P_Groudon Mock AMC 10

January 2020

Instructions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

1. What is the value of $(2020-(2000-(20-0)))-(2000-(20-(0-2020)))$?

(A) 0 **(B)** 20 **(C)** 80 **(D)** 1980 **(E)** 2040

2. In a list of 20 numbers, 5 of them have an average of 6, while the other 15 have an average of 10. What is the average of all 20 numbers?

(A) 9 **(B)** 9.2 **(C)** 9.5 **(D)** 9.8 **(E)** 10

3. How many ordered pairs (x, y) of non-negative integers satisfy $x+3y=30$?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

4. Box 1 contains 3 red balls and 2 blue balls, while Box 2 contains 2 red balls and 1 blue ball. A ball is randomly picked from each box. Given that the balls were different colors, what is the probability that the red ball came from Box 2?

(A) $\frac{2}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{4}{5}$

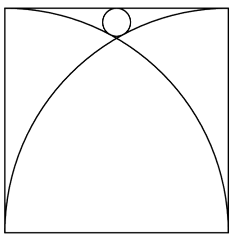
5. How many 3-digit positive integers have exactly one odd digit?

(A) 325 **(B)** 350 **(C)** 360 **(D)** 375 **(E)** 400

6. A parabola whose equation is in the form $y=ax^2+bx+c$ for some real numbers a, b, and c intersects the x-axis at $x=3$ and $x=7$. If the vertex of this parabola has y-coordinate 1, what is the y-intercept of the parabola?

(A) $-\frac{21}{2}$ **(B)** -7 **(C)** -6 **(D)** $-\frac{21}{4}$ **(E)** -5

7. In square $ABCD$ with side length 2, let ω_1 be a circle with its center at A and radius AB. Let ω_2 be a circle with its center at D and with radius DC. There exists a circle with radius r that is externally tangent to both ω_1 and ω_2 and tangent to BC. What is r?



(A) $\frac{1}{8}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

8. Let A denote the set of all positive integer divisors of 2100, and let B denote the set of all positive integer divisors of 360. How many positive integers are in at least one of A or B?

(A) 42 **(B)** 48 **(C)** 53 **(D)** 54 **(E)** 66

9. How many strictly increasing arithmetic sequences of prime numbers have at least three terms and contain both 29 and 53?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. How many ways can 2 As, 2 Bs, 1 C, 1 D, and 1 E be arranged in a line such that the two As are next to each other or the two Bs are next to each other, but not both at the same time?

(A) 240 **(B)** 450 **(C)** 480 **(D)** 540 **(E)** 600

11. Define $\lfloor x \rfloor$ to be the greatest integer less than or equal to x. Suppose that for some positive integer n, there are exactly 25 distinct integers m that satisfy $\lfloor \sqrt{m} \rfloor = n$. What is the sum of the digits of n?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 10

12. Let a, b, and c be the three distinct roots of $P(x) = x^3 - 5x^2 + 3x + 1$. Suppose that another cubic polynomial, $Q(x)$, has an x^3 coefficient of 1 and has roots ab, bc, and ac. What is $Q(1)$?

(A) -8 **(B)** -6 **(C)** -5 **(D)** -3 **(E)** -2

13. Suppose $\{a_k\}$ is a sequence defined by $a_{k+2} = a_{k+1} - a_k$ for all nonnegative integer values of k. If $a_0 = m$ and $a_2 = n$ for some integers m and n with $|m|, |n| \leq 10$, for how many ordered pairs (m, n) do we have $a_{2020} = 10$?

(A) 11 **(B)** 20 **(C)** 21 **(D)** 30 **(E)** 44

14. A paper trapezoid has side lengths $AB = 3, BC = 8, CD = 9$, and $DA = 9$ with AB parallel to DC. Let E be a point on line segment $\overline{BC}$ such that, when C is folded over the crease $\overline{DE}$, C coincides exactly with A. What is the length of CE?

