ADIHAYAMC 8

Actually Decently Interesting Humorous AoPS Youth Advanced Math Competition 8

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU WANT TO START.
2. You may only start the test after the test is released. Doing so before is literally impossible.
3. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
4. Private message your answers to each problem to **brianzjk** on AoPS. Use of copier, telephone, e-mail, World Wide Web, pig string toy, carrier pigeon, smoke signal, fax machines, sign language, semaphore, the dark web, RSA encryption, Discord Scam, AoPS Sheriff, or any other method of communication other than AoPS Private message is prohibited. Only answers properly entered in the private message form will be graded.
5. **SCORING:** There is no penalty for guessing. Your score is the number of correct answers.
6. No aids are permitted other than scratch paper, graph paper, rulers, compasses, and erasers. No calculators, smartwatches, pig string toys (other than pig string compass toys), consultings of Adihaya Jayasharmaramankumarguptareddybavarajugopal, fax machines, Teslas, or computing devices are allowed. No problems on the test will require the use of a calculator.
7. Figures are not necessarily drawn to scale.
8. Before beginning the test, you may choose to record certain information on the answer form.
9. When you give yourself the signal, begin working on the problems. You will have 40 minutes to complete the test.
10. When you finish the exam, you may sign your name on the AoPS Private Message.

The Actually Decently Interesting Humorous AoPS Youth Advanced Mock Competition 8 Committee (ADIHAYAMC 8 Committee) reserves the right to re-examine students before deciding whether to grant official status to their scores. The ADIHAYAMC8 Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, sexual reproduction, asexual reproduction, meiosis, mitosis, binary fission, self-replication, or communication of the problems or solutions of the ADIHAYAMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web, pig string toy, carrier pigeon, smoke signal, fax machines, sign language, semaphore, the dark web, RSA encryption, Discord Scam, or media of any type during this period is a violation of the competition rules and will result in disqualification and punishment from Adihaya Jayasharmaramankumarguptareddybavarajugopal.

1. Compute $6 \div 4(2^2 \cdot 10 \div 5(4))$.

   (A) $\frac{3}{64}$ **(B)** $\frac{3}{4}$ **(C)** 3 **(D)** 24 **(E)** 48

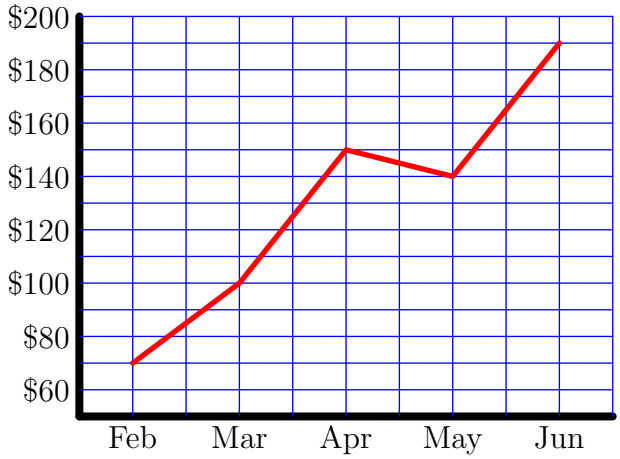
2. What is the second largest possible sum that can be achieved by adding two distinct elements of the set $\{-8, -3, 1, 5, 8, 10\}$?

   (A) -11 **(B)** -8 **(C)** 13 **(D)** 15 **(E)** 18

3. Let $\triangle ABC$ have side length $AB = 6$. Construct segment CD perpendicular to side AB with D on side AB. Given that $CD = 7$, then find the area of $\triangle ABC$.

   (A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

4. The following graph shows the stocks for the space company SpaceY. What is the ratio of the price of the stocks in June to the stocks in March?



   (A) $\frac{5}{14}$ **(B)** $\frac{10}{19}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{19}{10}$ **(E)** $\frac{14}{5}$

5. The final grade in a class is determined by the average of the grades of the six tests. If the grades on the first five tests were 70, 72, 74, 76, and 88, then what is the lowest grade the student can make on the last test if they want to have a final grade of at least 80?

   (A) 75 **(B)** 80 **(C)** 90 **(D)** 94 **(E)** 100

6. On a math test, all prodigies can solve 5 problems and all noobs can only solve 1 problem. Kevin TA sees that there are 111 total tests and 279 total problems solved. Given everyone is either a prodigy or a noob, how many prodigies took the test?

