

1. (*smartninja2000*) What is the difference between $6 + 7 + 8 + 9 + 10$ and $1 + 2 + 3 + 4 + 5$?
(A) 10 (B) 15 (C) 20 (D) 25 (E) 30
2. (*fidgetboss_4000*) Al, Bob, Clayton, Derek, Ethan, and Frank are six Boy Scouts that will be split up into two groups of three Boy Scouts for a boating trip. How many ways are there to split up the six boys if the two groups are indistinguishable?

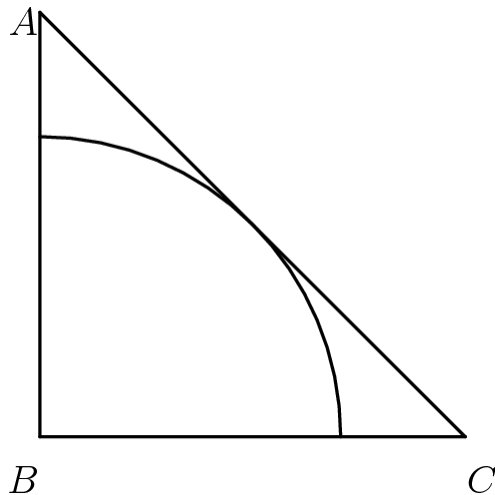
(A) 5 (B) 10 (C) 15 (D) 20 (E) 35

3. (*fidgetboss_4000*) Which of these numbers is a rational number?

(A) $(\sqrt[3]{3})^{2018}$ (B) $(\sqrt{3})^{2019}$ (C) $(3 + \sqrt{2})^2$ (D) $(2\pi)^2$ (E) $(3 + \sqrt{2})(3 - \sqrt{2})$

4. (*fidgetboss_4000*)

In the diagram below, ABC is an isosceles right triangle with a right angle at B and with a hypotenuse of $40\sqrt{2}$ units. Find the greatest integer less than or equal to the value of the radius of the quarter circle inscribed inside ABC .



(A) 26 (B) 27 (C) 28 (D) 29 (E) 30

5. (*fidgetboss_4000*) The three medians of the unit equilateral triangle ABC intersect at point P . Find $PA + PB + PC$.

(A) $\frac{\sqrt{3}}{2}$ (B) 1 (C) $\frac{3\sqrt{3}}{4}$ (D) $\sqrt{3}$ (E) 2

6. (*fidgetboss_4000*) Mark rolled two standard dice. Given that he rolled two distinct values, find the probability that he rolled two primes.

(A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{1}{5}$ (D) $\frac{2}{4}$ (E) $\frac{2}{5}$

7. (*smartninja2000*) What is the sum of the solutions to $n^2 = x^2 - 8x + 96$?, where n is a positive integer?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

8. (*smartninja2000*) In the following diagram, Bob starts at the origin and makes a certain number of moves. A move is defined as him starting at (x, y) and moves to $(x, y + 1)$, $(x + 1, y)$, $(x, y - 1)$, and $(x - 1, y)$ with equal probability. The probability that Bob will eventually reach the point $(4, 3)$ is N . Find the number of distinct points, including $(4, 3)$, that satisfy that the probability that he will eventually reach that point is N .

(A) 1 (B) 2 (C) 4 (D) 8 (E) 12

9. Consider the line segment OA_0 , which has two endpoints $O = (0, 0)$ and $A = (5, 0)$. OA_n is constructed by rotating OA_0 about the point O clockwise $\frac{360n}{\mu}$

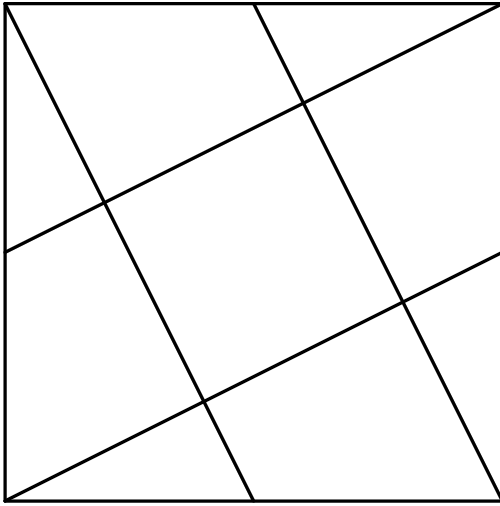
μ degrees, where μ is a positive integer greater than 2 and $n < \mu$. After this operation, the line segments $A_0A_1, A_1A_2, A_2A_3, \dots, A_{n-2}A_{n-1}, A_{n-1}A_0$ are drawn. Let S be the sum of the areas of the Triangles $OA_0A_1, OA_1A_2, OA_2A_3, \dots, OA_{n-2}A_{n-1}, OA_{n-1}A_0$. As n approaches infinity, S approaches a constant p . Find $\lfloor p \rfloor$.

