THE RULES

1. This test contains 25 questions, each a multiple choice with 5 answers. There is only one correct answer.
2. Correct answers are worth 6 points, questions left blank are worth 1.5 points, and questions answered incorrectly are worth nothing.
3. The time limit is 75 minutes.
4. No calculators are allowed. You may use protractors, compasses or other non-electronic drawing aids.
5. When you have finished, sumbit your answers in a PM to whatshisbucket. Unless you say otherwise, your score will be put in the public leaderboard.
6. If there is something in the test that you believe is an error, please report it ASAP in a PM to whatshisbucket.

Special thanks to eisirrational, blue8931, and sherlockming for making this test possible.

1. When one is in school, an hour feels like a year. My school day is 7 hours. How many whole days do I have to spend in school to feel like I have passed my life expectancy of 78.74 years?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

2. Hot dogs come in packages of 6 and buns come in packages of 14. What is the smallest number of total packages that must be bought in order to have the same number of hot dogs and buns?

(A) 10 (B) 12 (C) 15 (D) 20 (E) 30

3. What is the largest negative even integer a such that $a^2 + 1$ is composite?

(A) -2 (B) -4 (C) -6 (D) -8 (E) -10

4. There are 1700 people at my school. 40% are boys and 60% are girls. 30% of the boys do math and 40% of the girls do math. What fraction of the people who do math are boys?

(A) 1/6 (B) 1/4 (C) 1/3 (D) 1/2 (E) 2/3

5. If the interior angle of a regular polygon is $1977/11^\circ$, find the number of sides of the polygon.

(A) 110 (B) 200 (C) 330 (D) 660 (E) 1320

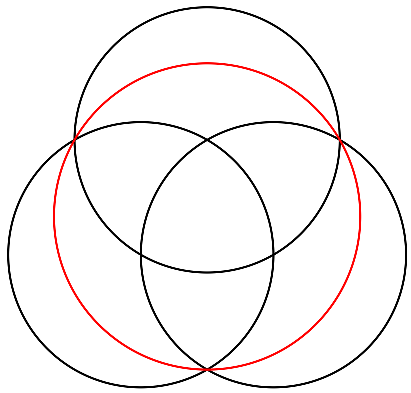
6. Find the sum of all positive integers n less than 20 such that n does not divide $(n-1)!$

(A) 77 (B) 79 (C) 81 (D) 83 (E) 87

7. Alice and Bob like to build walls. Alice can build a wall in 6 hours, and Bob can build a wall in 7 hours. Together, they can build a wall in 3 and a half hours. Alice and Bob are assigned two walls to build by their boss. How much time would be saved, in hours, if instead both working together on both walls, Alice and Bob each worked on their own wall until Alice finished, at which point she helped Bob until he finished?

(A) 1/3 (B) 1/2 (C) 3/4 (D) 1 (E) 3/2

8. Three circles of radius 1 are drawn, such that each passes through the centers of the other two circles. A fourth circle is drawn, such that it passes through three of the intersection points of the original three circles, and contains the other three intersection points. What is the area of this fourth circle?



(A) $2\pi/3$ (B) π (C) $4\pi/3$ (D) 2π (E) $8\pi/3$

9. For a positive integer n, define its roundness to be the sum of the exponents in the prime factorization of n. Find the sum of all positive integers less than 100 with a roundness of 5.

(A) 96 (B) 128 (C) 152 (D) 168 (E) 232

10. Let x be a real number such that $\log_2 x + \log_4 5 + \log_8 7x = \log_{\sqrt{2}} x^{5/6}$. Find x.

(A) 5 (B) 35 (C) $35\sqrt{5}$ (D) $35\sqrt{7}$ (E) $35\sqrt{35}$

11. A book has 300_{10} pages. They are numbered in base 6 starting from 1. What is the probability that a randomly selected page has a 1 in its base 6 numbering?

(A) 31/75 (B) 12/25 (C) 7/12 (D) 44/75 (E) 2/3

12. Let $\triangle ABC$ be an equilateral triangle of side length 1. Its inscribed circle is drawn. A second circle is drawn inside the triangle, externally tangent to that circle and tangent to sides AB and AC. A third circle is drawn, externally tangent to the second circle and tangent to AB and AC. This process is repeated infinitely many times. What is the sum of the areas of all the circles?

(A) $\pi/16$ (B) $\pi/12$ (C) $3\pi/32$ (D) $\pi/8$ (E) $\pi/6$

13. A sequence is defined such that $a_1 = 1$, $a_2 = 2$, and $a_{n+2} = a_{n+1}^{a_n}$ for $n \geq 1$. What is the 9th term of this sequence?

