# 1 Question 2

Given,

$$g = f * h$$

Where, $g$ is the derivative of the 1D image, $h$ is the convolution kernel for the gradient and $f$ is the original image.

## 1.1 PART 1

Given

$$g = f * h$$

Then, take Fourier transform on both sides. From parseval's theorem we get the following.

$$F(g) = F(f)F(h)$$

So, we can get $f$ by dividing both sides by $F(h)$ and taking inverse fourier transform. We get the following formula for $f$.

$$f = F^{-1}\frac{F(g)}{F(h)}$$

However, we will run into problems when $F(h)$ is zero. This is where we would need boundary conditions to get the values at points where $F(h) = 0$. For instance we will require the values of the pixels of $f$ at the boundaries so that we can manually solve for the pixel values of the original image.

## 1.2 PART 2

Now, we have the gradients of a 2D image in the X and Y directions and we need to find the original image. Given

$$g_x = f * h_x$$

$$g_y = f * h_y$$

Then, take Fourier transform on both sides. From parseval's theorem we get the following.

$$F(g_x) = F(f)F(h_x)$$

$$F(g_y) = F(f)F(h_y)$$

So, we can get $f$ by dividing both sides by $F(h_x)$ and $F(h_y)$ respectively and taking inverse fourier transform. We get the following formula for $f$.

$$f = F^{-1}\frac{F(g_x)}{F(h_x)}$$

$$f = F^{-1}\frac{F(g_y)}{F(h_y)}$$

So now we can calculate $f$ easily at all the non zero points of $F(h_x)$ and $F(h_y)$. If any one is zero, then we can use the other equation with the non zero denominator to find $f$. If both the denominators are zero, then we will have to rely on boundary conditions - such as knowing the pixel values of the original image $f$ at the boundaries so that the values at those pixels can be manually calculated by performing the convolution and solving the equations. ( for example if the kernels for X and Y direction were $[-1, 1]$ and $[-1, 1]^T$, then with the boundary values, we can get the image from the derivatives by adding the adjacent pixel values).