2020 AMC 8 MOCK TEST:
Code = ***EB34GDFX3BMN***

by ab2024, sigmaPiE, Anonymous Formatter, and credited users at the end

July 18, 2021

1 Rules

This is a 45 minute test; set a timer for 45 minutes once you begin.

This test is to assess your mathematical skills. If you are not honest, you will lose the chance to properly assess yourself and it will not pay off.

Do not google any of the problems or get help from outside resources or somehow view the answers. **You are allowed and encouraged to print this test.**

Calculators, rulers, or other devices that are not displaying the test are **not** allowed. However, a simple operation calculator is allowed on problem 8.

You may only talk to yourself during the duration of the test.

Please submit answers on google forms or at this link:

Submission Form

Do not cheat or share correct or incorrect answers with anybody else.

If you don't already, you must have blank scratch paper, a pencil, and an eraser right beside you in order for the test to be properly solved and scored. You may write down your answers on a separate sheet of paper.

1. How many distinct ways can I order the letters of MOCKAMC (MOCCKAM is the same as MOCCKAM)?

(A) 1260 **(B)** 2520 **(C)** 5040 **(D)** 10080 **(E)** $infinite$

2. What is the sum of all real p where $(p-4)^{49-p^2} = 1$?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 8

3. In a certain school with 200 people, the ratio of people who make spelling errors to people who don't make spelling errors is $2:3$. How many people make spelling errors?

(A) 20 **(B)** 40 **(C)** 80 **(D)** 120 **(E)** $Not\ enough\ information$

4. Betty has 20 marbles in a bag, identical ones of the same color. There are 7 blue marbles, 5 red marbles, and 8 yellow marbles. At least how many marbles will she have to add of the same color to have exactly a $\frac{1}{2}$ probability that she will pick the added color of marbles?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

5. How many non-congruent triangles have integer side lengths and perimeter less than 15?

(A) 4 **(B)** 10 **(C)** 18 **(D)** 27 **(E)** 28

6. Define the function $g(x)$ as $\sqrt{x^2-3} - \frac{4}{x-2}$. How many integers do not result in a real number output as an input for x?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

7. Two trains leave stations 60 miles apart at the same time heading toward one another on parallel tracks. Train $\mathcal{A}$ is traveling 30 miles per hour, while Train $\mathcal{B}$ is going 50 miles per hour. After how many minutes do they pass each other?

(A) 15 **(B)** 20 **(C)** 25 **(D)** 45 **(E)** 50

8. In the sequence $-1, 1, 4, 8, ...$, the nth term is n more than the previous term. What is the sum of the digits of the $2021st$ term in this sequence?

(A) 7 **(B)** 12 **(C)** 14 **(D)** 19 **(E)** 22

9. What is the largest integer value of n such that $\frac{2021!}{n^n}$ is an integer?

(A) 224 **(B)** 360 **(C)** 430 **(D)** 470 **(E)** 490

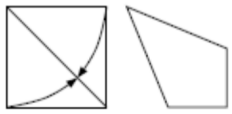
10. Let $f(n) = 2n(n+1)$ be a square number. For which function below is the function also a square number?

(A) $f(n+1)$ **(B)** $f(n^2)$ **(C)** $f(n-2)$ **(D)** $f(n+2)$ **(E)** $f(-n-1)$

11. If $x + y = 1$ and $x^2 + y^2 = 2$, what is the value of $x^3 + y^3$? Express your answer as a common fraction.

(A) $1/3$ **(B)** $2/3$ **(C)** $3/2$ **(D)** $5/2$ **(E)** $5/3$

12. A square piece of paper and two of its sides are folded to an inscribed quarter circle's intersection with the diagonal, as shown, to obtain a quadrilateral. What is the measure of the largest angle(s) of the quadrilateral?



(A) 112.5 **(B)** 120 **(C)** 123.75 **(D)** 135 **(E)** 150

13. How many positive integers less than $10,000$ have exactly three positive divisors?

(A)23 **(B)**24 **(C)**25 **(D)**26 **(E)**27

14. The side lengths of a certain non-degenerate right triangle are in a geometric sequence, where each of the sides are formed by multiplying a common ratio by the next shortest side. For example, the numbers $3, 9$, and 27 are in a geometric sequence with common ratio 3. The common ratio of the sides can be expressed as $\sqrt{\frac{\sqrt{x}+z}{y}}$, where x, y, and z are all positive integers. What is $x + y$?

(A)$1/2$ **(B)**$3/2$ **(C)**4 **(D)**$4\sqrt{2}$ **(E)**7

15. How many possible positive distinct pairs of unit fractions can you add to get the unit fraction $\frac{1}{8}$? A unit fraction is defined as having 1 as the numerator and a natural number in the denominator.

