1. What is (2018)(2022) − (2017)(2023)?

   (A) 1 **(B)** 5 **(C)** 9 **(D)** 2020 **(E)** 4039

   Answer: $\boxed{\textbf{(B)}}$

   Solution: We use difference of squares, which states that $(a-b)(a+b) = a^2 - b^2$, so $(2018)(2022) - (2017)(2023) = (2020-2)(2020+2) - (2020-3)(2020+3) = (2020^2 - 2^2) - (2020^2 - 3^2) = 3^2 - 2^2 = 5 \implies \boxed{\textbf{(B)}}$

2. $\triangle ABC$ has side lengths $AB = 28$, $BC = 45$, and $AC = 53$. What is the area of $\triangle ABC$?

   (A) $250\sqrt{6}$ **(B)** $360\sqrt{3}$ **(C)** $280\sqrt{5}$ **(D)** 630 **(E)** $450\sqrt{2}$

   Answer: $\boxed{\textbf{(D)}}$

   Solution: Notice that $28^2 + 45^2 = 53^2$. By the Pythagorean theorem, that means that $\triangle ABC$ is a right triangle. Therefore, the area of $\triangle ABC$ is $\frac{(28)(45)}{2} = 14 \times 45 = 630 \implies \boxed{\textbf{(D)}}$

3. Let N be the sum of the real roots of $5x^2 - 6x + 3 = 0$. What is N^2?

   (A) $-\frac{6}{5}$ **(B)** 0 **(C)** $\frac{6}{5}$ **(D)** $\frac{36}{25}$ **(E)** 36

   Answer: $\boxed{\textbf{(B)}}$

   Solution: By the quadratic formula, which states that the roots to the quadratic $ax^2 + bx + c = 0$ are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get that the roots to $5x^2 - 6x + 3 = 0$ are $\frac{6\pm\sqrt{6^2-4\cdot3\cdot5}}{10}$, which are not real. Therefore, the sum of the real roots is $0 \implies \boxed{\textbf{(B)}}$

4. How many ways are there to order the characters in the string 'aMAMC8' such that the '8' must be at the end of the string? (The 'a' and 'A' are distinguishable but the two M's are not.)

   (A) 12 **(B)** 24 **(C)** 48 **(D)** 60 **(E)** 120

   Answer: $\boxed{\textbf{(D)}}$

   Solution: This is the same as asking how many ways are there to order just 'aMAMC', since the '8' has to be at the end. To calculate this, we first consider when the two M's are distinguishable. Then the string is 'aM_1AM_2C'. There are just $5! = 120$ ways to order the characters. However, this overcounts because the M's are not distinguishable, so switching them shouldn't count as a different order. There are $2! = 2$ ways to switch the M's, so we must divide our original count by that, so the answer is $\frac{120}{2} = 60 \implies \boxed{\textbf{(D)}}$

5. Given that $\#x = \sqrt{x}$ and $x \heartsuit y = x^2 - 2xy + y^2$ for all real x and y, find

$$\#((4 \heartsuit [\#4]) \heartsuit (\#([\#4] \heartsuit 4))).$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Answer: $\boxed{\textbf{(C)}}$

Solution: First, we calculate the stuff inside the brackets. $\#4 = 2$, so we can simplify the expression to $\#((4 \heartsuit 2) \heartsuit (\#(2 \heartsuit 4)))$. Now notice that $x \heartsuit y = x^2 - 2xy + y^2 = (x-y)^2$. This means that $4 \heartsuit 2 = 2 \heartsuit 4 = (4-2)^2 = 4$. Now we can simplify the expression to $\#(4 \heartsuit (\#4))$. Now since $\#4 = 2$, we can simplify it to $\#(4 \heartsuit 2)$, which is just $\#4$, or $2 \implies \boxed{\textbf{(C)}}$

6. How many palindromes with 4 or less digits have 1 as their last digit?

Note: A *palindrome* is a number that can be read the same forwards and backwards. For example 1467641 and 1111111 are palindromes but 1467621 is not.

