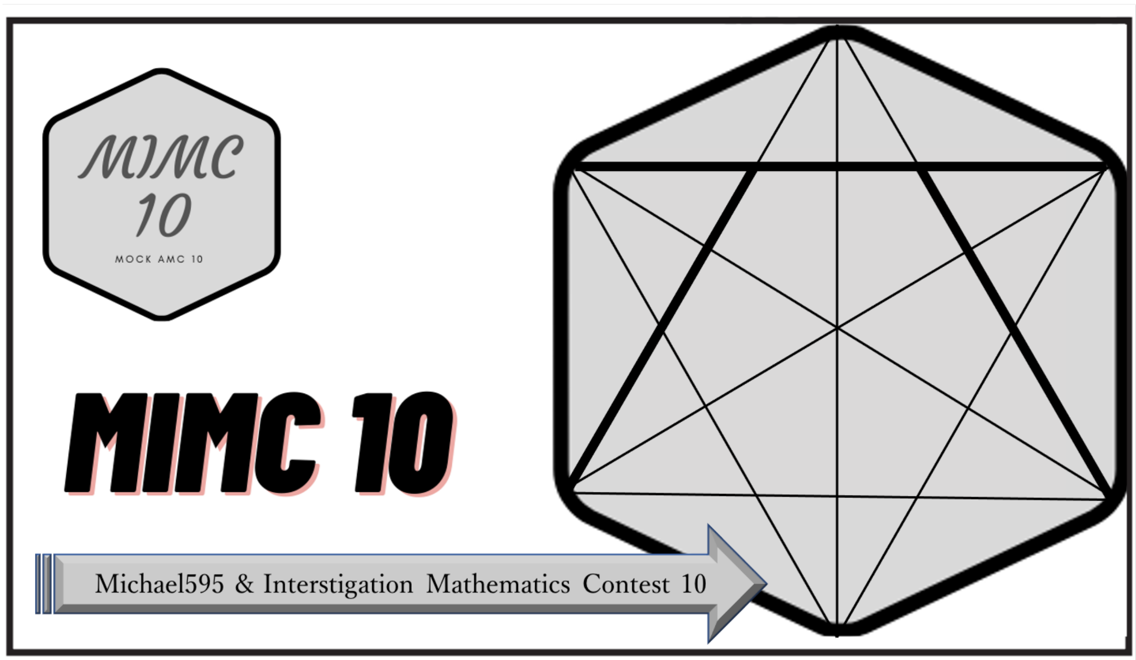


## INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil if you would like to get one from here. Check the blackened circles for accuracy and erase errors and stray marks completely. However, only answers on the MIMC Google Form found on the AoPS community page or here will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 75 minutes to complete the test.
8. When you finish the exam, fill in and submit the Google Form.
9. Enjoy the problems!

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**The MIMC Committee reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.**

The Committee will publish a projected AIME floor, Distinction and Distinguished Honor Roll, however, there will not be a mock AIME hosted by MIMC Committee.

1. What is the sum of $2^3 - (-3^4) - 3^4 + 1$?
**(A)** $-155$ **(B)** $-153$ **(C)** 7 **(D)** 9 **(E)** 171

2. Okestima is reading a 150 page book. He reads a page every $\frac{2}{3}$ minutes, and he pauses 3 minutes when he reaches the end of page 90 to take a break. He does not read at all during the break. After, he comes back with food and this slows down his reading speed. He reads one page in 2 minutes. If he starts to read at $2:30$, when does he finish the book?
**(A)** 4:33 **(B)** 5:30 **(C)** 5:33 **(D)** 6:30 **(E)** 7:33

3. Find the number of real solutions that satisfy the equation

$$(x^2 + 2x + 2)^{3x+2} = 1.$$

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. Stiskwey wrote all the possible permutations of the letters $AABBCCCD$ ($AABBCCCD$ is different from $AABBCCDC$). How many such permutations are there?
**(A)** 420 **(B)** 630 **(C)** 840 **(D)** 1680 **(E)** 5040

5. Given $x : y = 5 : 3, z : w = 3 : 2, y : z = 2 : 1$, Find $x : w$.
**(A)** 3:1 **(B)** 10:3 **(C)** 5:1 **(D)** 20:3 **(E)** 10:1

6. A worker cuts a piece of wire into two pieces. The two pieces, $A$ and $B$, enclose an equilateral triangle and a square with equal area, respectively. The ratio of the length of $B$ to the length of $A$ can be expressed as $a\sqrt[b]{c} : d$ in the simplest form. Find $a + b + c + d$.
**(A)** 9 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 15

7. Find the least integer $k$ such that $838_k = 238_k + 1536$ where $a_k$ denotes $a$ in base-$k$.
**(A)** 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

8. In the morning, Mr. Gavin always uses his alarm to wake him up. The alarm is special. It always rings in a cycle of ten rings. The first ring lasts 1 second, and each ring after lasts twice the time than the previous ring. Given that Mr. Gavin has an equal probability of waking up at any time, what is the probability that Mr. Gavin wakes up and end the alarm during the tenth ring?
**(A)** $\frac{511}{1023}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{512}{1023}$ **(D)** $\frac{257}{512}$ **(E)** $\frac{129}{256}$