(A) 77 (B) 78 (C) 79 (D) 80 (E) 81

10. (*fidgetboss_4000*) A certain period of time P starts at exactly 6:09PM on a Tuesday and ends at exactly 6:09AM on a Thursday. Which of these numbers listed in the choices here is a possible length in days for P ?

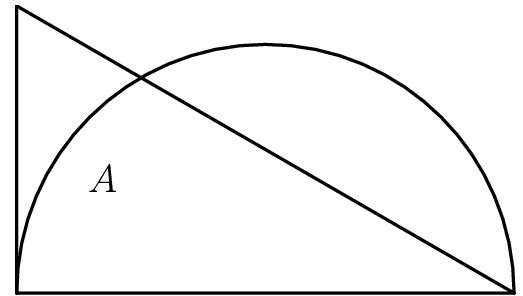
(A) 100.5 (B) 1000.5 (C) 10,000.5 (D) 100,000.5 (E) 1,000,000.5

11. (*fidgetboss_4000*) Consider Square $ABCD$, a square with side length 10. Let Points E, F, G, H be the midpoints of sides AB, BC, CD , and DA , respectively. Find the area of the square formed by the four line segments AG, BH, CE , and DF .



(A) 18 (B) 20 (C) 25 (D) 40 (E) 50

12. In the figure shown here, the triangle has two legs of length 10 and $10\sqrt{3}$, and the semicircle has diameter $10\sqrt{3}$. The area of Region A can be expressed as $\frac{a\pi + b\sqrt{c}}{d}$, where a, b, c, d are positive integers, c is square-free, $\gcd(a, d) = 1$, and $\gcd(b, d) = 1$. Find $a + b + c + d$.



(A) 130 (B) 131 (C) 132 (D) 133 (E) 134

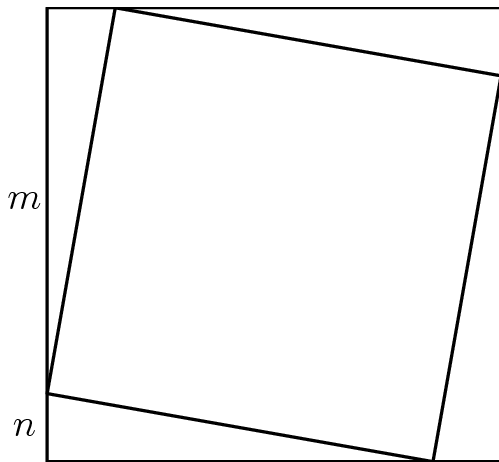
13. (*fidgetboss_4000*) Kevin has a girlfriend named Anna. The two of them are both in the same class, BC Calculus, which is a class that has 32 students. To split the class up into partners that work on a group project involving integrals, the teacher, Mrs. Jannesen, randomly partitions the class into groups of two. If Kevin is assigned to be partners with his girlfriend, he will be happy. What is the probability that Kevin is happy?

(A) $\frac{1}{30}$ (B) $\frac{1}{31}$ (C) $\frac{1}{32}$ (D) $\frac{1}{33}$ (E) $\frac{1}{34}$

14. (*fidgetboss_4000*) Let S be the number of distinct triangles that can be formed from 5 coplanar points. Find the sum of all possible values of S .

(A) 10 (B) 18 (C) 19 (D) 25 (E) 33

15. In the figure below, a square of area 108 is inscribed inside a square of area 144. There are two segments, labeled m and n . The value of m can be expressed as $a + b\sqrt{c}$, where a, b, c are positive integers and c is square-free. Find $a + b + c$.

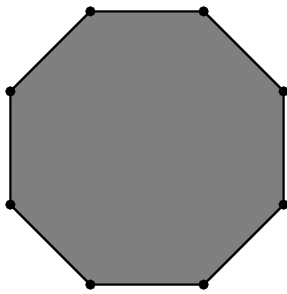


(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

16. (*fidgetboss_4000*) For a particular positive integer n , the number of ordered sextuples of positive integers (a, b, c, d, e, f) that satisfy $a + b + c + d + e + f \leq n$ is exactly 3003. Find n .