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

Answer: $\boxed{\textbf{(A)}}$

Solution: We proceed with casework based on the number of digits in the palindrome. If there is one digit, it must be 1, so there is only one option. If there are two digits, since the last digit is 1, the first digit must also be 1, so there is only one option, which is 11. If there are three digits, then it must be written in the form $1a1$, where a is an arbitrary digit. This gives 10 options. If there are 4 digits, then it must be written in the form $1aa1$, which also has 10 options. Thus, the answer is $1 + 1 + 10 + 10 = 22 \implies \boxed{\textbf{(A)}}$

7. Jack likes eating apples. He can eat 2 apples in 5 minutes. However, sometimes he gets tired and can only eat oranges, at a rate of 2 oranges per 4 minutes. Suppose he ate 36 apples in 100 minutes. How many oranges did he eat?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Answer: $\boxed{\textbf{(B)}}$

Solution: Since Jack ate 36 apples, he spent $\frac{36 \cdot 5}{2} = 90$ minutes eating apples. This is because he eats 2 apples in 5 minutes. Since he spends 100 minutes in total, he spends only 10 minutes eating oranges. Thus, the answer is $\frac{10 \cdot 2}{4} = 5 \implies \boxed{\textbf{(B)}}$

8. How many tuples of primes (a, b, c) exist such that the triangle with side lengths a, b, and c (if it exists) is a right triangle?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

Answer: $\boxed{\textbf{(A)}}$

Solution: There are 4 cases for the parity of (a, b), (even, even), (even, odd), (odd, even), and (odd, odd). If it is (even, even) or (odd, odd), then c is even by Pythagorean theorem. However, if c is even, then it must be 2 since it is prime, but 2 is too small (because of the triangle inequality)! Thus (a, b) must be (even, odd) or (odd, even), so one of a, b is 2. But this is not possible, because it doesn't satisfy the triangle inequality (which states that $a + b > c$), so there are 0 solutions $\Longrightarrow \boxed{\textbf{(A)}}$

9. When you take 20% of 120% of x, you get 50% of 150% of y. Given that $\frac{x}{y}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m + n$?

   (A) 8 **(B)** 9 **(C)** 17 **(D)** 33 **(E)** 37

   Answer: $\boxed{\textbf{(D)}}$

   Solution: Let's convert this to fractions. $\left(\frac{1}{5}\right)\left(\frac{6}{5}\right)x = \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)y$, or $\frac{6x}{25} = \frac{3y}{4}$. Cross-multiplying gives $24x = 75y$, $\frac{x}{y} = \frac{75}{24} = \frac{25}{8}, 25 + 8 = 33 \Longrightarrow \boxed{\textbf{(D)}}$

10. Find the sum of the distinct prime factors of 555555.

    (A) 39 **(B)** 63 **(C)** 71 **(D)** 76 **(E)** 84

    Answer: $\boxed{\textbf{(D)}}$

    Solution: Clearly, $555555 = 5 \times 111111$. The key thing to notice is that $111111 = 111000 + 111 = (111)(1000 + 1) = (111)(1001) = (3 \times 37)(7 \times 11 \times 13)$, so the answer is $5 + 3 + 37 + 7 + 11 + 13 = 76 \Longrightarrow \boxed{\textbf{(D)}}$

11. The set S is defined as $\{1, 2, 3 \ldots, 12\}$. Let A be the number of subsets of S such that the subset contains 6 or more elements. Let B be the number of subsets of S such that the subset contains 5 or less elements. Find $A - B$.

    (A) 792 **(B)** 924 **(C)** 1024 **(D)** 1440 **(E)** 1716

    Answer: $\boxed{\textbf{(B)}}$

    Solution: We can write A as $\binom{12}{6} + \binom{12}{7} + \binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}$, and B as $\binom{12}{0} + \binom{12}{1} + \binom{12}{2} + \binom{12}{3} + \binom{12}{4} + \binom{12}{5}$. Then, $A - B =$

$$\left(\binom{12}{6} + \binom{12}{7} + \binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}\right)$$

$$-\left(\binom{12}{0} + \binom{12}{1} + \binom{12}{2} + \binom{12}{3} + \binom{12}{4} + \binom{12}{5}\right)$$

$$= \binom{12}{6} + \left(\binom{12}{7} - \binom{12}{5}\right) + \left(\binom{12}{8} - \binom{12}{4}\right) + \left(\binom{12}{9} - \binom{12}{3}\right)$$
$$+ \left(\binom{12}{10} - \binom{12}{2}\right) + \left(\binom{12}{11} - \binom{12}{1}\right) + \left(\binom{12}{12} - \binom{12}{0}\right)$$
$$= \binom{12}{6}$$

using the fact that $\binom{n}{k} = \binom{n}{n-k}$ for $0 \le k \le n$. Thus the answer is $\binom{12}{6} = 924 \implies \boxed{\textbf{(B)}}$

Note: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

12. How many non-similar triangles with integer angle measures (in degrees) satisfy that the largest angle of the triangle is less than $70°$?

