



# MARH AMC

*ice creAM Cake (Mock AMC 8)*

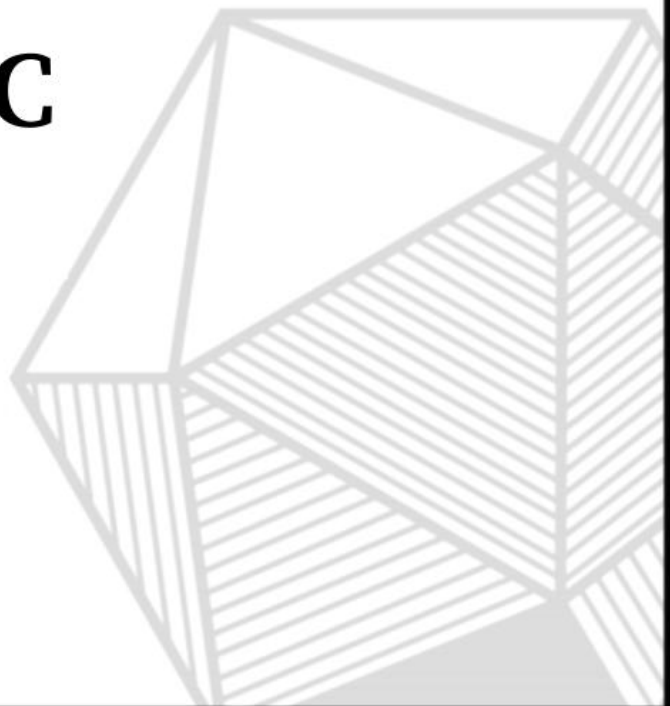
MARH ice creAM Cake (Mock AMC 8)

1<sup>st</sup> (Not Necessarily) Annual Mock

# AMC 8

Thursday, June 27, 2019

National Ice Cream Cake Day



## INSTRUCTIONS

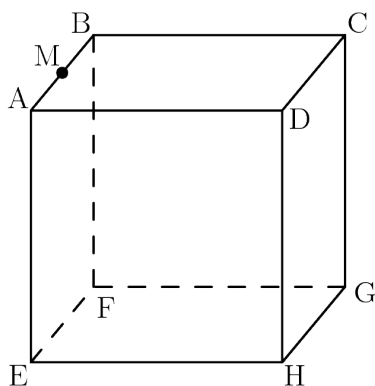
1. DO NOT PROCEED UNTIL YOU HAVE SET YOUR TIMER TO 40 MINUTES.
2. This is a twenty-five-question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Please record your answers anywhere, but only the ones submitted using the directions on the main post will be graded.
4. SCORING: You will receive 1 point for each correct answer and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compasses, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator. A timer is allowed.
6. Figures are not necessarily drawn to scale.
7. When you start your timer, begin working on the problems. You will have 40 minutes to complete the test.
8. **Please remember to submit your answers if you wish for them to be graded!**

The MAA Committee on the American Mathematics Competitions DOES NOT reserve the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also DOES NOT reserve the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the Mock AMC 8 during the period when students are eligible to participate does not jeopardize the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a not a violation of the competition rules; it is very encouraged. However, do not communicate during your test.

