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DAA

HOMEWORK -3 (WRITTEN)

Question-1

Array = [10, 7, 3, 8, 1, 9, 0]
loop till A.length

→ $j = 2$ key = $A[j] = 7$
$i = j-1 = 1$

while $i > 0$ and $A[i] > key$
$A[1] > 7$
$10 > 7$
true

$A[i+1] = A[i]$
$i = i-1 = 0$ $A[1] = \cancel{10}\ 7$

7	10	3	8	1	9	0

→ $j = 3$ key = $A[3] = 3$
while $i = 2 > 0$ and $A[2] = 10 > 3$
true

$A[3] = 10$

while $i = 1 > 0$ and $A[1] = 7 > 3$
$A[2] = 7$

$A[1] = 3$

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3	7	10	8	1	9	0

→ $j = 4$ $\quad$ $key = A[4] = 8$

~~i→3~~ $i = 3 > 0$ and $A[3] = 10 > 8$

$A[4] = 10$

$i = 2 > 0$ and $A[2] = 7 \not> 8$

$A[3] = 8$

3	7	8	10	1	9	0

→ $j = 5$ $\quad$ $key = A[5] = 1$

$i = 4 > 0$ and $A[4] = 10 > 1$ true

$A[5] = 10$

$i = 3 > 0$ and $A[3] = 8 > 1$ true

$A[4] = 8$

$i = 2 > 0$ and $A[2] = 7 > 1$ true

$A[3] = 7$

$i = 1 > 0$ and $A[1] = 3 > 1$ true

$A[2] = 3$

$i = 0 \not> 0$ break

$A[1] = 1$

1	3	7	8	10	9	0

→ $j = 6$ key $= A[6] = 9$

(key)

$i = 5 > 0$ and $A[5] = 10 > 9$ true

$A[6] = 10$

$i = 4 > 0$ and $A[4] = 8 \not> 9$ break

$A[5] = 9$

1	3	7	8	9	10	0

→ $j = 7$ key $= A[7] = 0$

(key)

