OAA OMC
Orange Math Competitions

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OMC 10

Orange Mathematics Competitions
Saturday, October 29, 2022

INSTRUCTIONS

1. DO NOT LOOK AT THE PROBLEMS UNTIL YOU ARE READY TO BEGIN.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem however you want. If you would like to create a more realistic test experience, then you may obtain an AMC 10 Answer Sheet from `https://www.maa.org/math-competitions/amc-1012/` and mark you answer to each problem on the AMC 10 Answer Sheet with a number 2 pencil. To simulate the real test, check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded in a real test. For the OMC, **you must submit your answers using the Submission Form. Only answers submitted to the Submission Form will be scored.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only pencils, erasers, rulers, and scratch paper are allowed as aids. No calculators, smartwatches, phones, computing devices, or resources such as Wolfram Alpha are allowed. No problems on the exam require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, you will ask yourself to record certain information on the answer form if you chose to obtain an AMC 10/12 Answer Sheet from `https://www.maa.org/math-competitions/amc-1012/`. You will have **75 MINUTES** to complete the test.
8. When you finish the exam, sign your name in the space provided at the top of the Answer Sheet should you choose to obtain one from `https://www.maa.org/math-competitions/amc-1012/`.
9. Enjoy the problems!

The Committee on the Orange Math Competitions reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

1. What is the value of $\sqrt{9}+\sqrt{16}+\sqrt{144}-\sqrt{9+16+144}$?

   (A) 0 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 7

2. Ashley drives from San Jose to San Diego to visit some relatives. She takes off at 9 : 48 AM and arrives at her destination at 6 : 15 PM. Given that San Jose and San Diego were in the same time zone, how long was Ashley's drive, in minutes?

   (A) 327 **(B)** 333 **(C)** 507 **(D)** 513 **(E)** 693

3. On a 20 question multiple choice exam, Sheldon gets exactly n questions correct. Each question on the exam was worth the same number of points. He notices that his score is exactly $n^2\%$. Given that Sheldon got at least one question right, what is the value of n?

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 20

4. A circular table has 1001 seats some of which are occupied. If another person were to take a seat, he or she would have to sit next to another person that is already seated. What is the smallest possible number of seats that can be already occupied?

   (A) 201 **(B)** 333 **(C)** 334 **(D)** 500 **(E)** 501

5. Three children, Ari, Rizzo, and Daeho, and their fathers are sitting in a row to watch a Minions movie. For safety concerns, both ends of the row should be occupied by adults. How many ways are there for the six individuals to be arranged?

   (A) 4 **(B)** 48 **(C)** 72 **(D)** 144 **(E)** 288

6. Suppose x and y are positive integers satisfying $x^y = 2^{120}$. What is the smallest possible value of $x+y$?

   (A) 44 **(B)** 46 **(C)** 48 **(D)** 56 **(E)** 64

7. Let $ABCD$ be a rectangle in the Cartesian plane such that the midpoint of AB and the midpoint of BC are $(-22,19)$ and $(14,67)$ respectively. What is the length of BD?

   (A) 48 **(B)** 60 **(C)** 84 **(D)** 96 **(E)** 120

8. Call a positive integer *rhit* if all its digits are nonzero and it is divisible by each of its digits. How many two-digit positive integers are *rhit*?

   (A) 9 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 18

9. A thirsty crow finds a large cylindrical container with radius 4 centimeters and height 20 centimeters that is filled halfway with water. If the crow drops three perfectly spherical rocks with radius 2 centimeters into the container, how many centimeters does the water level rise?

   (A) $\frac{2}{3}$ **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

10. Let a,b,c be three nonzero integers satisfying $7a+11b+13c=0$. What is the least possible value of $|a|+|b|+|c|$?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

11. Let $ABCD$ be a parallelogram with $AB = 1$ and $\angle ABC = 120$. The angle bisectors of $\angle ABC$ and $\angle CDA$ trisect the quadrilateral into three regions of equal area. If $AB \geq BC$, what is the value of BC?

    (A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{\sqrt{3}}{2}$ **(E)** 1

12. The three real roots of the polynomial $x^3 - 3x^2 + 2x - k$ are in geometric progression for some real number k. What is the value of k?

(A) $\frac{1}{8}$ **(B)** $\frac{8}{27}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{27}{8}$

13. For everyday in 2022, Ethan has a 50% chance of playing video games. For any day Ethan plays video games, there is a 20% chance his parents catch him, in which case Ethan does not play video games for the rest of the year. The probability Ethan is playing video games on the 100^{th} day in 2022 can be expressed as $\frac{m}{n}$ for positive relatively prime integers m and n. What is the remainder when $m+n$ is divided by 100?

(A) 0 **(B)** 1 **(C)** 11 **(D)** 89 **(E)** 99

14. How many positive integer ordered quintuplets (a, b, c, d, e) with $a, b, c, d, e \leq 10$ satisfy

$$3^a + 3^b + 3^c + 3^d + 3^e = 3^f$$

for some positive integer f?

