Repeated Measures

Neil J. Hatfield

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In this tutorial we will take a look at Repeated Measures ANOVA models. Keep in mind that there are **TWO** different types of Repeated Measures models: applying every treatment to each subject/measurement unit (a.k.a. "Within Subjects") and measuring each subject/measurement unit multiple times but each only gets one treatment (a.k.a "Nested Repeated Measures").

Make sure that you carefully examine the study design so that you correctly identify which type of Repeated Measures design you have. Here is the general structure of this guide/tutorial:

- Important Starting Considerations
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 - Approaches
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- Setting Up R
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Important Starting Considerations

There are a few important considerations you need to think through before you get too far into the process of analyzing Repeated Measures designs.

Conventions

In Repeated Measure designs, our measurement units are not as straightforward as in our other designs. To help signal this, we often use the term *subjects* to denote the living being (or object) who produces the multiple observations. This has two important consequences.

First, our last node in the Hasse diagram will no longer be the measurement units (i.e., objects/living beings) but rather the observations. Second, our subjects are almost always presumed to be randomly sampled from a broader population. Thus, they are *random effects* and need to be marked as such in our Hasse diagram.

Approaches

What I present here is **ONE** approach to using R to conduct analysis of Repeated Measures designs. This is one particular of analysis where there are dozens of different approaches which have their own champions. In working on putting together this resource for you, I easily came across dozens of different guides all using different formulations and different packages. This is my attempt to provide *one* set of approaches that is mostly consistent with all of our other approaches.

Identify Type of Repeated Measures

Repeated Measures Designs are almost exclusively used in experimental settings. To decide on which type of Repeated Measures Design you're facing, ask yourself the following questions:

- Who are the subjects?
- What are the treatments?
- How many different treatments does each subject get?
 - Answer: All of them \rightarrow Within Subjects Repeated Measures
 - Answer: Only one → Nested Repeated Measures

Wide and Long Formats of Data Frames

In both types of Repeated Measures Designs, we will need the data to be arranged in two formats: "wide" and "long". These terms refer to the construction of the data frame.

A "long data frame" is what we're must used to working with. Here, each row represents a unique combination of Subject & Treatment or Subject & Time Point. If we have n subjects, and g treatments (or t time points of measurement), we should have a total of $n \cdot g$ rows (alternatively, $n \cdot t$ rows). Our response is a single column. Visually, imagine your data frame as a rectangle that is taller than it is wide.

A "wide data frame" is a re-arrangement. Here, each subject gets one and only one row. Instead of a single response column, we have multiple response columns. In fact, we'll have a separate response column for each of the g treatments (or t time points of measurement). Now imagine your data frame as a rectangle that is wider than it is tall.

One of the first challenges you must tackle in analyzing Repeated Measures data is identifying which of these formats your data is currently in. Then creating a new data frame that is in the other format. In these situations, I tend to not include Data on the end of my object name; rather, I use either Long or Wide so that I have a reminder of which format I'm calling.

Transforming Data Frame Formats

Thankfully, we have some useful functions to help us. As part of the tidyverse, the package tidyr gives us the functions pivot_wider and pivot_longer. Given the imagery of the rectangles, you can imagine turning (pivoting) the long rectangle into the wide rectangle and vice versa. This imagery can help you keep in mind that pivot_wider takes a long data frame and makes a wide data frame. The pivot_longer function starts with a wide data frame and returns a long data frame.

```
# Generic Demo Code for creating a wide data frame
# Note: this code assumes you already read in a long format data frame
#IMPORTANT: you will need to the code to match the names in your actual situation
dataWide <- pivot_wider(</pre>
  data = dataLong, # "dataLong" is the name of the long format data frame
 names_from = group, #"group" is the name of the column that contains your treatments
  values_from = response #"response" is the name of the column with the response values
)
# Generic Demo Code for creating a long data frame
# Note: this code assumes you already read in a wide format data frame
#IMPORTANT: you will need to the code to match the names in your actual situation
dataLong <- pivot_longer(</pre>
  data = dataWide, # "dataWide" is the name of the wide format data frame
  cols = !subject, # Says to not use the "subject" column
  names to = "group", # This is the new column you want the treatments to go to
  names_transform = list(group = as.factor), # Makes the treatment column a factor
  values_to = "response" # A new column that will contain all of the response values
```

You'll be able to see the above code examples in action in the examples below.

Setting Up R

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Just as in the prior guides/tutorials, we have to first ensure that R is properly configured and prepared for our work. We will want to ensure that we load all of the appropriate packages, set our constraint, and load in any additional tools.

The core set of packages to load in include the following:

- tidyverse-for data cleaning/wrangling and the pipe, %>%
- knitr & kableExtra—for making professional looking tables
- parameters—to construct modern ANOVA tables, get omnibus effect sizes, and to switch out the type of Sums of Squares
- $\bullet \ \ {\tt hasseDiagram-to} \ \ {\tt construct} \ \ {\tt Hasse} \ \ {\tt diagrams}$
- $\bullet~$ car for nice QQ Plots
- psych-for easy descriptive statistics by group
- emmeans—for getting point estimates which attend to our models as well as doing post hoc analyses
- rstatix—to help with assessing sphericity
- lme4 and nlme-to help assess random effects

As a reminder, the following code does all of these things:

Within Subjects Repeated Measures Design

The Within Subjects Repeated Measures Design has the hallmark that each subject will be given each and every treatment. To prevent an order effect, we should randomize the order in which each subject gets the treatments.

Example Context-Taste Testing Beer

Beer is big business; the craft brewing industry contributed \$79.1 billion to the US Economy in 2018 and 550,000+ jobs (PA: \$6.335 billion).