$i = 6 > 0$ and $A[6] = 10 > 0$ true

$A[7] = 10$

$i = 5 > 0$ and $A[5] = 9 > 0$ true

$A[6] = 9$

$i = 4 > 0$ and $A[4] = 8 > 0$ true

$A[5] = 8$

$i = 3 > 0$ and $A[3] = 7 > 0$ true

$A[4] = 7$

$i = 2 > 0$ and $A[2] = 3 > 0$ true

$A[3] = 3$

$i = 1 > 0$ and $A[1] = 1 > 0$ true

$A[2] = 1$

$i = 0 \ngtr 0$

$A[1] = 0$

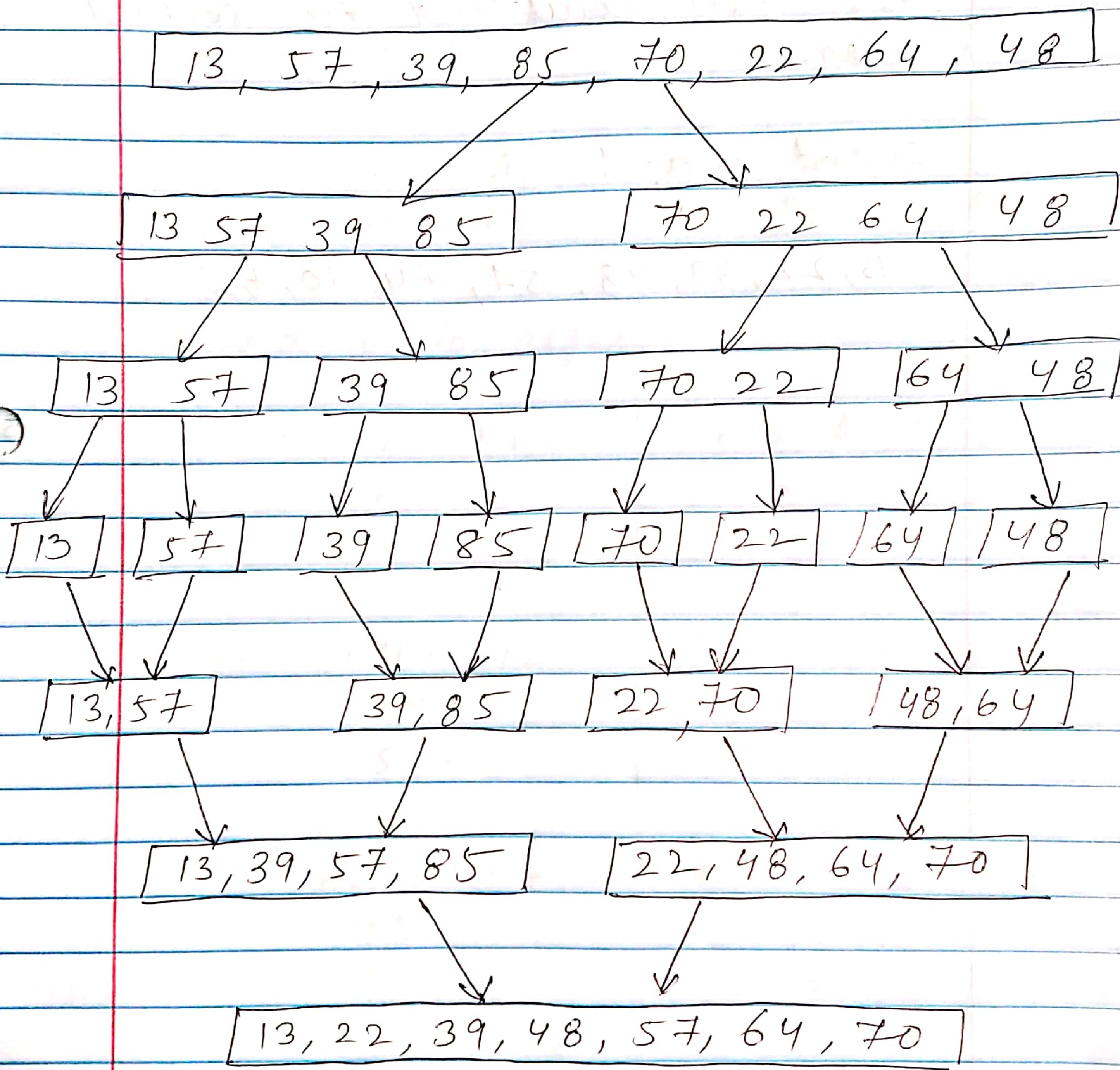
0	1	3	7	8	9	10

Sorted array is

$\{0, 1, 3, 7, 8, 9, 10\}$

Question-2

[13, 57, 39, 85, 70, 22, 64, 48]



Merge sort uses divide and conquer in which the array is divided into individual elements and then combined in sorted order.

Sorted array:

13, 22, 39, 48, 57, 64, 70, 85

Question-3

Strassen's method

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 1 & 3 \end{bmatrix}$

$\begin{bmatrix} A_{11} = 2 & A_{12} = 3 \\ A_{21} = 4 & A_{22} \ 5 \end{bmatrix}$ $\begin{bmatrix} B_{11} = 5 & B_{12} = 6 \\ B_{21} = 1 & B_{22} = 3 \end{bmatrix}$

According to algorithm,

$S_1 = B_{12} - B_{22}$
$= 6 - 3$
$= 3$

$S_2 = A_{11} + A_{12}$
$= 2 + 3$
$= 5$

$S_3 = A_{21} + A_{22}$
$= 4 + 5$
$= 9$

$S_4 = B_{21} - B_{11}$
$= 1 - 5$
$= -4$

$S_5 = A_{11} + A_{22}$
$= 2 + 5$
$= 7$

$S_6 = A_{11} + B_{22}$
$= 5 + 3$
$= 8$

$S_7 = A_{12} - A_{22}$
$= 3 - 5$
$= -2$

$S_8 = B_{21} + B_{22}$
$= 1 + 3$
$= 4$

$S_9 = A_{11} - A_{21}$
$= 2 - 4$
$= -2$

$S_{10} = B_{11} + B_{12}$
$= 5 + 6$
$= 11$

Compute seven matrix products.

$P_1 = A_{11} S_1$
$= 2 \times 3$
$= 6$

$P_2 = S_2 B_{22}$
$= 5 \cdot 3$
$= 15$

$P_3 = S_3 B_{11}$
$= 9 \cdot 5$
$= 45$

$P_4 = A_{22} S_4$
$= 5 \cdot (-4)$
$= -20$

$P_5 = S_5 \cdot S_6$
$= 7 \cdot 8$
$= 56$

$P_6 = S_7 \cdot S_8$
$= (-2) 4$
$= -8$

$P_7 = S_9 S_{10}$
$= (-2)\,11$
$= -22$

compute the desired matrices $C_{11}, C_{12}, C_{21}, C_{22}$ by adding / subtracting.

$C_{11} = P_5 + P_4 - P_2 + P_6$
$= 56 + (-20) - 15 + (-8)$
$= 13$

$C_{12} = P_1 + P_2$
$= 6 + 15$
$= 21$

$C_{21} = P_3 + P_4$
$= 45 + (-20)$
$= 25$

$C_{22} = P_5 + P_1 - P_3 - P_4$
$= 56 + 6 - 45 - (-22)$
$= 39$

$$\text{Array } C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 21 \\ 25 & 39 \end{bmatrix}$$

Question- 4

$A = [13, 9, 5, 7, 3, 1, 10, 6, 11, 2]$

Partion of Array $[p \ldots r]$

$p = 1$ $r = 10$.