   (A) 40 **(B)** 41 **(C)** 42 **(D)** 43 **(E)** 44

7. Find the number of orderings of the word **ADIHAYA**.

   (A) 5040 **(B)** 2520 **(C)** 840 **(D)** 720 **(E)** 630

8. In the school of CNCM, the students take tests, known as PoTDs. 3 out of every 5 people passed the Geometry PoTD and 1 out of every 2 people passed the Combinatorics PoTD. What is the smallest possible percentage of people that passed both PoTDs?

   (A) 0 **(B)** 4 **(C)** 10 **(D)** 60 **(E)** 110

9. There is a marching band with n people. The band director first tries to line the members up in rows of 5. However, there are 2 people left over. When the band director tries to line them up in

rows of 7, there are also 2 people left over. If there are more than 25 people in the band, then find the smallest possible value of n.

(A) 2 **(B)** 33 **(C)** 35 **(D)** 36 **(E)** 37

10. Adihaya is finishing up his proof of the Riemann Hypothesis. He can write the whole proof alone in four hours. However, he has hired his assistant Peebeehat, who can write the whole proof on his own in six hours. Adihaya starts by working on the proof alone at 2 : 00 PM, and at some point, Peebeehat joins him. Together, they finish the proof at 4 : 40. At what time did Peebeehat join Adihaya?

(A) 2 : 00 **(B)** 2 : 10 **(C)** 2 : 20 **(D)** 2 : 30 **(E)** 2 : 40

11. In a right triangle with right at B, $AB = 7$ and $AC = 25$. D is the point on BC such that $AD = 13$. Find CD. Express your answer in simplest radical form, if applicable.

(A) $24 - 2\sqrt{30}$ **(B)** $24 - 5\sqrt{5}$ **(C)** 13 **(D)** $25 - 2\sqrt{30}$ **(E)** $25 - 5\sqrt{5}$

12. Which of the following values is equivalent to $\frac{5^{100}-5^{99}}{20}$?

(A) 5^{100} **(B)** 5^{99} **(C)** 5^{98} **(D)** 5^{97} **(E)** 5^{96}

13. Given exactly one of the following is false, which one is false?

(A) This option is true **(B)** All vowels are true **(C)** D is the only false option **(D)** All consonants are true **(E)** Either A or B is true

14. How many integers n such that $35 \leq n \leq 350$ are divisible by either 5 or 7?

(A) 90 **(B)** 91 **(C)** 92 **(D)** 98 **(E)** 100

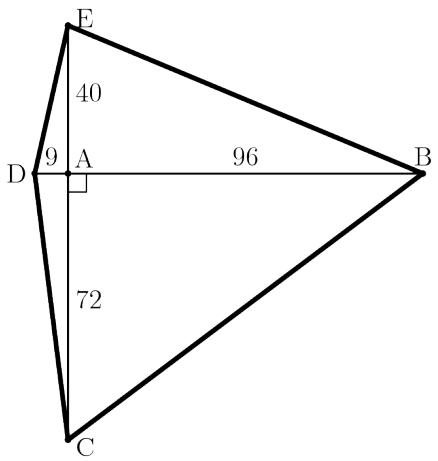
15. The sum of the number $3A25A$ and the number $364A1B$ is divisible by 11, where A and B are single digits. Find the number of ordered pairs (A, B).

(A) 3 **(B)** 8 **(C)** 9 **(D)** 13 **(E)** 15

16. You went to go get tested for the virus 91-DIVOC. There is a 20% chance that you have 91-DIVOC. The test for 91-DIVOC is 90% accurate. Given that you test positive, then what is the chance that you actually have 91-DIVOC? Round your answer to the nearest whole percentage, if applicable.

(A) 20% **(B)** 68% **(C)** 69% **(D)** 70% **(E)** 90%

17. In the following diagram, $AB = 96, AC = 72, AD = 9$, and $AE = 40$. All central angles are right angles. Find the area of the quadrilateral $BCDE$.



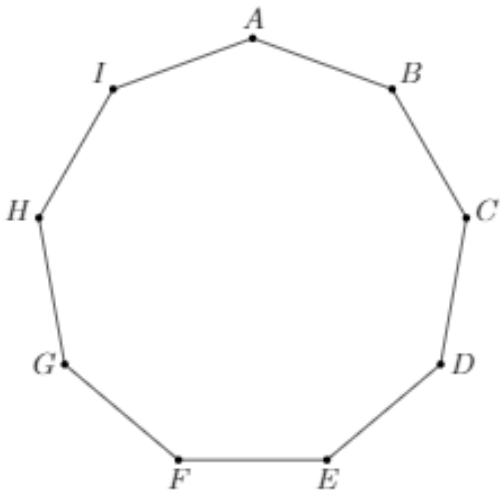
(A) 12760 **(B)** 11760 **(C)** 6380 **(D)** 5880 **(E)** 2940

18. The number 45 in base x is equal to the number 54 in base y, or $45_x = 54_y$. Find the smallest possible value of $x + y$.

