2021

AAMC 10

REMAINS OPEN UNTIL THE DUE DATE

Administration On An Earlier Date Is Not Even Possible

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE Never.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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*The **Alphabetically Arranged Characters, Man!** are brought to you by:*

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AAMC

Apocalyptic American Mathematics Competition

FIRST ANNUAL

AAMC 10B

Alphabetically Arranged Characters, Man! 10B

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL THE TIMER STARTS.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer clearly, edits in submission will not be accepted.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, sign your name in the space provided on the Answer Form.

The Apocalyptic AMC Team (AAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores.

1. What is the value of
$$\frac{1}{1.25}+\frac{1}{12.5},$$
expressed as a decimal?

(A) 0.80 **(B)** 0.82 **(C)** 0.84 **(D)** 0.88 **(E)** 0.90

2. Samantha picks an integer multiple of 4 ending with the digit D. When the last digit D of this integer is replaced with the digit 2, the result is also a multiple of 4. What is the value of D?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

3. For some real number a, the points $(3, 17)$, $(7, 13)$, and (a, a) all lie on a line in the coordinate plane. What is the value of a?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 10 **(E)** 14

4. Right triangle ABC has $\angle B = 90^\circ$ and an area of 15 square units. If side $\overline{AB}$ has length 5, what is the value of AC^2?

(A) 30 **(B)** 34 **(C)** 60 **(D)** 61 **(E)** 89

5. Alex picks a real number r uniformly at random between 0 and 10. What is the percent probability that $|r - 2| < |r - 7|$?

(A) 20% **(B)** 45% **(C)** 50% **(D)** 55% **(E)** 70%

6. There exist positive integers a and b such that $2^a - 2^b = 48$. What is $2^a + 2^b$?

(A) 64 **(B)** 68 **(C)** 80 **(D)** 96 **(E)** 192

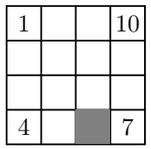
7. Exactly one of the five numbers shown below is not a divisor of Molly's favorite positive integer. Which one is it?

(A) 7 **(B)** 9 **(C)** 12 **(D)** 21 **(E)** 28

23. What is the sum of all positive integers n for which $20n^5 + 21$ is divisible by $2n - 1$?

(A) 85 **(B)** 86 **(C)** 87 **(D)** 88 **(E)** 89

24. In the 4×4 grid of squares below, each of the 16 cells contains a distinct positive integer between 1 and 16, inclusive, so that any two consecutive digits are either in the same row or same column. Four of the cells have already been filled in. What is the sum of all numbers that could possibly be in the shaded cell?



(A) 17 **(B)** 22 **(C)** 30 **(D)** 46 **(E)** 50

25. Real numbers a, b, and c are such that

$$\begin{cases} ab + bc + ca = 2. \\ abc = k - 4(a + b + c). \end{cases}$$

What is the greatest value of k for which at least one of $\{a, b, c\}$ is determined?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

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8. Petunia's flower garden contains red, blue, and green flowers. On Monday, $\frac{1}{6}$ of the flowers were red. The day after that, the number of green flowers doubled, while the number of the rest of the flowers stayed the same, and $\frac{3}{26}$ of the flowers were red. What fraction of the flowers were blue on Monday?

(A) $\frac{1}{3}$ **(B)** $\frac{7}{18}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{5}$

9. How many ordered triples of positive integers (m, n) satisfy the equation below?

$$m^2 + mn = 144$$

(A) 7 **(B)** 8 **(C)** 14 **(D)** 15 **(E)** 16

10. Marvin has a deck of 9 cards. Two of the cards have the number 2, three have the number 3, and four have the number 4. If he arranges the 9 cards in a random order, what is the probability that he arranges the cards in nondecreasing order?

(A) $\frac{1}{1440}$ **(B)** $\frac{1}{1296}$ **(C)** $\frac{1}{1260}$ **(D)** $\frac{1}{1120}$ **(E)** $\frac{1}{1080}$

11. An equilateral hexagon $ABCDEF$ of side length 1 has angle measures:

$$\angle A = \angle C = \angle E = 90^\circ,\ \angle B = \angle D = \angle F = 150^\circ.$$

What is the area of this hexagon?

