Putting Units 1-3 Into Practice

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The Context

We are members of the statistical consulting firm, STAT461 Inc., and we have been approached by a new client, J & N Toys.

J & N Toys manufactures a variety of children's plastic toys and would like to limit how much scrap content (i.e., defective toys) that gets generated in each production run. They have considered the introduction of a statistical process control (SPC) as well as an engineering process control (EPC) in order to achieve this aim at each of their nine manufacturing plants (located in different parts of three different regions). To assess the effects of these quality control practices, they have asked us to design and carry out an experiment over the course of a six-month period.

Of primary interest is the impact of using SPC, SPC + EPC, or no quality control on defect rate (measured as decrease in the number of defective toys per 1000 toys) at the end of the six month period.

What is the Statistical Research Question?

Does a quality control process decrease the defect rate for plastic toys manufactured by J & N Toys?

Study Design

With our SRQ in mind, let us design an experiment which will allow us to answer the SRQ.

Key Elements

- Response: Defect rate decrease (number of defective toys / 1000)
- Measurement Unit: Manufacturing Plants (9; census)
- Factor: Quality Control Process (3 levels, fixed)
- Experimental Design:
  1) Experimental Units: Manufacturing Plant (3 per treatment)
  2) Treatments: Statistical Process Control (SPC), Statistical Process Control and Engineering Process Control (SPC + EPC), and no quality control process (Null Treatment)
  3) Treatment Assignment: Create cards with each plant's name and place the cards into a bag. Shake the bag and draw out cards one at a time without replacement. The first three drawn will get the Null treatment, next three SPC, and the last three will get SPC + EPC.
- What kind of design? Completely Randomized Design (Single Factor)

Study Design Description

To answer the research question "Does a quality control process decrease the defect rate for plastic toys manufactured by J & N Toys?", we will propose a completely randomized design with a single factor (quality control method). We will use all nine of the manufacturing plants in the study and measure the change in the defect rate (number of defective toys per 1000 toys) from the start of the study to the end. A positive

value will indicate that production decreased the defect rate; negative value will indicate an increase in the defect rate.

We will use three quality control methods in the study: a statistical process control (SPC), a statistical process control partnered with an engineering process control (SPC + EPC), and no process control methods (null treatment). We will apply each treatment to three of nine plants. To do so, we will place the name of each plant on an otherwise identical card and then place the cards into a bag. After thoroughly shaking the bag, we will draw the cards out, one at a time, placing the cards in a row according to drawn order. The first three cards drawn will get the null treatment, the second three drawn will get SPC, and the last three will get SPC + EPC. We will measure the defect rate twice: once at the start of the six-month trial period and once at the end. The difference (start – end) will be recorded.

Exploratory Data Analysis

Figure 1 shows side-by-side box plots for our nine plants' defect rates. While eight of the nine plants showed a decrease in their defect rates, one plant did experience an increase of 2.1 defective toys/1000 toys. That being said, we do not see any potential outliers in Figure 1 according to the *IQR* rule of thumb. Further, each plot take up about equal vertical spacing indicating similar levels of variation. There are some differences in the relative densities: the upper 50% of Null Treatment and SPC are more compact and thus denser than the corresponding portion of SPC + EPC. For SPC + EPC, the lower 50% is the denser portion.