Getting a craft beer scored can be quite the achievement. In a single blind tasting, judges are given a chilled, properly poured beer and told the style category. They then judge the beer on Aroma (24 pts), Appearance (6 pts), Flavor (40 pts), Mouthfeel (10 pts), and Overall Impression (20 pts).

We have decided to put several State College beers to the test:

- Barnstormer (IPA, Happy Valley Brewing Company)
- Craftsman (Brown, HVBC)
- Red Mo (Red, Otto's)
- King Richard Red (Amber, Robin Hood)

For this study, we have used a lottery to select six individuals to act as judges. Each judge will be presented with samples of the four beers and they will score each beer. The order in which each judge samples/scores the beers will be determined by the research team drawing labeled tokens without replacement.

Fit the Model

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 - Form the Models
- Assessing Assumptions
 - Gaussian Residuals
 - Gaussian Subject Effects
 - Homoscedasticity
 - Independence of Observations up to Subject
 - No Interaction between Subjects and Factor
 - Sphericity
- Results
 - Omnibus

- * Relative Efficiency
- * Adjusting for Violation of Sphericity
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For this particular study, we can express the Repeated Measures-Within Subjects design with the following Hasse diagram (Figure 1).

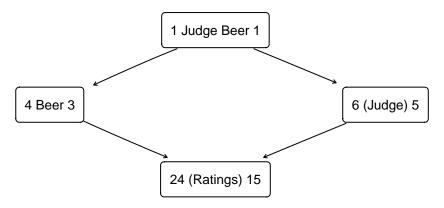


Figure 1: Hasse Diagram for Beer Judging

Notice that this layout looks a lot like a Randomized Complete Block Design (RCBD). For a one-way (single factor) experiment with repeated measures on all treatments, we end up with an essentially identical approach.

Data and Format of the Data Table

The following are the data for this situation:

```
# Beer Data
beer <- data.frame(
    judge = as.factor(rep(x = LETTERS[1:6], each = 4)),
    beer = as.factor(rep(
        x = c("Barnstormer", "King Richard Red", "Craftsman", "Red Mo"),
        times = 6)),
score = c(
    50, 60, 70, 70,
    38, 45, 58, 60,</pre>
```

```
45, 48, 60, 58,
65, 65, 75, 75,
55, 60, 70, 65,
48, 53, 68, 63
```

For the methods we're going to look at, we need our data in the long format: each row is unique observation. Fortunately, our data are already in this format.

If our data were in the wide format (i.e., each row refers to a single subject and *all* of their observations), we would see something like the following for a data table:

Long to Wide Format Transformation

For assessing Compound Symmetry/Sphericity, we will need our data in the wide format. We will use the pivot_wider function from the tidyr package to transform our data into a wide format.

```
beerWide <- pivot_wider(
   data = beer,
   names_from = beer,
   values_from = score
)
beerWide</pre>
```

```
## # A tibble: 6 x 5
##
     judge Barnstormer 'King Richard Red' Craftsman 'Red Mo'
                  <dbl>
                                                  <dbl>
                                                            <dbl>
##
     <fct>
                                       <dbl>
## 1 A
                     50
                                          60
                                                     70
                                                               70
## 2 B
                     38
                                          45
                                                     58
                                                               60
## 3 C
                     45
                                          48
                                                     60
                                                               58
                                                     75
## 4 D
                     65
                                          65
                                                               75
## 5 E
                     55
                                          60
                                                     70
                                                               65
## 6 F
                     48
                                          53
                                                     68
                                                               63
```

Wide to Long Format Transformation

If your data happen to be in the wide format, we can switch to the long format via the pivot_longer function from the tidyr package (part of the tidyverse).

```
beerLong <- pivot_longer(
   data = beerWide,
   cols = !judge, # This says to use all except the judge column
   names_to = "beer", # what column to place the beer names in
   names_transform = list(beer = as.factor), # turn beer to a factor
   values_to = "score" # what column to put the values in
)
head(beerLong, n = 5) # Display the first 5 rows.</pre>
```

```
## # A tibble: 5 x 3
##
     judge beer
                             score
     <fct> <fct>
##
                             <dbl>
## 1 A
           Barnstormer
                                50
## 2 A
                                60
           King Richard Red
## 3 A
           Craftsman
                                70
           Red Mo
## 4 A
                                70
## 5 B
           Barnstormer
                                38
```

Explore the Data

Just as with all of our other models, you should explore the data through data visualizations and descriptive statistics.

Fit the Models

We are going to need to fit three (3) models for Within Subjects designs:

- 1) We will fit a One-way ANOVA + Block (RCBD) model for our ANOVA table.
- 2) We will fit a Mixed Effects model for estimating judge effects and assess assumptions about the judge factor.
- 3) We will use a Nested Approach via the rstatix package for assessing Compound Symmetry/Sphericity assumption.

```
# Omnibus Model (for our ANOVA table)
beerOmni <- aov(
  formula = score ~ judge + beer,
  data = beerLong
)
# Random Effect Model (Assumption Check and Point Estimation)
beerMixed <- lme4::lmer(</pre>
  formula = score ~ (1|judge) + beer,
  data = beerLong
)
# Compound Symmetry/Sphericity Assessment
beerSphere <- rstatix::anova_test(</pre>
  data = beerLong,
  formula = score ~ beer + Error(judge %in% beer)
)
## The %in% tells R that judge should be treated as nested in beer
```

Assess the Assumptions

For Within Subjects Repeated Measures designs, we have the following assumptions:

- 1) Our residuals follow a Gaussian distribution,
- 2) Subject effects follow a Gaussian distribution (just like a Random Effect),
- 3) Homoscedasticity around the model,
- 4) Independence of Observations up to Subject; that is, observations should be independent between subjects much like for RCBDs,
- 5) No interaction between Subjects and Factor (just like RCBDs), and
- 6) We have Sphericity.

