Method of Exhaustion
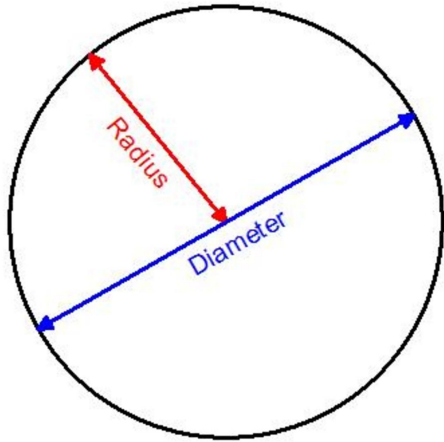
Nate Stemen 10/29/21

Microteaching Session

How do we know what the area of a circle is?

- A = π r², but we haven't always known π!
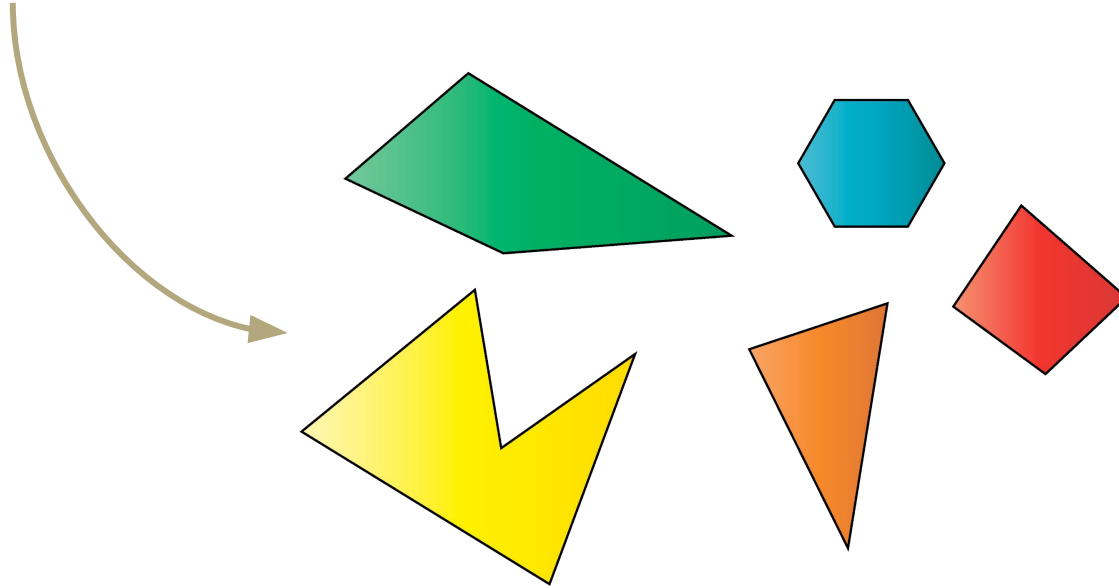
Raise your (physical/zoom) hand if

- You've heard of the method of exhaustion.

- You've taken a limit

- You've taken calculus

What is a regular polygon?

A polygon that is equiangular and equilateral

What is a regular polygon?

A polygon that is equiangular and equilateral

What is a regular polygon?

A polygon that is equiangular and equilateral.
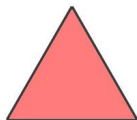
Equal interior angles

Equal length sides

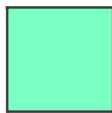# What is a regular polygon?