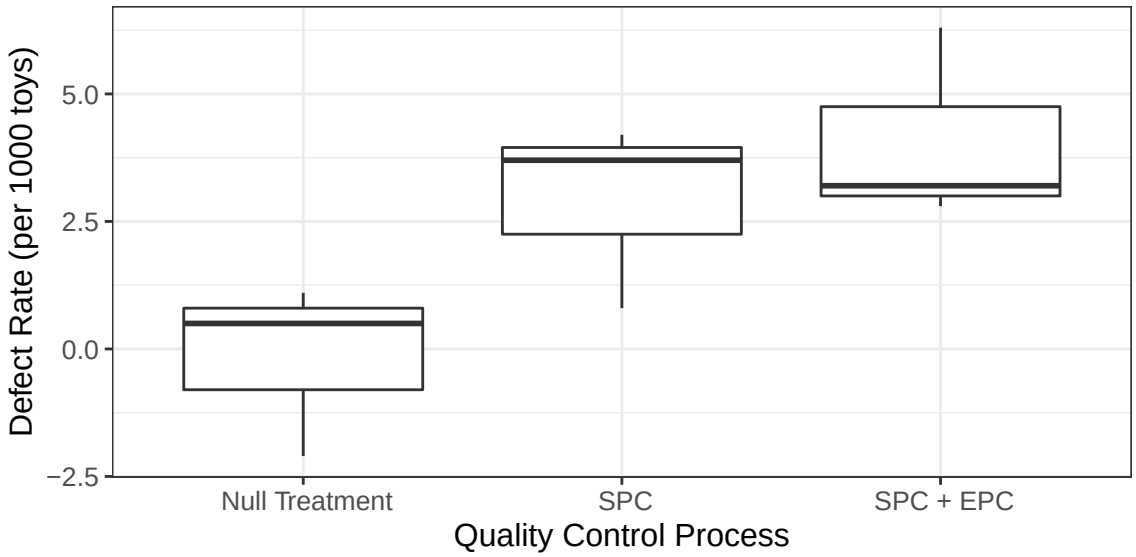

Figure 1: Box Plots for Defect Rate by Quality Control Process

Looking at the histogram (Figure 2), we can see that our nine plants are relatively tightly packed together in terms of their changes in defect rates. We can see a few more differences when we examine Table 1. We can see that plants using SPC are routinely about 3 defective parts/1000 better off than those that did not use any quality control method (see the values of the five number summary). There is a similar boost over the null treatment for SPC + EPC condition. In terms of the values of the *sample arithmetic means*, we can see that the conditions involving process control methods had a better performance than the plants which did not. However, this could be result of the single plant which did experience an increase in defect rate. All groups have similar values of the *sample arithmetic standard deviation* as well as *sample excess kurtosis* indicating that they all have similar levels of variation within their groups and rate of potential outliers. The values of *sample skewness* are a touch concerning: while none of them are terribly far from zero, two of the groups have slight negative skewness while the third group (SPC + EPC) has slight positive skewness. This might be a concern when assessing the Gaussian assumption.

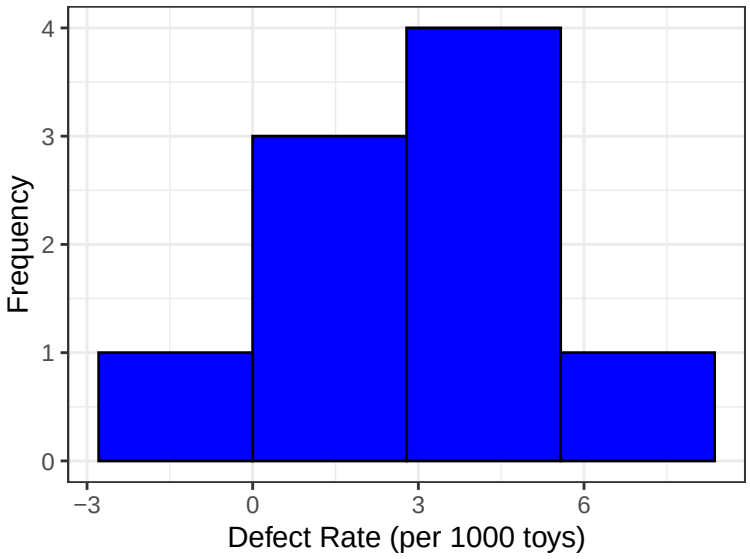

Figure 2: Histogram of Defect Rate

Table 1: Summary Statistics for Defect Rates by Quality Control Process

	n	Min	Q1	Median	Q3	Max	MAD	SAM	SASD	Sample Skew	Sample Ex. Kurtosis
Null Treatment	3	-2.1	-0.80	0.5	0.80	1.1	0.890	-0.167	1.701	-0.332	-2.333
SPC	3	0.8	2.25	3.7	3.95	4.2	0.741	2.900	1.836	-0.353	-2.333
SPC + EPC	3	2.8	3.00	3.2	4.75	6.3	0.593	4.100	1.916	0.366	-2.333