Gaussian Residuals

Use a QQ plot:

```
# QQ plot for residuals
car::qqPlot(
    x = residuals(beerMixed),
    distribution = "norm",
    envelope = 0.90,
    id = FALSE,
    pch = 20,
    ylab = "Residuals (score)"
)
```

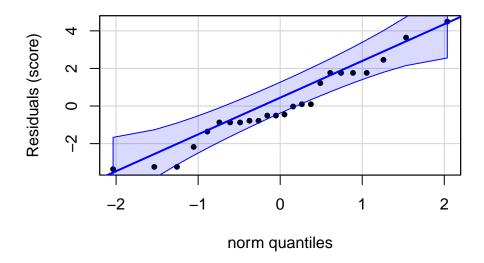


Figure 2: QQ Plot of Beer Judging Residuals

Gaussian Treatment Effects

Again, use a QQ plot.

```
# Judge Effects
car::qqPlot(
    x = unlist(
        lme4::ranef(
            object = beerMixed,
            whichel = c("judge")
        )
    ),
    distribution = "norm",
    envelope = 0.90,
    id = FALSE,
    pch = 20,
    ylab = "Judge Effects"
)
```

Homoscedasticity

Use a Tukey-Anscombe Plot

```
# Tukey-Anscombe Plot
ggplot(
   data = data.frame(
     residuals = residuals(beerMixed),
     fitted = fitted.values(beerMixed)
),
   mapping = aes(x = fitted, y = residuals)
) +
   geom_point(size = 2) +
   geom_hline(
     yintercept = 0,
```

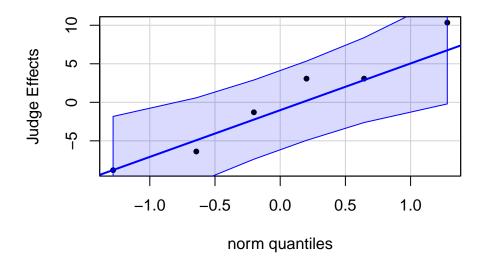


Figure 3: QQ Plot for Judge Effects

```
linetype = "dashed",
  color = "grey50"
) +
geom_smooth(
  formula = y ~ x,
  method = stats::loess,
  method.args = list(degree = 1),
  se = FALSE,
  size = 0.5
) +
theme_bw() +
xlab("Fitted values (score)") +
ylab("Residuals (score)")
```

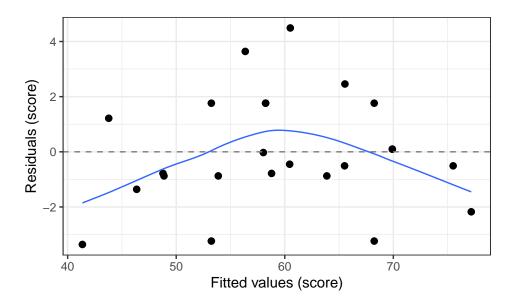


Figure 4: Tukey-Anscombe Plot for Beer Judging Study

Independence of Subjects

If you happen to know measurement order, you can look at an Index plot. However, be sure that you incorporate the subject attribute into the plot (in this situation, the judge). This can help highlight that whether any patterns are due to our subjects (and therefore anticipated) or if we have any additional patterns threats. Treat this situation just like checking for independence of subjects in a RCBD.

Interaction of Subject and Factor

We will want to make an interaction plot to look for consistency between the judges and the beers; again, think about what we would do with a RCBD.

```
# Interaction Plot
ggplot(
 data = beer,
 mapping = aes(
    x = beer,
    y = score,
    color = judge,
    group = judge
)
 geom_point(size = 2) +
 geom_line() +
 ggplot2::theme_bw() +
 xlab("Beer") +
 ylab("Score") +
 labs(
    color = "Judge"
```

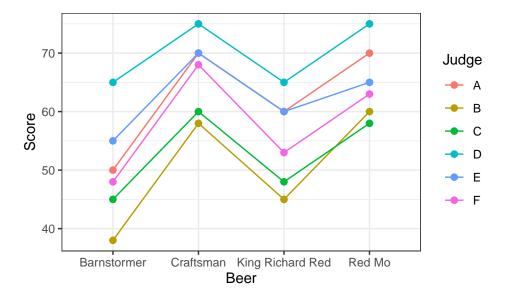


Figure 5: Interaction Plot for Beer and Judge

Sphericity

The idea behind sphericity is that we have essentially the same levels of variation for the differences between treatments. While there is a visual method we can use here, we do need to do so with some caution. Much like looking for homoscedasticity, we'll want to see if any difference has excessively different variation than another difference. Unlike homoscedasticity there is no

rule of thumb/guideline (e.g., more than twice) for sphericity. Thus, this is one assumption where we will supplement with a formal test: Mauchly's Test of Sphericity.

```
# Sphericity Plot
sphericityPlot(
  dataWide = beerWide, # Data needs to be in wide format
  subjectID = "judge" # character name of the subject column
)
```

