MAA AMC

American Mathematics Competitions

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1st (and probably last)

aMAMC 8

asbodke's Mock AMC 8

Saturday, July 25, 2020

INSTRUCTIONS

1. DO NOT BEGIN THE TEST UNTIL YOU START A TIMER FOR 40 MINUTES.
2. This is a 25 question multiple choice test. For each question, only one answer choice is correct.
3. PM asbodke your answers. Make sure to check for mistakes as you will not be able to change them once they are sent. Edited messages will be disqualified (unless you have a legitimate reason).
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 40 minutes to complete the test once you start the timer.

1. What is $(2018)(2022) - (2017)(2023)$?

   (A) 1 **(B)** 5 **(C)** 9 **(D)** 2020 **(E)** 4039

2. $\triangle ABC$ has side lengths $AB = 28$, $BC = 45$, and $AC = 53$. What is the area of $\triangle ABC$?

   (A) $250\sqrt{6}$ **(B)** $360\sqrt{3}$ **(C)** $280\sqrt{5}$ **(D)** 630 **(E)** $450\sqrt{2}$

3. Let N be the sum of the real roots of $5x^2 - 6x + 3 = 0$. What is N^2?

   (A) $-\frac{6}{5}$ **(B)** 0 **(C)** $\frac{6}{5}$ **(D)** $\frac{36}{25}$ **(E)** 36

4. How many ways are there to order the characters in the string 'aMAMC8' such that the '8' must be at the end of the string? (The 'a' and 'A' are distinguishable but the two M's are not.)

   (A) 12 **(B)** 24 **(C)** 48 **(D)** 60 **(E)** 120

5. Given that $\blacktriangledown x = \sqrt{x}$ and $x \heartsuit y = x^2 - 2xy + y^2$ for all real x and y, find

   $$\blacktriangledown((4 \heartsuit [\blacktriangledown 4]) \heartsuit (\blacktriangledown([\blacktriangledown 4] \heartsuit 4))).$$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

6. How many palindromes with 4 or less digits have 1 as their last digit?

   Note: A *palindrome* is a number that can be read the same forwards and backwards. For example 1467641 and 1111111 are palindromes but 1467621 is not.

   (A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

7. Jack likes eating apples. He can eat 2 apples in 5 minutes. However, sometimes he gets tired and can only eat oranges, at a rate of 2 oranges per 4 minutes. Suppose he ate 36 apples in 100 minutes. How many oranges did he eat?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. How many tuples of primes (a, b, c) exist such that the triangle with side lengths a, b, and c (if it exists) is a right triangle?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

9. When you take 20% of 120% of x, you get 50% of 150% of y. Given that $\frac{x}{y}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m + n$?

   (A) 8 **(B)** 9 **(C)** 17 **(D)** 33 **(E)** 37

10. Find the sum of the distinct prime factors of 555555.

    (A) 39 **(B)** 63 **(C)** 71 **(D)** 76 **(E)** 84

11. The set S is defined as $\{1, 2, 3 \ldots, 12\}$. Let A be the number of subsets of S such that the subset contains 6 or more elements. Let B be the number of subsets of S such that the subset contains 5 or less elements. Find $A - B$.

    (A) 792 **(B)** 924 **(C)** 1024 **(D)** 1440 **(E)** 1716

12. How many non-similar triangles with integer angle measures (in degrees) satisfy that the largest angle of the triangle is less than $70°$?

    (A) 73 **(B)** 74 **(C)** 75 **(D)** 76 **(E)** 77

13. Square $ABCD$ has side length 4. Let M be the midpoint of side BC and N be the midpoint of side CD. What is the area of quadrilateral $ABMN$?

    (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

14. Round $(1 + \sqrt{2})^5$ to the nearest integer.

    (A) 80 **(B)** 81 **(C)** 82 **(D)** 83 **(E)** 84

15. The ancient Gruks had 4 letters: A, B, C, and D. They could create a word by taking an arbitrary string of A's, B's, and C's and they could also accent the A's by adding a D after the A (but the A didn't have to be accented). However, a D couldn't be added anywhere else in the word. For example, BADC and ADAD are correct 4-letter words, but ACDB is not since the D is after a C. How many 4-letter words did the ancient Gruks have?

