Error mitigation for short-depth quantum circuits

Nate Stemen

Oct 2, 2020

Todays Paper

Error mitigation for short-depth quantum circuits

Kristan Temme, Sergey Bravyi and Jay M. Gambetta
IBM T.J. Watson Research Center, Yorktown Heights NY 10598
(Dated: November 7, 2017)

Two schemes are presented that mitigate the effect of errors and decoherence in short-depth quantum circuits. The size of the circuits for which these techniques can be applied is limited by the rate at which the errors in the computation are introduced. Near-term applications of early quantum devices, such as quantum simulations, rely on accurate estimates of expectation values to become relevant. Decoherence and gate errors lead to wrong estimates of the expectation values of observables used to evaluate the noisy circuit. The two schemes we discuss are deliberately simple and don't require additional qubit resources, so to be as practically relevant in current experiments as possible. The first method, extrapolation to the zero noise limit, subsequently cancels powers of the noise perturbations by an application of Richardson's deferred approach to the limit. The second method cancels errors by resampling randomized circuits according to a quasi-probability distribution.

Why do we need error mitigation?

PHYSICAL REVIEW A VOLUME 51, NUMBER 2 FEBRUARY 1995

Maintaining coherence in quantum computers

W. G. Unruh*
Canadian Institute for Advanced Research, Cosmology Program, Department of Physics, University of British Columbia, Vancouver, Canada V6T 1Z1
(Received 10 June 1994)

The effects of the inevitable coupling to external degrees of freedom of a quantum computer are examined. It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling be small, but the time taken in the quantum calculation must be less than the thermal time scale $\hbar/k_B T$. For longer times the condition on the strength of the coupling to the external world becomes much more stringent.

Why do we need error mitigation?

Scheme for reducing decoherence in quantum computer memory

Peter W. Shor*

AT&T Bell Laboratories, Room 2D-149, 600 Mountain Avenue, Murray Hill, New Jersey 07974

(Received 17 May 1995)

Recently, it was realized that use of the properties of quantum mechanics might speed up certain computations dramatically. Interest has since been growing in the area of quantum computation. One of the main difficulties of quantum computation is that decoherence destroys the information in a superposition of states contained in a quantum computer, thus making long computations impossible. It is shown how to reduce the effects of decoherence for information stored in quantum memory, assuming that the decoherence process acts independently on each of the bits stored in memory. This involves the use of a quantum analog of error-correcting codes.

Aren't we done?

Well, mitigation methods can be/are *expensive*

Focus of this paper

- Meaningful impact for current, and near term quantum computers

Extrapolation to the zero noise limit

- View computation as the evolution of a Hamiltonian
- Hamiltonian $K(t) = \sum_\alpha J_\alpha(t) P_\alpha$ where $J_\alpha(t)$ are "time dependent coupling coefficients" and $P_\alpha \in \{\mathbf{1}, X_j, Y_j, Z_j\}$.

$$\partial_t \rho(t) = -\mathrm{i}[K, \rho] + \lambda \mathcal{L}(\rho)$$

- $\lambda \mathcal{L}(\rho)$ being the error term with $\lambda \ll 1$
  - expected to be constant in time, and independent of J_α's
- Ultimately we care about $E(\lambda) = \mathrm{tr}\left(A \rho_\lambda(T)\right)$ where T is the "final time".
- Expand this as a "Born series"

$$E(\lambda) = \mathrm{tr}\left(A \rho_0(t)\right) + \sum_{k=1}^{n} a_k \lambda^k + R_{n+1}(\lambda, \mathcal{L}, T)$$

Extrapolation to the zero noise limit

How can we improve our expectation value?

- Ideal system is $\lambda = 0$
- If we could run the system with different error rates λ_i, $i \in \{1, 2, \ldots, n\}$ then we might be able to "extrapolate to the zero noise limit"
  - Experimentalists can't just tune their noise down
  - $J_\alpha^j(t) = c_j^{-1} J_\alpha(c_j^{-1} t)$ and evolve until $T_j = c_j T$ with $c_j > 1$
  - Can show $\rho_\lambda^j(T_j) = \rho_{c_j \lambda}(T)$ where ρ^j is the state that evolves with rescaled Hamiltonian.

Combine the results from each run as $\hat{E}(\lambda) = \sum_j \gamma_j E(c_j \lambda)$ where the γ_j's must satisfy

$$\sum_{j=0}^{n} \gamma_j = 1 \qquad\qquad \sum_{j=0}^{n} \gamma_j c_j^k = 0$$

Our estimates for the expectation value off by $\mathcal{O}(\lambda^{n+1})$ if rescaling is run $n+1$ times.

Probabalistic error cancellation

- Back to the circuit model, let $\Omega = \{\mathcal{O}_1, \ldots, \mathcal{O}_m\}$ be our noisy operations.
- $\alpha = (\alpha_1, \ldots, \alpha_L)$ implements a circuit of length L as $\mathcal{O}_\alpha = \mathcal{O}_{\alpha_L} \cdots \mathcal{O}_{\alpha_2} \mathcal{O}_{\alpha_1}$.

$$E_A(\alpha) = \mathrm{tr}\left[A\mathcal{O}_\alpha(|0\rangle\langle 0|^{\otimes n})\right]$$

- If $\Gamma = \{U_1, \ldots, U_k\}$ is a set of ideal gates, then our goal is to construct

$$U_\beta = \gamma_\beta \sum_{\alpha \in \Omega_L} P_\beta(\alpha)\sigma_\beta(\alpha)\mathcal{O}_\alpha$$

$$U_2 = 5\left(p_1\tilde{X}_1 - p_2\tilde{Y}_1 + p_3\tilde{Z}_1\right)$$

where $\sigma_\beta(\alpha) = \pm 1$, $P_\beta(\alpha)$ is a probability distribution and $\gamma_\beta \geq 1$.

Probabilistic error cancellation

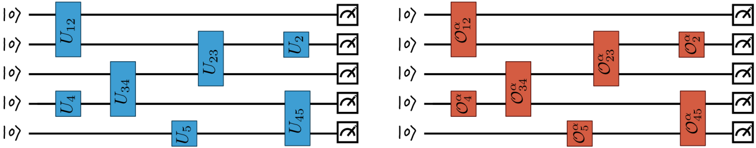

Figure 1: left circuit is the ideal, right circuit is approximated

Conclusion

Methods described here require no additional resource and work directly with physical qubits.

Zero noise extrapolation

- need control over time evolution of system
- "hinges on the assumption of a large time-scale separation between the dominant noise and the controlled dynamics"

Probabilistic error cancellation

- full characterization of the noisy gates