Eisirrational's 2017 Mock AMC 10

1. What is $1+2-3-4+5+6-7-8+\cdots-2015-2016+2017+2018$?

(A) 2 **(B)** 2016 **(C)** 2018 **(D)** 2019 **(E)** 2021

2. Today it will rain with a 25% probability. If it is true that for any day that rains, there will be a 70% chance of raining the day after. If it doesn't rain, then there will be a 30% chance of raining the day after. What is the probability that the day after tomorrow is a rainy day?

(A) 28% **(B)** 46% **(C)** 54% **(D)** 62% **(E)** 80%

3. Al and Bob have allowances. On Monday, Al had 17 dollars. That same day, Bob had 3 dollars. Every day after that, Al received 3 dollars, while Bob received 8 dollars. One day, after receiving their day's allowances, Al and Bob counted their money. It turned out that Bob had 21 dollars more than Al. What day of the week was it?

(A) Sunday **(B)** Monday **(C)** Tuesday **(D)** Wednesday **(E)** Thursday

4. The area of a right triangle with integer side lengths in an arithmetic progression is 54. Find the perimeter of the triangle.

(A) 12 **(B)** 21 **(C)** 36 **(D)** 78 **(E)** 108

5. A palindrome is a number read the same from backwards to forwards. How many 3-digit palindromes are multiples of 7?

(A) 6 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 12

6. Adam eats $\frac{1}{2}$ of a pizza, Bob eats $\frac{1}{3}$ of the pizza, and Carl eats $\frac{5}{8}$ of what is left. If there are now 3 slices of pizza left in the box, how many slices did Bob eat?

(A) 16 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 48

7. 3 people went on vacation. They spent some money, and at the end they agreed to share the costs. A gave B 32 dollars, and B gave C 27 dollars. Then they realized that C needed to give 15 dollars back to A. After this, each person paid the same amount of money. If in total they paid 216 during the vacation (note including the money they paid among themselves), how much money did B pay?

(A) 55 **(B)** 67 **(C)** 72 **(D)** 77 **(E)** 82

8. A pool is filled with water from two hoses; one hose can fill the pool in 3 hours, and the other can fill it in 4 hours. Unknown to the workmen though, there is a hole in the pool, which, when none of these hoses are on, can empty all the water in the pool in 2 hours. If it takes x hours to fill the pool, while the workmen had expected y hours, what is x/y?

(A) $\frac{12}{7}$ **(B)** 4 **(C)** 7 **(D)** $\frac{54}{7}$ **(E)** 12

9. An elastic ball is launched off a corner of a rectangular elastic container at an angle of 30 degrees off the short side. It bounces off the opposite long wall, and then back to the first long wall, and so on. If it bounces exactly 4 times on the long sides before hitting a short side, and it hits the short side at its midpoint, what is the ratio of the long side to the short side?

(A) $2\sqrt{3}$ **(B)** $3\sqrt{2}$ **(C)** $2\sqrt{2}$ **(D)** 3 **(E)** $\frac{3\sqrt{3}}{2}$

10. Let AB be a tangent drawn from A to circle O. Point A is outside circle O, and B is on circle O. OA intersects the circle at C, and $AC < AO$. If $AB = 7$, $AC = 5$, what is r, the radius of circle O?

(A) $\frac{12}{5}$ **(B)** 3 **(C)** $\frac{18}{7}$ **(D)** $\frac{27}{7}$ **(E)** $\frac{49}{5}$

11. A sequence is such that $a_1 = 1, a_2 = 3$, and $a_{n+2} = \frac{a_{n+1}^2 - 7}{a_n}$ for all $n \geq 1$. Find the average value of a_n^2 over integers n.

(A) 4 **(B)** $\frac{14}{3}$ **(C)** 0 **(D)** 3 **(E)** 7

12. There are 9 beads in a drawer; 4 identical red beads, 3 identical green beads, and 2 identical blue beads. I randomly draw 3 beads without replacement from the drawer. What is the probability that I first draw 2 indentical beads, then a different bead (in that order)?