    (A) 109 **(B)** 121 **(C)** 132 **(D)** 133 **(E)** 149

16. What is $1^3 - 2^3 + 3^3 - 4^3 \ldots + 15^3$?

    (A) 1472 **(B)** 1504 **(C)** 1664 **(D)** 1728 **(E)** 1856

17. How many positive integers n less than 1000 satisfy exactly one of the following properties?

    - n is not a multiple of 3.
    - n is not a multiple of 5.

    (A) 400 **(B)** 401 **(C)** 466 **(D)** 467 **(E)** 533

18. Given that x and y are positive real numbers such that $x + y = 5$, and that the minimum value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{xy}$ can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m + n$.

    (A) 5 **(B)** 13 **(C)** 27 **(D)** 49 **(E)** 61

19. Bob is trying to get into a university. There are 5 universities that he is applying to, with ratings of 1, 2, 3, 4, and 5 and acceptance rates of 100%, 80%, 60%, 40%, and 20% respectively. If Bob gets accepted into multiple universities, he will choose the one with the highest rating. Given that the expected value of the rating of the university Bob gets into can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, find the remainder when $m+n$ is divided by 5.

    Note: The *expected value* is the "average" of the possible outcomes; it is the sum of all the (value)(probability)'s. For example, the expected value of a 6-sided die roll is $\frac{1}{6}(1)+\frac{1}{6}(2)+\frac{1}{6}(3)+\frac{1}{6}(4)+\frac{1}{6}(5)+\frac{1}{6}(6)=\frac{7}{2}$.

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

20. How many integers n satisfy

    $$n=\frac{5m^2+14m+5}{m^2+1}$$

    for some real m?

    (A) 11 **(B)** 15 **(C)** 23 **(D)** 29 **(E)** infinitely many

21. Rectangle $ABCD$ satisfies $AB=4$ and $BD=5$. Let M be the on diagonal BD such that $BM=2$. Line AM intersects BC at N. Find the area of triangle DMN.

    (A) 2 **(B)** $\frac{12}{5}$ **(C)** 3 **(D)** $\frac{16}{5}$ **(E)** $\frac{18}{5}$

22. There exists exactly one solution (x,y,z) that satisfies the following system:

    $$x<y<z$$
    $$x+y+z=3$$
    $$xy+yz+zx=-12$$
    $$xyz=4$$

    Given that z can be written as $\frac{a+\sqrt{b}}{c}$ in simplest form and c is positive, find $a+b+c$.

    (A) 36 **(B)** 37 **(C)** 38 **(D)** 39 **(E)** 40

23. What is the remainder when 13^{64} is divided by 2194?

    (A) 135 **(B)** 356 **(C)** 689 **(D)** 1851 **(E)** 2123

24. A point A is outside of a circle ω with center O and radius greater than 1. Line AO intersects ω at B, and $AB=1$. A secant from A intersects ω at C and D with C closer to A than D such that $CD=2$. A line passing through O intersects the midpoint M of CD. The tangent from A to ω intersects ω at E. Given that $OM=1$, find AE^2.

Note: In this context, the word "secant" may be replaced by the word "line".

(A) $2\sqrt{2}+1$ **(B)** $2\sqrt{3}+1$ **(C)** $\sqrt{3}+1$ **(D)** 6 **(E)** $5+2\sqrt{2}$

25. The sequence $a_{n(n\geq 1)}$ is defined so a_n is the largest integral value k such that 2^k divides $n!$ for every integral value of $n \geq 1$. The first 5 terms of the sequence are $0, 1, 1, 3, 3$. Find the sum of the first 100 terms of the sequence.

(A) 4730 **(B)** 4731 **(C)** 4732 **(D)** 4733 **(E)** 4734