Mock 2018 American Mathematics Competitions (AMC 10)

Author: scrabbler94
Test solved by: KenV, Z_Math404

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers. Calculators are not allowed on the test. No problems on the test will *require* the use of a calculator.
4. Figures are not necessarily drawn to scale.
5. You will have **75 minutes** to complete the test.
6. To submit your answers, private message (PM) them to scrabbler94 no later than Tuesday, January 16, 2018, 11:59 pm EST. A running leaderboard will be kept; if you wish to remain anonymous, please state so when submitting your answers.

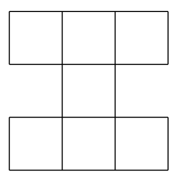
1. At a store, candy bars sell for \$1 each, or six for \$5. What is the greatest number of candy bars that Ashton can buy with \$34?

   (A) 34 **(B)** 36 **(C)** 39 **(D)** 40 **(E)** 42

2. A taxi fare costs \$3.00 for the first mile, then \$0.20 for each tenth of a mile thereafter. A \$5.00 surcharge is added for each additional passenger. Andrew and Bob wish to take a 6.4-mile taxi ride from their school to the airport. How many dollars will they save if they take the same taxi instead of two different taxis?

   (A) \$6.80 **(B)** \$8.80 **(C)** \$10.80 **(D)** \$13.80 **(E)** \$18.80

3. Seven congruent squares are attached together as shown below. If the perimeter of the figure is 48 units, what is the area of the figure, in square units?



   (A) 21 **(B)** 28 **(C)** 42 **(D)** 63 **(E)** 112

4. Alice, Beth, Charlie, David, Ellie, and Frank are seated at a round table containing six seats. Alice and Beth sit next to each other. Charlie sits directly across from David. Ellie sits immediately to Charlie's left. Alice is not seated next to David. Who is seated directly across from Frank?

   (A) Alice **(B)** Beth **(C)** Charlie **(D)** David **(E)** Ellie

5. The number $\frac{3^{3^3}}{3^3}$ is equal to

   (A) 3^3 **(B)** 3^6 **(C)** 3^9 **(D)** 3^{24} **(E)** 3^{27}

6. The perimeter and area of a semicircle, in units and square units respectively, are numerically equal. What is the length of the radius of the semicircle, in units?

   (A) 2 **(B)** $2+\dfrac{2}{\pi}$ **(C)** $2+\dfrac{4}{\pi}$ **(D)** 4 **(E)** $4+\dfrac{4}{\pi}$

7. The product of three different prime numbers, each less than 15, is one less than a perfect square. What is the sum of the three prime numbers?

   (A) 16 **(B)** 17 **(C)** 19 **(D)** 21 **(E)** 28

8. Mark rolls two fair, six-sided dice with faces numbered $1, 2, \ldots, 6$ and records the product of the two resulting numbers. The probability that the product is a composite number is

   (A) $\dfrac{25}{36}$ **(B)** $\dfrac{7}{9}$ **(C)** $\dfrac{29}{36}$ **(D)** $\dfrac{5}{6}$ **(E)** 1

9. The infinite string ABBCCCDDDDEEEEEAAAAAA... consists of one A, two B's, ..., five E's, six A's, seven B's, and so on. What is the 2018^{th} letter in this string?

   (A) A **(B)** B **(C)** C **(D)** D **(E)** E

10. Let $s(n)$ denote the sum of the digits of a positive integer n. For example, $s(47) = 4 + 7 = 11$. John is given a three-digit integer n and is told to compute $n + s(n)$. John obtains 835 as the answer. Find the remainder when n is divided by 9.

    (A) 1 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 8

11. How many ordered pairs (m, n) of positive integers satisfy the equation $m + n + \dfrac{m}{n} = 19$?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. Two perpendicular lines ℓ_1 and ℓ_2 in the xy-plane intersect at the point $(7, 5)$. If the sum of the slopes of ℓ_1 and ℓ_2 is 10, find the sum of their y-intercepts.

(A) -60 **(B)** -55 **(C)** -50 **(D)** -40 **(E)** 80

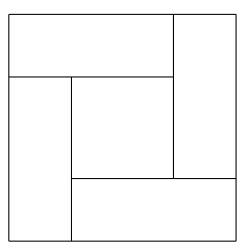
13. Recall that year N is a leap year if and only if N is divisible by 400, or if N is divisible by 4 but not 100. Given that January 1, 2018 was a Monday, what day of the week will January 1, 2118 be?

