
FedeX333X
OmicronGamma
KenV
Kvedula2004

pdf Creator:
OmicronGamma
KenV