1. Evaluate  $\frac{(201+9)-(20+19)}{2019}$ .  
(A)  $\frac{1}{2019}$     (B)  $\frac{57}{673}$     (C)  $\frac{209}{2019}$     (D)  $\frac{1}{3}$     (E) 1
2. Unit square  $ABCD$  has point  $E$  on the midpoint of  $AB$ , point  $F$  on the midpoint of  $BC$ , point  $G$  on the midpoint of  $CD$ , and point  $H$  on the midpoint of  $AD$ . Find the area of  $AEFGH$ .  
(A)  $\frac{3}{8}$     (B)  $\frac{1}{2}$     (C)  $\frac{5}{8}$     (D)  $\frac{3}{4}$     (E)  $\frac{7}{8}$
3. Two fair 12-sided dice are rolled. The faces of each die are labeled with the numbers from 1 through 12. Find the number of possible values of the sum of the bottom faces of the two dice.  
(A) 20    (B) 21    (C) 22    (D) 23    (E) 24
4. Amy, Billy, and Cathy are competing in a 1-mile race. Amy gets a time of exactly 420 seconds and beats Billy by 60 seconds. Billy beats Cathy by 60 seconds. What is the ratio of Amy's speed to Cathy's speed?  
(A)  $\frac{5}{7}$     (B)  $\frac{7}{9}$     (C)  $\frac{9}{8}$     (D)  $\frac{64}{49}$     (E)  $\frac{9}{7}$
5. What is the smallest positive integer number greater than 1 that has a remainder of 1 when divided by each of the first 5 positive even numbers?  
(A) 121    (B) 241    (C) 361    (D) 481    (E) 601
6. Bob's phone has really bad battery. When he is not using it, his phone's battery will not discharge, but when he is using it, it will discharge at a constant rate. His phone's battery will also charge at a constant rate. When he is using his phone that is at 100 percent and charging it at the same time, his battery will last 50 minutes. When he is using his phone that is at 100 percent but not charging it, his battery will last 20 minutes. Find the amount of time, in minutes, necessary to charge Bob's phone from 0 percent to 100 percent when he is not using it.  
(A) 15    (B)  $22\frac{1}{2}$     (C)  $33\frac{1}{3}$     (D) 45    (E) 50
7. If  $a\clubsuit b = |a^2 - b^2|$ , evaluate  $(10\clubsuit(5\clubsuit 6))$ .  
(A) 12    (B) 21    (C) 161    (D) 4071    (E) 5589
8. Let  $m = \frac{1}{4} \cdot \frac{2}{5} \cdot \frac{3}{6} \cdot \frac{4}{7} \cdot \frac{5}{8} \cdot \dots \cdot \frac{2016}{2019}$ , where  $m$  can be expressed as  $\frac{a}{b}$ , with  $a$  and  $b$  being relatively prime positive integers. Find the remainder when  $a + b$  is divided by 5.  
(A) 0    (B) 1    (C) 2    (D) 3    (E) 4
9. The Smith family has 3 children. They recently went to Falmart to buy candies to distribute among their distinguishable children. If they bought 6 identical candies and each child must have at least 1, how many possible ways can Mrs. and Mr. Smith distribute the candies?  
(A) 3    (B) 6    (C) 10    (D) 15    (E) 21

10. Bob is asked to find the sum of the first six positive perfect squares and the first five positive perfect cubes. However, he accidentally finds the sum of the first five positive perfect squares and the first six positive perfect cubes. What is the positive difference between the answer he should have gotten and the answer he calculated?
- (A) 20      (B) 30      (C) 100      (D) 180      (E) 191
11. Square  $ABCD$  with side length 2 is circumscribed by circle  $O$ . Circle  $P$  is drawn with diameter  $BC$ . Find the area of the overlap of circles  $O$  and  $P$ .
- (A)  $\pi - 1$       (B)  $\pi$       (C)  $\frac{3\pi}{2} - 1$       (D)  $2\pi - 2$       (E)  $2\pi - 1$
12. How many positive integers under 1000 are relatively prime to 2019? Note that 1 is relatively prime to all positive integers and the prime factorization of 2019 is  $3^1 \cdot 673^1$ .
- (A) 665      (B) 666      (C) 667      (D) 997      (E) 998
13. In cube  $ABCDEFGH$  with edges of length 2, as shown, let the midpoint of  $\overline{AB}$  be  $M$ . Find the positive difference between the area of  $\triangle ACG$  and the area of  $\triangle AMG$ .



- (A) 0      (B)  $\sqrt{2}$       (C)  $2\sqrt{2} - 1$       (D)  $2\sqrt{2}$       (E)  $2\sqrt{2} + 1$
14. It is known that exactly two of the following four statements are true. Find the sum of the numbers of the statements that are true.
- Statement 1: Statement 2 is true.  
 Statement 2: Neither Statement 1 nor Statement 4 are true.  
 Statement 3: Statement 2 is false.  
 Statement 4: Statement 3 is true.
- (A) 3      (B) 4      (C) 5      (D) 6      (E) 7
15. Lucky Ben flips 10 identical coins and somehow gets all heads while arranging them in a straight line. Unfortunately, his little sister comes over and flips over 4 not necessarily different coins (while still keeping the coins in a straight line). How many arrangements of heads and tails are possible? (One possible arrangement is HHHHHHTHTH.)
- (A) 24      (B) 210      (C) 255      (D) 256      (E) 324