9. Find the largest number in the choices that divides $11^{11} + 13^2 + 126$.
**(A)** 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

10. If $x + \frac{1}{x} = -2$ and $y = \frac{1}{x^2}$, find $\frac{1}{x^4} + \frac{1}{y^4}$.

**(A)** $-2$ **(B)** $-1$ **(C)** 0 **(D)** 1 **(E)** 2

11. How many factors of 16! is a perfect cube or a perfect square?

**(A)** 158 **(B)** 164 **(C)** 180 **(D)** 1280 **(E)** 3000

12. Given that $x^2 - \frac{1}{x^2} = 2$, what is $x^{16} - \frac{1}{x^8} + x^8 - \frac{1}{x^{16}}$?

**(A)** 1120 **(B)** 1188 **(C)** 3780 **(D)** $840\sqrt{2}$ **(E)** $1260\sqrt{2}$

13. Given that Giant want to put 12 green identical balls into 3 different boxes such that each box contains at least two balls, and that no box can contain 7 or more balls. Find the number of ways that Giant can accomplish this.

**(A)** 0 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 19

14. James randomly choose an ordered pair $(x, y)$ which both $x$ and $y$ are elements in the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$, $x$ and $y$ are not necessarily distinct, and all of the equations:

$$x + y$$

$$x^2 + y^2$$

$$x^4 + y^4$$

are divisible by 5. Find the probability that James can do so.

**(A)** $\frac{1}{25}$ **(B)** $\frac{2}{45}$ **(C)** $\frac{11}{225}$ **(D)** $\frac{4}{75}$ **(E)** $\frac{13}{225}$

15. Paul wrote all positive integers that's less than 2021 and wrote their base 4 representation. He randomly choose a number out the list. Paul insist that he want to choose a number that had only 2 and 3 as its digits, otherwise he will be depressed and relinquishes to do homework. How many numbers can he choose so that he can finish his homework?

**(A)** 30 **(B)** 62 **(C)** 64 **(D)** 84 **(E)** 126

16. Find the number of permutations of $AAABBC$ such that at exactly two $A$s are adjacent, and the $B$s are not adjacent.

**(A)** 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25

17. The following expression

$$\sum_{k=1}^{60}\binom{60}{k}+\sum_{k=1}^{59}\binom{59}{k}+\sum_{k=1}^{58}\binom{58}{k}+\sum_{k=1}^{57}\binom{57}{k}+\sum_{k=1}^{56}\binom{56}{k}+\sum_{k=1}^{55}\binom{55}{k}+\sum_{k=1}^{54}\binom{54}{k}+...+\sum_{k=1}^{3}\binom{3}{k}-2^{10}$$

can be expressed as $x^y - z$ which both $x$ and $y$ are relatively prime positive integers. Find $2^x(xy + 2x + z)$.

**(A)** 4632 **(B)** 4844 **(C)** 4860 **(D)** 4864 **(E)** 8960

18. What can be a description of the set of solutions for this: $x^2 + y^2 = |2x + |2y||$?

**(A)** Two overlapping circles with each area $2\pi$.

**(B)** Four not overlapping circles with each area $4\pi$.

**(C)** There are two overlapping circles on the right of the $y$-axis with each area $2\pi$ and the intersection area of two overlapping circles on the left of the $y$-axis with each area $2\pi$.

**(D)** Four overlapping circles with each area $4\pi$.

**(E)** There are two overlapping circles on the right of the $y$-axis with each area $4\pi$ and the intersection area of two overlapping circles on the left of the $y$-axis with each area $4\pi$.

19. $(0.51515151...)_n$ can be expressed as $(\frac{6}{n})$ in base 10 which $n$ is a positive integer. Find the sum of the digits of $n^3$.

**(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** Does Not Exist

20. Given that $y = 24 \cdot 34 \cdot 67 \cdot 89$. Given that the product of the even divisors is $a$, and the product of the odd divisors is $b$. Find $a : b^4$.

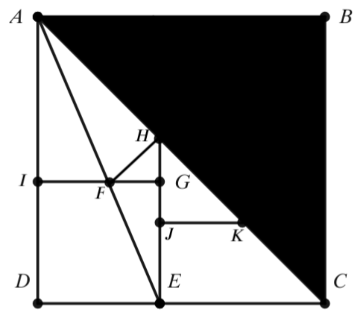
**(A)** 512:1 **(B)** 1024:1 **(C)** $2^{64}$:1 **(D)** $2^{80}$:1 **(E)** $2^{160}$:1

21. How many solutions are there for the equation $\lfloor x \rfloor^2 - \lceil x \rceil = 0$. (Recall that $\lfloor x \rfloor$ is the largest integer less than $x$, and $\lceil x \rceil$ is the smallest integer larger than $x$.)