(A) 73 **(B)** 74 **(C)** 75 **(D)** 76 **(E)** 77

Answer: $\boxed{\textbf{(C)}}$

Solution: We prove the following claim.

Claim: The number of non-similar triangles with integer angle measures that satisfy the largest angle of the triangle is n, for $60 \le n < 90$, is $\left\lfloor \frac{3n}{2} - 89 \right\rfloor$, where $\lfloor x \rfloor$ is the largest integer that is less than or equal to x.

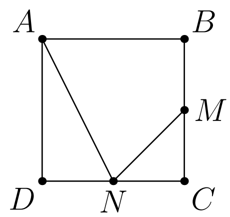
Proof: We proceed by casework based on the second-largest angle (which we shall call a). If $60 \le n < 90$, then a can range from $\left\lceil \frac{180-n}{2} \right\rceil$ to n, (where $\lceil x \rceil$ is the smallest integer that is greater than or equal to x) because if $a > n$, then n isn't the largest angle, and if $a < \left\lceil \frac{180-n}{2} \right\rceil$, then a isn't the second-largest angle anymore. Subtracting the two and adding 1, we get the desired result.

Now, we just add these from 60 to 69. A simpler way to do it without much computation is to split it into 2 cases: n is even, and n is odd. The sum of the values for even n is $\frac{3\cdot 60}{2} - 89 + \frac{3\cdot 62}{2} - 89 + \cdots + \frac{3\cdot 68}{2} - 89$, which is just $3(30+31+32+33+34) - 89\cdot 5 = 3(32\cdot 5) - 89\cdot 5 = 96\cdot 5 - 89\cdot 5 = 7\cdot 5 = 35$. The sum of the values for odd n is $\frac{3\cdot 61-1}{2} - 89 + \frac{3\cdot 63-1}{2} + \cdots + \frac{3\cdot 69-1}{2} - 89$, which is $\frac{3(61+63+65+67+69)-5}{2} - 89\cdot 5 = \frac{3(65\cdot 5)-5}{2} - 89\cdot 5 = \frac{194\cdot 5}{2} - 89\cdot 5 = 97\cdot 5 - 89\cdot 5 = 8\cdot 5 = 40$, thus the answer is $35 + 40 = 75 \implies \boxed{\textbf{(C)}}$

13. Square $ABCD$ has side length 4. Let M be the midpoint of side BC and N be the midpoint of side CD. What is the area of quadrilateral $ABMN$?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

Answer: $\boxed{\textbf{(C)}}$



Solution: Denote [<Insert polygon here>] as the area of <Insert polygon here>. Then, $[ABMN] = [ABCD] - [ADN] - [MCN]$, and $[ABCD] = 4 \cdot 4 = 16, [ADN] = \frac{4 \cdot 2}{2} = 4, [MCN] = \frac{2 \cdot 2}{2} = 2$, so the answer is $16 - 4 - 2 = 10 \implies \boxed{\textbf{(C)}}$

14. Round $\left(1+\sqrt{2}\right)^5$ to the nearest integer.

    (A) 80 **(B)** 81 **(C)** 82 **(D)** 83 **(E)** 84

    Answer: $\boxed{\textbf{(C)}}$

    Solution 1: Notice that adding $\left(1-\sqrt{2}\right)^5$ will not change the answer at all. This is because the result is an integer, and $\left(1-\sqrt{2}\right)^5 > -\frac{1}{2}$. Now we can expand $\left(1+\sqrt{2}\right)^5 + \left(1-\sqrt{2}\right)^5$ using the binomial theorem. Notice that all the terms with a $\sqrt{2}$ will cancel, since the left expansion will have a positive coefficient, but the right will have a negative coefficient. Thus, we only care about the integer parts. Expanding, we get $(20+20+1)+(20+20+1) = 82$, so our answer is $82 \implies \boxed{\textbf{(C)}}$

