1 Mock Combo AMC 12 III Problems - 75 Minutes

1. What is the value of

$$1+1+2+1+2+3+1+2+3+4+1+2+3+4+5+1+2+3+4+5+6?$$

(A) 35 **(B)** 50 **(C)** 56 **(D)** 60 **(E)** 70

2. How many lattice points on the (x, y) coordinate plane are exactly 5 units away from the origin?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 16

3. Let m be the least amount of money, in cents, that cannot be formed with 6 quarters, 2 dimes, 1 nickel, and 4 pennies. Find the sum of digits of m.

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

4. The base 6 number

$$\underline{1}\ \underline{5}\ \underline{a}\ \underline{4}\ \underline{b}\ _6,$$

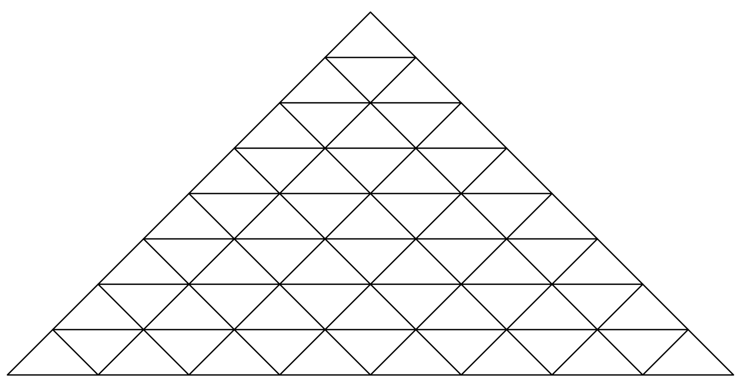
where a and b are not necessarily distinct digits, is divisible by 7. What is the sum of all possible distinct values of $a+b$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

5. Let S be the set of all 4-digit base 10 palindromes, and let $T = \{a_1, a_2, a_3, ...a_{10}\}$ be a 10-element subset of S. What is the maximum possible value of $\gcd(a_1, a_2, a_3, ..., a_{10})$?

(A) 55 **(B)** 66 **(C)** 88 **(D)** 99 **(E)** 132

6. How many triangles are in the figure below?



(A) 120 **(B)** 170 **(C)** 190 **(D)** 204 **(E)** 240

7. How many ways can one arrange 2 bishops on an 8×8 chessboard such that they are on the same diagonal?

(A) 84 **(B)** 140 **(C)** 168 **(D)** 280 **(E)** 336

8. The AMC 12 is a 25-question, 75-minute test where each correctly answered question is worth 6 points, each incorrectly answered question is worth 0 points, and each blank question is worth 1.5 points. If N is the least AMC 12 score greater than 90 in which the number of correctly answered questions can be uniquely determined, find the sum of digits of $\frac{N}{1.5}$.

(A) 8 **(B)** 11 **(C)** 12 **(D)** 14 **(E)** 17

9. 20 fans of the anime The Quintessential Quintuplets walk into a room, in which there are 5 main prominent characters: Ichika, Nino, Miku, Yotsuba, and Itsuki. There are a fans in the room that think Ichika is the best character, b fans in the room that think Nino is the best character, c fans in the room that think Miku is the best character, d fans in the room that think Yotsuba is the best character, and e fans in the room that think Itsuki is the best character. What is the greatest possible number of 10-element subsets of the the 20 people in the room such that for each of the five characters, the subset contains exactly 2 people whose favorite is that character?

