1. Search the Web for Cumani colour edge detector and write a report on it. Analyse how it may be better than doing Sobel edge detection three times on an RGB image.

   Ans.

   Cumani color edge detector :

Cumani operator, suggested by Cumani in 1989, utilizes the second order partial derivatives to find edges in color images. In Cumani, the color image, C, is treated as a two dimensional vector field as in equation given below;

$$C(x,y) = (C_R(x,y) + C_G(x,y) + C_B(x,y)) \quad \text{(i)}$$

In his work, Cumani suggests a new norm, the squared local contrast, which is a quadratic norm of directional derivatives toward the unit vector n = (n_1 . N_2) and ins given as;

$$S(p;\hat{n}) = K.n_1^2 + 2.F\, n_1.n_2 + H.n_2^2 \quad \text{(ii)}$$

where,

$$K=\sum_{i=1}^{3}\frac{\delta C_i}{\delta x}\cdot\frac{\delta C_i}{\delta x} \quad \text{(iii)}$$

$$F=\sum_{i=1}^{3}\frac{\delta C_i}{\delta x}\cdot\frac{\delta C_i}{\delta y} \quad \text{(iv)}$$

$$H=\sum_{i=1}^{3}\frac{\delta C_i}{\delta y}\cdot\frac{\delta C_i}{\delta y} \quad \text{(v)}$$

Cumani states that [4], the eigen values of the matrix in (vi) coincide with extreme points of the norm defined above and can be obtained if the unit vector $\hat{n}$ is the corresponding eigenvector, which enables the equations in (vii, viii) to be written in order to find these extreme eigen values and their corresponding eigenvectors.

$$A=\begin{bmatrix} K & F \\ F & H \end{bmatrix} \quad \text{(vi)}$$

$$\lambda_{+-}=\frac{K+H}{2}\pm\sqrt{\frac{(K+H)^2}{4}+F^2} \quad \text{(vii)}$$

$$n_{+-}=(\cos(\Theta_{+-}),\sin(\Theta_{+-})) \quad \text{(viii)}$$

$$\theta_{+-}=\begin{cases} \frac{\pi}{4}, if\ (K-H)=0 \text{ and } F>0, \\ -\frac{\pi}{4}, if\ (K-H)=0 \text{ and } F<0, \\ undefined, if\ K=H=F=0, \text{and} \\ \frac{1}{2}\tan^{-1}(\frac{2F}{K-H}), otherwise \end{cases} \quad \text{(ix)}$$

$$\theta_{-}=(\cos(\Theta_{+-}),\sin(\Theta_{+-})) \quad \text{(x)}$$

After the calculation of the extreme eigen values and eigenvectors, the minimum extremes can be omitted because they give the edges with the weakest magnitude and those are not the points of interest. Therefore, only the variables with + indices remain.

The extreme eigen values λ_+ are defined previously as the maximum of the norm $S(p;\hat{n}_+)$ defined in (ii). Therefore, the maxima of the eigen values λ_+ is searched for next, deriving the eigen value function in (vii). The zeros of this derivation are sought in order to find the edges, which corresponds to looking at the zero crossings of the derivative of the norm defined in (ii).

$$D_s(p;\hat{n}) := \nabla \lambda_+ . n_+ = K_x . n_1^3 + (K_y + 2. F_x) . n_1^2 . n_2 + (H_x + 2. F_y) . n_1 . n_2^2 + H_y . n_2^3 \qquad \text{(xi)}$$

This clearly illustrates that the equation in (xi) is a form of second order directional derivative, whose zero crossings are sought to detect edges in an image just like in gray-level methods.

For the 2nd part of the question,
Applying Sobel on each component of a color image means treating them independently without considering any relationship among them. Whereas Cumani being a vector valued technique considers those component in a combined manner duering the process of detection. So, hopefully its retaining more information about the image during the process.

