Art of Problem Solving

Autumn Mock AMC 10!

Problems

1 What is $2018 \div 20 + 18$ to the nearest integer?

(A) 53 **(B)** 100 **(C)** 119 **(D)** 132 **(E)** 2018

2 The answer to this question when added to 1 then divided by 7 and then added to 5 is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

3 Simplify the product $\frac{1}{3} \cdot \frac{3}{5} \cdot \frac{5}{7} \dots \frac{237}{239} \cdot \frac{239}{241}$

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{135}$ **(D)** $\frac{5}{239}$ **(E)** $\frac{1}{241}$

4 Phoebe's class takes a quiz, which has 15 questions worth 6 points each. The teacher notices that the average score of everyone in the class except Phoebe is 60 points. If Phoebe is included, the class average becomes 62. Which of the following is a possible number of students in the class?

(A) 4 **(B)** 10 **(C)** 12 **(D)** 17 **(E)** 21

5 Jason wants to walk from one corner of the school to the other to get from the parking lot to the entrance; however, he can't go through the school. If he walks, he can reach the other corner in 10 minutes. If he runs 5 kilometers per hour faster, he can make it in 4 minutes. What is the distance in meters he has to walk (or run) in order to get to the entrance of the school?

(A) 500 **(B)** $\frac{5000}{9}$ **(C)** 600 **(D)** $\frac{6000}{9}$ **(E)** 700

6 Jim is walking down the street when he sees some keys. There are 3 indistinguishable bronze keys, 2 indistinguishable silver keys, and 1 gold key. How many different ways can Jim lay the keys in a row, if the gold key cannot be next to a silver key, and the order of the keys matters, from left to right?

(A) 18 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 60

7 Mrs. Applegum wants her students to be in a line when they walk out to recess. She wants a "responsible" kid in the very front of the line. If there are 3 "responsible" kids, and 5 normal students, how many ways are there to form a line?

(A) 5040 **(B)** 10080 **(C)** 15120 **(D)** 25200 **(E)** 50400

8 Equilateral triangle ABC has side length 6. Points D, E, F lie within the lines AB, BC and AC such that $BD = 2AD$, $BE = 2CE$, and $AF = 2CF$. Let N be the numerical value of the area of triangle DEF. Find N^2.

(A) 8 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 27

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9 Tim's school has a competition where they are trying to find the area of the square playground. The only hint they have is that the length of the diagonal of the playground (in meters) is equal to the sum of the infinite geometric series: $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} \cdots$. What is the area of the playground?

(A) $\frac{8}{25}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{8}{9}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{4}{3}$

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10 Abinav is in a dark room with socks (he's a weird kid). He has 8 blue socks, 14 red socks, and x yellow socks. He chooses 3 socks without replacement. The probability that he chooses 3 straight red socks is $\frac{1}{9}$. What is x?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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11 Bob's teacher wrote the greatest power of two which divides both 50! and 80!. What was the exponent in the number Bob's teacher wrote? (A power of two is a number that can be expressed in the form 2^n for a nonnegative integer n.)

(A) 38 **(B)** 47 **(C)** 78 **(D)** 125 **(E)** 397

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12 Consider $\triangle ABC$ with $AB = 13, BC = 15, CA = 14$. If M is the midpoint of BC and P is a point on AC such that $MP \perp AC$, find MP.

(A) $\frac{21}{4}$ **(B)** 6 **(C)** $\sqrt{42}$ **(D)** $\frac{13}{2}$ **(E)** $3\sqrt{5}$

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13 How many ordered pairs (x, y) of positive integers satisfy the equation $x^2 + y^2 - 2018x - 2018y + xy - 2018 = 1 - xy$?

(A) 1008 **(B)** 1009 **(C)** 2017 **(D)** 2018 **(E)** 2019

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14 A rectangle $ABCD$ has $AB = 6$ and $BC = 8$. Let M be the midpoint of $\overline{AD}$ and let N be the midpoint of $\overline{CD}$. Let $\overline{BM}, \overline{BN}$ intersect $\overline{AC}$ at X, Y. Find XY.

(A) $\frac{4}{3}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{10}{3}$ **(D)** $\frac{13}{3}$ **(E)** $\frac{17}{3}$

15 The marble guy has a collection of ten distinguishable marbles equal in size: one of these weigh 1 gram, two others weigh 4 grams each, three more weigh 9 grams each, and the other four marbles weigh 16 grams.

