

$$* \quad \frac{df(x=x_0)}{dx} = f[x_0, x_1] + \sum_{j=2}^N f[x_0, \dots, x_j] \cdot (x_0 - x_1) \dots (x_0 - x_{j-1}).$$

$$P(x) = f[x_0] + f[x_0, x_1] \cdot (x - x_0) + f[x_0, x_1, x_2] \cdot (x - x_0)(x - x_1) \dots f[x_0, \dots, x_N] \cdot (x - x_0) \dots (x - x_{N-1}).$$

$$\text{here; } P(x_0) = f[x_0], \dots, P(x_{N-1}) = f[x_{N-1}].$$

$$\text{Thus: } \left. \frac{df(x)}{dx} \right|_{x=x_0} = \left. \frac{d}{dx} P(x) \right|_{x=x_0}$$

$$\therefore \left. \frac{df(x)}{dx} \right|_{x=x_0} = \left. \frac{d}{dx} \left[ f[x_0] + f[x_0, x_1] \cdot (x - x_0) \dots f[x_0, \dots, x_N] \cdot (x - x_0) \dots (x - x_{N-1}) \right] \right|_{x=x_0}$$

$$\text{For Any Term:} \\ \left. \frac{d}{dx} \left[ f[x_0, \dots, x_N] \cdot (x - x_0) \dots (x - x_{N-1}) \right] \right|_{x=x_0} \\ = f[x_0, \dots, x_N] \cdot \left[ 1 \cdot (x - x_1) \dots (x - x_{N-1}) + \right. \\ \left. \text{Correct} \right]$$

$$(x - x_0) \cdot 1 \cdot (x - x_2) \dots (x - x_{N-1}) + (x - x_0)(x - x_1) \dots (x - x_{N-2}) \cdot 1 \Big|_{x=x_0}$$

$$\Rightarrow f[x_0, \dots, x_N] \left[ (x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_{N-1}) + 0 + \right.$$

$$0 \dots + 0 \Big] (x = x_0).$$

$$\Rightarrow 0 + f[x_0, x_1] + f[x_0, x_1, x_2] (x_0 - x_1) \dots f[x_0, x_1, \dots, x_N] (x_0 - x_1) \dots (x_0 - x_{N-1}).$$

$$\Rightarrow \frac{d}{dx} f(x) \Big|_{x=x_0} = f[x_0, x_1] + \sum_{j=2}^n f[x_0, \dots, x_j] \cdot (x_1 - x_0) \dots (x_0 - x_{j-1})$$