Original image

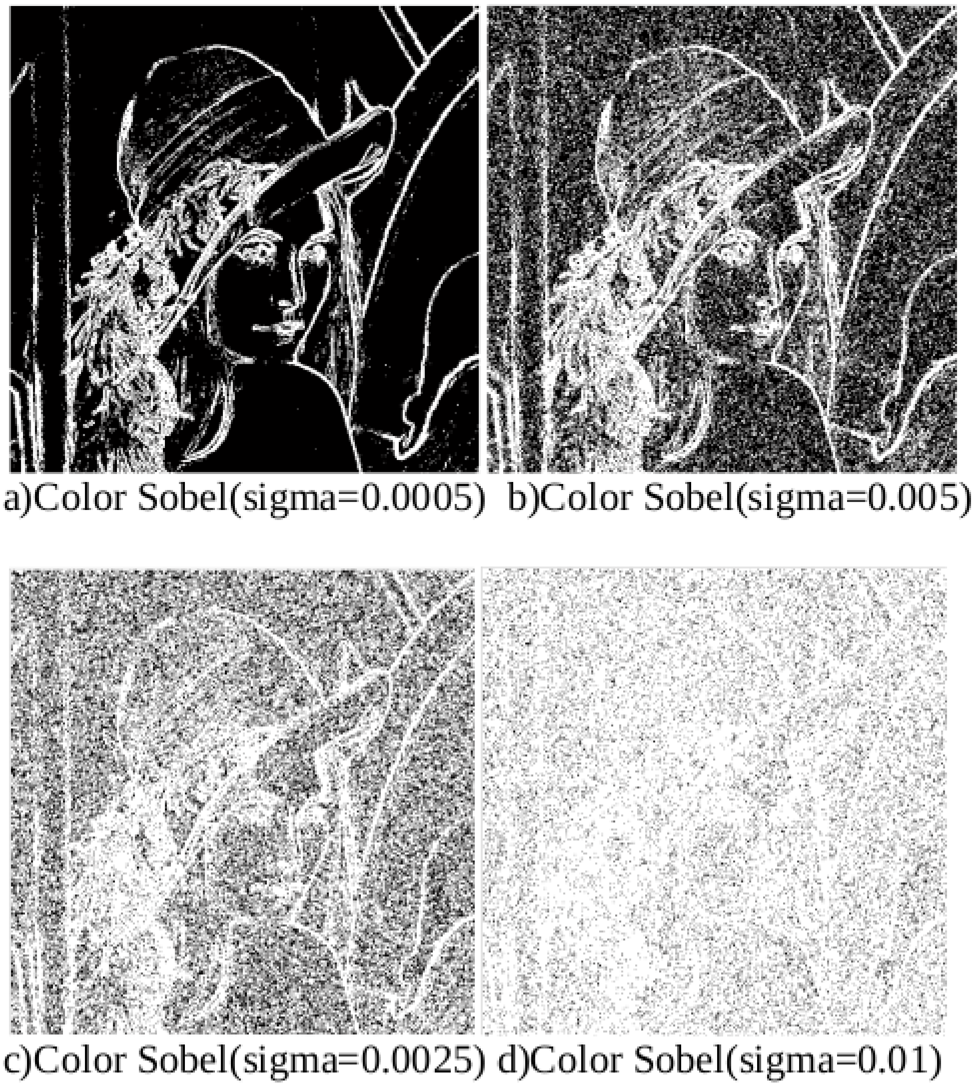
Color edge detection :