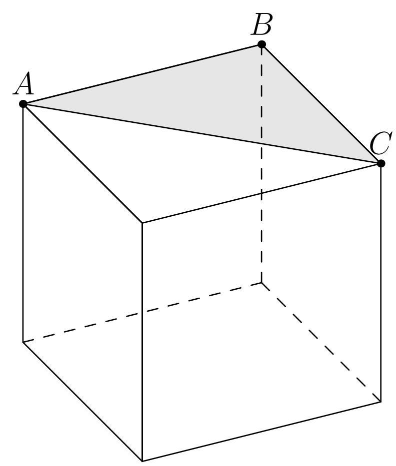
(A) 6480 **(B)** 7776 **(C)** 12960 **(D)** 100000 **(E)** 248832

10. What is the coefficient of x^{10} in the polynomial

$$(1+x+x^2+...+x^{10})(1+x)^{10}?$$

(A) 252 **(B)** 256 **(C)** 512 **(D)** 1013 **(E)** 1024

11. Three ants are initially on three vertices of the cube that form a triangle with area $\frac{1}{2}$. Every second, one of the ants (which is randomly determined) moves to a random adjacent vertex that is not occupied by an ant. What is the expected value of seconds until the three ants are on vertices that form a triangle with the largest possible area?



(A) 2 **(B)** $2\frac{1}{2}$ **(C)** 3 **(D)** $3\frac{1}{2}$ **(E)** 4

12. Three 20-sided dice (each with faces numbered 1 through 20, inclusive) are weighted such for each die, that the probability of rolling an n, where $1 \leq n \leq 20$, is proportional to n. Given this information, let the probability that rolling the three dice produces a sum less than or equal to 20 be N, which can be expressed in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. What is the largest prime factor of p?

(A) 7 **(B)** 11 **(C)** 17 **(D)** 19 **(E)** 23

13. Let n be a positive integer in base 10. Convert n to hexadecimal, and start with $k = 1$. For every digit of the hexadecimal form of n that is a letter, multiply k by 2. After all the digits the base 16 number are iterated through, call the final value of k the score of n. Let N be the sum of the scores of n over the range $1 \leq n \leq 2^{2022}$. Compute the remainder when the number of factors of N is divided by 100.

(A) 35 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 49

14. Let S be the set of all solutions to the equation $z^{167} = 1$ on the complex plane, and let z_0 be the solution that has the maximal possible real part r_0 among all of the solutions that have a positive complex part. Then, recursively define the sequence $\{r_n\}$ as

$$r_{n+1} = r_0(2r_n^2 - 1) - \sqrt{(1 - r_0^2)(1 - (2r_n^2 - 1)^2)}.$$

How many distinct values are contained in the set of values $r_0, r_1, r_2, r_3, ..., r_{2022}$?

(A) 9 **(B)** 16 **(C)** 17 **(D)** 83 **(E)** 166

15. The positive integer divisors of 441 are randomly arranged in a circle. What is the probability that the product of every pair of adjacent divisors is divisible by 21?

(A) $\frac{1}{21}$ **(B)** $\frac{1}{20}$ **(C)** $\frac{2}{21}$ **(D)** $\frac{41}{420}$ **(E)** $\frac{1}{10}$

16. Fred is going bowling, but he is a terrible bowler. The bowling lane is 50 meters long and 2.5 meters wide and is surrounded by 50 meter long gutters on both sides. For every meter forward Fred's bowling ball travels, given that it was previously in the center of the lane, there is a $\frac{1}{4}$ chance of it going 1 meter to the left, $\frac{1}{4}$ chance of it going to the 1 meter to the right, and $\frac{1}{2}$ chance of it remaining in the center. Given that the bowling ball was previously not in the center of the lane, for every meter forward it travels, there is a $\frac{3}{8}$ chance of it sliding into the gutter, a $\frac{1}{8}$ chance of it moving 1 meter back to the center, and a $\frac{1}{2}$ chance of it remaining in its horizontal place. If the ball goes into the gutter, it will permanently stay there. If the ball is not in the gutter by the end of the 50 meters, Fred will score some points by knocking over some pins. Given that Fred starts by launching his ball in the center of the lane, which of the following is closest to the probability that he will score a nonzero number of points?