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** $\infty$

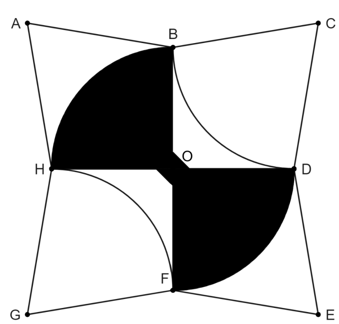
[Scroll Down for #22-25]

22. In the diagram, $ABCD$ is a square with area $6+4\sqrt{2}$. $AC$ is a diagonal of square $ABCD$. Square $IGED$ has area $11-6\sqrt{2}$. Given that point $J$ bisects line segment $HE$, and $AE$ is a line segment. Extend $EG$ to meet diagonal $AC$ and mark the intersection point $H$. In addition, $K$ is drawn so that $JK//EC$. $FH^2$ can be represented as $\frac{a+b\sqrt{c}}{d}$ where $a,b,c,d$ are not necessarily distinct integers. Given that $gcd(a,b,d)=1$, and $c$ does not have a perfect square factor. Find $a+b+c+d$.




**(A)** 5 **(B)** 15 **(C)** 61 **(D)** 349 **(E)** 2009

23. On a coordinate plane, point $O$ denotes the origin which is the center of the diamond shape in the middle of the figure. Point $A$ has coordinate $(-12,12)$, and point $C$, $E$, and $G$ are formed through $90°$, $180°$, and $270°$ rotation about the origin $O$, respectively. Quarter circle $BOH$ (formed by the arc $BH$ and line segments $BO$ and $GH$) has area $25\pi$. Furthermore, another quarter circle $DOF$ formed by arc $DF$ and line segments $OF$, $OD$ is formed through a reflection of sector $BOH$ across the line $y=x$. The small diamond centered at $O$ is a square, and the area of the little square is 2. Let $x$ denote the area of the shaded region, and $y$ denote the sum of the area of the regions $ABH$ (formed by side $AB$, arc $BH$, and side $HA$), $DFE$ (formed by side $ED$, arc $DF$, and side $FE$) and sectors $FGH$ and $BCD$. Find $\frac{x}{y}$ in the simplest radical form.



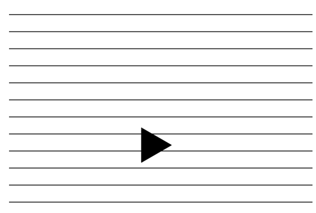

**(A)** $\frac{50\pi+1}{280}$ **(B)** $\frac{50\pi\sqrt{2}+\sqrt{2}}{560}$ **(C)** $\frac{50\pi+1}{140+100\pi}$ **(D)** $\frac{50\pi+1}{280+100\pi}$ **(E)** $\frac{50\pi^2+700\pi\sqrt{2}+3001\pi-70\sqrt{2}+60}{2\pi^2+240\pi+6920}$

24. One semicircle is constructed with diameter $AH = 4$ and let the midpoint of $AH$ be $M$. Construct a point $O$ on the side of segment $AH$ (closer to segment $AH$ than arc $AH$) such that the distance from $A$ to $O$ is $2\sqrt{5}$, and that $OM$ is perpendicular to the diameter $AH$. Three more such congruent semicircles are formed through multiple $90^\circ$ rotations around the point $O$. Name the 6 endpoints of the diameters $B$, $C$, $D$, $E$, $F$, $G$ in a circular direction from $A$ to $H$. Another four congruent semicircles are constructed with diameters $AB, CD, EF, GH$, and that the distance from the diameters to the point $O$ are less than the distance from the arcs to the point $O$. Connect $AC$, $CD$, $DO$, $OG$, and $GA$. Find the ratio of the area of the pentagon $ACDOG$ to the total area of the shape formed by arcs $AB$, $BC$, $CD$, $DE$, $EF$, $FG$, $GH$, $HA$.

**(A)** $\frac{14+10\pi}{17}$ **(B)** $\frac{13+\sqrt{2}}{28}$ **(C)** $\frac{4+\sqrt{2}}{7+3\pi}$ **(D)** $\frac{13}{28+6\pi}$ **(E)** $\frac{13}{30\pi}$

25. Suppose that a researcher hosts an experiment. He tosses an equilateral triangle with area $\sqrt{3}\ cm^2$ onto a plane that has a strip every 1 $cm$ horizontally. Find the expected number of intersections of the strips and the sides of the equilateral triangle.

**(A)** 4 **(B)** $\frac{12}{\pi}$ **(C)** $\frac{2+3\sqrt{3}}{2}$ **(D)** $\frac{4+\sqrt{3}}{2}$ **(E)** $\frac{12+4\sqrt{2}-2\sqrt{3}}{4}$



# ADDITIONAL INFORMATION

**1. The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.**

**2. The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.**

**Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:**

**Interstigation (Problem Writer)**

**Michael595 (Problem Writer)**

**Fidgetboss_4000 (Testsolver)**

**Skyscraper (Suggester)**