Statistical Inference Methods

Given our study design being a completely randomized design with a single factor, our goal will be to use a One-way ANOVA approach to answer the statistical research question. This method is appropriate in that we are wanting to explore the impact of our categorized factor (quality control method) on the continuous change in defect rate. Further, the Hasse diagram in Figure 3 shows us that we can conceptualize our study as an additive model with sufficient *degrees of freedom* (the numbers on right of each node) to allow for estimation of terms. We will use a factor effects model with the constraint that the treatment effects add to zero.

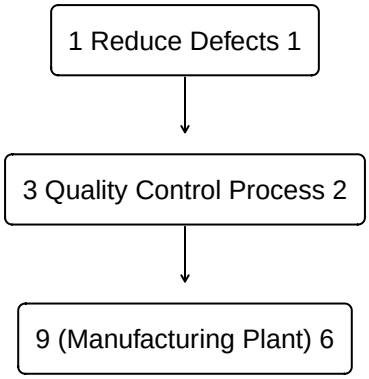

Figure 3: Hasse Diagram for Process Control Study

Hypotheses and Method

Given our statistical research question ("Does a quality control process decrease the defect rate for plastic toys manufactured by J & N Toys?"), we may frame our two hypotheses as:

$$H_0 : y_{ij} = \mu_{..} + \epsilon_{ij}$$
$$H_1 : y_{ij} = \mu_{..} + \alpha_i + \epsilon_{ij}$$

for some $\alpha_i \neq 0$.

To decide between the hypotheses and answer our research question, we will use the parametric shortcut of the One-way ANOVA F test with an Unusualness Threshold/Level of Significance of 10% $UT = 0.1$.

Assumptions

There are three assumptions to the parametric shortcut (One-way ANOVA F test): our residuals follow a Gaussian distribution, homoscedasticity, and independence of observations.

Figure 4 shows the Quantile-Quantile plot of our model's residuals against a Gaussian distribution. All nine of our observations are within the dashed envelope, thus suggesting that our residuals are consistent with a Gaussian distribution. This does alleviate the concerns we raised earlier about skewness from Table 1.

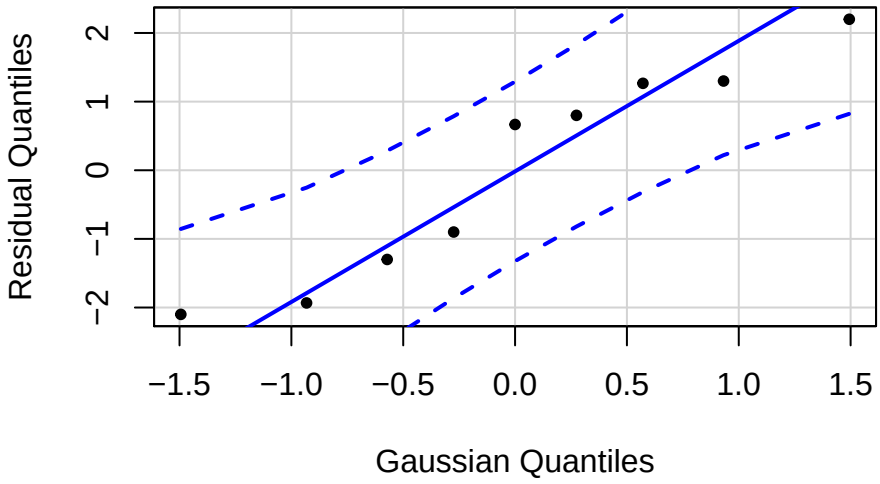

Figure 4: QQ Plot for Residuals from the Toy Quality Control Study

To assess homoscedasticity, we'll turn to Figure 5. We can see that each strip in the plot takes up approximately the same amount of vertical space and there are no worrisome patterns (e.g., a funnel from left to right) detectable in the plot. This suggests that our data satisfy the homoscedasticity assumption.