with Sobel operator

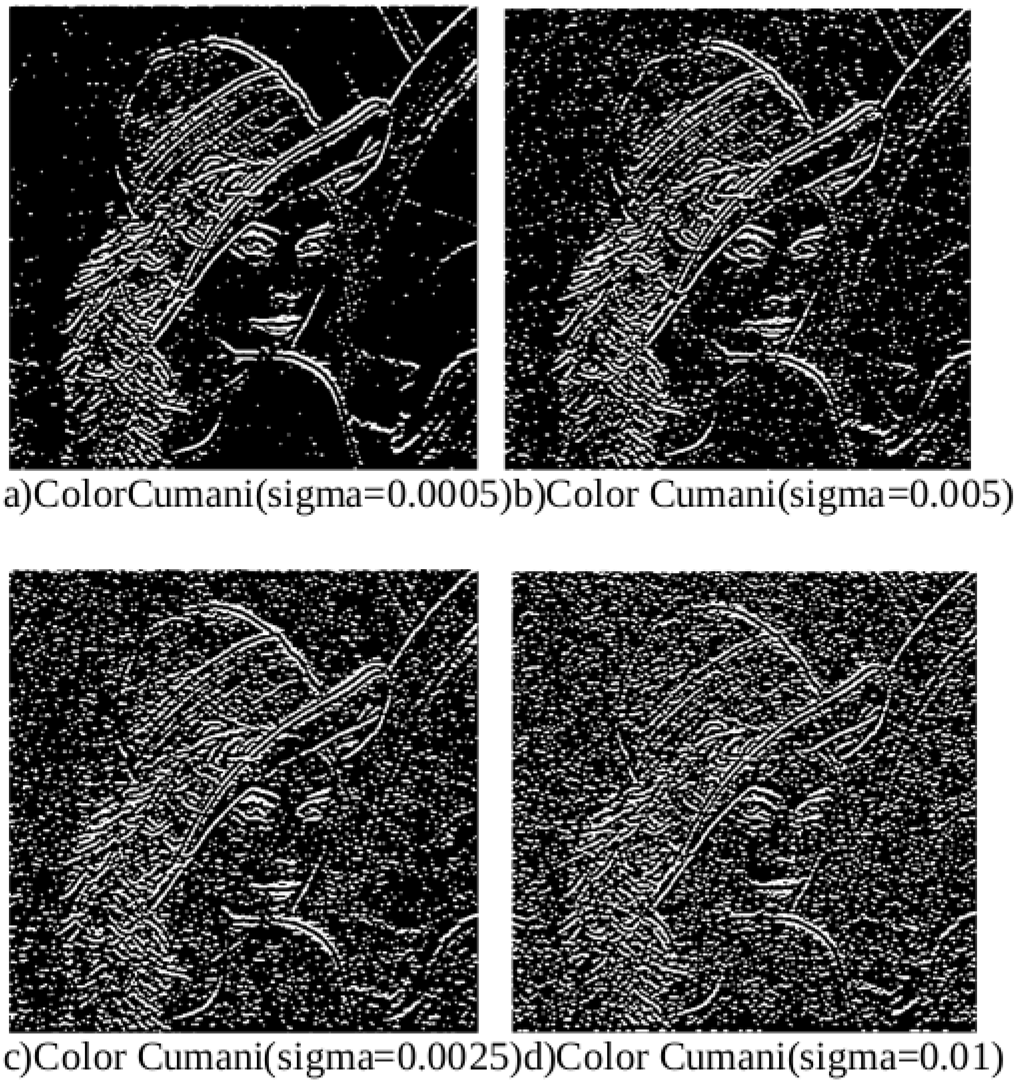
with Cumani operator

But on the contarary though Cumani operator is giving more details in the patterns caused by texture of the hair (and also in its reflection in mirror), it seems that sobel is doing better if you notice the region around lips, nose and eyes.

But things get more interesting if nose is present in the input image. You can see that the image has almose diappeared when the noise is been increased. Whereas for Cumani operator we can still get the actual edges though significant ammount of noise is present in it. So while considering noise susceptibility Cumani is a better choice than Sobel.



Monochromatic Sobel applied on noisy image
High succeptibility to noise



Cumani operator applied on noisy images
Result is less susceptible to noise

References:

1. COMPARING COLOR EDGE DETECTION TECHNIQUES
Conference Paper · November 2012
https://www.researchgate.net/publication/323005580
Hasan İhsan Turhan
Middle East Technical University

2. Implement `colour ranging` operation in RGB space on colour images.

   Ans. The implementation is given in ***rangingRGB.py*** inside code directory**.**
   Sample Input: *orchid.ppm*

Output: Removed all G component using ranging operation. Hence producing RB representaion of the input.

rangingRGB_(RB)orchid.bmp

3. Implement `colour ranging` operation in HSV space on colour images.

   Ans. Python implementation is given in ***rangingHSV.py*** inside code directory.