    Solution 2: Expanding $\left(1+\sqrt{2}\right)^5$ using the binomial theorem gives $1+5\sqrt{2}+20+20\sqrt{2}+20+4\sqrt{2} = 41+29\sqrt{2}$. Approximating $\sqrt{2}$ as 1.4, we get an answer of 81.6, but the answer should be slightly greater than that (since $1.4^2 = 1.96 < 2$), or $\approx 82 \implies \boxed{\textbf{(C)}}$

15. The ancient Gruks had 4 letters: A, B, C, and D. They could create a word by taking an arbitrary string of A's, B's, and C's and they could also accent the A's by adding a D after the A (but the A didn't have to be accented). However, a D couldn't be added anywhere else in the word. For example, BADC and ADAD are correct 4-letter words, but ACDB is not since the D is after a C. How many 4-letter words did the ancient Gruks have?

    (A) 109 **(B)** 121 **(C)** 132 **(D)** 133 **(E)** 149

    Answer: $\boxed{\textbf{(A)}}$

    Solution: We proceed with casework based on the number of D's in the word. If there are no D's, then there are $3^4 = 81$ total words possible since each letter can be an A, B, or C. If there is 1 D, then the D must be after an A, so an 'AD' must be in the word. There are 3 places to keep the 'AD' (AD__, _AD_, and __AD). Then, there are 3^2 possible combinations for each case, since each _ can be replaced by an A, B, or C. Thus, there are $3 \cdot 9 = 27$ ways where there is 1 D. If there are 2 D's, then there is only 1 way (ADAD), since a D must be after an A. Thus, the answer is $81 + 27 + 1 = 109 \implies \boxed{\textbf{(A)}}$

16. What is $1^3 - 2^3 + 3^3 - 4^3 \cdots + 15^3$?

    (A) 1472 **(B)** 1504 **(C)** 1664 **(D)** 1728 **(E)** 1856

    Answer: $\boxed{\textbf{(E)}}$

    Solution: First, we calculate $1^3 + 2^3 + 3^3 + \cdots + 15^3$. This is simply $\left(\frac{15\cdot 16}{2}\right)^2 = 14400$ using the fact that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{(n)(n+1)}{2}\right)^2$. Now, we notice that $1^3 - 2^3 + 3^3 - 4^3 \cdots + 15^3 = (1^3 + 2^3 + 3^3 + \cdots + 15^3) - 2(2^3 + 4^3 + 6^3 + \cdots + 14^3) = 14400 - 16(1^3 + 2^3 + 3^3 + \cdots + 7^3) = 14400 - 16\left(\frac{7\cdot 8}{2}\right)^2 = 14400 - 12544 = 1856 \implies \boxed{\textbf{(E)}}$

17. How many positive integers n less than 1000 satisfy exactly one of the following properties?

    - n is not a multiple of 3.
    - n is not a multiple of 5.

    (A) 400 **(B)** 401 **(C)** 466 **(D)** 467 **(E)** 533

    Answer: $\boxed{\textbf{(A)}}$

    Solution: Since one of the statements is true, one of the statements is false, so exactly one of the following statements are true:

    - n is a multiple of 3.
    - n is a multiple of 5.

    The amount of n's that satisfy that n is a multiple of 3 is 333, and for 5, it is 199 (make sure you remember that it is asking for integers *less* than 1000), but this counts the intersection twice instead of 0 times. Thus, we have to subtract twice the intersection, which is just the multiple of 15's. There are 66 multiples of 15 less than 1000, so the answer is $333 + 199 - 2(66) = 400 \implies \boxed{\textbf{(A)}}$

18. Given that x and y are positive real numbers such that $x + y = 5$, and that the minimum value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{xy}$ can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m + n$.