We will turn our attention to the last assumption of the parametric shortcut: the independence of observations. We must note that while the assignment of treatments is not independent, this assumption is about whether the manufacturing plants impact one another. We can tackle this assumption in several ways.

First, we will note that the nine plants are spaced apart from one another; together occupying three different regions. Even though plants might be in the same region, we can reason that they would not be spaced close enough to each other to cause production problems.

Second, we did record the order we took measurements, thus, we can examine an index plot (Figure 6) to see if there are any worrisome patterns based on the order in which we took measurements. There are no worrisome patterns that is detectable. This in conjunction with our prior reasoning indicate that we can be satisified that the independence of observations is met.

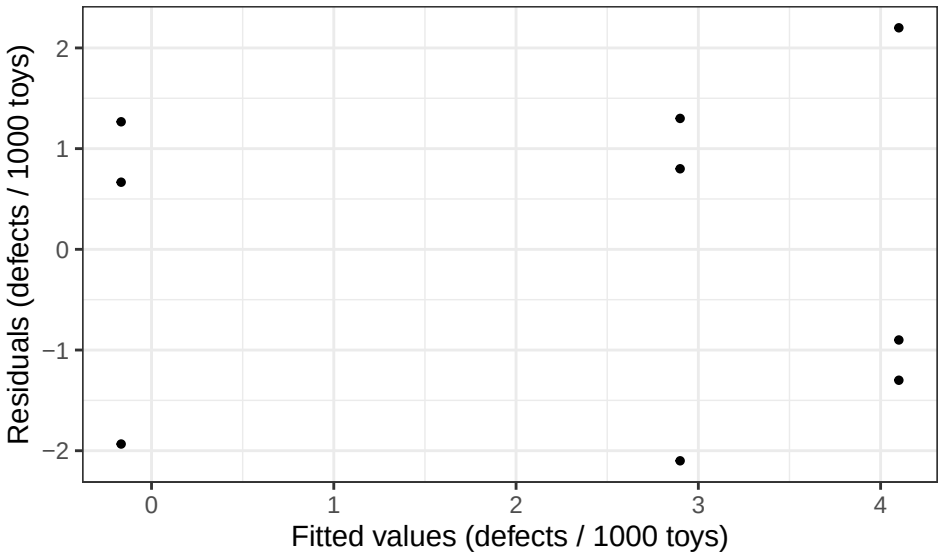

Figure 5: Strip Chart of the Residuals from the Toy Quality Control Study

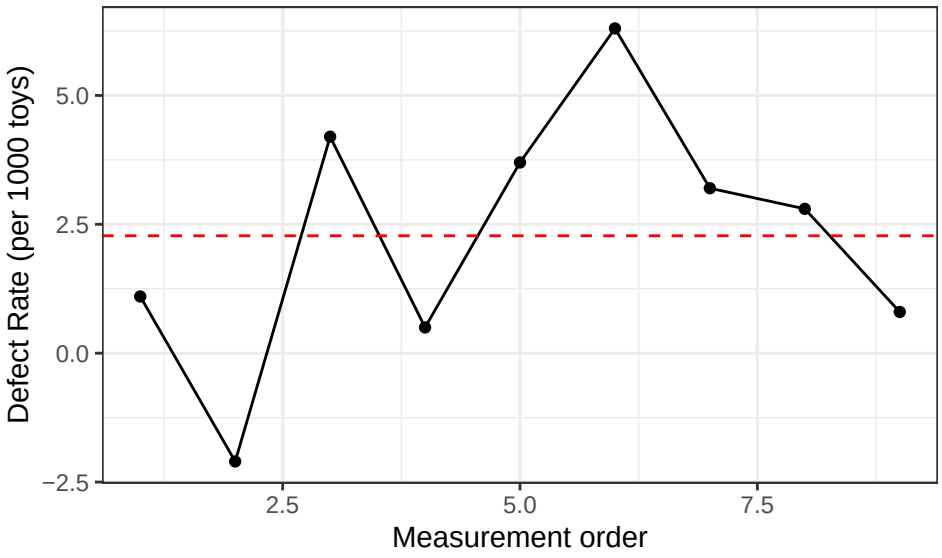

Figure 6: Index Plot for Defect Rate from the Toy Quality Control Study

Results

With the assumptions and base requirements of the One-way ANOVA F test checked, we can turn to the results of our analysis as displayed in Table 2. We see that $F \approx 4.39$, indicating that our quality control process captured approximately 4.39 times as much variation in the change in defect rate as our residuals. Under the null hypothesis where the quality control processes do not impact a defect rate, we would anticipating seeing this ratio or an even larger one 6.7% of the time. Given our Unusualness Threshold is 10%, we will note that we have an unusual event in our results for the null