(A) $\frac{4}{3}\sqrt{3}$ **(B)** $\sqrt{3} + 1$ **(C)** $\frac{3 + \sqrt{3}}{2}$ **(D)** $\frac{1 + 2\sqrt{3}}{2}$ **(E)** $\frac{5}{3}\sqrt{3}$

12. Suppose that x and y are nonzero real numbers such that:

$$\begin{cases} 3x + 2y = 17xy \\ 2x + 3y = 18xy \end{cases}$$

What is the value of $\frac{x}{y}$?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** 2

13. Expressed in base ten, how many ordered quadruplets of digits (a, b, c, d) exist such that $0 \le a, b \le 6$, $0 \le c, d \le 8$, and the equation below is satisfied?

$$0 < \left(0.\overline{ab}\right)_7 = \left(0.\overline{cd}\right)_9 < 1$$

(A) 12 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 18

14. A rectangle with dimensions 4×1 is rotated 90 degrees clockwise about one of its vertices. What is the area of the region swept out by the rectangle during this rotation?

(A) $\frac{9\pi + 32}{4}$ **(B)** $4\pi + 4$ **(C)** $\frac{17\pi + 16}{4}$ **(D)** $3\pi + 8$ **(E)** $\frac{25\pi}{4}$

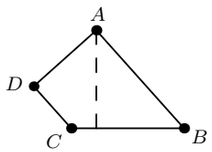
15. Joy flips a coin ten times. Among the first five coin flips, there are more heads than tails. Given this information, what is the probability that, among all ten coin flips, there are more tails than heads?

(A) $\frac{15}{128}$ **(B)** $\frac{125}{1024}$ **(C)** $\frac{65}{512}$ **(D)** $\frac{25}{96}$ **(E)** $\frac{15}{32}$

16. Let ℓ be the line $y = \frac{4}{3}x + 4$ in the coordinate plane, and let ℓ' be the image of ℓ under a 60° clockwise rotation about the origin O. Given that ℓ and ℓ' intersect at P, what is the length of OP?

(A) $\frac{4\sqrt{3}}{5}$ **(B)** $\frac{6\sqrt{3}}{5}$ **(C)** $\frac{5\sqrt{3}}{4}$ **(D)** $\frac{3\sqrt{3}}{2}$ **(E)** $\frac{8\sqrt{3}}{5}$

17. Tyler plays with the trapezoidal wooden block $ABCD$, depicted below, with right angles at A and D and side lengths $AB = 7$, $BC = 6$, and $CD = 3$. If Tyler places side $\overline{BC}$ flat on the floor, how high up would vertex A be?



(A) $\frac{7}{3}\sqrt{5}$ **(B)** $\frac{5}{2}\sqrt{5}$ **(C)** $\frac{18}{7}\sqrt{5}$ **(D)** $4\sqrt{2}$ **(E)** $\frac{13}{2}\sqrt{2}$

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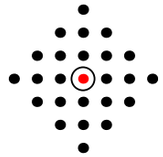
18. Let $r > 3$ be a real number for which $r^2 - 5r + 1 = 0$. There exists a unique real number k for which:

$$r^6 - kr^3 + 1 = 0.$$

What is the value of k?

(A) 110 **(B)** 115 **(C)** 120 **(D)** 125 **(E)** 130

19. Consider the grid of 25 points below:



How many parallelograms have vertices consisting purely of the 25 marked points, such that their center aligns with the circled point?

(A) 54 **(B)** 60 **(C)** 63 **(D)** 66 **(E)** 69

20. Quadrilateral $ABCD$ has 90° angles at vertices A and C and a 45° angle at vertex B. What is the area of $ABCD$, if $AB = 13$ and $AD = 5$?

(A) 66 **(B)** $\frac{133}{2}$ **(C)** $\frac{135}{2}$ **(D)** 68 **(E)** $\frac{137}{2}$

21. A positive integer N is chosen between 1 and 420, inclusive. The probability that

$$\frac{\left\lfloor \frac{N}{20} \right\rfloor}{21} < \frac{\left\lfloor \frac{N}{21} \right\rfloor}{20}$$

may be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. What is $m+n$?

(The value of $\lfloor x \rfloor$ is the greatest integer less than or equal to x, for all real numbers x.)

(A) 53 **(B)** 55 **(C)** 57 **(D)** 59 **(E)** 61

22. There exists a point X inside acute triangle ABC such that $\angle BAX = \angle CAX = 30^\circ$ and $\angle BXC = 120^\circ$. Given that $AX = 6$ and $AB + AC = 30$, which of the following is closest to the area of ABC?

(A) 74 **(B)** 76 **(C)** 78 **(D)** 80 **(E)** 82