(A) 3 **(B)** $\frac{15}{4}$ **(C)** 4 **(D)** $\frac{13}{3}$ **(E)** $\frac{24}{5}$

15. A pyramid $ABCDE$ with a square base $ABCD$ and apex E has side lengths $AB = BC = CD = DA = 4$ and $EA = EB = EC = ED = 2\sqrt{5}$. Let $EDCFG$ be a pyramid congruent to $ABCDE$, with square base $CDFG$ and apex E. Given that the interiors of the two pyramids do not overlap, what is the distance between $\overline{AB}$ and $\overline{FG}$?

(A) 6 **(B)** $\frac{13}{2}$ **(C)** $3\sqrt{5}$ **(D)** $4\sqrt{3}$ **(E)** 7

16. The curves with equations $x^2 + y^2 = a$ and $x + 2y = a$ meet at exactly one point, where a is a nonzero real number. The point of intersection can be written as (p, q). What is the value of $p + q$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

17. Let x_n represent the number x written in base n. For example, $25_8 = 21_{10}$. Find the number of ordered pairs of positive integers (m, n) that satisfy $148_m = 183_n + 100_{11}$.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

18. A regular tetrahedron $ABCD$ has 6 edges, each of which is painted red with probability $\frac{1}{2}$ or blue with probability $\frac{1}{2}$. An ant starts at vertex A and may walk along any red edge, but it cannot walk along a blue edge. What is the probability that it is possible for the ant to visit all the vertices of $ABCD$?

(A) $\frac{1}{2}$ **(B)** $\frac{17}{32}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{19}{32}$ **(E)** $\frac{5}{8}$

19. Triangle $\triangle ABC$ with centroid G has $AB = 8$. If BG and CG are perpendicular, what is the maximum possible area of $\triangle ABC$?

(A) 20 **(B)** $\frac{45}{2}$ **(C)** $\frac{95}{4}$ **(D)** 24 **(E)** $\frac{74}{3}$

20. Scalene triangle $\triangle ABC$ has area 1. Points D and E lie on segment BC such that $BD = DE = EC$. Point F is the midpoint of AC. The area of the triangle bounded by lines AD, AE, and BF can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 23 **(B)** 28 **(C)** 31 **(D)** 36 **(E)** 39

21. How many ways are there to place 3 Os and 3 Xs in a 3×4 rectangular grid, given that each row and column may contain at most one O and at most one X? Each cell of the grid may not contain more than one letter.

(A) 216 **(B)** 264 **(C)** 288 **(D)** 336 **(E)** 360

22. Suppose 10 distinct points $P_1, P_2, \ldots, P_9, P_{10}$ are placed in a plane such that no three points are collinear and no three points form a right triangle. Among the angles that can be formed with some set of 3 of these 10 points, let N be the minimum number of acute angles that can be formed. What is the sum of the digits of N?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

23. Define sequences p_k, q_k, and r_k for all positive integers k by $p_k = \sqrt[3]{(k+1)^2(k-1)}$, $q_k = \sqrt[3]{(k+1)(k-1)^2}$, and $r_k = \sqrt[3]{2 - 3(p_k - q_k)}$. If the sum $r_1 + r_2 + ... + r_{214} + r_{215}$ can be expressed in the form $\sqrt[3]{m} + n$ for some positive integers m and n, what is $m + n$?

(A) 220 **(B)** 221 **(C)** 222 **(D)** 223 **(E)** 224

24. For each positive integer n, define $\phi(n)$ to be the number of positive integers less than or equal to n which are relatively prime to n. For example, $\phi(10) = 4$ and $\phi(23) = 22$. For how many positive integers $a < 2020$ does there exist a positive integer b such that $4\phi(ab) = 7\phi(a)\phi(b)$?

(A) 72 **(B)** 73 **(C)** 95 **(D)** 96 **(E)** 288

25. A triangle $\triangle ABC$ has a right angle at B. A point D lies on segment AC such that $CD = 6$. The circle ω with diameter CD intersects AB at two distinct points, E and F, such that $AE < AF$. If $AE = 2$ and $DE = EF$, the length BF can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 33 **(B)** 34 **(C)** 35 **(D)** 36 **(E)** 37