hypothesis. Thus, we will reject the null hypothesis and decide to act as if quality control processes impact the change in defect rate at the toy manufacturing plants. Based upon the effect sizes (Omega Sq., Eta Sq., and Epsilon Sq.) in Table 2, we can point out that our alternative model appears to account for a larger proportion of varition in resonse (approximately 45%).

Table 2: ANOVA Table for Toy Quality Control Study

Source	SS	df	MS	F	p-value	Omega Sq.	Eta Sq.	Epsilon Sq.
process	29.0489	2	14.5244	4.3866	0.067	0.4294	0.5939	0.4585
Residuals	19.8667	6	3.3111					

Table 3 provides the estimates for the Grand Mean as well as the individual treatment effects. In general, the plants appear to attempt to decrease the defect rate (2.28 defects/1000/plant). The introduction of quality control processes appears to provide additional gains on this front. However, these point estimates are not sufficient to make any claims that SPC + EPC is better than SPC or that the null treatment truly is the worst of the three.

Table 3: Point Estimates from the Toy Quality Control Study

	Estimates (defects/1000/plant)
Grand Mean	2.28
Null Treatment	-2.44
SPC	0.62
SPC + EPC	1.82

Discussion

Given the study and our results, we would recommend that J & N Toys begin making plans to place quality control processes in their remaining factories. Our results show that quality control processes have an impact the change in defect rate for manufacturing plants, accounting for approximately 45% of the variation we observed. While this is a large proportion of variance explained, we need to plan some further explorations to see if there is a significant different between the various treatments (i.e., SPC vs. SPC + EPC) as this could provide information which could help save costs in the long run.