(A) 1 **(B)** 2 **(C)** 4 **(D)** 7 **(E)** 8

16. What is the value of $\left(-\frac{3}{12}-\frac{3}{20}\right)-\left(\frac{3}{30}+\frac{3}{42}\right)-...-\left(\frac{3}{(n+2)(n+3)}+\frac{3}{(n+3)(n+4)}\right)-$ ... for an integer n greater than 0? Assume the sequence does not end.

(A) $-1/2$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** $1/2$

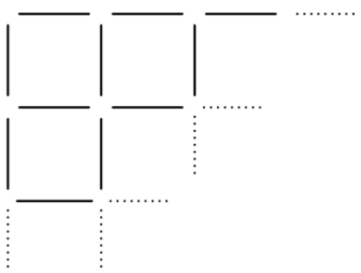
17. Three different numbers are chosen from the set $\{1, 2, 3, 4, 5, 6, 7\}$. What is the probability that their sum is a multiple of 4? Express your answer as a percentage to the nearest whole number.

(A) 15% **(B)** 25% **(C)** 26% **(D)** 35% **(E)** 50%

18. Line segment $\overline{AB}$ is tangent to circle O at point P. Line segment BX is tangent to circle O at X, and A, X and the center of O are collinear. If the ratio of AP to the radius of O is $\frac{4}{3}$, what is the ratio of the radius of O to PB?

(A) $1/3$ **(B)** $1/2$ **(C)** $3/5$ **(D)** $2/3$ **(E)** 1

19. A square arrangement of 144 toothpicks is created such that there are the same number of toothpicks on each side of the square. How many toothpicks are on each side?



(A) 8 **(B)** 12 **(C)** 16 **(D)** 24 **(E)** 36

20. An airplane starts at one airport, flying at 600 mph, and arrives at the airport 7 minutes late. Starting from the same airport, the same airplane flying at 350 mph arrives 2 hours late. How many miles are between the airports?

(A) 250 **(B)** 670 **(C)** 1352 **(D)** 1582 **(E)** 94920

21. What is the average of all base 10 integers that have 4 digits in both base 7 and base 13?

(A) 4597/2 **(B)** 2299 **(C)** 2230 **(D)** 4599/2 **(E)** 256

22. A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 2015, 2014, and 2013 tokens, respectively. How many rounds will there be in the game, including the round where they run out of tokens?

(A) 300 **(B)** 1000 **(C)** 2013 **(D)** 3016 **(E)** 6037

23. Jeremy has a deck with 6 red cards and 3 black cards. His will randomly pull a card face up, then flip an unfair coin to decide if he'll return it to the deck. $\frac{3}{4}$ of the time, the coin will say to keep the card in the deck. $\frac{1}{4}$ of the time, the coin will tell Jeremy not to return it to the deck. He'll do this two more times, looking at a total of 3 cards. What is the sum of the numerator and denominator of the fractional probability that all three of his cards are red?

(A) 107 **(B)** 502 **(C)** 1000 **(D)** 13789 **(E)** 15023

24. A number is considered *groovy* if it has the same alternating sequences (at least 2 untruncated sequences) of more than 1 digit throughout its digits. For example, 342342342 is groovy, and so is 131313, but 33333, 456789, and 3456734 are not. If the number of digits in the number is not divisible by the number of digits before a repetition, we may truncate it. For example, 45645645 is a groovy number because it has gotten truncated, but 456456457 is not. How many 7 digit numbers are groovy numbers?

(A) 972 **(B)** 981 **(C)** 990 **(D)** $99,990$ **(E)** $1,000,000$

25. A box of 24 identical candy bars is opened and candy bars are handed out to six different children. Every child receives at least one candy bar, and the number of candy bars received by any two children does not differ by more than one. Some candy bars may remain in the box. How many distinct distributions of the candy bars are possible?

(A) 128 **(B)** 190 **(C)** 240 **(D)** 255 **(E)** 360

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Congratulations on finishing! Please submit your answers here, you will be graded as soon as you do.

Honor Roll : Distinguished Honor Roll = 15:20

Acknowledgements

@SigmaPiE for problem 15, and inspiration for problem 6.

Anonymous Formatter for half of the formatting.

@bobthegod78 for solution diagram for problem something.

@LemonBerry for a tiebreaker problem.

@JustinLee2017 for confirm-solving the problems.

Credit to Math Kangaroo for problem 12.

Credit to AoPS for problem 25

Happy new year, everyone!