2023 Mock AMC 10

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Instructions:

1. This is a 25-question multiple-choice test to be completed in **75 minutes**. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
3. No aids are permitted other than pencils, pens, blank scratch paper, rulers, compasses, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
4. Figures are not necessarily drawn to scale.
5. Submit your answers by sending a PM (private message) to QIDb602 in the following format:

   XXXXX
   XXXXX
   XXXXX
   XXXXX
   XXXXX

6. A leaderboard with scores will be posted. If you wish to remain anonymous in the leaderboard, please indicate that in the PM.
7. If there are errors, please also indicate them in the PM. Do not discuss errors publicly.
8. Please do not discuss the test outside of the private discussion forum until the submission window is over.

1. What is the value of

$$\frac{2^2 - 0^2 + 2^2 - 3^2}{2 - 0 + 2 - 3} - \frac{2 - 0 + 2 - 3}{2^2 - 0^2 + 2^2 - 3^2}$$

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

2. John and Roger left from town A to town B on a 300 mile long road at 8:00 am and 9:00 am respectively, driving at constant speeds of 60 and 80 miles per hour respectively. When John and Roger met, John increased his speed to a constant 90 miles per hour. When did John arrive at town B?

(A) 12:00 pm **(B)** 12:40 pm **(C)** 12:45 pm **(D)** 1:30 pm **(E)** 1:40 pm

3. Kite $ABCD$ has $AB = AD = 15$, $CB = CD = 20$, and $AC = 25$. What is BD?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 25 **(E)** 32

4. Mindy wants to memorize her multiplication facts from 1×1 to 12×12 using a multiplication table that has rows and columns labeled with factors, such that the products form the body of the table. Of the 144 numbers in the body of the table, how many are less than 100?

(A) 133 **(B)** 134 **(C)** 135 **(D)** 136 **(E)** 137

5. The ancient Greeks believed that all matter was made of at least one of the following four elements: fire, earth, water, and air. If this were true, how many types of matter were possible? (Assume there are no other elements, and the relative amount of each element is of no concern.)

(A) 4 **(B)** 15 **(C)** 16 **(D)** 23 **(E)** 24

6. The solution set of the equation

$$xy(x-y)(x+y)(x^2+y^2-1)(x^2+y^2-4)=0$$

is a set of circles and lines that divides the xy-coordinate plane into n different regions. What fraction of these n regions have finite area?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{5}{6}$

7. Let $ABCDEF$ be a regular hexagon. If the sum of the distances from A to $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{EF}$ (when all are extended) is $4\sqrt{3}$, what is the side length of the hexagon?

(A) 1 **(B)** $\frac{2\sqrt{3}}{3}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** 2

8. Nina and Maria started running around a circular track at the same time. Nina ran her first lap in 100 seconds and slowed down by 3 seconds with each subsequent lap. Maria ran her first lap in 160 seconds and sped up by 2 seconds with each subsequent lap. They both stopped after running the same number of laps, finishing their last lap at the same time. How long, in seconds, did Maria take to run her last lap?

(A) 112 **(B)** 114 **(C)** 116 **(D)** 118 **(E)** 120

9. A Martian analog clock has 3 minutes per hour and 3 seconds per minute. Its hour, minute, and second hands all rotate at constant rates of $\frac{1}{3}$ of a full circle per respective time unit. It is currently Martian noon. The next time two of the angles between the three hands are equal, how many Martian seconds will have elapsed? (An angle only counts if there are no hands between the two hands that form the angle.)

(A) $\frac{27}{16}$ **(B)** $\frac{9}{5}$ **(C)** $\frac{27}{14}$ **(D)** $\frac{9}{4}$ **(E)** $\frac{27}{10}$

10. Let $P(x)$ be the probability that the sum of the values that show up when two fair 6-sided dice are rolled is equal to x. Let $Q(x)$ be the probability that the sum of the values that show up when a fair 5-sided die and a fair 7-sided die are rolled is equal to x. What is the smallest x for which $P(x) > Q(x)$? (Assume that for an n-sided die, the numbers on the die are integers from 1 through n.)

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

11. What is the greatest common divisor of all integers that can be represented as $k^6 - k^2$, where k is a positive integer?

    (A) 10 **(B)** 12 **(C)** 20 **(D)** 30 **(E)** 60

12. Let A denote the region consisting of all points in the xy-coordinate plane such that either $|x| \leq 1$ or $|y| \leq 1$ (or both). Let A' denote A rotated $45°$ around the origin. What is the area of the intersection of A and A'?

    (A) $8\sqrt{2} - 4$ **(B)** 8 **(C)** $8\sqrt{2} - 2$ **(D)** $4 + 4\sqrt{2}$ **(E)** $8\sqrt{2}$

13. Which of the following numbers yields the smallest remainder when divided by 2023?

    (A) 1978^2 **(B)** 1979^2 **(C)** 1980^2 **(D)** 1981^2 **(E)** 1982^2

14. Hamza has 2023 cows and 119 of them are infected with the Cow-ronavirus. He tested a randomly chosen cow for the disease. The test is 90% accurate on cows who are infected and 95% accurate on cows who are healthy. If the test result shows healthy, what is the probability the cow is actually infected?

    (A) $\frac{1}{170}$ **(B)** $\frac{1}{161}$ **(C)** $\frac{1}{160}$ **(D)** $\frac{1}{153}$ **(E)** $\frac{1}{152}$

15. x is a 2023-digit positive integer in base 10. When x is converted to base 20, it has d digits. Which of the following intervals must d be within?