(A) $\frac{1}{10^8}$ **(B)** $\frac{1}{10^7}$ **(C)** $\frac{1}{10^6}$ **(D)** $\frac{1}{10^5}$ **(E)** $\frac{1}{10^4}$

17. How many distinct monic quintic polynomials (with not necessarily real roots a_1, a_2, a_3, a_4, a_5) with nonnegative integer coefficients are there such that $-a_1a_2a_3a_4a_5$ is an integer power of 2 and

$$\frac{(2a_1 - 1)(2a_2 - 1)(2a_3 - 1)(2a_4 - 1)(2a_5 - 1)}{a_1a_2a_3a_4a_5} = 225?$$

(A) 2925 **(B)** 3654 **(C)** 4495 **(D)** 5525 **(E)** 8555

18. The number 1093 is prime. What is the sum of all integer values of $1 \le x \le 1093$ such that 7 is the least positive integer k such that $x^k - 1$ is divisible by 1093?

(A) 1092 **(B)** 1093 **(C)** 2184 **(D)** 2185 **(E)** 2186

19. Let $f_k(n) = \underbrace{2^{2^{2^{\cdot^{\cdot^{\cdot^{2^n}}}}}}}_{\text{k twos}}$. Let K be the least value of k such that the remainder when $f_k(n)$ is divided by 131 is a constant M for every positive integer n. What is $M + K$?

(A) 39 **(B)** 40 **(C)** 41 **(D)** 44 **(E)** 45

20. Let $a_0 = n$ and let a_{k+1} equal either $\left\lfloor \frac{a_k}{2} \right\rfloor$ or $\left\lfloor \frac{a_k}{2} \right\rfloor + 1$. Let $f(n)$ denote the number of distinct sequences $a_0, a_1, a_2, ..., a_{\lfloor \log_2(n) \rfloor}$ such that $a_{\lfloor \log_2(n) \rfloor} = 1$. For how many distinct integer values of $1 \le n \le 100000$ does $f(n) = 220$?

(A) 7 **(B)** 8 **(C)** 14 **(D)** 15 **(E)** 16

21. Let p be a prime, and let $E(p)$ denote the expected value of the least positive integer k such that $n^k - 1$ is divisible by p for a randomly chosen integer in the range $1 \le n \le p - 1$. Which of the following primes has the highest value of $\frac{E(p)}{p}$?

(A) 71 **(B)** 73 **(C)** 83 **(D)** 103 **(E)** 107

22. Let n be a randomly selected positive integer in the range $130 \le n < 2^{2022}+130$. What is the probability that $\binom{n}{130}$ is divisible by 2^{128}?

(A) $\frac{1}{2^{127}}$ **(B)** $\frac{5}{2^{129}}$ **(C)** $\frac{1}{2^{126}}$ **(D)** $\frac{5}{2^{128}}$ **(E)** $\frac{1}{2^{125}}$

23. How many 4-element subsets of the 16-element set $\{1, 3, 5, 7, ..., 31\}$ are there such that the product of its elements is one more than a multiple of 32?

(A) 116 **(B)** 117 **(C)** 118 **(D)** 119 **(E)** 120

24. What is the least value of n such that

$$\sum_{k=0}^{n} k^3 \binom{n}{k} = 1^3 \cdot \binom{n}{1} + 2^3 \cdot \binom{n}{2} + 3^3 \cdot \binom{n}{3} + \ldots + n^3 \cdot \binom{n}{n}$$

is divisible by 2021?

(A) 470 **(B)** 473 **(C)** 1548 **(D)** 1551 **(E)** 2018

25. Call a quintuplet of integers (i, n, m, y, t) quintessential if:

- $1 \le i < n < m < y < t \le 40$,
- There exists two distinct integers p and q in the quintuplet such that 41 divides $p^2 + q^2$
- There do not exist two distinct integers p and q in the quintuplet such that 41 divides $p^2 - q^2$

For instance, the quintuplet $(i, n, m, y, t) = (1, 2, 3, 4, 5)$ is quintessential since $4^2 + 5^2 = 41$. How many quintessential quintuplets are there?

(A) 7440 **(B)** 215040 **(C)** 238080 **(D)** 261120 **(E)** 264000

2 Feedback(Optional)

26. How would you judge the quality?

27. What is is the difficulty of this test from 1 to 10, with 1 being AMC 8 and 10 being AIME?

28. Which of your problems is your favorite?