2021

AAMC 10A

REMAINS OPEN UNTIL THE DUE DATE

****Administration On An Earlier Date Is Not Even Possible****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE Never.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10A CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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AAMC

Apocalyptic American Mathematics Competitions

FIRST ANNUAL

AAMC 10A

Ah, Another Mock Contest... 10A

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL THE TIMER STARTS.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer clearly, edits in submission will not be accepted.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, sign your name in the space provided on the Answer Form.

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1. Anna, Bella, and Carly are running for school president. If Anna earned twice as many votes as Bella, and Bella earned three times as many votes as Carly, what percentage of votes did Bella win?

   (A) 10% **(B)** 15% **(C)** 20% **(D)** 24% **(E)** 30%

2. For how many integers N is $20 \le \sqrt{N} \le 21$?

   (A) 20 **(B)** 21 **(C)** 41 **(D)** 42 **(E)** 84

3. In isosceles triangle ABC with $AB = AC$, point D lies on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$. Given that $\triangle ABD$ is isosceles, what is the degree measure of $\angle C$?

   (A) 45° **(B)** 60° **(C)** 67.5° **(D)** 72° **(E)** 75°

4. Ponce rolls a four-sided die $(1-4)$, a six-sided die $(1-6)$, and an eight-sided die $(1-8)$, all of which are perfectly fair. What is the probability that all three dice show the same number?

   (A) $\frac{1}{24}$ **(B)** $\frac{1}{32}$ **(C)** $\frac{1}{36}$ **(D)** $\frac{1}{48}$ **(E)** $\frac{1}{64}$

5. Pentagon $ABCDE$ has perimeter 36. If the perimeter of quadrilateral $ABCD$ is 29, and the perimeter of triangle DEA is 17, what is the length of AD?

   (A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

6. Alexis starts at the park, walks 80 meters south, 340 meters in some direction, and then 160 meters east. Given that her ending location is k meters directly north from the park, what is the value of k?

   (A) 200 **(B)** 210 **(C)** 220 **(D)** 230 **(E)** 240

7. Three distinct positive integers have a mean of 20, a range of 21, and a median of 17. What is their maximum?

   (A) 28 **(B)** 29 **(C)** 30 **(D)** 31 **(E)** 32

23. Let N be the number of positive integers $a < 1000$ for which the sum of the digits of a is one-half the sum of the digits of $11a$. What is the sum of the digits of N?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

24. The cubic equation $x^3 - 7x^2 + 3x + 2$ has irrational roots r, s, and t, with $r \geq s \geq t$. There exists a unique set of rational numbers A, B, and C, such that the cubic $x^3 + Ax^2 + Bx + C$ has $r + s$ as a root. What is $A + B + C$?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

25. Point P lies on side $\overline{AD}$ of square $ABCD$ with side length 3 such that $AP = 1$ and $DP = 2$. Point Q is the unique point on segment $\overline{BP}$ such that the measure of angle $\angle DQC$ is maximized. If PQ can be expressed as $\sqrt{m} - \sqrt{n}$ for positive integers m and n, what is the value of $m + n$?

(A) 86 **(B)** 90 **(C)** 94 **(D)** 98 **(E)** 102

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8. Define the operation $\ominus$ so that:

$$x \ominus y = \max\{0, x - y\}.$$

What is the sum of all real values of x for which:

$$x \ominus (20 \ominus x) = 21$$

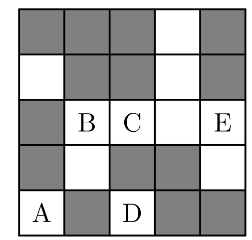
is satisfied?

(A) 21 **(B)** $\frac{41}{2}$ **(C)** $\frac{63}{2}$ **(D)** 41 **(E)** $\frac{83}{2}$

9. For some point P on the coordinate plane, reflecting P about the line $y = 4$ is the same as translating it down 4 units, and reflecting P about the line $x = 10$ is the same as translating it right 10 units. What is the sum of the coordinates of P?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

10. Penny places five quarters into distinct cells of the grid below so that no two quarters lie in the same row or column, and no quarter is on a shaded cell. Which labeled cell must contain a quarter, regardless of how Penny places the quarters?