(A) Sunday **(B)** Tuesday **(C)** Wednesday **(D)** Friday **(E)** Saturday

14. Quadrilateral $ABCD$ has $AB = 3$, $BC = 4$, $CD = 12$, $\angle B = 90°$, and $\angle C = 120°$. The area of quadrilateral $ABCD$ is

(A) $15 + 12\sqrt{3}$ **(B)** $18 + 18\sqrt{3}$ **(C)** 36 **(D)** $15 + 54\sqrt{3}$ **(E)** $30 + 51\sqrt{3}$

15. The figure below shows a square that is divided into four congruent rectangles and a smaller square. The area of the smaller square equals the area of each of the other four rectangles. Find the ratio of the length of the longer side to the length of the shorter side of one of these rectangles.



(A) $\dfrac{3+\sqrt{5}}{3}$ **(B)** $\dfrac{2+2\sqrt{5}}{3}$ **(C)** $\sqrt{5}$ **(D)** $\dfrac{5}{2}$ **(E)** $\dfrac{3+\sqrt{5}}{2}$

16. A right pyramid has a square base of side length 10 cm, and height 12 cm. A sphere is inscribed inside the pyramid, tangent to all five faces of the pyramid. Find the radius of the sphere, in centimeters.

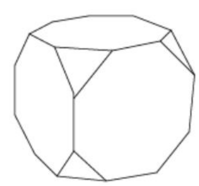
(A) $\frac{8}{3}$ **(B)** $\frac{10}{3}$ **(C)** $\frac{11}{3}$ **(D)** 4 **(E)** $\frac{13}{3}$

17. Find the sum of all real values x satisfying $9^x = 3^{x+2} - 1$.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 9

18. A truncated cube is a solid formed by slicing eight tetrahedra from the corners of a cube, as shown below. How many space diagonals does the truncated cube have?

(A *space diagonal* is a segment connecting two vertices of a polyhedron that is contained in the interior of the polyhedron)



(A) 108 **(B)** 120 **(C)** 156 **(D)** 240 **(E)** 276

19. Compute the value of

$$\left\lfloor \frac{1}{2} + \sqrt{1} \right\rfloor + \left\lfloor \frac{1}{2} + \sqrt{2} \right\rfloor + \left\lfloor \frac{1}{2} + \sqrt{3} \right\rfloor + \ldots + \left\lfloor \frac{1}{2} + \sqrt{2018} \right\rfloor.$$

Note: $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

(A) 31035 **(B)** 31080 **(C)** 60360 **(D)** 60405 **(E)** 60450

20. In trapezoid $ABCD$, sides AB and CD are parallel. Denote by M the midpoint of side BC, and P the intersection of segments AM and BD. If $AB = 10$, $BC = 8$, and $CD = DA = 5$, determine the area of triangle BMP.

(A) 2 **(B)** $\frac{12}{5}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{16}{5}$ **(E)** 4

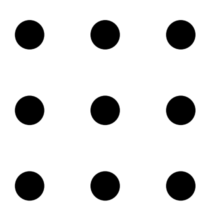
21. The polynomial $P(x) = 2x^3 + 6x^2 - 3$ has three distinct real roots r_1, r_2, and r_3. Let $Q(x)$ denote the cubic polynomial with leading coefficient 1 such that $Q(r_i + \frac{1}{r_i} + 1) = 0$ for $i = 1, 2, 3$. Find the value of $Q(3)$.

(A) $-\frac{50}{3}$ **(B)** $-\frac{25}{6}$ **(C)** 0 **(D)** $\frac{25}{6}$ **(E)** $\frac{50}{3}$

22. Three circles Ω_A, Ω_B, Ω_C are mutually externally tangent and have radii 5, 5, and 8 respectively. A fourth circle Ω is constructed so that Ω_A, Ω_B, and Ω_C are internally tangent to Ω. The radius of Ω is

(A) 12 **(B)** $\frac{25}{2}$ **(C)** $\frac{40}{3}$ **(D)** 14 **(E)** $\frac{50}{3}$

23. Nine points are arranged to form a 3×3 lattice grid, as shown below. All lines passing through at least two of these nine points are drawn in the plane. The number of points for which at least two lines intersect is



(A) 53 **(B)** 61 **(C)** 69 **(D)** 77 **(E)** 93

24. Six teams play a round robin tennis tournament, where each team plays every other team exactly once in a match. The probability that any team wins any given match is 50%, independently of other matches, and no ties occur. Find the probability that every team wins at least one match and loses at least one match.

(A) $\frac{5}{8}$ **(B)** $\frac{335}{512}$ **(C)** $\frac{163}{256}$ **(D)** $\frac{175}{256}$ **(E)** $\frac{95}{128}$

25. A positive integer $N \geq 1$ is called *interesting* if there exists an integer $2 \leq k \leq 10$ and a length k sequence $a_1, \ldots, a_k$ satisfying the following conditions:

- $a_1 = 1$ and $a_k = N$
- For all integers n between 2 and k inclusive, a_n is equal to either $a_{n-1} + 1$ or $2a_{n-1}$

Find the number of interesting integers.

(A) 142 **(B)** 143 **(C)** 144 **(D)** 145 **(E)** 146