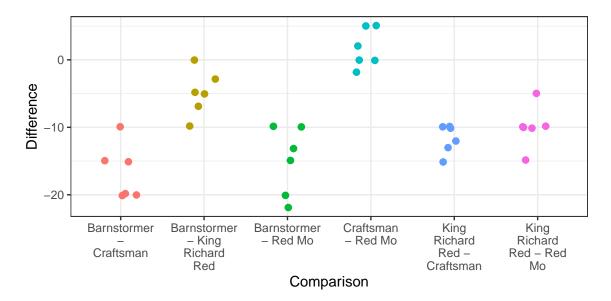


Figure 6: Sphericity Plot for Beer Judging

Look for any groups that have very different amounts of variation

The null hypothesis for Mauchly's Test is that there is **no** violation of Sphericity (Compound Symmetry); under this hypothesis, Mauchly's Test Statistic, W follow a χ^2 with 2 degrees of freedom. To see the results of Mauchly's test, we will use the following code:

```
# Mauchly's Test
beerSphere$`Mauchly's Test for Sphericity` %>%
    dplyr::select(Effect, W, p) %>%
    knitr::kable(
        digits = 4,
        col.names = c("Effect", "Mauchly's W", "p"),
        caption = "Mauchly's Sphericity Test",
        align = c('l', "c", "c"),
        booktab = TRUE
) %>%
    kableExtra::kable_styling(
        bootstrap_options = c("striped", "condensed"),
        font_size = 12,
        latex_options = c("HOLD_position")
)
```

Table 1: Mauchly's Sphericity Test

Effect	Mauchly's W	р
beer	0.365	0.597

Thus, we have W = 0.365 with a p-value of 0.597 (use your overall Type I Error Risk to set the Unusualness Threshold here). This is one of those strange cases where we want to fail to reject the null hypothesis.

Results

Omnibus Test

For our omnibus test, we will use our RCBD equivalent design:

```
beerTable1 <- parameters::model_parameters(</pre>
    model = beerOmni, # the RCBD with all fixed effects
    omega_squared = "partial",
    eta_squared = "partial",
    epsilon_squared = "partial"
)
beerTable1$p <- lapply(</pre>
 X = beerTable1$p,
  FUN = pvalRound
knitr::kable(
  x = beerTable1,
  digits = 4,
  col.names = c("Source", "SS", "df", "MS", "F", "p-value",
                "Partial Omega Sq.", "Partial Eta Sq.", "Partial Epsilon Sq."),
  caption = "ANOVA Table for Beer Judging Study",
  align = c('l', rep('c', 8)),
  booktab = TRUE
) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("scale_down", "HOLD_position")
```

Table 2: ANOVA Table for Beer Judging Study

Source	SS	df	MS	F	p-value	Partial Omega Sq.	Partial Eta Sq.	Partial Epsilon Sq.
judge	1045.833	5	209.1667	32.8534	< 0.0001	0.869	0.9163	0.8884
beer	1150.000	3	383.3333	60.2094	< 0.0001	0.881	0.9233	0.9080
Residuals	95.500	15	6.3667					

Notice that I slipped the p-values are excessively small and were coming out as 0 after rounding. We interpret the terms in this table exactly as we have been all semester.

Efficiency of Repeated Measures

Since the One-way Within Subjects subjects is like a RCBD, we can get a measure of the efficiency of such a design versus a CRD.

```
# Use the block relative efficiency function
block.RelEff(
   aov.obj = beerOmni,
   blockName = "judge",
   trtName = "beer"
)
```

```
## [1] "The relative efficiency of the block, judge, is 7.715."
```

Thus, we would need 8 times as many ratings for each beer as what we used in order to get the same level of information. That would mean that we would need around 48 scores for each beer.

What if Sphericity is Violated?

If Sphericity is violated (i.e., Mauchly's Test leads you to reject the null hypothesis), we are not out of luck. When we fit the model to check for Sphericity, we also automatically got two corrected tests: the Greenhouse-Geisser and the Huynh-Feldt corrections:

```
correctedTable <- beerSphere$`Sphericity Corrections` %>%
  dplyr::select(GGe, `p[GG]`, HFe, `p[HF]`)
correctedTable$`p[GG]` <- lapply(</pre>
  X = correctedTable$`p[GG]`,
  FUN = pvalRound
)
correctedTable$`p[HF]` <- lapply(</pre>
  X = correctedTable$`p[HF]`,
  FUN = pvalRound
)
knitr::kable(
  x = correctedTable,
  digits = 4,
  col.names = c("Greenhouse-Geisser", "p-value", "Huynh-Feldt", "p-value"),
  caption = "Sphericity Corrections",
  align = "c",
  booktab = TRUE
) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font size = 12,
    latex_options = c("HOLD_position")
```

Table 3: Sphericity Corrections

Greenhouse-Geisser	p-value	Huynh-Feldt	p-value
0.613	< 0.0001	0.952	< 0.0001

For the Corrections, the *p*-values are the adjusted *p*-values for the **omnibus** test. Thus, we would say that there is an effect due to our repeated measures model (i.e., there is a difference in the scores of the beer).

Post Hoc Analysis

For Post Hoc Analysis, we will want to make use of the emmeans package and make sure that we're looking at the correct aspect of our model. We already assume that there's some difference in the judges, thus we are really just after the marginals of our other factor(s). In this situation, the type of beer.

```
# Using emmeans
beerPH <- emmeans::emmeans(
  object = beerMixed, # Notice the use of the mixed model here
  specs = pairwise ~ beer,
  adjust = "tukey",
  level = 0.92
)</pre>
```

Point Estimates You can get point estimates for the Beer effects:

Table 4: Marginal Means-Tukey 92% Adjustment

Type of Beer	Marginal Mean	SE	DF	Lower Bound	Upper Bound
Barnstormer	50.1667	3.084	5.9383	43.6635	56.6698
Craftsman	66.8333	3.084	5.9383	60.3302	73.3365
King Richard Red	55.1667	3.084	5.9383	48.6635	61.6698
Red Mo	65.1667	3.084	5.9383	58.6635	71.6698

Pairwise Comparisons You can also do the standard pairwise comparisons of the beer types.