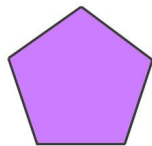A polygon that is equiangular and equilateral

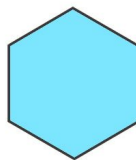Equal interior angles   Equal length sides
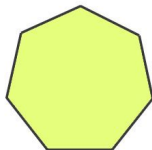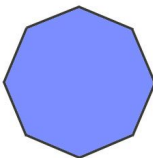
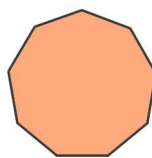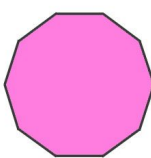Triangle   Quadrilateral   Pentagon   Hexagon
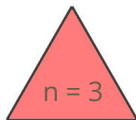
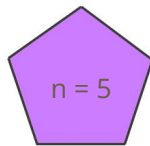Heptagon   Octagon   Nonagon   Decagon
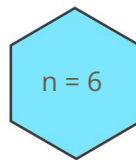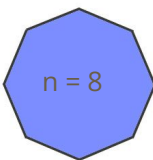
What is an n-gon?
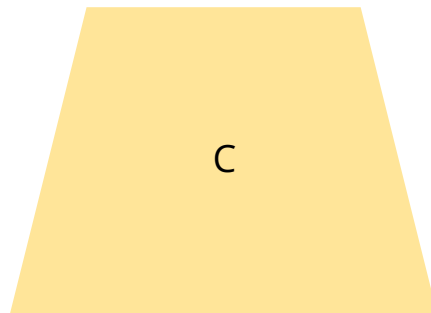
A regular polygon that has n sides.
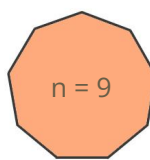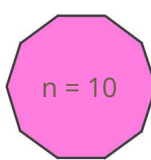
Is this a regular polygon?

Answer in the chat. E.g "A: yes, B: yes, C: yes"
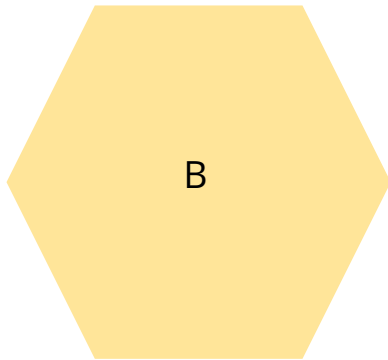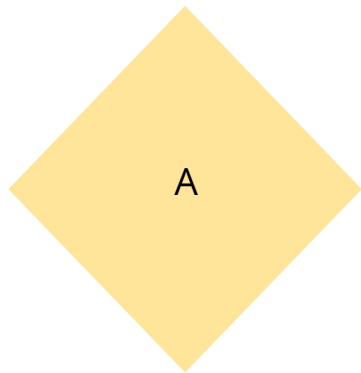
Circumscribed and inscribed polygons
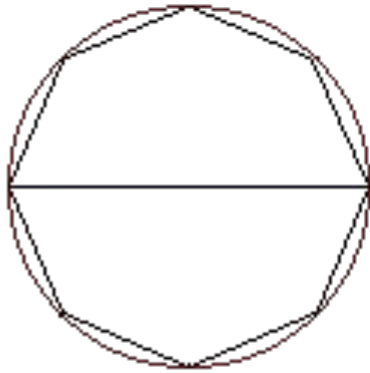
Inscribed polygon
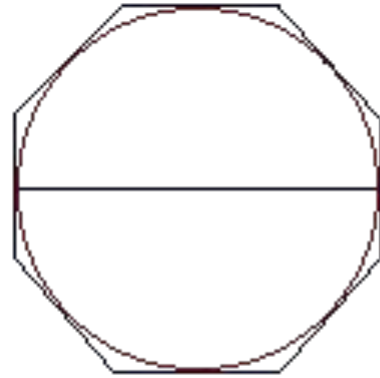
Circumscribed polygon

