MAH AMC

Halloween Mock AMC 8

2nd (Not Necessarily) Annual Mock

AMC 8

Friday October 16, 2020
Probably not Halloween

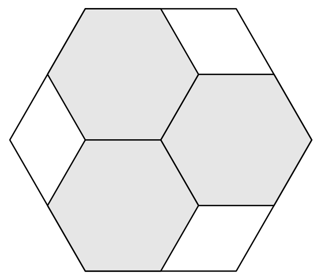
INSTRUCTIONS

1. DO NOT PROCEED UNTIL YOU HAVE SET YOUR TIMER TO 40 MINUTES.
2. This is a twenty-five-question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Please record your answers anywhere, but only the ones submitted using the directions on the main post will be graded.
4. SCORING: You will receive 1 point for each correct answer and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compasses, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator. A timer is allowed.
6. Figures are not necessarily drawn to scale.
7. When you start your timer, begin working on the problems. You will have 40 minutes to complete the test.
8. **Please remember to submit your answers if you wish for them to be graded!**

1. Frank and Greg like eating cookies. At first, the number of cookies Frank has is a perfect cube, and Greg also has a whole number of cookies. After Frank eats two of those cookies (but Greg eats none), he realizes that he now has exactly 3 times as many cookies as Greg. Which of the following could be the number of cookies Frank had before he ate two of the cookies?

(A) 27 **(B)** 64 **(C)** 125 **(D)** 216 **(E)** 343

2. What fraction of the original regular hexagon's area is shaded?



(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{5}{6}$

3. Let $ABCDEFGH$ be a regular octagon with side length 1. The *incircle* is the circle that is tangent to each of the sides. The *circumcircle* is the circle that passes through each of the vertices. What is the positive difference between the areas of the circumcircle and incircle of $ABCDEFGH$?

(A) $\frac{\pi}{16}$ **(B)** $\frac{\pi}{8}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{2}$ **(E)** π

4. Ava, Brandon, Caleb, and Delaney are all driving from their school to the grocery store and drive at the same speed. There are two ways to go from their school to the library, and there are two ways to go from the library to the grocery store. Each of them passes by the library but take different paths. Suppose that Ava takes the shortest time to get to the grocery store, and Delaney takes the longest. If Ava takes 8 minutes, Brandon takes 10, and Caleb takes 13, then how many minutes does Delaney take?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

5. For a positive integer n, let $\omega(n)$ be the number of distinct prime factors of n and let $\Omega(n)$ be the number of total prime factors of n. For example, $\omega(12) = \omega(2^2 \cdot 3) = 1 + 1 = 2$ and $\Omega(12) = \Omega(2^2 \cdot 3) = 2 + 1 = 3$. What is $\Omega(8400) - \omega(8400)$?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 8

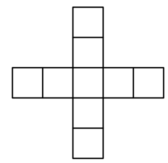
6. Let $a☼b = \frac{a!}{b!}$ for positive integers a, b with $a > b$. If, for some positive integer n,

$$n☼(n-2020) = 2((n-1)☼(n-2021)),$$

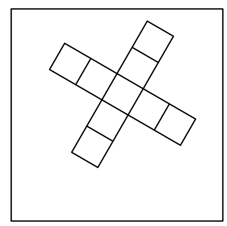
then compute n.

(A) 3030 **(B)** 4040 **(C)** 5050 **(D)** 6060 **(E)** 7070

7. Bill has a plus sign shaped figure like the one below (each square is a unit square):



and he wishes to put it inside a 7 by 7 square. For example,



is a valid placement. Find the area of the region consisting of all of the points P such that there exists a placement of the plus-sign shaped figure such that the center of the "+" is located at P.

(A) 4 **(B)** $67 - 42\sqrt{2}$ **(C)** 9 **(D)** $\dfrac{123 - 70\sqrt{2}}{2}$ **(E)** $\dfrac{107 - 42\sqrt{2}}{2}$

8. Chris rolls two fair, six-sided dice. What is the probability that the sum of the numbers rolled is neither even nor a prime?

