CMC Mock AMC 8

November 4, 2020

Instructions

1. DO NOT READ THE PROBLEMS UNTIL THE TIMER HAS BEGUN.
2. This is an **individual contest** only.
3. This is a 25 question multiple choice test. The answer to each question will be one of the following: A, B, C, D or E. The problems are generally in increasing order of difficulty, although this is not guaranteed.
4. No **aids** are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smart-watches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. The test is **40 minutes** long.
6. SCORING: You will receive 1 point for a correct answer, and 0 points for an incorrect answer.
7. As a result, it is to your advantage to answer all of the problems.
8. Have fun!

1. Chris is starting a word company. He sells letters based on their position in the alphabet, i.e. A costs \$1, B costs \$2, etc. How much does the word "DAB" cost?

   (A) \$4 **(B)** \$5 **(C)** \$6 **(D)** \$7 **(E)** \$8

2. Find the sum $1^{-5}+1^{-4}+1^{-3}+1^{-2}+1^{-1}+1^{0}+1^{1}+1^{2}+1^{3}+1^{4}+1^{5}$.

   (A) 0 **(B)** 1 **(C)** 5 **(D)** 10 **(E)** 11

3. What is the value of $(4+14+24+34+44+54)+(56+46+36+26+16+6)$?

   (A) 320 **(B)** 360 **(C)** 420 **(D)** 450 **(E)** 480

4. Chris has X friends. He gives $1/2$ of them to Aditya. He then gives $1/3$ of his remaining friends to Tiger. What is the minimum possible value of X?
   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. Henry was thinking of a two-digit number but forgot what it was. He remembered that it was a multiple of 5, and the sum of its digits was twice the second digit. What is the product of its digits?

   (A) 0 **(B)** 5 **(C)** 10 **(D)** 25 **(E)** 50

6. Find the sum of the digits of the product of the numbers shown:

   $$(-8)\cdot(-7)\cdot(-6)\cdots 12\cdot 13\cdot 14$$

   (A) 0 **(B)** 9 **(C)** 18 **(D)** 27 **(E)** 36

7. The Cupertino Math Pentagon (our military base) has 20 windows. How many ways are there for an intruder to enter through one window and leave through another?

   (A) 39 **(B)** 95 **(C)** 190 **(D)** 285 **(E)** 380

8. Flora has two spinners, one with the numbers 1-3 and one with the numbers 1-4, and all possible values for each spinner are equally likely. She spins both spinners simultaneously. What is the probability that the sum of the numbers that the spinners land on is even?

   (A) $\frac{1}{3}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{2}{3}$

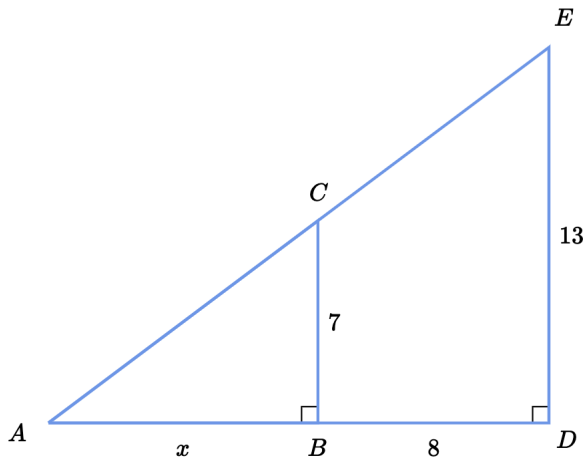
9. David's store sells mops for \$10.00 each. He has a discount for CMC members for 20% off, and a discount for students for \$1 off. After applying both these discounts in the optimal order, how much would a mop cost for you?

   (A) \$4.80 **(B)** \$6.80 **(C)** \$7.00 **(D)** \$7.20 **(E)** \$8.00

10. Jay has 3 quarters and 19 pennies. How many different values in cents could be obtained from combinations of at least one or more of his coins?