Area of circle
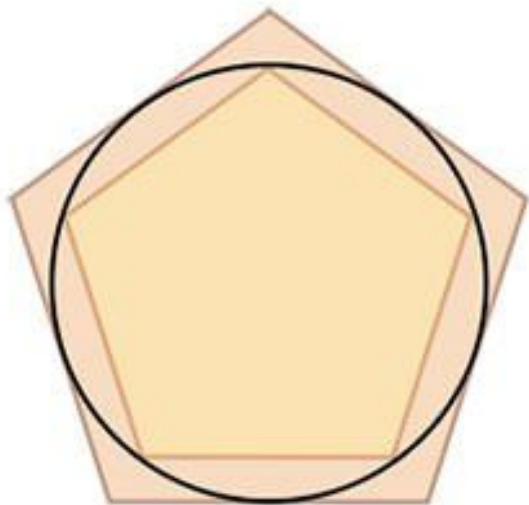
- Calculate area of inner (inscribed) polygon

- Calculate area of outer (circumscribed) polygone

- Average the two to get an approximation for the area of the circle

- If radius = 1, then $A_{\text{circle}} = \pi r^2 = \pi$

Area of n-gon
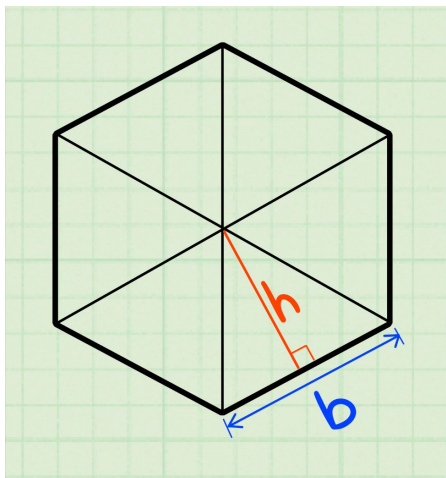
$$A_\Delta = \frac{1}{2}bh$$

$$A = \frac{n}{2}bh$$

Is the area of the inscribed polygons increasing or decreasing?
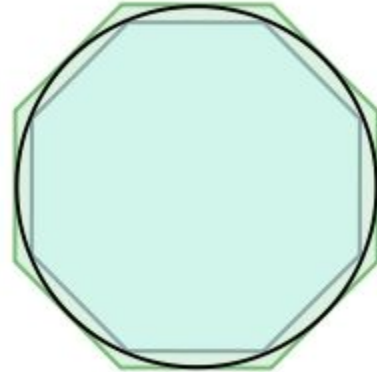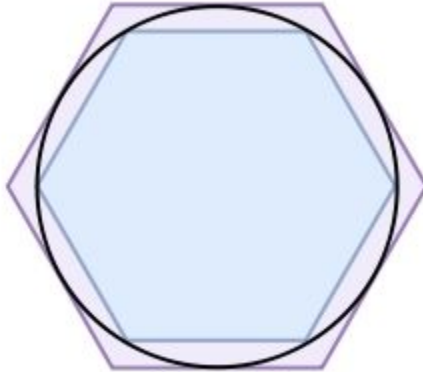
Is the area of the circumscribed polygons increasing or decreasing?

Let's approximate the area of a circle with pentagons!

$$A_\triangle = \frac{1}{2} bh$$

$$A = \frac{n}{2} bh$$

$$A_{\text{circle}} = \pi r^2 = \pi$$

1.453

1.175

1

How well does this work?