(A) $\frac{55}{84}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{55}{252}$ **(D)** $\frac{55}{504}$ **(E)** $\frac{29}{84}$

13. What is the area of an equiangular hexagon with 4 consecutive sides being $2, 3, 4, 1$?

(A) $15\sqrt{3}$ **(B)** $\frac{35\sqrt{3}}{4}$ **(C)** $\frac{21\sqrt{3}}{2}$ **(D)** $\frac{45\sqrt{3}}{4}$ **(E)** $\frac{49\sqrt{3}}{4}$

14. Caymon lists an increasing sequence of all the natural numbers that are divisible by $2, 3$, but not 5, starting from 2. What is the 2017th number in her list?

(A) 4032 **(B)** 2744 **(C)** 4321 **(D)** 3556 **(E)** 3782

15. In triangle ABC, $AB = BC = 5$, and $AC = 6$. Drop the altitude BP, then drop the perpendicular from P to BC (meeting at Q), and then from Q to AC, then to BC, then to AC, and so on, continuing indefinitely. What is the total length drawn?

(A) $\frac{15}{2}$ **(B)** $\frac{48}{5}$ **(C)** 10 **(D)** $\frac{54}{5}$ **(E)** $\frac{71}{6}$

16. Let $P(x)$ be a degree-10 polynomial with leading coefficient 20 such that $P(1) = 17, P(2) = 27, P(3) = 37, ...P(10) = 107$. Then $P(0)$ can be expressed as $a * b! + c$, where a and c are as small as possible, and a, b, and c are all positive integers. Find $a + b + c$.

(A) 18 **(B)** 37 **(C)** 116 **(D)** 227 **(E)** 360

17. Right triangle ABC as a right angle at B, and $AB = 20, BC = 21$. Let I be the incenter. If BI intersects AC at E, and the incircle is tangent to AC at D, then find DE.

(A) $\frac{6}{41}$ **(B)** $\frac{8}{41}$ **(C)** $\frac{12}{41}$ **(D)** $\frac{20}{41}$ **(E)** $\frac{24}{41}$

18. Alice and Bob both flip a fair coin. Alice's coin is fair, but Bob's has a probability of $\frac{1}{3}$ of it landing on heads. Each stops after they flipped their first head. What is the probability that after they stopped, Alice will have flipped more times than Bob?

(A) $\frac{1}{3}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{2}{9}$ **(E)** $\frac{1}{6}$

19. Let F_n be the nth Fibonacci number, defined as $F_{n-1} + F_n = F_{n+1}$ and $F_1 = F_2 = 1$.

Find the value of $\frac{F_1+F_2+F_3}{2^1} + \frac{F_2+F_3+F_4}{2^2} + \frac{F_3+F_4+F_5}{2^3} + ... + \frac{F_n+F_{n+1}+F_{n+2}}{2^n} + \dots$

(A) 10 **(B)** 4 **(C)** 6 **(D)** 18 **(E)** 22

20. If $m(m + 3) = n(n + 1) + 192$, m and n are distinct positive integers, then find last two digits of $m \cdot n$.

(A) 12 **(B)** 35 **(C)** 52 **(D)** 56 **(E)** 72

21. If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, where a, b, c are pairwise distinct, and $\frac{a^3+b^3+c^3}{3} = a^2 + b^2 + c^2$, then what is $a + b + c$?

(A) 1 **(B)** $\frac{4}{3}$ **(C)** 2 **(D)** 3 **(E)** cannot be determined

22. In equilateral triangle ABC, P is placed inside of it so that angle APB is $150°$, and $AP = 5$, $BP = 12$. Find CP.

(A) 7 **(B)** 11 **(C)** 13 **(D)** $\frac{72}{5}$ **(E)** $\frac{205}{12}$

23. Prices for tickets to a theater costs \$7, \$11, and \$13 for children, seniors, and adults. One noon, the theater counted their money, and found that there were exactly 3 possible number of children, seniors, and adults that came possible. What is the smallest possible revenue for the theater?

(A) \$65 **(B)** \$72 **(C)** \$83 **(D)** \$96 **(E)** \$98

24. How many 4 digit integers $\overline{abcd}$ are there so that the subtraction $\overline{bcd} - \overline{abc}$ requires exactly 1 regrouping (and results in a positive answer)?

(A) 1086 **(B)** 2016 **(C)** 2172 **(D)** 3024 **(E)** 2520

25. Find the number of terms in the expansion of $(x^{20} + x^{17} + x^{13})^{10} + (x^{13} + x^{11} + x^8)^{10}$.
(A) 48 **(B)** 56 **(C)** 64 **(D)** 84 **(E)** 100