Table 5: Marginal Means-Tukey 92% Adjustment

Comparison	Difference	SE	DF	t Statistic	p-value
Barnstormer - Craftsman	-16.6667	1.4568	15	-11.4407	0.0000
Barnstormer - King Richard Red	-5.0000	1.4568	15	-3.4322	0.0174
Barnstormer - Red Mo	-15.0000	1.4568	15	-10.2966	0.0000
Craftsman - King Richard Red	11.6667	1.4568	15	8.0085	0.0000
Craftsman - Red Mo	1.6667	1.4568	15	1.1441	0.6692
King Richard Red - Red Mo	-10.0000	1.4568	15	-6.8644	0.0000

Effect Sizes

For Post Hoc Effect Sizes, you'll need to use the emmeans package and my probSup function:

```
tempEMM <- emmeans::emmeans(</pre>
 object = beerMixed,
 specs = "beer"
# Pass the stored marginals into the effect size function
cohenTemp <- emmeans::eff_size(</pre>
 object = tempEMM,
 sigma = sigma(beerMixed),
 edf = df.residual(beerMixed)
)
# Create a data frame, add on the probability of superiority
# Send that data frame into a nice table
as.data.frame(cohenTemp) %>%
 dplyr::mutate(
    ps = probSup(effect.size),
    .after = effect.size
 dplyr::select(contrast, effect.size, ps) %>%
 knitr::kable(
    digits = 3,
    col.names = c("Comparison", "Cohen's d", "Probability of Superiority"),
    align = "lcc",
    caption = "Effect Sizes for Beer",
    booktab = TRUE
 ) %>%
 kableExtra::kable styling(
    bootstrap_options = c("striped", "condensed"),
    font size = 12,
    latex_options = "HOLD_position"
```

Table 6: Effect Sizes for Beer

Comparison	Cohen's d	Probability of Superiority
Barnstormer - Craftsman	-6.605	0.000
Barnstormer - King Richard Red	-1.982	0.081
Barnstormer - Red Mo	-5.945	0.000
Craftsman - King Richard Red	4.624	0.999
Craftsman - Red Mo	0.661	0.680
King Richard Red - Red Mo	-3.963	0.003

Nested Repeated Measures Context

The second kind of Repeated Measures ANOVA design deals with taking multiple measurements from our measurement units on the same attribute over time. Unlike the Within Subjects design, each measurement unit here only gets **ONE** treatment. A some what handy way to help you decide if you're in a Nested Repeated Measures design is to see if you can think about the situation as being like a Pre-/Post-Testing situation. This classic situation involves testing/measuring everyone before we apply treatments, then apply the treatments (each person only gets one), and then testing/measuring everyone again afterwards. If you can fit the situation in to the pre/post design, you're a Nested Repeated Measures design.

For this example, we are going to look at the impact of two advertising campaigns on the volume of sales of athletic shoes over time. Ten similar test markets were selected at random to participate in this study. The two advertising campaigns were similar in all respects except that a different national sports personality was used in each. Sales data were collected for three two-week periods (before-t1, during-t2, and after-t3)

Hasse Diagram

If you use the Hasse Diagram app, you'll need to watch out for a couple of things:

- 1) You'll need to remove the interaction of Time Point X Market Nested Campaign. (Use Markets X Time as your measurement unit.)
- 2) The degrees of freedom will be off for the Markets. The app isn't subtracting the degrees of freedom for Campaign. Thus, you'll have to manually adjust the code until I can get a fix in place.
- 3) Remember to carry your degrees of freedom fix through to the final node.

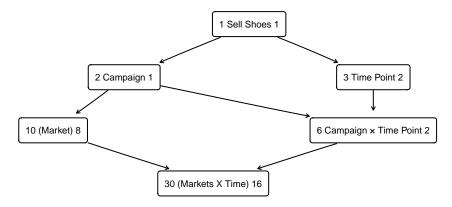


Figure 7: Hasse Diagram for Shoe Advertisement Study

Data

The shoe sales data comes to use in the wide format. We will want to make a long format version as well.

```
shoesWide <- read.table(</pre>
  file = "https://raw.github.com/neilhatfield/STAT461/master/dataFiles/shoes.csv",
  header = TRUE,
  sep = ","
)
shoesWide$campaign <- as.factor(shoesWide$campaign)</pre>
shoesWide$market <- as.factor(shoesWide$market)</pre>
# Make a long version of the data
shoesLong <- tidyr::pivot_longer(</pre>
  data = shoesWide,
  cols = dplyr::starts_with("t"),
  names_to = "time",
  names_ptypes = list("time" = factor()),
  values_to = "sales"
)
# Recode times to before, during, after and puts an ordering
shoesLong$time <- dplyr::recode_factor(</pre>
  shoesLong$time,
  "t1" = "before",
  "t2" = "during",
```

```
"t3" = "after",
.ordered = TRUE
)
```

Explore the Data

As usual, you'll want to explore the data. With Repeated Measures data, remember that can the explore the data by looking across subjects at a particular time point or across time points (following a particular subject) or both.

Fit the Models

We will fit two models: one that we'll use for the omnibus test and one we'll use for sphericity checking.

Let's begin with our omnibus testing model. Unfortunately, there is not a nice, clean way to get a well organized table here without doing some manipulation; more on this in the results section.

The key here is that you will want to make sure that you listen/watch for a particular warning message: Error() model is singular. This is because our final interaction term (subject x time) uses up all remaining degrees of freedom so we will not have a traditional Residuals/Error term. That is the crux of this particular message.

```
# Omnibus Model
shoesModel <- aov(
  formula = sales ~ campaign * time + Error(market %in% campaign),
  data = shoesLong
)</pre>
```

Due to the lack of a typical Residuals/Error term, we will need to use an alternative route for getting things like residuals or fitted values for our assumption checking. For our second model, we will turn to the nlme package.

```
# Use the nlme package to fit a model that we can use for assumption checking
shoesAssumptions <- nlme::lme(
   data = shoesLong,
   fixed = sales ~ campaign * time,
   random = ~ 1|market
)</pre>
```

Assessing the Assumptions

The methods here are the same as for the Within Subjects Repeated Measures design.