(A) 2^{16} (B) 2^{256} (C) 2^{280} (D) $2^{2^{256}}$ (E) $2^{2^{280}}$

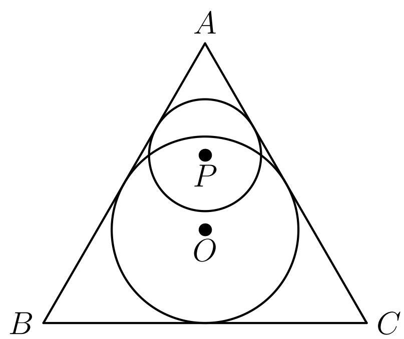
14. A function f is defined such that $f(2) = 2$, and $f(n) + f(n+1) = n(n+1)$ for all integers n. Compute $f(100)$.

(A) 100 (B) 1650 (C) 4900 (D) 5000 (E) 10000

15. What is the sum of all complex numbers z which are solutions to $z^n = nz$ for some $n \in \{2, 3, 4, ..., 10\}$?

(A) 0 (B) 2 (C) $3 + 2i$ (D) $-2i$ (E) 54

16. Let circle O have radius 1 and be inscribed in equilateral triangle $\triangle ABC$. Draw another circle P not congruent to O, with a diameter with both endpoints on the circumference of O, such that P is tangent to both AB and AC. Find the area of P.



(A) $\pi/4$ (B) $9\pi/25$ (C) $\pi/2$ (D) $16\pi/25$ (E) $25\pi/36$

17. For a positive integer n, let $k(n)$ be the product of the digits of n when written in base 4. Find the number of integers n between 1 and 256 inclusive, such that $k(n) > 0$ and $k(n)$ is divisible by 4.

(A) 22 (B) 35 (C) 41 (D) 52 (E) 59

18. Let $f(x) = ax + b$. Find the number of integer solutions (a, b) such that $f(f(3)) = 2$ and $a \neq 0$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) There are infinitely many

19. My watch is broken, causing it to stop every hour, on the hour. After it stops, I will notice at a random time less than one hour after the stoppage and restart the watch. What is the probability that during a period of 3 hours starting when my watch stops at noon, my watch runs for at least 2 hours?

(A) 1/12 (B) 1/9 (C) 1/6 (D) 1/4 (E) 1/3

20. Alice has infinitely many empty boxes in a row. The nth box can hold up to n balls. Each minute, Alice places a ball in the first box that is not completely full. She then removes all of the balls in any previous boxes. How many balls are in boxes after the 2016th minute?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 15

21. Alice, Bob, and Cheryl are attending a dinner party with five other people. While eating, the guests and host will sit at a rectangular table. Each longer side of the table will seat three people, and each shorter side will seat one person. Alice and Bob are not allowed to sit on the same side of the table. Also, at least one of the short sides of the rectangle must be filled by one of the other five guests. Considering rotations as distinct arrangements, how many allowable seating arrangements are there?

(A) 12240 (B) 13680 (C) 24480 (D) 27360 (E) 32040

22. David repeatedly flips a fair coin. Find the expected value of the total number of heads he will flip before flipping two consecutive tails.

(A) 2 (B) 5/2 (C) 3 (D) 4 (E) 9/2

23. How many subsets containing at least two elements exist of the set $\{2, 3, 4, ..., 15\}$ such that when ordered from least to greatest, no two consecutive elements are relatively prime, and no set of three consecutive elements share a common divisor greater than 1?

(A) 34 (B) 41 (C) 53 (D) 54 (E) 55

24. An ant moves in the plane by the following rules:
a. The ant moves by a distance of 1 each second on a line parallel to either the x or y-axis. It moves every second.
b. The ant is equally likely to make a horizontal motion as it is to make a vertical motion.
c. If the ant is at an x-coordinate of $n > 0$, it is $n + 1$ times as likely to move left as it is to move right. If it has an x-coordinate $n < 0$, it is $-(n - 1)$ times as likely to move right as it is to move left. If the ant's x-coordinate is equal to 0, it is equally likely to move left or right.
The same applies for the y-coordinate, i.e. it is more likely to move towards the origin. For example, if the ant were at $(2, -3)$ it would have a 1/3 chance of moving left, a 1/6 chance of moving right, a 1/8 chance of moving down, and a 3/8 chance of moving up. The ant starts at the origin. What is the probability, that during the first four moves, the ant only moves right and up?

(A) 1/64 (B) 7/320 (C) 9/320 (D) 3/64 (E) 9/160

25. A SET deck consists of 81 cards. Each card contains a set of symbols with four attributes: color (red, green, purple), shading (empty, striped, or solid), shape (oval, squiggle, or rhombus), and number of symbols (one, two, or three). Each symbol on any one card is identical, and all the cards are unique. A SET occurs when three cards satisfy the following property for each of the four attributes:
They are either: All the same (all three striped)
OR They are all different (red, green, and purple symbols).
In order for three cards to form a SET, they must satisfy this property for all four attributes. For example, if three cards contain two ovals and a squiggle, then they are automatically not a set, regardless of their other attributes. Find the maximum possible number of SETs in 6 distinct cards. One card may appear in more than one set.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6