2023 Mock AMC 12

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Instructions:

1. This is a 25-question multiple-choice test to be completed in **75 minutes**. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
3. No aids are permitted other than pencils, pens, blank scratch paper, rulers, compasses, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
4. Figures are not necessarily drawn to scale.
5. Submit your answers by sending a PM (private message) to QIDb602 in the following format:

   XXXXX
   XXXXX
   XXXXX
   XXXXX
   XXXXX

6. A leaderboard with scores will be posted. If you wish to remain anonymous in the leaderboard, please indicate that in the PM.
7. If there are errors, please also indicate them in the PM. Do not discuss errors publicly.
8. Please do not discuss the test outside of the private discussion forum until the submission window is over.

1. What is the value of

$$\frac{2^2-0^2+2^2-3^2}{2-0+2-3}-\frac{2-0+2-3}{2^2-0^2+2^2-3^2}$$

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

2. Kite $ABCD$ has $AB = AD = 15$, $CB = CD = 20$, and $AC = 25$. What is BD?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 25 **(E)** 32

3. Mindy wants to memorize her multiplication facts from 1×1 to 12×12 using a multiplication table that has rows and columns labeled with factors, such that the products form the body of the table. Of the 144 numbers in the body of the table, how many are less than 100?

(A) 133 **(B)** 134 **(C)** 135 **(D)** 136 **(E)** 137

4. Five men and five women are standing in a line. Two men see a woman in the spot in front of them. w women see a man in the spot in front of them. What is the sum of all possible values of w?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. Let $ABCDEF$ be a regular hexagon. If the sum of the distances from A to $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{EF}$ (when all are extended) is $4\sqrt{3}$, what is the side length of the hexagon?

(A) 1 **(B)** $\frac{2\sqrt{3}}{3}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** 2

6. A Martian analog clock has 3 minutes per hour and 3 seconds per minute. Its hour, minute, and second hands all rotate at constant rates of $\frac{1}{3}$ of a full circle per respective time unit. It is currently Martian noon. The next time two of the angles between the three hands are equal, how many Martian seconds will have elapsed? (An angle only counts if there are no hands between the two hands that form the angle.)

(A) $\frac{27}{16}$ **(B)** $\frac{9}{5}$ **(C)** $\frac{27}{14}$ **(D)** $\frac{9}{4}$ **(E)** $\frac{27}{10}$

7. Let $P(x)$ be the probability that the sum of the values that show up when two fair 6-sided dice are rolled is equal to x. Let $Q(x)$ be the probability that the sum of the values that show up when a fair 5-sided die and a fair 7-sided die are rolled is equal to x. What is the smallest x for which $P(x) > Q(x)$? (Assume that for an n-sided die, the numbers on the die are integers from 1 through n.)

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

8. What is the greatest common divisor of all integers that can be represented as $k^6 - k^2$, where k is a positive integer?

(A) 10 **(B)** 12 **(C)** 20 **(D)** 30 **(E)** 60

9. Given a 5×5 grid composed of unit squares, how many ways are there to shade 12 unit squares such that no two shaded unit squares touch along a side? (Rotations and reflections of the same shading are considered separate.)

(A) 12 **(B)** 13 **(C)** 14 **(D)** 25 **(E)** 26

10. Which of the following numbers yields the smallest remainder when divided by 2023?

(A) 1978^2 **(B)** 1979^2 **(C)** 1980^2 **(D)** 1981^2 **(E)** 1982^2

11. Hamza has 2023 cows and 119 of them are infected with the Cow-ronavirus. He tested a randomly chosen cow for the disease. The test is 90% accurate on cows who are infected and 95% accurate on cows who are healthy. If the test result shows healthy, what is the probability the cow is actually infected?

(A) $\frac{1}{170}$ **(B)** $\frac{1}{161}$ **(C)** $\frac{1}{160}$ **(D)** $\frac{1}{153}$ **(E)** $\frac{1}{152}$

12. Let the base n representation of positive integer k be $\underline{a}\,\underline{b}_n$ for some positive integers $a \geq 2$, $b \geq 1$, and $n \leq 10$. Suppose the base is increased by 2, the digits are decreased by 1, and the numerical value remains k. How many possible k are there?

(A) 18 **(B)** 20 **(C)** 21 **(D)** 24 **(E)** 25

13. Suppose positive integer k satisfies $\text{lcm}(44100, k) = k \cdot \gcd(44100, k) \neq 44100$. What is the sum of the digits of the smallest possible value of k?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

14. Quadrilateral $ABCD$ has $\angle ABC = \angle ADC = 90^\circ$, $AB = 3$, and $BC = 4$. Let the point of intersection of $\overline{AC}$ and $\overline{BD}$ be E. If $AE = 1$, the area of quadrilateral $ABCD$ can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 19 **(B)** 29 **(C)** 32 **(D)** 43 **(E)** 54

15. Which of the following values of x causes the following expression to evaluate to an integer?

$$6^{\log_6 12} 12^{\log_6 12} 18^{\log_6 12} 24^{\log_6 12} x^{\log_6 12}$$

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

16. Given randomly chosen real numbers h and k within the interval $[-1, 1]$, let P be the probability that the following system of equations has no real solutions. Which of the following is closest to P? (NOTE: $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.732$)

