$*$ Integral :

$$\int_{x_0}^{x_3} dx\, f(x) = \frac{3h}{8}\left(f_0 + 3f_1 + 3f_2 + f_3\right)$$

$$\int_{x_0}^{x_N} dx \cdot f(x) = \int_{x_0}^{x_3} dx \cdot f(x) + \int_{x_3}^{x_6} dx \cdot f(x)$$

$$+ \int_{x_6}^{x_9} dx \cdot f(x) \cdots \cdots \int_{x_{N-3}}^{x_N} dx\, f(x)$$

$$\int_{x_0}^{x_N} dx \cdot f(x) = \frac{3h}{8}\Big[f_0 + 3f_1 + 3f_2 + f_3 + f_3 + 3f_4 + 3f_5$$

$$+ f_6 + f_6 + 3f_7 + \cdots \cdots f_{N-3} + 3f_{N-2} + 3f_{N-1} + f_N \Big]$$

$$\int_{x_0}^{x_N} dx \cdot f(x) = \frac{3h}{8}\Big[f_0 + 3f_1 + 3f_2 + 2f_3 + 3f_4$$

$$+ 3f_5 + 2f_6 + 3f_7 \cdots \cdots 2f_{N-3} + 3f_{N-2} + 3f_{N-1}$$
$$+ f_N \Big]$$

$\#$ $\sum_n n \cdot f_i$

and $n = 1$ for $i == 0$ and $i == N-1$.

$n = 2$ for $i \% 3 == 0$.

and $n = 3$ for others.