(A) A **(B)** B **(C)** C **(D)** D **(E)** E

11. For how many sets of three distinct positive integers is the absolute difference between each pair of integers also in the set?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

12. Let $ABCD$ be a rectangle with $AB = 5$ and $BC = 8$. There exists a circle tangent to sides $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ of the rectangle that meets side $\overline{DA}$ at points X and Y. What is XY^2?

(A) 48 **(B)** 50 **(C)** 54 **(D)** 56 **(E)** 60

13. A racetrack is divided into two sections of equal length. Farley can drive the first section at 18 miles per hour and the second section at a faster k miles per hour. By the time she is 50% done with the race, Farley has only driven 35% of the length of the track. What is the value of k?

(A) 27 **(B)** 30 **(C)** 36 **(D)** 45 **(E)** 51

14. Let $f(x)$ be a function such that

$$f(x) = f(x-1) + 3$$

for all reals x. What is the value of $f(f(x+3)) - f(f(x))$?

(A) 3 **(B)** 9 **(C)** 27 **(D)** $f(x)+9$ **(E)** $3f(x)$

15. When the two equations below are graphed in the coordinate plane, they enclose a region of finite area. What is the area of this region?

$$\begin{cases} y = |x+8| + 4 \\ y = -|x+15| + 21 \end{cases}$$

(A) 60 **(B)** 72 **(C)** 80 **(D)** 90 **(E)** 120

16. The table below shows Melinda's twelve grades over the four quarters of a school year. Each quarter is associated with a grade equal to the average of all grades in that quarter, and Melinda's final grade is the average of all of her quarter grades. Before the final grade is calculated, Melinda can remove exactly one of her grades from the table. Which grade should she remove to maximize her final grade?

Q1	Q2	Q3	Q4
100	92	90	47
70	93	96	59
100	94	54	73

(A) 47 **(B)** 54 **(C)** 70 **(D)** 92 **(E)** do not remove any

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17. Let M and N be the midpoints of sides $\overline{AB}$ and $\overline{AC}$, respectively, of $\triangle ABC$. Given that $BN = 9$, $CM = 12$, and $BC = 10$, what is the area of $\triangle ABC$?

(A) 48 **(B)** $48\sqrt{2}$ **(C)** 72 **(D)** $48\sqrt{3}$ **(E)** $60\sqrt{2}$

18. Let N be the unique positive integer satisfying:

$$N \leq \frac{1981 \times 2001 \times 2021}{2000} \leq N+1.$$

What is the sum of the digits of N?

(A) 8 **(B)** 9 **(C)** 17 **(D)** 18 **(E)** 19

19. How many sequences of five nonzero digits are such that the product of any two consecutive digits is a perfect square?

(A) 279 **(B)** 280 **(C)** 281 **(D)** 282 **(E)** 283

20. Call a pair of positive integers (m, n) *tweenies* if and only if their least common multiple is 30. What is the sum of all possible products of two positive integers that are *tweenies*?

(A) 1800 **(B)** 1920 **(C)** 1960 **(D)** 2160 **(E)** 2196

21. Given that there is exactly one point P in the same plane as rectangle $ABCD$ with $AB = 6$ such that $\angle APB = 60^\circ$ and $\angle CPD = 90^\circ$, what is the sum of all possible lengths of BC?

(A) $3\sqrt{3}-3$ **(B)** 6 **(C)** $4\sqrt{3}$ **(D)** $3\sqrt{3}+3$ **(E)** $6\sqrt{3}$

22. Cherry makes a slight fold along the diagonal $\overline{AC}$ of a square $ABCD$ of side length 6. She then places base $\triangle ABD$ onto a flat table, such that $\triangle ABD$ is equilateral. How high up is vertex C from the table?

(A) $2\sqrt{5}$ **(B)** $2\sqrt{6}$ **(C)** $3\sqrt{3}$ **(D)** $2\sqrt{7}$ **(E)** $4\sqrt{2}$