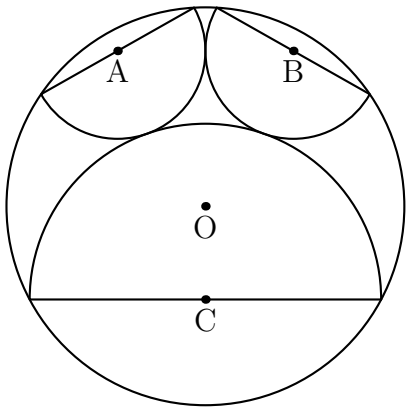
$$x^2 + y^2 = 4$$
$$|x - h| + |y - k| = \sqrt{2}$$

(A) 0.305 **(B)** 0.315 **(C)** 0.325 **(D)** 0.335 **(E)** 0.345

17. Suppose complex number z satisfies $\text{Im}(z^6) = \text{Im}((z+1)^3) = 0$, $\text{Im}(z^m) \neq 0$ for all integers $0 < m < 6$, and $\text{Im}((z+1)^n) \neq 0$ for all integers $0 < n < 3$. What is the greatest possible value of z^6?

(A) -27 **(B)** $-\frac{27}{64}$ **(C)** $\frac{27}{64}$ **(D)** 1 **(E)** 27

18. In the following diagram, semicircles A and B have radii of 1 while semicircle C has a radius of 2. The endpoints of the diameters of semicircles A, B, and C lie on circle O. In addition, semicircles A, B, and C are pairwise externally tangent to each other. The radius of circle O can be written in the form $\frac{\sqrt{x}}{y}$, where x and y are positive integers and y is as small as possible. What is $x + y$?



(A) 49 **(B)** 86 **(C)** 134 **(D)** 191 **(E)** 314

19. Anita used a 4-digit combination lock to lock her bicycle, with each digit ranging from 0 to 9 and leading zeros permitted. However, she did not do a good job of scrambling the digits.

A *turn* is defined as increasing a particular digit by 1, except that turning a 9 would make it a 0. Anita had the correct combination, and then turned each digit either $n - 1$, n, or $n + 1$ times, where n is a positive integer. If the lock reads "2023", how many possible correct combinations are there? (For example, "2112" is a possible correct combination, where $n = 10$ and the digits are turned 10, 9, 11, and 11 times respectively.)

(A) 630 **(B)** 640 **(C)** 650 **(D)** 660 **(E)** 670

20. Buildings B_1, B_2, B_3, B_4, and B_5 have positive height. For $2 \leq n \leq 5$, building B_n is $4^{n-2}k$ times the height of building B_{n-1}, where k is some positive real constant. If building B_x has the same height as the average height of the five buildings, how many of those five buildings could B_x possibly be?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

21. Define an *n-stretchable* integer as a positive, three-digit integer that is evenly divisible by n, and remains evenly divisible by n no matter how many times the middle digit is repeated. For example, 369 is 3-stretchable because 369, 3669, 36669, etc. are all evenly divisible by 3. How many 7-stretchable integers are there?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

22. Right circular cone α with apex A has a base radius of 17 and a height of 17. Right circular cone β with apex B has a base radius of 17 and a height of 34. If α and β were joined at the base, the maximum possible side length of a cube inside the combined solid with one of its space diagonals on $\overline{AB}$ can be written in the form $\sqrt{m}-\sqrt{n}$, where m and n are positive integers. What is $m+n$?

(A) 441 **(B)** 459 **(C)** 477 **(D)** 495 **(E)** 513

23. Let $\mathbb{S}$ denote the set of all subsets of the set $\{x, y, z\}$. A function $f : \mathbb{S} \to \mathbb{S}$ satisfies

$$f(A \cup B) = f(A) \cup f(B)$$

for all A and B in $\mathbb{S}$. How many such functions f are possible?

(A) 343 **(B)** 512 **(C)** 585 **(D)** 729 **(E)** 4096

24. Let $P(x)$ be a polynomial of degree 5 which satisfies

$$P(2^i) = i$$

for all integers $0 \leq i \leq 5$. The coefficient of x^4 in $P(x)$ can be written in the form $-\frac{a}{b}$, where a and b are relatively prime positive integers. What is the sum of the digits of b?

(A) 7 **(B)** 15 **(C)** 17 **(D)** 19 **(E)** 20

25. Suppose that real number x satisfies

$$5(\tan^6 x + \tan^4 x + \tan^2 x) + 2^{60} \cdot \sin^{10} x + \cos^2 x = 2^{60}$$

What is the sum of the digits of $\tan^2 x$?

(A) 18 **(B)** 19 **(C)** 25 **(D)** 26 **(E)** 31

END OF TEST

We would appreciate your honest feedback. The following questions are optional but encouraged. They will not affect your score and do not need to be completed within the allotted 75 minute time frame.

1. On a scale of 0 to 10, with 0 being very poor quality and 10 being excellent quality, how would you rate the quality of this test?
2. On a scale of 0 to 10, with 0 being very easy, 10 being very difficult, and 5 being comparable to an average AMC 12 from the last five years, how would you rate the difficulty of this test?
3. Do you have any further comments or suggestions on specific problems?