    (A) [1, 1519] **(B)** [1520, 1539] **(C)** [1540, 1559] **(D)** [1560, 1579]

    (E) [1580, ∞)

16. Jerome is on the game show and has 23 closed doors in front of him. Behind 22 randomly chosen doors are \$23 each, and behind the other door is a thief. He may open as many or as few doors as he likes. If all the doors he opens contain money, he gets to keep all the money. However, if he opens the door that contains the thief, he loses all the money and cannot open any more doors. Using an optimal strategy, what is the expected value of his earnings?

    (A) \$132.00 **(B)** \$132.25 **(C)** \$144.00 **(D)** \$156.00 **(E)** \$156.25

17. Suppose positive integer k satisfies $\text{lcm}(44100, k) = k \cdot \gcd(44100, k) \neq 44100$. What is the sum of the digits of the smallest possible value of k?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

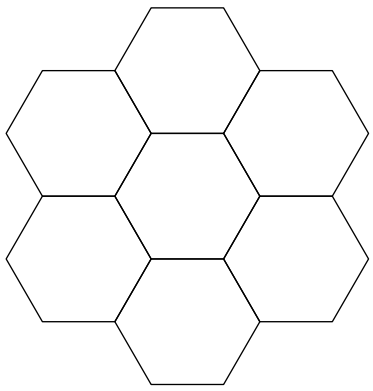
18. Quadrilateral $ABCD$ has $\angle ABC = \angle ADC = 90^\circ$, $AB = 3$, and $BC = 4$. Let the point of intersection of $\overline{AC}$ and $\overline{BD}$ be E. If $AE = 1$, the area of quadrilateral $ABCD$ can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 19 **(B)** 29 **(C)** 32 **(D)** 43 **(E)** 54

19. Let $P(x)$ be a polynomial $x^4 - 5x^3 + 6x^2 + ax + b$ for some real constants a and b. Suppose $P(x)$ has exactly two distinct roots. What is the difference between the largest and smallest possible values of a root of $P(x)$?

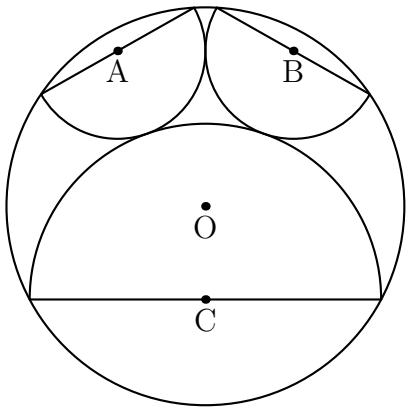
(A) $\frac{5}{2}$ **(B)** 3 **(C)** $\frac{7}{2}$ **(D)** 4 **(E)** $\frac{9}{2}$

20. There are seven hexagons in the following diagram. Two hexagons are colored red, one hexagon is colored orange, one hexagon is colored yellow, one hexagon is colored green, one hexagon is colored blue, and one hexagon is colored violet. The red hexagons cannot touch each other. A coloring is considered indistinguishable from another if the figure can undergo a series of reflections and/or rotations for the colorings to look identical. How many distinguishable colorings are possible?



(A) 45 **(B)** 60 **(C)** 75 **(D)** 90 **(E)** 120

21. In the following diagram, semicircles A and B have radii of 1 while semicircle C has a radius of 2. The endpoints of the diameters of semicircles A, B, and C lie on circle O. In addition, semicircles A, B, and C are pairwise externally tangent to each other. The radius of circle O can be written in the form $\frac{\sqrt{x}}{y}$, where x and y are positive integers and y is as small as possible. What is $x + y$?



(A) 49 **(B)** 86 **(C)** 134 **(D)** 191 **(E)** 314

22. Let (a_n) be a sequence of reals which starts $a_1 = -3$ and $a_2 = 336$. For all integers $n > 2$, the sequence is defined by the recursive relation

$$a_n a_{n-2} - a_{n-1}^2 = 7a_{n-1}a_{n-2}$$

There exists an integer k for which $a_k = 0$ and $a_i \neq 0$ for all $0 < i < k$. What is k?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

23. Define an *n-stretchable* integer as a positive, three-digit integer that is evenly divisible by n, and remains evenly divisible by n no matter how many times the middle digit is repeated. For example, 369 is 3-stretchable because 369, 3669, 36669, etc. are all evenly divisible by 3. How many 7-stretchable integers are there?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

24. Right circular cone α with apex A has a base radius of 17 and a height of 17. Right circular cone β with apex B has a base radius of 17 and a height of 34. If α and β were joined at the base, the maximum possible side length of a cube inside the combined solid with one of its space diagonals on $\overline{AB}$ can be written in the form $\sqrt{m}-\sqrt{n}$, where m and n are positive integers. What is $m+n$?

    (A) 441 **(B)** 459 **(C)** 477 **(D)** 495 **(E)** 513

25. Let $\mathbb{S}$ denote the set of all subsets of the set $\{x, y, z\}$. A function $f : \mathbb{S} \to \mathbb{S}$ satisfies

$$f(A \cup B) = f(A) \cup f(B)$$

    for all A and B in $\mathbb{S}$. How many such functions f are possible?

    (A) 343 **(B)** 512 **(C)** 585 **(D)** 729 **(E)** 4096

END OF TEST

We would appreciate your honest feedback. The following questions are optional but encouraged. They will not affect your score and do not need to be completed within the allotted 75 minute time frame.

1. On a scale of 0 to 10, with 0 being very poor quality and 10 being excellent quality, how would you rate the quality of this test?
2. On a scale of 0 to 10, with 0 being very easy, 10 being very difficult, and 5 being comparable to an average AMC 10 from the last five years, how would you rate the difficulty of this test?
3. Do you have any further comments or suggestions on specific problems?