Gaussian Residuals

Use a QQ plot:

```
# QQ plot for residuals
car::qqPlot(
    x = residuals(shoesAssumptions),
    distribution = "norm",
    envelope = 0.90,
    id = FALSE,
    pch = 20,
    ylab = "Residuals (sales)"
)
```

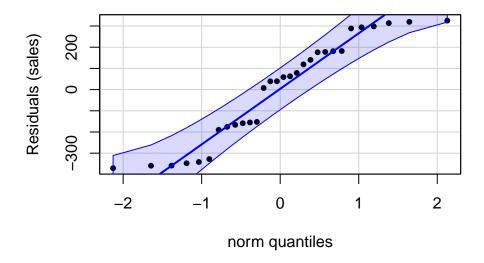


Figure 8: QQ Plot of Shoe Sales Study

Gaussian Treatment Effects

Again, use a QQ plot.

```
# Market Effects
car::qqPlot(
    x = unlist(
        lme4::ranef(
        object = shoesAssumptions,
        whichel = c("market")
    )
),
distribution = "norm",
envelope = 0.90,
id = FALSE,
pch = 20,
ylab = "Market"
)
```

Homoscedasticity

Use a Tukey-Anscombe Plot

```
# Tukey-Anscombe Plot
ggplot(
   data = data.frame(
      residuals = residuals(shoesAssumptions),
      fitted = fitted.values(shoesAssumptions)
),
   mapping = aes(x = fitted, y = residuals)
) +
   geom_point(size = 2) +
   geom_hline(
      yintercept = 0,
```

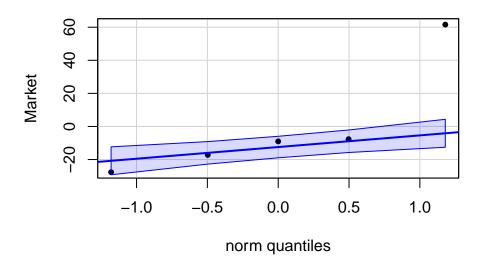


Figure 9: QQ Plot for Market Effects

```
linetype = "dashed",
  color = "grey50"
) +
geom_smooth(
  formula = y ~ x,
  method = stats::loess,
  method.args = list(degree = 1),
  se = FALSE,
    size = 0.5
) +
theme_bw() +
xlab("Fitted values (sales)") +
ylab("Residuals (sales)")
```

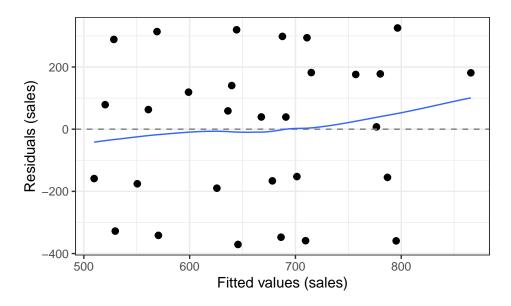


Figure 10: Tukey-Anscombe Plot for Shoe Sales Study

Independence of Subjects

One of the interesting things about Nested Repeated Measures is that we inherently know at least *some* of the the measurement order. We might not know which store got measured before which other store, but we know the sequence of measurements for each store. Thus, we can make the following plot:

```
ggplot(
  data = shoesLong,
  mapping = aes(
    x = time,
    y = sales,
    color = paste(campaign, market, sep = ":"),
    group = paste(campaign, market, sep = ":")
  )
) +
  geom_point() +
  geom_line() +
  theme_bw() +
  xlab("Time Point (relative to campaign)") +
  ylab("Sales (coded)") +
  labs(
    color = "Campagin:Market"
```

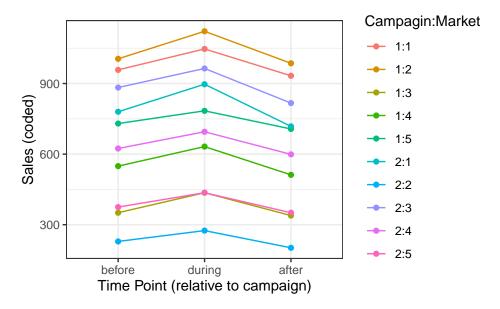


Figure 11: Shoe Sales by Time and Store

Now, this plot is great for letting us compare the effects over time. This is not necessarily the greatest for letting us see if our subjects are independent of one another. The consistency of the effects over time do indicate that the inherent dependency of each subject's measurements is consistent across time. If a particular market had a huge increase in sales after the campaign, that would suggest something strange happened.

For Nested Repeated Measures we will ultimately want to fall back on our study design to make a solid justification. This is why I've been pressing you all course long on including details/being specific.

Interaction of Subject and Factor

A similar plot to the previous Time Series plot is to construct a plot known as "Growth Curves". In essence, we want to see how the response changes for each subject over time... but we're going to separate the data a bit more cleanly so we can see if and how the factor might interact with our subjects.

```
# Growth Curves
ggplot(
 data = shoesLong,
 mapping = aes(
    x = time,
    y = sales,
    color = market,
    group = market
)
  geom_point(size = 2) +
 geom_line() +
 facet_wrap(facets = ~campaign) +
 ggplot2::theme_bw() +
 xlab("Time Period") +
 ylab("Sales (coded)") +
 labs(
    color = "Market"
```

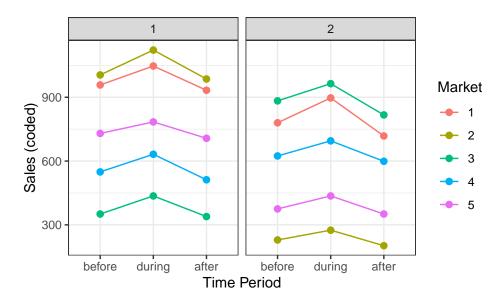


Figure 12: Growth Curves for Shoe Sales Study

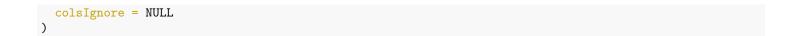
Notice that we used the facet_wrap on campaign to split the time series plot into separate panels/facets for each campaign. If we see the same behaviors in both facets, then there is no worrisome interaction between subjects and our factor.