(A) 5 **(B)** 13 **(C)** 27 **(D)** 49 **(E)** 61

Answer: $\boxed{\textbf{(D)}}$

Solution: We can write $\frac{1}{x}+\frac{1}{y}+\frac{1}{xy}$ as $\frac{x+y+1}{xy}=\frac{6}{xy}$. This is minimized when xy is maximized. To maximize xy, we can use AM-GM, which states that $\frac{x+y}{2}\geq\sqrt{xy}$. In this case, $x+y=5$, so $\frac{5}{2}\geq\sqrt{xy}$, or $xy\leq\frac{25}{4}$. Thus, xy is maximized at $\frac{25}{4}$ (when $(x,y)=\left(\frac{5}{2},\frac{5}{2}\right)$). Then, $\frac{6}{xy}=\frac{24}{25}, 24+25=49 \implies \boxed{\textbf{(D)}}$

19. Bob is trying to get into a university. There are 5 universities that he is applying to, with ratings of 1, 2, 3, 4, and 5 and acceptance rates of 100%, 80%, 60%, 40%, and 20% respectively. If Bob gets accepted into multiple universities, he will choose the one with the highest rating. Given that the expected value of the rating of the university Bob gets into can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, find the remainder when $m+n$ is divided by 5.

    Note: The *expected value* is the "average" of the possible outcomes; it is the sum of all the (value)(probability)'s. For example, the expected value of a 6-sided die roll is $\frac{1}{6}(1)+\frac{1}{6}(2)+\frac{1}{6}(3)+\frac{1}{6}(4)+\frac{1}{6}(5)+\frac{1}{6}(6)=\frac{7}{2}$.

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

    Answer: $\boxed{\textbf{(B)}}$

    Solution: We look at the probability that he gets into each university. The probability he gets into the university with rating 1 is just the probability he doesn't get into any other university, which is $\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)\left(\frac{1}{5}\right)=\frac{24}{625}$. The probability that he gets into the university with rating 2 is the probability he doesn't get into the universities with ratings 3, 4, and 5, times the probability he gets accepted into the university with rating 2. This is $\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)\left(\frac{4}{5}\right)=\frac{96}{625}$. We can repeat for the others. For the university with rating 3, it is $\frac{36}{125}$, for 4, it is $\frac{8}{25}$, and for 5, it is $\frac{1}{5}$. We can verify that these are the correct probabilities since they all add to 1. Thus by the definition of expected value, our answer is $\left(\frac{24}{625}\right)\cdot 1+\left(\frac{96}{625}\right)\cdot 2+\left(\frac{36}{125}\right)\cdot 3+\left(\frac{8}{25}\right)\cdot 4+\left(\frac{1}{5}\right)\cdot 5=\frac{2181}{625}, 2181+625=2806 \implies \boxed{\textbf{(B)}}$

20. How many integers n satisfy

    $$n=\frac{5m^2+14m+5}{m^2+1}$$

    for some real m?

    (A) 11 **(B)** 15 **(C)** 23 **(D)** 29 **(E)** infinitely many

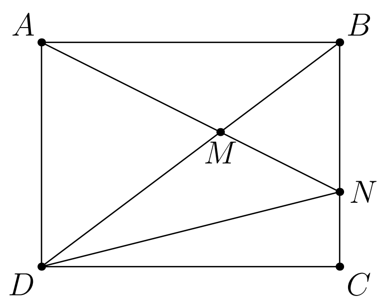
    Answer: $\boxed{\textbf{(B)}}$

Solution: We calculate m in terms of n. Multiplying by m^2+1, we get $nm^2+n = 5m^2+14m+5, (5-n)m^2+14m+(5-n)=0, m = \frac{-14\pm\sqrt{196-4(5-n)^2}}{10-2n}$. For n to exist for some **real** $m, 196-4(5-n)^2$ must be ≥ 0. Expanding, we get $96+40n-4n^2 \geq 0, n^2-10n-24 \leq 0, (n-12)(n+2) \leq 0$. Thus, $n-12$ must be negative or 0 and $n+2$ is positive or 0. This means that $-12 \leq n \leq 2$, and the answer is $2-(-12)+1 = 15 \implies \boxed{\textbf{(B)}}$

21. Rectangle $ABCD$ satisfies $AB = 4$ and $BD = 5$. Let M be the on diagonal BD such that $BM = 2$. Line AM intersects BC at N. Find the area of triangle DMN.