16. Paul is standing on a point at a plane. He moves north 10 meters, then west 20 meters, then south 30 meters, then east 40 meters, then north 50 meters, and so on. How far will Paul be from his starting point after he has moved a total of 360 meters?  
(A)  $8\sqrt{10}$     (B)  $12\sqrt{5}$     (C) 40    (D)  $40\sqrt{2}$     (E) 360
17. In Eggville, eggs can only be purchased in containers of 3 and 10. How many different quantities of eggs from 1 to 100, inclusive, cannot be purchased?  
(A) 7    (B) 9    (C) 12    (D) 15    (E) 17
18. Let  $f(x, y) = \sqrt{xy}$  and  $g(x, y, z) = \sqrt[3]{xyz}$ . Evaluate  $g(f(3, 4), f(2, 5), f(10, 3))$ .  
(A) 3    (B)  $\sqrt[3]{60}$     (C) 4    (D)  $2\sqrt[3]{10}$     (E)  $3\sqrt[3]{3}$
19. A spaceship is flying through a galaxy with a crew in dire need of fresh water. Each day, they stop by a planet to try and find water. On the first day, they have a  $\frac{1}{2}$  chance of finding water. Each day, their chance of finding water is  $\frac{1}{3}$  of the previous day. Which of the following is the probability that the crew finds water within the first four days closest to?  
(A)  $\frac{2}{5}$     (B)  $\frac{1}{2}$     (C)  $\frac{3}{5}$     (D)  $\frac{7}{10}$     (E)  $\frac{3}{4}$
20. There is a special wall in Falmart that has a 10% chance of turning humans that touch it into turkeys, and a 90% chance of turning turkeys that touch it into humans. What is the probability that Bob (a human) ends up as a turkey after touching the wall 3 times?  
(A)  $\frac{81}{1000}$     (B)  $\frac{9}{100}$     (C)  $\frac{1}{10}$     (D)  $\frac{3}{20}$     (E)  $\frac{729}{1000}$
21. Alice is making a rectangular garden bordering a river. Every side of the garden will be fenced except the one facing the river to prevent thieves from stealing her vegetables. She has 60 meters of fencing. What is the maximum possible area of her garden?  
(A) 350    (B) 400    (C) 420    (D) 450    (E) 500
22. Bob has a special calculator that changes its output based on the time of day. If it is morning, the output will be  $f(x) = x^2 + 5$ . If it is not morning, the output will be  $f(x) = 6x - 4$ . Bob enters a number during the morning and enters the same number during the afternoon. Both times, he gets the same answer. What number did he enter?  
(A)  $-9$     (B)  $-3$     (C) 3    (D) 6    (E) 9
23. For how many positive integer values of  $x$  is the median of  $\{2, 20, 201, 2019, x\}$  greater than the median of  $\{1, 17, 289, 4913, x\}$ ?  
(A) 0    (B) 19    (C) 20    (D) 180    (E) 201
24. Tanner has two identical cylindrical barrels with radii of 10 feet and heights of 15 feet. One barrel is upright, and the other is turned on its side. If both barrels are filled with oil up to a height of 5 feet, what is the positive difference between the amounts of oil in the two barrels in cubic feet?

(A) 0      (B)  $375\sqrt{3}$       (C)  $250\pi$       (D)  $500\pi$       (E)  $375\sqrt{3} + 400\pi$

25. Define the sequence  $A$ , where the  $n^{\text{th}}$  term is  $a_n$  for all positive integers  $n$ . For all integers  $n \geq 3$ ,  $a_n = 2a_{n-1} - 2a_{n-2}$ . If  $a_1 = 1$  and  $a_{2019} = 9 \cdot 2^{1009}$ , then  $a_{2020}$  can be expressed as  $x \cdot 2^y$  where both  $x$  and  $y$  are integers and  $x$  is not a multiple of 2. Compute  $x + y$ .

(A) 1009      (B) 1010      (C) 1011      (D) 1012      (E) 1013

We hope you enjoyed the 2019 ice creAM Cake Mock AMC 8. Please direct any questions, comments, suggestions, or errors to PROA200, Ooof, and Whee by PM or email [icecreamcake.mockamc@gmail.com](mailto:icecreamcake.mockamc@gmail.com). If you take this within the time-frame from 6/27/2019 to 9/23/2019, then please submit via your preferred method. Thank you!

Much thanks to **claw**, **jeffisepic**, **logz** and **smartninja2000** for testsolving.

Best Regards,  
PROA200, Ooof, and Whee