Keep in mind that Market 3 in Campaign 1 is **not** the same market as Market 3 in Campaign 2. They just happen to be the *third* market inside each campaign.

Sphericity

The idea behind sphericity is that we have essentially the same levels of variation for the differences between treatments. While there is a visual method we can use here, we do need to do so with some caution. Much like looking for homoscedasticity, we'll want to see if any difference has excessively different variation than another difference. Unlike homoscedasticity there is no rule of thumb/guideline (e.g., more than twice) for sphericity. Thus, this is one assumption where we will supplement with a formal test: Mauchly's Test of Sphericity.

```
# Sphericity Plot
sphericityPlot(
  dataWide = shoesWide,
  subjectID = c("market", "campaign"),
```



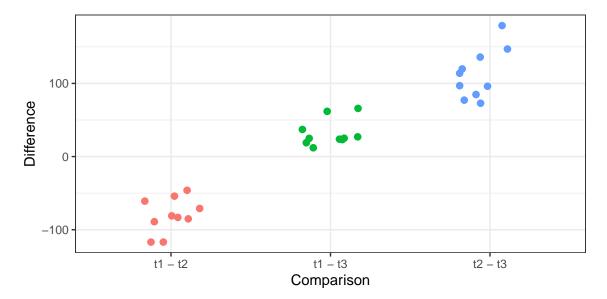


Figure 13: Sphericity Plot for Shoe Sales Study

Look for any groups that have very different amounts of variation

The nlme package does not support Mauchly's Test for Sphericity.

Results

Omnibus Test

As mentioned, getting a nice looking table is not as straightforward with the Nested Repeated Measures ANOVA problems.

```
shoesTemp <- summary(shoesModel)</pre>
shoesOmni <- rbind(</pre>
  shoesTemp$`Error: market:campaign`[[1]],
  shoesTemp$`Error: Within`[[1]]
)
row.names(shoesOmni) <- c("campaign", "market", "time", "campaign:time", "market:time")</pre>
shoesOmni["market", "F value"] <- shoesOmni["market", "Mean Sq"] /</pre>
  shoesOmni["market:time", "Mean Sq"]
shoesOmni["market", "Pr(>F)"] <- pf(</pre>
  q = shoesOmni["market", "F value"],
  df1 = shoesOmni["market", "Df"],
  df2 = shoesOmni["market:time", "Df"],
  lower.tail = FALSE
)
shoesOmni %>%
  tibble::rownames_to_column(
    var = "Source"
  ) %>%
  knitr::kable(
    digits = 4,
```

```
col.names = c("Source", "df", "SS", "MS", "F", "p-value"),
  caption = "ANOVA Table for Athletic Shoes Study",
  align = c('l',rep('c',5)),
  booktab = TRUE
) %>%
kableExtra::kable_styling(
  bootstrap_options = c("striped", "condensed"),
  font_size = 12,
  latex_options = c("HOLD_position")
)
```

Table 7: ANOVA Table for Athletic Shoes Study

Source	df	SS	MS	F	p-value
campaign	1	168150.5333	168150.5333	0.7336	0.4166
market	8	1833680.9333	229210.1167	640.3113	0.0000
time	2	67073.0667	33536.5333	93.6862	0.0000
campaign:time	2	391.4667	195.7333	0.5468	0.5892
market:time	16	5727.4667	357.9667		

Post Hoc

Here you might want to look at both Campaign and Time Effects:

```
# Using emmeans
shoesCampaignPH <- emmeans::emmeans(
   object = shoesModel,
   specs = pairwise ~ campaign,
   adjust = "tukey",
   level = 0.99
)

shoesTimePH <- emmeans::emmeans(
   object = shoesModel,
   specs = pairwise ~ time,
   adjust = "tukey",
   level = 0.99
)</pre>
```

Point Estimates You can get point estimates for both effects:

Table 8: Marginal Means-Tukey 99% Adjustment

Campaign	Marginal Mean	SE	DF	Lower Bound	Upper Bound
1	739.4000	123.615	8	324.6237	1154.176
2	589.6667	123.615	8	174.8904	1004.443

Table 9: Marginal Means-Tukey 99% Adjustment

Time Point	Marginal Mean	SE	DF	Lower Bound	Upper Bound
before during	648.4 728.8	87.5454 87.5454		355.1807 435.5807	941.6193 1022.0193
after	616.4	87.5454	8.05	323.1807	909.6193

Pairwise Comparisons You can also do the standard pairwise comparisons

Table 10: Campaign Comparison-Tukey 99% Adjustment

Comparison	Difference	SE	DF	t Statistic	p-value
1 - 2	149.7333	174.818	8	0.8565	0.4166

Table 11: Time Point-Tukey 99% Adjustment

Comparison	Difference	SE	DF	t Statistic	p-value
before - during	-80.4	8.4613	16	-9.5021	0.0000
before - after	32.0	8.4613	16	3.7819	0.0044
during - after	112.4	8.4613	16	13.2840	0.0000

Effect Sizes

We will not worry about effect sizes here.