(A) 2 **(B)** $\frac{12}{5}$ **(C)** 3 **(D)** $\frac{16}{5}$ **(E)** $\frac{18}{5}$

Answer: $\boxed{\textbf{(B)}}$



Solution 1: Since $\angle AMD = \angle BMN$ and $\angle MAD = \angle MNB, \triangle AMD \sim \triangle NMB$ with ratio $\frac{5-2}{2} = \frac{3}{2}$. Thus, Since $AD = 3$ from the Pythagorean theorem, $BN = \frac{2}{3}\cdot 3 = 2$. Denote [<Insert polygon here>] as the area of <Insert polygon here>. Then, $[DMN] = [ABCD]-[NMB]-[AMB]-[AMD]-[DNC] = 12-[BNM]-[ABM]-[DNC]$. $[DNC]$ is just $\frac{DC\cdot CN}{2} = \frac{4\cdot(3-2)}{2} = 2$. To find $[NMB]$ and $[AMD]$, we use their similarity ratios. The sum of their altitudes is 4, and the height of AMD is $\frac{3}{2}$ the height of NMB. Solving this, we get that the height of AMD is $\frac{3}{5}\cdot 4 = \frac{12}{5}$ and the height of NMB is $\frac{2}{5}\cdot 4 = \frac{8}{5}$, so $[AMD] = \frac{3\cdot\frac{12}{5}}{2} = \frac{18}{5}$ and $[NMB] = \frac{2\cdot\frac{8}{5}}{2} = \frac{8}{5}$. $[AMB] = [ABD]-[AMD]$, and $[ABD] = \frac{4\cdot 3}{2} = 6$. Thus, $[AMB] = 6-\frac{18}{5} = \frac{12}{5}$. The answer is then $12-\frac{8}{5}-\frac{12}{5}-\frac{18}{5}-2 = \frac{12}{5} \implies \boxed{\textbf{(B)}}$

Solution 2: We use coordinates. Let $D = (0,0)$ and $C = (4,0)$. We aim to find the coordinates of M and N. Since $BM = 2$ and $BD = 5$, M is $\frac{3}{5}$ up segment BD. Since $B = (4,3), M = \left(4\cdot\frac{3}{5}, 3\cdot\frac{3}{5}\right) = \left(\frac{12}{5}, \frac{9}{5}\right)$. To find the coordinates of N, we find the equation of line AM. The y-intercept is simply 3, and the slope is $-\frac{1}{2}$. Thus, the equation of line AM is $y = -\frac{1}{2}x+3$, and since N is on $x = 4$, we can substitute $x = 4$ into our equation to get $y = 1$. Thus, the coordinates of N is $(4,1)$. Now, we can proceed with finding the areas as in the previous solution, or we can use the Shoelace Formula, to get $\frac{1}{2}\left|\frac{12}{5}-\frac{36}{5}\right|=\frac{12}{5} \implies \boxed{\textbf{(B)}}$

22. There exists exactly one solution (x, y, z) that satisfies the following system:

$$x < y < z$$

$$x + y + z = 3$$

$$xy + yz + zx = -12$$

$$xyz = 4$$

Given that z can be written as $\frac{a+\sqrt{b}}{c}$ in simplest form and c is positive, find $a+b+c$.

(A) 36 **(B)** 37 **(C)** 38 **(D)** 39 **(E)** 40

Answer: $\boxed{\textbf{(E)}}$

Solution: Vieta's Formulas tells us that x, y and z are the roots of the polynomial $x^3 - 3x^2 - 12x - 4$. By the Rational Root Theorem, all rational roots must be one of $\pm 1, \pm 2, \pm 4$. We test all of these and see that -2 works. Using synthetic division to divide $x^3 - 3x^2 - 12x - 4$ by $x + 2$, we get $x^2 - 5x - 2$. Using the quadratic formula, we get our other roots as $\frac{5\pm\sqrt{33}}{2}$. Since the greatest root is $\frac{5+\sqrt{33}}{2}$, that is z, and our answer is $5 + 33 + 2 = 40 \implies \boxed{\textbf{(E)}}$

23. What is the remainder when 13^{64} is divided by 2194?

(A) 135 **(B)** 356 **(C)** 689 **(D)** 1851 **(E)** 2123

Answer: $\boxed{\textbf{(E)}}$

Solution: Define $a \pmod b$ as the remainder when a is divided by b. As a corollary, $-a \pmod b$ is equal to $b - a \pmod b$. Then,

$$13^{64} \equiv \left(13^3\right)^{21} \cdot 13 \equiv (2197)^{21} \cdot 13 \equiv 3^{21} \cdot 13 \equiv \left(3^7\right)^{21} \cdot 13 \equiv (2187)^{21} \cdot 13$$
$$\equiv (-7)^3 \cdot 13 \equiv -343 \cdot 13 \equiv -4459 \equiv 2123 \pmod{2194},$$