Algorithm :

portition (A, p, r)
$x = A[r]$
$i = p - 1$
for $j = p$ to $r - 1$
if $A[j] \leq x$
$i = i + 1$
exchange $A[i]$ with $A[j]$
exchange $A[i+1]$ with $A[r]$
return $i+1$.

$p = 1$ $r = 10$

$x = A[r] = A[10] = 2$

$i = 0$

→ $j = p = 1$

$A[j] \leq x$

$13 \leq 2$ false, j++

~~exchange A[i+1] with A[r]~~

~~A[1] with A[10]~~

13	9	5	7	3	1	10	6	11	2

→ $j = 2$

$A[j] \leq x$

$9 \leq 2$ false, j++

13	**9**	5	7	3	1	10	6	11	2

→ $j = 3$

$A[j] \leq x$

~~A~~ $5 \leq 2$ false, j++

exchanging ....

13	9	5	7	3	1	10	6	11	2

→ $j=4$

$A[j] \leq x$

$7 \leq 2$ false, $j{+}{+}$.

13	9	5	7	3	1	10	6	11	2

→ $j=5$

$A[j] \leq x$

$3 \leq 2$ false, $j{+}{+}$

13	9	5	7	3	1	10	6	11	2

→ $j=6$

$A[j] \leq x$

$1 \leq 2$ true

$i = i+1$

$i = 0+1$

$i = 1$

$A[i] = 13$

$A[j] = 1$

exchanging $A[i]$ with $A[j]$

~~A[i] = 1~~ $\quad$ A[1] = 1

~~1 9 5 A[j] = 13~~ $\quad$ A[6] = 13

1	9	5	7	3	13	10	6	11	2

→ $j = 7$

$A[j] \leq x$

$A[7] \leq 2$

$10 \leq 2$ false, $j{+}{+}$

1	9	5	7	3	13	10	6	11	2

→ $j = 8$

$A[j] \leq x$

$A[8] \leq 2$

$6 \leq 2$ false, $j{+}{+}$.

1	9	5	7	3	13	10	6	11	2

Question-4

$A = [13, 9, 5, 7, 3, 1, 10, 6, 11, 2]$

→ $j = 9$

$A[j] \le x$

$A[9] \le 2$

$11 \le 2$ false.

exchange

$A[i+1] = A[2] = 9$

$A[r] = A[10] = 2$

$A[2] = 2$

$A[10] = 9$

1	2	5	7	3	13	10	6	11	9

return $i+1 = 2\,3$

Therefore $q = 2$, so values on the left of 2 are sorted independently from values to the right of 2.

Question - 5

Algorithm :

find (Array A) // sorted array.
size = sizeof (A)
if (size == 1) {
A[0] = 1
}
search_function (A, 0, size-1)

search_function (A, min, max)
if (max >= min)
mid = (max + min) / 2
if (mid == A[mid])
return mid
else if mid > A[mid]
return search_function (
A, (mid+1), max)
else
return search_function (
A, low, mid-1)

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We can use recursive since we have array in sorted order.

Analysing run time complexity.

run time of n elements is $T(n)$.

Dividing the array will result in $T(n/2)$ and for stand alone statements run time would be $\theta(1)$.

$$T(n) = T\left(n/2\right) + 1$$

$$a = 1 \quad b = 2$$

$$f(n) = 1 = \theta\left(n^{\log_2 1}\right) = \theta(n^0) = \theta(1)$$

~~applying applying~~

~~applying~~ applying case 2,

$$T(n) = \theta(n^0 \log n) =$$

$$\theta(\log n).$$

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Proving corectness of algorithm.

If $A[i] = i$ is present at any index, it will generate the result and we can get i.

Initialization : If initial array A contains i, the the sub arrays will also contain i.

Maintanances to find the i, we divide the array into two sub arrays

We check three conditions, if element = A[mid], or if is on the A[0 ....p] or A[r.... n] part of the array,

If it is present in the array it will be present in the sub array.

Termination : It will check both the subarray and i is found at the last position of the 2nd sub array.

Hence we proved the loop invariant for the above code.

Question-6

$$P(X) = \frac{1}{X}$$

$P(1) = 1$ $\quad P(2) = \frac{1}{2}$ $\quad P(3) = \frac{1}{3}$

$P(4) = \frac{1}{4}$ $\quad P(5) = \frac{1}{5}$ $\quad P(6) = \frac{1}{6}$

P(number greater than 3) $= P(4) + P(5) + P(6)$

$$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

$$= \frac{15 + 12 + 10}{60}$$

$$= \frac{37}{60}$$

$$= 0.616.$$