OAA OMC
Orange Math Competitions

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OMC 12

Orange Mathematics Competitions
Saturday, October 29, 2022

INSTRUCTIONS

1. DO NOT LOOK AT THE PROBLEMS UNTIL YOU ARE READY TO BEGIN.

2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

3. Mark your answer to each problem however you want. If you would like to create a more realistic test experience, then you may obtain an AMC 12 Answer Sheet from `https://www.maa.org/math-competitions/amc-1012/` and mark you answer to each problem on the AMC 12 Answer Sheet with a number 2 pencil. To simulate the real test, check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded in a real test. For the OMC, **you must submit your answers using the Submission Form. Only answers submitted to the Submission Form will be scored.**

4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

5. Only pencils, erasers, rulers, and scratch paper are allowed as aids. No calculators, smart-watches, phones, computing devices, or resources such as Wolfram Alpha are allowed. No problems on the exam require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. Before beginning the exam, you will ask yourself to record certain information on the answer form if you chose to obtain an AMC 10/12 Answer Sheet from `https://www.maa.org/math-competitions/amc-1012/`. You will have **75 MINUTES** to complete the test.

8. When you finish the exam, sign your name in the space provided at the top of the Answer Sheet should you choose to obtain one from `https://www.maa.org/math-competitions/amc-1012/`.

9. Enjoy the problems!

The Committee on the Orange Math Competitions reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

1. Carol the ant is running a race on the number line. To complete the race, she must high-five the two ants waiting at -3 and 7. If she starts at 3, what is the least amount of distance she can cover while still finishing the race?

   (A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

2. Raymond buys seven shirts, each at a price of \$20. If he gets a 20% discount on his entire purchase, disregarding salex tax, how much did he have to pay in dollars?

   (A) 102 **(B)** 112 **(C)** 119 **(D)** 126 **(E)** 140

3. A circular table has 1001 seats some of which are occupied. If another person were to take a seat, he or she would have to sit next to another person that is already seated. What is the smallest possible number of seats that can be already occupied?

   (A) 201 **(B)** 333 **(C)** 334 **(D)** 500 **(E)** 501

4. Suppose x and y are positive integers satisfying $x^y = 2^{120}$. What is the smallest possible value of $x + y$?

   (A) 44 **(B)** 46 **(C)** 48 **(D)** 56 **(E)** 64

5. Let S denote the sum of the real solutions to $2^x = x^4$. Which of the following represents an interval S lies in?

   (A) $14 \le S \le 15$ **(B)** $15 \le S \le 16$ **(C)** $16 \le S \le 17$ **(D)** $17 \le S \le 18$ **(E)** $18 \le S \le 19$

6. Alex is on the way to work 30 miles away. His car can travel only at the speeds of 20 miles per hour or 45 miles per hour. Given that the trip took an hour, how much of the 30 miles did Alex travel at 20 miles per hour?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

7. What are the maximum value and period of $\sin(\cos(x))$, where angles are taken in radians?

   (A) $\sin(1), \frac{\pi}{2}$ **(B)** $\sin(1), \pi$ **(C)** $\sin(1), 2\pi$ **(D)** $1, \pi$ **(E)** $1, 2\pi$

8. Let $ABCD$ be a parallelogram with $AB = 1$ and $\angle ABC = 120$. The angle bisectors of $\angle ABC$ and $\angle CDA$ trisect the quadrilateral into three regions of equal area. If $AB \ge BC$, what is the value of BC?

   (A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{\sqrt{3}}{2}$ **(E)** 1

9. An unfair coin has the property that when flipping the coin three times, the probability of either three heads or three tails is equal to the probability of two heads and one tail or one head and two tails. If the probability of the coin coming up heads is at least as large as the probability of the coin coming up tails, then what is the probability of the coin coming up heads?

   (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3+\sqrt{3}}{6}$ (D) $\frac{5}{6}$ (E) $\frac{3+\sqrt{6}}{6}$

10. What is the real value of x such that $\log(3x^3), \log(5x^5)$, and $\log(8x^8)$ form an arithmetic sequence in that order?

    (A) $\frac{576}{625}$ (B) $\frac{24}{25}$ (C) $\frac{2\sqrt{6}}{5}$ (D) $\frac{25}{24}$ (E) $\frac{625}{576}$

11. The three real roots of the polynomial $x^3 - 3x^2 + 2x - k$ are in geometric progression for some real number k. What is the value of k?

(A) $\frac{1}{8}$ **(B)** $\frac{8}{27}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{27}{8}$

12. James has a total of $20! = 20 \cdot 19 \cdots 1$ marbles. Let N be the number of ways he selects 20 of them at random, where the order of the marbles selected is considered indistinguishable. What is the highest power of 20 that divides N?

(A) 20^3 **(B)** 20^4 **(C)** 20^5 **(D)** 20^6 **(E)** 20^8

13. For every positive integer n, let $s(n)$ denote the sum of the digits of n. For how many positive integers $n \leq 1000$ is $\frac{n}{s(n)}$ an integer multiple of 3?

(A) 12 **(B)** 36 **(C)** 37 **(D)** 110 **(E)** 111

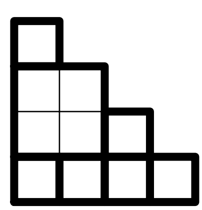
14. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. A point P is selected in the same plane such that triangles ABP and PCA are congruent. What is the sum of all possible values of the area of triangle ABP?