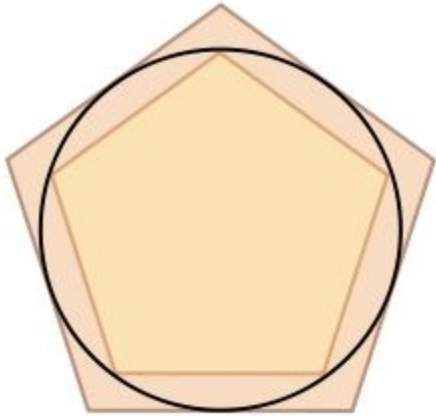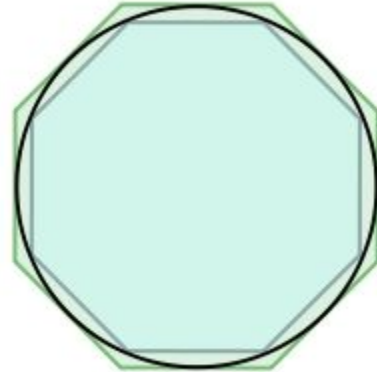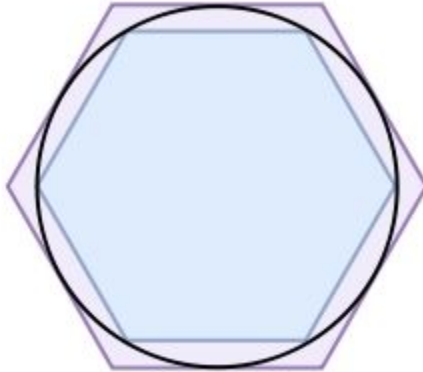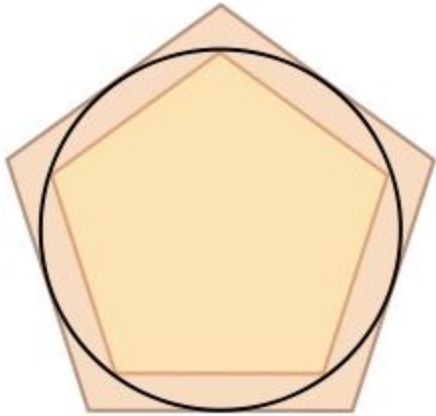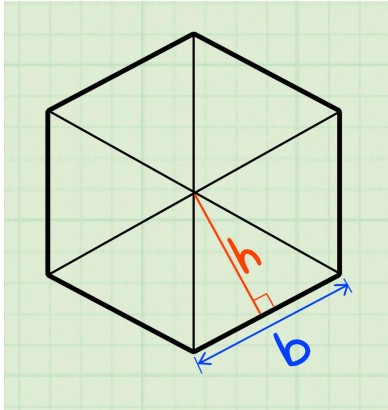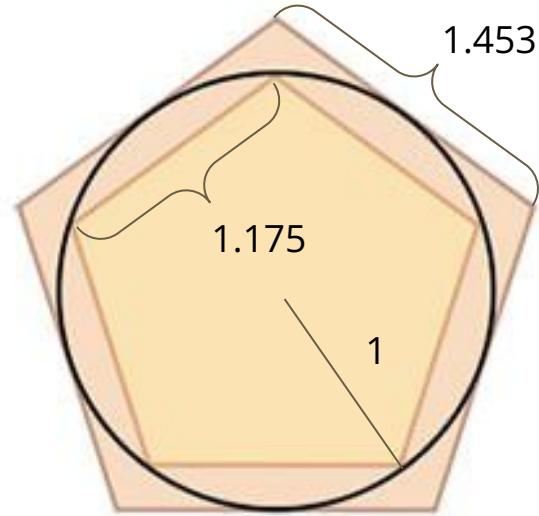| Sides	Inside	Outside	Pi guess	% Accuracy	Notes
4	2.828427125	4	3.414213562	91%	
8	3.061467459	3.313708499	3.187587979	98.535923322%	
16	3.121445152	3.182597878	3.152021515	99.668039026%	
32	3.136548491	3.151724907	3.144136699	99.919020520%	99.9% already!
64	3.140331157	3.144118385	3.142224771	99.979879075%	
128	3.141277251	3.14222363	3.14175044	99.994977489%	
256	3.141513801	3.141750369	3.141632085	99.998744854%	
512	3.14157294	3.141632081	3.141602511	99.999686244%	Five nines!
...					
524288	3.141592654	3.141592654	3.141592654	100.000000000%	Excel's limit

Liu Hui (265 - 420 AD)

Precursor to Calculus

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$