(A) $\dfrac{1}{36}$ **(B)** $\dfrac{1}{9}$ **(C)** $\dfrac{5}{36}$ **(D)** $\dfrac{2}{9}$ **(E)** $\dfrac{1}{4}$

9. Byan and Rai each roll a fair, six-sided die. What is the probability that Byan rolls a number at least double of Rai's?

(A) $\dfrac{1}{6}$ **(B)** $\dfrac{1}{4}$ **(C)** $\dfrac{5}{18}$ **(D)** $\dfrac{1}{3}$ **(E)** $\dfrac{11}{18}$

10. There are two concentric circles both with center O with radius 2020 and 2021. Find the smallest possible distance OP such that for all positive real numbers r, there exists a point Q that is either strictly inside both circles or neither circle and is r distance away from P.

(A) 0.5 **(B)** 1 **(C)** 2 **(D)** 505 **(E)** 1010

11. Nonzero real numbers a and b satisfy that $a + b$ is seven times $a - b$ and $a \times b$ is seven times $a \div b$. Then there are two possible values for the expression $(a + b) + (a - b) + (a \times b) + (a \div b)$. Find the sum of these two values.

(A) $\dfrac{32}{3}$ **(B)** 21 **(C)** $\dfrac{64}{3}$ **(D)** 42 **(E)** 49

12. Let x, y, and z be real numbers. If the following equations are true, find $x \cdot y \cdot z$.

$$x + 2y + 3z = 49$$
$$2x + 3y + z = 50$$
$$3x + y + 2z = 51$$

(A) 576 **(B)** 600 **(C)** 640 **(D)** 720 **(E)** 846

13. You and Ben are playing a game. First, strictly inside a disc of radius 8, Ben chooses a disc of radius 3 uniformly and at random. Now, you win if you are able to choose another disc of radius 3 strictly inside the larger disc but not overlapping with Ben's. Otherwise, you lose. What is the probability that you win?

(A) $\frac{21}{25}$ **(B)** $\frac{15}{16}$ **(C)** $\frac{24}{25}$ **(D)** $\frac{35}{36}$ **(E)** $\frac{63}{64}$

14. Sunny has a weighted 6-sided die that rolls an even number with probability $\frac{1}{3}$ and rolls a prime number with probability $\frac{1}{2}$. If the dice rolls a 2 with probability $\frac{1}{9}$, what is the probability that the dice rolls a 1?

(A) $\frac{1}{18}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{2}{9}$ **(E)** $\frac{5}{18}$

15. Let x be the smallest positive integer such that the value

$$\sqrt[3]{3\sqrt{\frac{5x}{6}}}$$

is an integer. Suppose that $x = 2^a \cdot 3^b \cdot 5^c$ for nonnegative integers a, b, and c. Find $a + b + c$.

(A) 10 **(B)** 11 **(C)** 17 **(D)** 18 **(E)** 21

16. Charlie has a 9 by 12 grid of unit squares. Initially, they are all colored white. In a move, he can choose a row or column and switch the colors of all of the squares in that row or column between white and black. That is, a white square in that row/column will turn black, and vice versa. Let N be the number of colorings that are attainable after some sequence of moves. Find the largest power of 2 that divides N.

(A) 2^{10} **(B)** 2^{20} **(C)** 2^{21} **(D)** 2^{22} **(E)** 2^{42}

17. If p and q are the roots of $x^2 + 12x + k$, and $\frac{p}{2}$ and $2q$ are the roots of $x^2 + 8x + k$, evaluate k.

(A) $-\frac{1792}{9}$ **(B)** $-\frac{2464}{25}$ **(C)** $\frac{128}{9}$ **(D)** $\frac{224}{25}$ **(E)** $\frac{896}{25}$

18. For how many two digit positive integers n is n^n divisible by 24^{24}?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

19. Timmy has a large bag of stones which consists of

    - 1 stone of mass 1 gram,
    - 2 stones of mass 2 grams,
    - 3 stones of mass 3 grams,
    - ...
    - 10 stones of mass 10 grams,
    - 11 stones of mass 11 grams.