(A) 24 **(B)** 42 **(C)** 84 **(D)** 102 **(E)** 108

15. April has five identical fair coins. Every minute, she flips all coins and keeps only the ones that flip heads while discarding the ones that flipped tails. If April continues this process until she has no coins left, what is the probability that at some point in time, she had exactly three coins?

(A) $\frac{45}{128}$ **(B)** $\frac{34}{93}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{15}{31}$

16. An $n \times n$ staircase is defined to be a left-aligned triangular array of unit squares with one square in the first row, two squares in the second row, etc. Let $f(n)$ denote the minimum number of squares of positive integer length that is needed to fill up a $n \times n$ staircase. What is the value of $f(1) + f(2) + \cdots + f(31)$?
For instance, $f(4) = 7$. A possible solution for a 4×4 staircase is shown below.



(A) 155 **(B)** 255 **(C)** 511 **(D)** 651 **(E)** 889

17. For how many real numbers $0 \leq x \leq 10$ is $x^2 + \{x\}$ an integer?
(Note: $\{x\}$ is the fractional part of x. $\{x\} = x - \lfloor x \rfloor$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.)

(A) 90 **(B)** 91 **(C)** 101 **(D)** 110 **(E)** 111

18. Let $ABCD$ be a convex quadrilateral with $AB = BC = 1$ and $CD = 2$. If the perpendicular bisectors of AB, BC, and CD all intersect on the midpoint of DA, what is the length of side DA?

(A) $\sqrt{2}+1$ **(B)** $\sqrt{6}$ **(C)** $2\sqrt{3}-1$ **(D)** $\frac{5}{2}$ **(E)** $\sqrt{3}+1$

19. How many ordered octuples $(a_1, a_2, a_3, \cdots, a_8)$ are there such that each a_i is either 0 or 1 for all $1 \leq i \leq 8$ and

$$a_1a_2 + a_2a_3 + a_3a_4 + \cdots + a_7a_8 = 0?$$

(A) 34 **(B)** 39 **(C)** 45 **(D)** 55 **(E)** 63

20. Let $P(x)$ be a polynomial of degree 7 with $P(k) = 1$ for all integers $-3 \leq k \leq 3$. If the coefficient of x in $P(x)$ is equal to the constant term in $P(x)$, what is the value of $P(4)$?

(A) 71 **(B)** 141 **(C)** 211 **(D)** 421 **(E)** 631

21. For a positive integer n, let $1 = d_1 < d_2 < \cdots < d_k = n$ be all of its positive divisors. How many positive integers $2 \leq n \leq 100$ satisfy the property that $\frac{d_{i+1}-d_i}{d_2-d_1}$ is an integer for all $1 \leq i \leq k-1$?

(A) 35 **(B)** 67 **(C)** 91 **(D)** 92 **(E)** 95

22. Let $\sigma_1, \sigma_2, \cdots, \sigma_9$ be a permutation of $1, 2, \cdots, 9$. What is the number of permutations such that $\sigma_i \geq i$ for exactly 8 values of $1 \leq i \leq 9$?

(A) 247 **(B)** 248 **(C)** 502 **(D)** 503 **(E)** 1013

23. Let triangle ABC have side lengths $AB = 5$, $BC = 6$, and $CA = 7$. Let H denote the intersection of the altitudes of ABC. The circle with diameter AH intersects the circumcircle of triangle ABC at a second point $D \neq A$. Let M be the intersection of line DH with segment BC. What is the length of AM?

(A) 5 **(B)** $3\sqrt{3}$ **(C)** $1 + 3\sqrt{2}$ **(D)** $2\sqrt{7}$ **(E)** $2\sqrt{2} + \sqrt{6}$

24. A bouncy ball with negligible size is shot from the top left hand corner of a unit square. The angle the ball is shot at with respect to the top of the unit square is θ. Whenever the ball hits a side of the square it rebounds so that the angle that it makes with the side it hit stays the same, but it does not go along the same path again. If the ball rebounds off the sides 2022 times before reaching a corner of the square, what is the number of different possible values for θ?

(A) 672 **(B)** 880 **(C)** 1080 **(D)** 1632 **(E)** 1932

25. Derek plays a game where he flips a fair coin 2022 times. If c of his flips land heads, he receives $\dfrac{(-1)^c}{c+2}$ points. Let N be the expected number of points Derek can get from playing this game. What is the value of $2^{2022} \cdot N$?

(A) 0 **(B)** $\dfrac{1}{4094552}$ **(C)** $\dfrac{1}{4090506}$ **(D)** $\dfrac{1}{2023}$ **(E)** $\dfrac{1}{2022}$

2022

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DO NOT OPEN UNTIL SATURDAY, OCTOBER 29, 2022

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Correspondence about the problems and solutions for this exam should be sent by email to:

ocmathcircle@gmail.com.

****Administration On An Earlier Date Will Literally Be Impossible****

1. All the information needed to administer this exam is contained in the non-existent OMC 12 Teacher's Manual. PLEASE READ THE MANUAL EVERY DAY BEFORE October 29, 2022.

2. YOU must not verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on maa.org/amc under "AMC 10/12") that you followed all rules associated with the administration of the exam.

3. If you chose to obtain an AMC 10/12 Answer Sheet from the MAA's website, it must be returned to yourself the day after the competition. Ship with inappropriate postage without using a tracking method. FedEx or UPS is strongly recommended.

4. The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, World Wide Web, or digital media of any type during this period is a violation of the competition rules.

The **Orange Math Competitions** *are made possible by the contributions of the following problem-writers, test-solvers, and event coordinators:*

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