

Solutions to Mock AMC 810

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1 Solutions

1. We add one hour, or 60 mins, to get to 5:30. Then we add another 47 mins to get to 6:17. Now we just add $47+60$ to get: 107
2. We just multiply $15*80$ to get the sum of all the scores when the absent student took the test, and then subtract the sum of the original 14 scores which is $14*79$. We get: 94
3. We just divide both diagonals by 2 getting 20 and 21. Using Pythag, we get 29 as the side length for the rhombus. To find the perimeter, we just multiply by 4 to get: 116.
4. We just find the number of ways to arrange 6 people ($6!$) and then divide by the number ways we can place one person (6). We get: 120.
5. We just figure out the number of people who take both subjects. That is just $179+149-308$ which is 20. Now, we subtract 20 from the number of students who take math $7/8$ and we get: 129.
6. Using the Exterior Angle Theorem, we just add 48 and 76 to get: 124.
7. There are 9 single digit number so we subtract 9 to get 516. Then there are 90 2 digit number so we subtract $90*2$ or 180 to get 336. Then we divide by 3 and get 112 3 digit page numbers. To find the number of pages, we just do $100+112-1$ and we get: 211.
8. First he loses 20 percent. That brings him to 28.8 percent. Then he gains 40 percent and finally has an approval rating of: 40.32 percent.
9. The store first reduces by 40 percent bringing them to a rating of 60 percent. Now they gain back that 40 percent but it is relative to the 60 percent. $40/60*100=67$ percent approximately. $67-40$ is simply: 27 percent.
10. Using Pythag, we get $97^2 + 37^2$ which is 10778. Using approximation, we get that the square root of it as around 103.8. Now we add 97 and 37 and subtract 103.8 to get an answer of: 30.2.

11. Since the diagonal is 13, and the area using the ratios is 60, then k should be $\frac{60}{13^2}$ or: $\frac{60}{169}$.
12. Using a very handy theorem, the number of tiles a person crosses in an x by y floor is $x+y-\text{GCD}(x, y)$. We plug in x and y to get $16+19-1$ which is: 34.
13. We know what $11!$ using the hint. Then we divide by $4!$, $4!$, and $2!$ because of repeating letters. We get: 34,650.
14. The number has to be a perfect square and the least perfect square under 16,000 is 15,876, which is the answer.
15. We prime factorize 561 into $3 \cdot 11 \cdot 17$. Then we know that the number of perfect squares is 7^3 and the number of perfect cubes is 5^3 . We add them and then subtract 3^3 because of overcounting. We get an answer of: 441.
16. We multiply the area of the semicircle by 2 and then take the square root. We get 50 as the side length. Since the diameter is the length and the radius the width, we get $50 \cdot 100$ or 5000.
17. Question Removed
18. Using Pythag, we get an equation of $16^2 + x^2 = (x + 2)^2$. We simplify this to get $4x + 4 = 256$ and we get x as 63 and $x+2$ as 65. We take the average which is 64.
19. You can look at the options and try them out from least to greatest. You can also prime factorize 36 and see all the ways to make a number with 36 factors. We get the same answer in both ways: 1260.
20. We find that that the smaller angle is 94. We know that the 2 angles sum to 360, so the second angle is $360 - 94$. That is 266. Now we just find the difference which is $266 - 94$ or: 172.
21. We split the isosceles into 2 right triangles. We find that the height is 20 using pythag. Now we just find the altitude in the right triangle which is just $15 \cdot 20 / 25$ or 12. That is the radius of the semicircle so the area of the semicircle is 72π . The area of the triangle was 300. So the area of the space outside the circle is: $300 - 72\pi$.
22. If we look at the options, the only number which multiplied by 4 has the units digit that the original number's first digit had, then we see that it is 219,978
23. We use stars and bars to solve this. First we break 29 up into $1+1+1+1+1 \dots$. Now we just take out 5 of the ones because the numbers can't be zero. Now we just use stars and bars with the numbers 24 and 5. That is just $28C4$ and the answer is: 20475

24. We just divide both side lengths by 2 to get a right triangle. We use Pythag on 35 and 12 to get the side length for the rhombus. Then we just find the altitude coming from the hypotenuse which is $35 \cdot 12 / (35^2 + 12^2)$. Rounded to the nearest hundredth, that is: 11.35.
25. There is a really handy formula to figure out the sum of the digits of all the numbers with n digits. The formula is Sum of digits(n digit numbers) = $45 \cdot 10^{n-1} + 10 \cdot \text{Sum of digits}(n-1 \text{ digit numbers})$. Using this, we get the sum of all the 1 digit numbers as 45, 2 digit numbers as $45 \cdot 10^1 + 10 \cdot 45$. This keeps going on and we add all the answers to get 27,000,000. But we forgot 1 million which has sum of digits 1. So now we have 27,000,001. We take the sum of digits of that number to get: 10.