    Timmy randomly draws two stones, without replacement (each stone has the same probability of being drawn). What is the probability that the sum of the weights of the two stones is exactly 12 grams?

    (A) $\frac{70}{1089}$ **(B)** $\frac{28}{429}$ **(C)** $\frac{13}{198}$ **(D)** $\frac{1}{15}$ **(E)** $\frac{2}{15}$

20. Let ABC be an isosceles triangle with $AB = AC$. Let D, E, and F be the feet of the A-, B-, and C- altitudes respectively. If $BD = CD = 1$ and $m\angle EDF = 90°$, then compute AD.

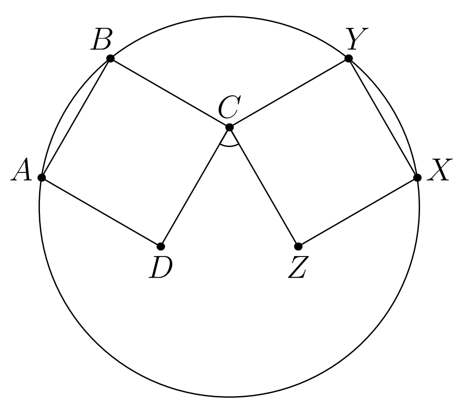
    (A) 1 **(B)** $\frac{1+\sqrt{2}}{2}$ **(C)** $1+\sqrt{2}$ **(D)** $2\sqrt{2}$ **(E)** $2+\sqrt{2}$

21. For how many integers n with $1 \leq n \leq 100$ is $2^n + n^2$ exactly one more than a multiple of 5?

    (A) 0 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 20

22. Two unit squares, $ABCD$ and $XYCZ$, are inside circle Ω and have points A, B and X, Y on circle Ω, as shown below. They share vertex C and angle $\angle DCZ$ is $60°$. Find the area of circle Ω.

    (A) $\frac{(4-\sqrt{3})\pi}{3}$ **(B)** $\frac{4\pi}{3}$ **(C)** $\frac{3\pi}{2}$ **(D)** $\frac{29\pi}{16}$ **(E)** $\frac{(4+\sqrt{3})\pi}{3}$



23. Define ℓ_{XY} to be the perpendicular bisector of segment XY. Let Γ be a circle with radius $R > 1$. Suppose that there exists three distinct circles γ_A, γ_B, and γ_C centered at points A, B, and C, respectively, each with radius 1 such that

    - γ_A, γ_B, and γ_C are each internally tangent to Γ.
    - ℓ_{AB} does not intersect the interiors of γ_A, γ_B, or γ_C.
    - ℓ_{CA} does not intersect the interiors of γ_A, γ_B, or γ_C.
    - ℓ_{BC} does not intersect the interiors of γ_A, γ_B, or γ_C.

    Find the minimum (possibly non-integer) possible value of R.

    (A) 3 **(B)** $2\sqrt{3}$ **(C)** 4 **(D)** $3\sqrt{2}$ **(E)** $3+\sqrt{2}$

24. A pool containing 2020 liters of water is being drained by 2020 pumps which drain at speeds of 1 liter per hour, 2 liters per hour, ..., 2020 liters per hour. However, the pump that drains at n liters per hour stops draining once there remains $n-1$ liters in the pool. Find the total amount of time, in hours, required to drain the pool.

    (A) $\frac{2019}{1010}$ **(B)** $\frac{4040}{2021}$ **(C)** $\frac{2021}{1011}$ **(D)** $\frac{4041}{2021}$ **(E)** 2

25. At a party, there are five people (Alan, Andrew, Abby, Vincent, and Victoria). There are 4 kinds of burgers, 2 of which are vegetarian-friendly. There are two of each kind of burger (and hence 8 burgers total). How many ways are there to give one burger to each person if Vincent and Victoria are vegetarians (i.e. they can only have the vegetarian-friendly burger), while the other three people can receive any kind of burger? Note that burgers of the same type are indistinguishable.

    (A) 60 **(B)** 72 **(C)** 96 **(D)** 120 **(E)** 132