One day, the marble guy decides to randomly select four of the marbles from his collection to be donated to the Marble Foundation. He notices that the total weight (in grams) of the six marbles he had left is divisible by 5. What is the probability of this happening?

$\textbf{(A) } \frac{2}{5} \qquad \textbf{(B) } \frac{3}{7} \qquad \textbf{(C) } \frac{16}{35} \qquad \textbf{(D) } \frac{10}{21} \qquad \textbf{(E) } \frac{8}{15}$

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16 For a positive integer r, let T_r be the rth term of an arithmetic sequence. For some positive integers m and n (with $m \neq n$), we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$. Find T_{mn}

$\textbf{(A) } 0 \qquad \textbf{(B) } \frac{1}{2} \qquad \textbf{(C) } 1 \qquad \textbf{(D) } \frac{3}{2} \qquad \textbf{(E) } 2$

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17 At how many points do the graphs of the equations

$$\begin{cases} x^3 - (x-y)^2 + xy = 0 \\ y^3 - (y-x)^2 + xy = 0 \end{cases}$$

intersect?

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 4$

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18 How many positive integers n are there such that when $n!$ is expressed in base 8, it has less than 12 digits?

$\textbf{(A) } 10 \qquad \textbf{(B) } 12 \qquad \textbf{(C) } 13 \qquad \textbf{(D) } 14 \qquad \textbf{(E) } 15$

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19 James has 7 marbles; 3 yellow ones, 2 red ones, and 2 blue ones. He arranges the marbles in a row from left to right. What is the probability that no two adjacent marbles are the same color?

$\textbf{(A) } \frac{16}{105} \qquad \textbf{(B) } \frac{17}{105} \qquad \textbf{(C) } \frac{37}{210} \qquad \textbf{(D) } \frac{19}{105} \qquad \textbf{(E) } \frac{41}{210}$

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20 In regular octagon $ABCDEFGH$ with side length 2, the intersection of AF and CH is marked as K. The distance from the centroid of $\triangle KEC$ to G can be expressed as $\frac{\sqrt{a+b\sqrt{c}}}{d}$. What is the value of $a+b+c+d$?

$\textbf{(A) } 85 \qquad \textbf{(B) } 90 \qquad \textbf{(C) } 95 \qquad \textbf{(D) } 100 \qquad \textbf{(E) } 105$

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21 How many of the solutions to $x^1 + x^2 + x^3 + \cdots + x^{59} = 0$ are not solutions to $x^2 + x^4 + x^6 + x^8 = 0$?

(A) 51 **(B)** 55 **(C)** 57 **(D)** 58 **(E)** 59

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22 Jackson is taking a Probability class which has three equally weighted tests - two midterms and a final.
In this class, an A equates to any percentage score 90 or above, a B to a score at least 80 but less than 90, and so forth, until an F which equates to any score under 60. On the first test, he scores a B, and on the second test, he scores an A.

Jackson forgot his exact percentage scores on the first two tests, but wants to secure an A in the class, so he studies long and hard for the final. What is the probability that he will be able to do so, given that he scores an A on the final? (The test scores need not be integers.)

(A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{7}{8}$ **(E)** $\frac{11}{12}$

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23 Determine the last two digits of $\lfloor(\sqrt{5}+\sqrt{6})^{4000}\rfloor$, where $\lfloor n \rfloor$ is defined as the largest positive integer less than or equal to n.

(A) 01 **(B)** 02 **(C)** 04 **(D)** 10 **(E)** 25

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24 Let $ABCD$ be a square and point P be placed in $ABCD$ such that $AP = 3$, $BP = 6$, and $CP = 9$. The side length of $ABCD$ can be expressed as $a\sqrt{b + c\sqrt{d}}$ in simplest radical form. Find $a + b + c + d$.

(A) 9 **(B)** 11 **(C)** 12 **(D)** 14 **(E)** 17

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25 A monic polynomial $P(x)$ satisfies $(x - 2^{2018})P(2x) = 2^{2018}(x-1)P(x)$ for all real x. Let c denote the coefficient of x in the expansion of $P(x)$. Compute the last digit of $|c|$.

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

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