(A) 0 **(B)** 9 **(C)** 90 **(D)** 100 **(E)** 180

15. James has a total of $20! = 20 \cdot 19 \cdots 1$ marbles. Let N be the number of ways he selects 20 of them at random, where the order of the marbles selected is considered indistinguishable. What is the highest power of 20 that divides N?

(A) 20^3 **(B)** 20^4 **(C)** 20^5 **(D)** 20^6 **(E)** 20^8

16. What is the number of positive integer values of $n \leq 2025$ such that

$$x + \lfloor \sqrt{x} \rfloor + \lfloor \sqrt[3]{x} \rfloor = n.$$

has no real solution for x?

(A) 54 **(B)** 55 **(C)** 56 **(D)** 57 **(E)** 58

17. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. A point P is selected in the same plane such that triangles ABP and PCA are congruent. What is the sum of all possible values of the area of triangle ABP?

(A) 24 **(B)** 42 **(C)** 84 **(D)** 102 **(E)** 108

18. April has five identical fair coins. Every minute, she flips all coins and keeps only the ones that flip heads while discarding the ones that flipped tails. If April continues this process until she has no coins left, what is the probability that at some point in time, she had exactly three coins?

(A) $\frac{45}{128}$ **(B)** $\frac{34}{93}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{15}{31}$

19. Ada is bored and decides to expand and simplify

$$f(x, y, z) = (x + y^2 + z^3)^K - (x^3 + y^2 + z)^K$$

for some positive integer K. If the number of monomials in Ada's expansion of $f(x, y, z)$ is greater than or equal to 300, what is the smallest possible value of K?
(Note: A monomial is a single nonzero term.)

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

20. For how many real numbers $0 \leq x \leq 10$ is $x^2 + \{x\}$ an integer?
(Note: $\{x\}$ is the fractional part of x. $\{x\} = x - \lfloor x \rfloor$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.)

(A) 90 **(B)** 91 **(C)** 101 **(D)** 110 **(E)** 111

21. Let $ABCD$ be a convex quadrilateral with $AB = BC = 1$ and $CD = 2$. If the perpendicular bisectors of AB, BC, and CD all intersect on the midpoint of DA, what is the length of side DA?

    (A) $\sqrt{2}+1$ **(B)** $\sqrt{6}$ **(C)** $2\sqrt{3}-1$ **(D)** $\frac{5}{2}$ **(E)** $\sqrt{3}+1$

22. How many positive four-digit integers $\overline{abcd}$ with $a \neq 0$ satisfy

$$a^{b+c+d} = (a+b)^{c+d} = (a+b+c)^d?$$

    (For example, 2022 works as $2^{0+2+2} = (2+0)^{2+2} = (2+0+2)^2$.)

    (A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

23. For a positive integer n, let $1 = d_1 < d_2 < \cdots < d_k = n$ be all of its positive divisors. How many positive integers $2 \leq n \leq 100$ satisfy the property that $\frac{d_{i+1}-d_i}{d_2-d_1}$ is an integer for all $1 \leq i \leq k-1$?

    (A) 35 **(B)** 67 **(C)** 91 **(D)** 92 **(E)** 95

24. Finn the hunter and a rabbit are at the points $(0,0)$ and $(2002, 2000)$ respectively on the coordinate plane. Every minute, the rabbit randomly chooses to move one unit up, down, right, or left. Immediately after the rabbit moves, Finn moves one unit up, down, right, or left as to minimize the distance between the two. In the case where Finn has more than one path that would minimize the distance, he randomly selects one of them. The expected number of minutes until the rabbit and Finn are on the same point on the plane can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. What is the remainder when $m+n$ is divided by 1000?

    (A) 11 **(B)** 47 **(C)** 191 **(D)** 675 **(E)** 731

25. A bouncy ball with negligible size is shot from the top left hand corner of a unit square. The angle the ball is shot at with respect to the top of the unit square is θ. Whenever the ball hits a side of the square it rebounds so that the angle that it makes with the side it hit stays the same, but it does not go along the same path again. If the ball rebounds off the sides 2022 times before reaching a corner of the square, what is the number of different possible values for θ?

    (A) 672 **(B)** 880 **(C)** 1080 **(D)** 1632 **(E)** 1932

2022

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DO NOT OPEN UNTIL SATURDAY, OCTOBER 29, 2022

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Correspondence about the problems and solutions
for this exam should be sent by email to:

ocmathcircle@gmail.com.

****Administration On An Earlier Date Will Literally Be Impossible****

1. All the information needed to administer this exam is contained in the non-existent OMC 10 Teacher's Manual. PLEASE READ THE MANUAL EVERY DAY BEFORE October 29, 2022.

2. YOU must not verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on maa.org/amc under "AMC 10/12") that you followed all rules associated with the administration of the exam.

3. If you chose to obtain an AMC 10/12 Answer Sheet from the MAA's website, it must be returned to yourself the day after the competition. Ship with inappropriate postage without using a tracking method. FedEx or UPS is strongly recommended.

4. The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, World Wide Web, or digital media of any type during this period is a violation of the competition rules.

The **Orange Math Competitions**
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