    (A) 3 **(B)** 19 **(C)** 22 **(D)** 79 **(E)** 80

11. Solve for x in this triangle:



    (A) $\frac{56}{13}$ **(B)** $\frac{28}{13}$ **(C)** $\frac{21}{4}$ **(D)** $\frac{28}{3}$ **(E)** 10

12. Find $p+q$ if

$$\frac{1}{3}\cdot\frac{2}{4}\cdots\frac{97}{99}\cdot\frac{98}{100}=\frac{p}{q}$$

and $\frac{p}{q}$ is in simplest form.

(A) 2450 **(B)** 4950 **(C)** 4951 **(D)** 9900 **(E)** 9902

13. How many times from 1:00 PM to 5:00 PM is the time on a digital clock a palindrome? (The digits are the same when read from left-to-right or right-to-left; for example, 1:01 is a palindrome but 1:02 is not).

(A) 6 **(B)** 9 **(C)** 18 **(D)** 24 **(E)** 36

14. How many possible values of b are there if a, b, c, d, e satisfy

$$\begin{aligned}3a+b+c+d+e&=54\\a+3b+c+d+e&=42\\a+b+3c+d+e&=12\\a+b+c+3d+e&=45\\e+b+c+d+3a&=67\end{aligned}$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

15. David has a 3x3 Rubik's cube. It has a surface area of 54. Now, he removes its 8 corners. What is the surface area of the resulting figure he ends up with?

(A) 38 **(B)** 46 **(C)** 48 **(D)** 54 **(E)** 60

16. Triangle ABC is similar to triangle BCD. If AB/BC=2, what is CD/BC?

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 144

17. A special coin has one-third chance of landing on heads. If Chris tosses the coin three times, what is the probability he gets head exactly once?
(A) $\frac{1}{27}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{4}{27}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{4}{9}$

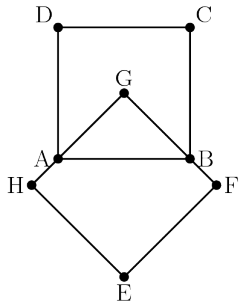
18. A rectangle has area 60 and perimeter 34. If each of its sides are increased by 6 units, what will be the resulting area?

(A) 84 **(B)** 96 **(C)** 108 **(D)** 144 **(E)** 198

19. Aditya, Chris, and Tiger are discussing the results of their last math contest. Tiger shows Aditya and Chris his test, but Aditya and Chris don't show theirs to anyone. Chris says, 'I didn't get the lowest score between us three,' and Aditya adds, 'I didn't get the highest score.' What is the ranking of the three students from the highest score to the lowest score?

(A) Tiger, Chris, Aditya **(B)** Tiger, Aditya, Chris
(C) Chris, Aditya, Tiger **(D)** Chris, Tiger, Aditya
(E) Aditya, Tiger, Chris

20. Squares $ABCD$ and $EFGH$ both have area 100 and G is the center of square $ABCD$. What is the area of the region inside square $EFGH$ but outside square $ABCD$?



(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

21. At Cupertino Math School, there are four math courses: Algebra, Geometry, Number Theory, and Counting. There are also three science courses: Biology, Physics, and Chemistry. How many possible schedules can he receive? A schedule consists of two math courses and one science course, arranged in three consecutive time slots.

(A) 24 **(B)** 36 **(C)** 48 **(D)** 72 **(E)** 108

22. Let $ABCD$ be a rectangle with $AB = 8$ and $BC = 6$. Let E be a point on AB such that $AE = 2$, and let the intersection of AC and DE be P. Find the area of $\triangle CPD$.

(A) $\frac{36}{5}$ **(B)** 8 **(C)** $\frac{48}{5}$ **(D)** 12 **(E)** $\frac{96}{5}$

23. Jeffrey, James and Jonah, who are not friends with each other, bring their three friends to watch a movie together. When they arrive to the movie theater, they see that there are only a row of 6 seats remaining. How many ways can the 6 of them seat themselves if Jeffrey, James, and Jonah do not want to sit next to each other?

(A) 12 **(B)** 24 **(C)** 36 **(D)** 72 **(E)** 144

24. For a decagon (10 sides), how many pairs of diagonals intersect inside the polygon?

(A) 210 **(B)** 330 **(C)** 980 **(D)** 990 **(E)** 5040

25. Find the units digit of $7^{7^{7^{\cdots 7}}}$, where there are 77 sevens total (including the bottom seven).

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9