Code Appendix

```
# Setting Document Options
knitr::opts_chunk$set(
  echo = FALSE,
  warning = FALSE,
  message = FALSE,
  fig.align = "center"
)

# Add additional packages by name to the following list
packages <- c("tidyverse", "knitr", "kableExtra",
              "hasseDiagram", "car", "psych", "parameters")
lapply(
  X = packages,
  FUN = library,
  character.only = TRUE
)

# Set constraint
options("contrasts" = c("contr.sum", "contr.poly"))

options(knitr.kable.NA = "")

# Load the data
toyData <- read.table(
  file = "https://raw.github.com/neilhatfield/STAT461/master/dataFiles/toyProcessControl.dat",
  header = TRUE,
  sep = ","
)

# We want to ensure that R knows what our factor is
toyData$process <- as.factor(toyData$process)

# Box plots
ggplot(
  data = toyData,
  mapping = aes(x = process, y = defectRate)
) +
  geom_boxplot() +
  theme_bw() +
  xlab("Quality Control Process") +
  ylab("Defect Rate (per 1000 toys)") +
  theme(
    text = element_text(size = 12)
  )

ggplot(
  data = toyData,
  mapping = aes(x = defectRate)
) +
  geom_histogram(
    binwidth = function(x){
      ifelse(
```

```
        IQR(x) == 0,
        0.1,
        2 * IQR(x) / (length(x)^(1/3))
      )},
    boundary = 0,
    closed = "left",
    fill = "blue",
    color = "black"
  ) +
  theme_bw() +
  xlab("Defect Rate (per 1000 toys)") +
  ylab("Frequency")


groupStats <- psych::describeBy(
  x = toyData$defectRate,
  group = toyData$process,
  na.rm = TRUE,
  skew = TRUE,
  ranges = TRUE,
  quant = c(0.25, 0.75),
  IQR = TRUE,
  mat = TRUE,
  digits = 4
)

# Set row names as type of cookie; select useful columns
groupStats <- groupStats %>%
  tibble::remove_rownames() %>%
  tibble::column_to_rownames(
    var = "group1"
  ) %>%
  dplyr::select(
    n, min, Q0.25, median, Q0.75, max, mad, mean, sd, skew, kurtosis
  )

groupStats %>%
  knitr::kable(
    caption = "Summary Statistics for Defect Rates by Quality Control Process",
    digits = 3,
    format.args = list(big.mark = ","),
    align = rep('c', 11),
    col.names = c("n", "Min", "Q1", "Median", "Q3", "Max", "MAD", "SAM", "SASD", "Sample Skew",
                  "Sample Ex. Kurtosis"),
    format = "latex",
    booktabs = TRUE
  )  %>%
  kableExtra::kable_styling(
    font_size = 12,
    latex_options = c("scale_down", "HOLD_position")
  )

# Create your Hasse Diagram here
```

```r
modelLabels <- c("1 Reduce Defects 1", "3 Quality Control Process 2", "9 (Manufacturing Plant) 6")
modelMatrix <- matrix(
  data = c(FALSE, FALSE, FALSE, TRUE, FALSE, FALSE, TRUE, TRUE, FALSE),
  nrow = 3,
  ncol = 3,
  byrow = FALSE
)
hasseDiagram::hasse(
 data = modelMatrix,
 labels = modelLabels
)

# Apply your chosen method for the sampling distribution here

toyModel <- aov(
  formula = defectRate ~ process,
  data = toyData,
  na.action = "na.omit"
)

car::qqPlot(
  x = toyModel$residuals,
  distribution = "norm",
  envelope = 0.90,
  id = FALSE,
  pch = 20,
  ylab = "Residual Quantiles",
  xlab = "Gaussian Quantiles"
)

ggplot(
  data = data.frame(
    residuals = toyModel$residuals,
    fitted = toyModel$fitted.values
  ),
  mapping = aes(x = fitted, y = residuals)
) +
  geom_point(size = 1) +
  theme_bw() +
  xlab("Fitted values (defects / 1000 toys)") +
  ylab("Residuals (defects / 1000 toys)")

ggplot(
  data = toyData,
  mapping = aes(x = measOrder, y = defectRate)
) +
  geom_point(size = 1.5) +
  geom_line() +
  theme_bw() +
  geom_hline(
    yintercept = mean(toyData$defectRate),
    linetype = "dashed",
    color = "red"
```

```
  ) +
  xlab("Measurement order") +
  ylab("Defect Rate (per 1000 toys)")

# Create a professional looking ANOVA Table, if applicable
parameters::model_parameters(
  model = toyModel,
  omega_squared = "raw",
  eta_squared = "raw",
  epsilon_squared = "raw"
) %>%
  knitr::kable(
  digits = 4,
  col.names = c(
    "Source", "SS", "df", "MS", "F", "p-value",
    "Omega Sq.", "Eta Sq.", "Epsilon Sq."),
  caption = "ANOVA Table for Toy Quality Control Study",
  format = "latex",
  booktabs = TRUE,
  align = c("l", rep("c", 8))
  ) %>%
  kableExtra::kable_styling(
    font_size = 10,
    latex_options = c("scale_down", "HOLD_position")
  )

# List Point Estimates
pEst <- dummy.coef(toyModel)
pEst <- unlist(pEst)
names(pEst) <- c("Grand Mean", "Null Treatment", "SPC",
                 "SPC + EPC")

data.frame("Estimate" = pEst) %>%
  knitr::kable(
  digits = 2,
  caption = "Point Estimates from the Toy Quality Control Study",
  col.names = kableExtra::linebreak(
    x = c("Estimates\n(defects/1000/plant)"),
    align = "c"
  ),
  format = "latex",
  booktabs = TRUE,
  align = "c",
  escape = FALSE
  ) %>%
  kableExtra::kable_styling(
    font_size = 12,
    latex_options = c("HOLD_position")
  )
```