   Sample Input: *makaw-bad.ppm*

Output (for h_c = 0, h_bw = 22, s_c = 150, v_c = 50)

rangingHSV_(0-22-150-50)makaw.bmp

4. Implement vector median filter and any one of the basic vector edge detectors. Show example images to demonstrate that these are better than the grayscale versions for colour images.

   Ans. The code is given in **vmf.py** file under code directory.

   Sample Input : *childern-faded.ppm*

   Output 1 (considering canberra distance between vectors): ***vmf_canberra_children_faded.bmp***

Output 2 (considering euclidian distance): ***vmf_euclidian_children_faded.bmp***

Output 3 (considering Manhattan distance): ***vmf_manhattan_children_faded.bmp***

It seems that taking Canberra distance is giving slightly better result, cleaner and less distorted.

Input 2: *clock.ppm*

Output 1 (with Canberra distance): ***vmf_canberra_clock.bmp***

The edge detection implementations are given in

a. ***ved_color.py*** for color image

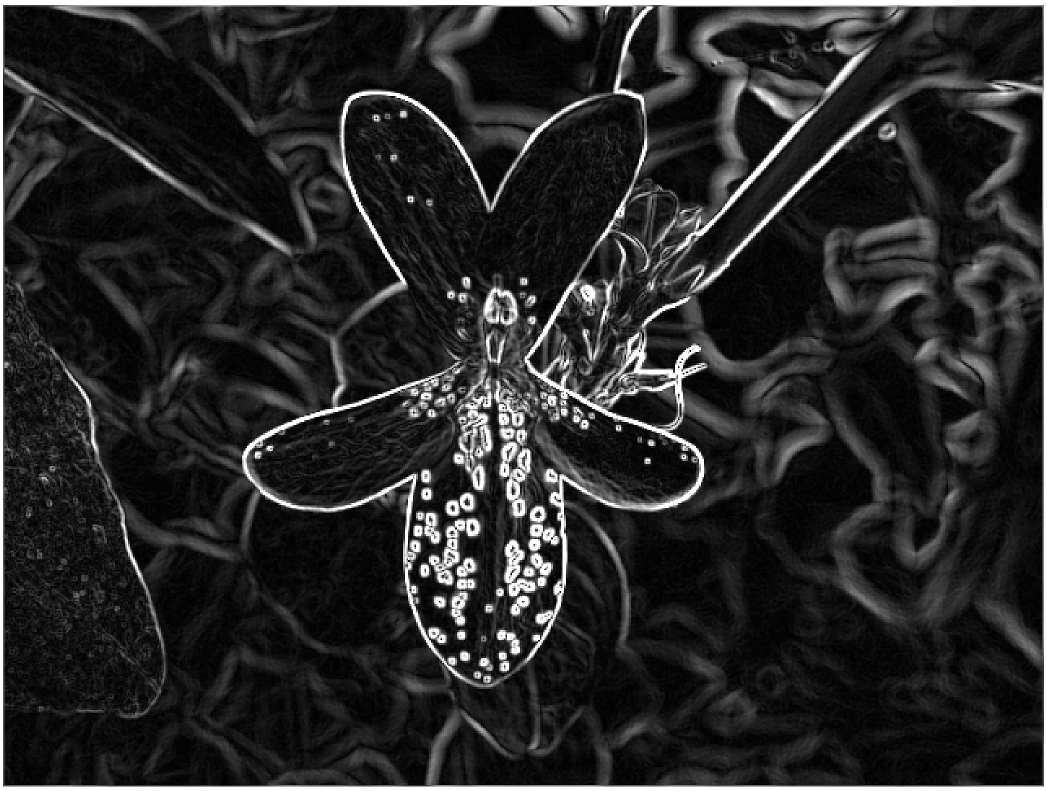
b. ***ved_gray.py*** for grayscale image

Inputs: ***orchid.ppm*** and ***orange-flower.ppm***

Outputs:

ved_color_prewit_orchid.bmp

ved_gray_sobel_orchid.bmp



Outputs: ***ved_gray_sobel_orange-flower.bmp***

ved_color_prewit_orange-flower.bmp