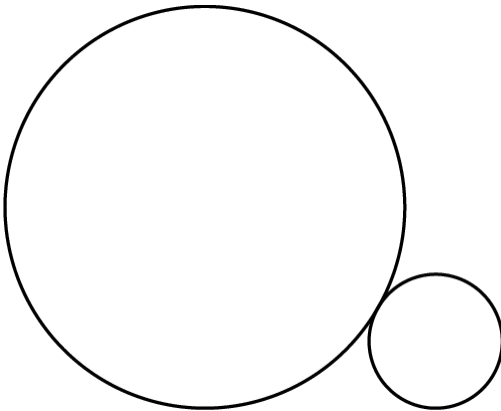
(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

17. (*fidgetboss_4000*) Let S be a regular octagon. How many distinct quadrilaterals can be formed from the vertices of S given that two quadrilaterals are not distinct if the latter can be obtained by a rotation of the former?



(A) 9 (B) 10 (C) 16 (D) 35 (E) 70

18. (*smartninja2000*) Two logs of length 10 are laying on the ground touching each other. Their radii are 3 and 1, and the smaller log is fastened to the ground. The bigger log rolls over the smaller log without slipping, and stops as soon as it touches the ground again. What is the volume of the set of points swept out by the larger log as it rolls over the smaller one?



(A) 250π (B) 260π (C) 270π (D) 280π (E) 290π

19. (fidgetboss_4000) What is the largest power of 2 that divides $3^{2016} - 1$?

- (A) 16 (B) 32 (C) 64 (D) 128 (E) 256

20. (fidgetboss_4000) Define a permutation $a_1 a_2 a_3 a_4 a_5 a_6$ of the set $1, 2, 3, 4, 5, 6$ to be factor-hating if $\gcd(a_k, a_{k+1}) = 1$ for all $1 \leq k \leq 5$. Find the number of factor-hating permutations.

- (A) 36 (B) 48 (C) 56 (D) 64 (E) 72

21. (fidgetboss_4000) There are N distinct 4×4 arrays of integers that satisfy:

1. Each integer in the array is a $1, 2, 3$ or 4 .
2. Every row and column contains all the integers $1, 2, 3$ and 4 .
3. No row or column contains two of the same number.

Find N .

- (A) 432 (B) 576 (C) 864 (D) 1,152 (E) 1,296

22. (fidgetboss_4000) Let $S = \{r_1, r_2, r_3, \dots, r_\mu\}$ be the set of all possible remainders when $15^n - 7^n$ is divided by 256, where n is a positive integer and μ is the number of elements in S . The sum $r_1 + r_2 + r_3 + \dots + r_\mu$ can be expressed as $p^q r$, where p, q, r are positive integers and p and r are as small as possible. Find $p + q + r$.

- (A) 40 (B) 41 (C) 42 (D) 43 (E) 44

23. (*fidgetboss_4000*) Four real numbers x_1, x_2, x_3, x_4 are randomly and independently selected from the range $[0, 9]$. Let the Sets S_1, S_2, S_3, S_4 contain all of the real numbers in the range $[x_1, x_1 + 1], [x_2, x_2 + 1], [x_3, x_3 + 1]$, and $[x_4, x_4 + 1]$, respectively. The probability that the four aforementioned sets are disjoint can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

(A) 95 (B) 96 (C) 97 (D) 98 (E) 99

24. (*fidgetboss_4000*) At a political discussion meeting held by Stalin, FDR, and Hitler, four communists (Stalin's team), four capitalists (FDR's team), and four fascists (Hitler's team) sit around a round table with 12 seats. To encourage political debate, there is a rule that no two people of the same political stance may sit adjacent to each other. Let N be the number of distinct seating arrangements following the rule. Find $\frac{N}{(4!)^3}$.

(A) 804 (B) 876 (C) 948 (D) 984 (E) 1,020

25. (*fidgetboss_4000*) Let
$$S_{n,k} = \sum_{a=0}^n \binom{a}{k} \binom{n-a}{k}.$$
 Find the remainder when
$$\sum_{n=0}^{200} \sum_{k=0}^{200} S_{n,k}$$
 is divided by 1000.

(A) 374 (B) 375 (C) 503 (D) 750 (E) 751