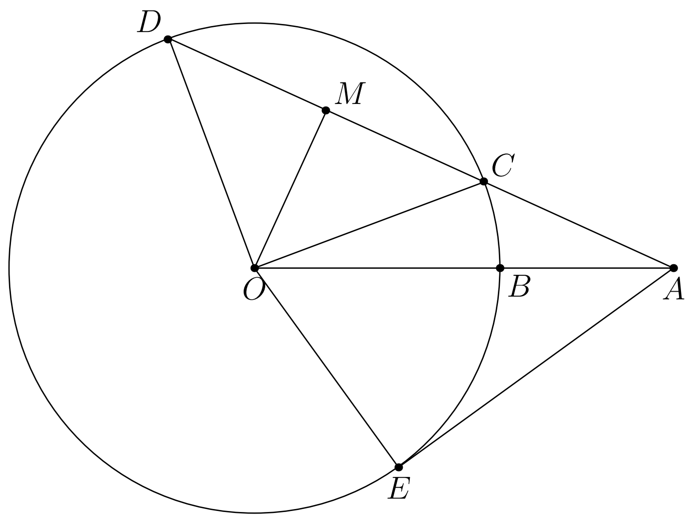
thus the answer is $2123 \implies \boxed{\textbf{(E)}}$

24. A point A is outside of a circle ω with center O and radius greater than 1. Line AO intersects ω at B, and $AB = 1$. A secant from A intersects ω at C and D with C closer to A than D such that $CD = 2$. A line passing through O intersects the midpoint M of CD. The tangent from A to ω intersects ω at E. Given that $OM = 1$, find AE^2.

Note: In this context, the word "secant" may be replaced by the word "line".

(A) $2\sqrt{2}+1$ **(B)** $2\sqrt{3}+1$ **(C)** $\sqrt{3}+1$ **(D)** 6 **(E)** $5+2\sqrt{2}$

Answer: $\boxed{\textbf{(A)}}$



Solution: We first prove that $OM \perp CD$. Since $OMD \cong OMC$ by SSS, we get that $\angle OMD = \angle OMC = 90°$, so we are done. Since $OM = MC = 1$, by the Pythagorean Theorem, the radius of the circle (OC) is $\sqrt{2}$. Now, $AO = AB + OB = 1 + \sqrt{2}$, and $OE = \sqrt{2}$, thus $AE^2 = AO^2 - OE^2 = 2\sqrt{2} + 1 \implies \boxed{\textbf{(A)}}$

25. The sequence $a_{n(n\geq 1)}$ is defined so a_n is the largest integral value k such that 2^k divides $n!$ for every integral value of $n \geq 1$. The first 5 terms of the sequence are $0, 1, 1, 3, 3$. Find the sum of the first 100 terms of the sequence.

(A) 4730 **(B)** 4731 **(C)** 4732 **(D)** 4733 **(E)** 4734

Answer: $\boxed{\textbf{(B)}}$

Solution: Legendre's Formula tells us that $a_n = \left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n}{8} \right\rfloor + \cdots$ Thus, the sum of the first 100 terms is $\left\lfloor \frac{1}{2} \right\rfloor + \left\lfloor \frac{2}{2} \right\rfloor + \left\lfloor \frac{3}{2} \right\rfloor + \cdots + \left\lfloor \frac{1}{4} \right\rfloor + \left\lfloor \frac{2}{4} \right\rfloor + \left\lfloor \frac{3}{4} \right\rfloor + \cdots + \left\lfloor \frac{1}{8} \right\rfloor + \left\lfloor \frac{2}{8} \right\rfloor + \left\lfloor \frac{3}{8} \right\rfloor + \cdots + \cdots$ The first part is $0 + 2(1 + 2 + \cdots + 49) + 50 = 2500$. The next part is $3(0) + 4(1 + 2 + \cdots 24) + 25 = 1225$. If we keep going, we get $2500 + 1225 + 588 + 270 + 111 + 37 = 4731 \implies \boxed{\textbf{(B)}}$