    Note: For bases b other than 10, the rightmost digit has unit value 1, the second-rightmost digit has unit value b, the 3rd-rightmost digit has unit value b^2, and so on. For example, the number $\overline{a_n a_{n-1} \ldots a_2 a_1 a_0}_b$ has value $a_n \cdot b^n + a_{n-1} \cdot b^{n-1} + \cdots + a_2 \cdot b^2 + a_1 \cdot b + a_0$.

    (A) 2 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 20

19. In the following diagram, $ABCDEFGHI$ is a regular nonagon-shaped building with side length 60 feet. There is a goat tied to a rope at A. The rope is 120 feet long. If the goat cannot go into the building, what is the area the goat can reach, in simplest form?



    (A) 12000π **(B)** 9600π **(C)** 8000π **(D)** 4800π **(E)** 4000π

20. Find the value of

$$\frac{1}{2^2 - 1^2} \cdot \frac{3}{3^2 - 2^2} \cdot \frac{5}{4^2 - 3^2} \cdots \frac{4039}{2021^2 - 2020^2}$$

    (A) $\frac{1}{4043}$ **(B)** $\frac{1}{4041}$ **(C)** $\frac{1}{4039}$ **(D)** $\frac{1}{3}$ **(E)** 1

21. Bob is a undecidable voter. Every 3 days, he gets an ad to vote for Donald Trump. Every Wednesday, he gets an ad to vote for Joe Biden. Every 8 days, he gets an ad to vote for Bernie Sanders who was let into the final race by winning a rap battle against both Donald Trump and Joe Biden. In 2020, how many days does he not receive any ads if all 3 advertised on January 1st? Assume that the ads continue throughout the entire year (even after election day).

    (A) 181 **(B)** 182 **(C)** 183 **(D)** 184 **(E)** 185

22. Find the value of $x + y + z$ given that

$$x^2 + y^2 + z^2 - 14x - 8y + 4z + 69 = 0.$$

    (A) 6 **(B)** 9 **(C)** 11 **(D)** 12 **(E)** 42

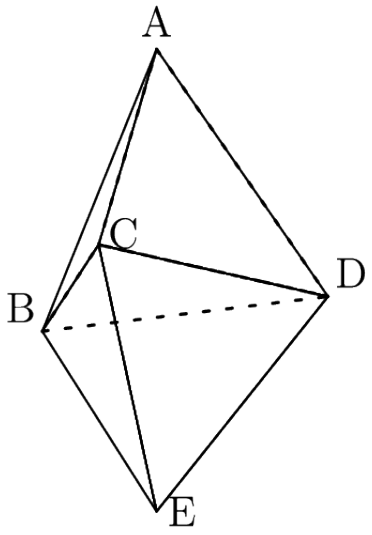
23. Brian likes drawing socks from a drawer, but the number of socks he draws must always be a multiple of 69. Among the socks that he has drawn, they are divided into 23 groups, each group containing $3k$ socks and with a unique color among the others. As k ranges from 1 to 420, for each case, Brian may pick n_k number of socks to guarantee that he picks a whole group. For example, if Brian picks 69 socks, then each group will have 3 socks with the same color, and we denote n_1 as the number of socks Brian has to pick to guarantee all 3 socks of the same color drawn. All the possible n_k are elements in set S, in ascending order. Find the 10th element.

(A) 598 **(B)** 599 **(C)** 600 **(D)** 668 **(E)** 669

24. Let set **S** contain the values of r_n, where $4^{2n} + 16^{2n} \equiv r_n \pmod{91}$. Furthermore, $0 < r_n < 91$. Find the sum of the 100 values of **S** that correspond to when n ranges from 1 to 100.

(A) 690 **(B)** 6069 **(C)** 6096 **(D)** 6690 **(E)** 6696

25. We define a ADITETRA-gon as two tetrahedrons glued together with one shared base on top of each other. Given that in a regular ADITETRA-gon $A - BCD - E$ where BCD is the shared triangle face, each of the side lengths has length of 4, the the distance from A to E may be expressed as $\frac{a\sqrt{b}}{c}$, where $\gcd(a, b, c) = 1$. Find abc.



(A) 60 **(B)** 72 **(C)** 96 **(D)** 144 **(E)** 198