Code Appendix

```
# Setting Document Options
knitr::opts_chunk$set(
 echo = FALSE,
 warning = FALSE,
 message = FALSE,
 fig.align = "center"
)
packages <- c("tidyverse", "knitr", "kableExtra",</pre>
              "parameters", "hasseDiagram", "car",
              "psych", "lme4", "nlme", "emmeans", "rstatix")
lapply(packages, library, character.only = TRUE)
options(knitr.kable.NA = "")
options(contrasts = c("contr.sum", "contr.poly"))
source("https://raw.github.com/neilhatfield/STAT461/master/rScripts/ANOVATools.R")
# Generic Demo Code for creating a wide data frame
# Note: this code assumes you already read in a long format data frame
#IMPORTANT: you will need to the code to match the names in your actual situation
dataWide <- pivot_wider(</pre>
data = dataLong, # "dataLong" is the name of the long format data frame
 names_from = group, #"group" is the name of the column that contains your treatments
  values_from = response #"response" is the name of the column with the response values
)
# Generic Demo Code for creating a long data frame
# Note: this code assumes you already read in a wide format data frame
#IMPORTANT: you will need to the code to match the names in your actual situation
dataLong <- pivot_longer(</pre>
 data = dataWide, # "dataWide" is the name of the wide format data frame
 cols = !subject, # Says to not use the "subject" column
 names_to = "group", # This is the new column you want the treatments to go to
 names_transform = list(group = as.factor), # Makes the treatment column a factor
  values_to = "response" # A new column that will contain all of the response values
)
# Demo code to set up R
## Load packages
packages <- c("tidyverse", "knitr", "kableExtra",</pre>
              "parameters", "hasseDiagram", "car",
              "psych", "emmeans", "rstatix", "lme4", "nlme")
lapply(packages, library, character.only = TRUE)
options(knitr.kable.NA = "")
options(contrasts = c("contr.sum", "contr.poly"))
source("https://raw.github.com/neilhatfield/STAT461/master/rScripts/ANOVATools.R")
# Hasse Diagram for Beer Judging
modelLabels <- c("1 Judge Beer 1", "4 Beer 3", "6 (Judge) 5", "24 (Ratings) 15")
modelMatrix <- matrix(</pre>
```

```
data = c(FALSE, FALSE, FALSE, TRUE, FALSE, FALSE, FALSE, TRUE, FALSE, FALSE,
           FALSE, TRUE, TRUE, TRUE, FALSE),
 nrow = 4,
 ncol = 4,
  byrow = FALSE
)
hasseDiagram::hasse(
data = modelMatrix,
labels = modelLabels
# Beer Data
beer <- data.frame(</pre>
  judge = as.factor(rep(x = LETTERS[1:6], each = 4)),
  beer = as.factor(rep(
    x = c("Barnstormer", "King Richard Red", "Craftsman", "Red Mo"),
    times = 6)),
  score = c(
    50, 60, 70, 70,
    38, 45, 58, 60,
    45, 48, 60, 58,
    65, 65, 75, 75,
    55, 60, 70, 65,
    48, 53, 68, 63
  )
)
beerWide <- pivot_wider(</pre>
 data = beer,
 names_from = beer,
  values_from = score
beerWide
beerLong <- pivot_longer(</pre>
  data = beerWide,
 cols = !judge, # This says to use all except the judge column
  names_to = "beer", # what column to place the beer names in
 names_transform = list(beer = as.factor), # turn beer to a factor
  values_to = "score" # what column to put the values in
)
head(beerLong, n = 5) # Display the first 5 rows.
# Omnibus Model (for our ANOVA table)
beerOmni <- aov(
  formula = score ~ judge + beer,
  data = beerLong
)
# Random Effect Model (Assumption Check and Point Estimation)
beerMixed <- lme4::lmer(</pre>
  formula = score ~ (1|judge) + beer,
  data = beerLong
)
# Compound Symmetry/Sphericity Assessment
beerSphere <- rstatix::anova_test(</pre>
```

```
data = beerLong,
  formula = score ~ beer + Error(judge %in% beer)
)
## The %in% tells R that judge should be treated as nested in beer
# QQ plot for residuals
car::qqPlot(
  x = residuals(beerMixed),
  distribution = "norm",
  envelope = 0.90,
 id = FALSE,
  pch = 20,
  ylab = "Residuals (score)"
)
# Judge Effects
car::qqPlot(
  x = unlist(
    lme4::ranef(
      object = beerMixed,
      whichel = c("judge")
  ),
  distribution = "norm",
  envelope = 0.90,
  id = FALSE,
  pch = 20,
  ylab = "Judge Effects"
# Tukey-Anscombe Plot
ggplot(
  data = data.frame(
   residuals = residuals(beerMixed),
    fitted = fitted.values(beerMixed)
  mapping = aes(x = fitted, y = residuals)
) +
  geom_point(size = 2) +
  geom_hline(
    yintercept = 0,
    linetype = "dashed",
    color = "grey50"
  ) +
  geom_smooth(
    formula = y \sim x,
    method = stats::loess,
    method.args = list(degree = 1),
    se = FALSE,
    size = 0.5
  ) +
  theme bw() +
  xlab("Fitted values (score)") +
  ylab("Residuals (score)")
# Interaction Plot
ggplot(
  data = beer,
```

```
mapping = aes(
    x = beer,
    y = score,
    color = judge,
    group = judge
) +
  geom_point(size = 2) +
  geom_line() +
  ggplot2::theme_bw() +
  xlab("Beer") +
  ylab("Score") +
  labs(
    color = "Judge"
# Sphericity Plot
sphericityPlot(
  dataWide = beerWide, # Data needs to be in wide format
  subjectID = "judge" # character name of the subject column
)
## Look for any groups that have very different amounts of variation
# Mauchly's Test
beerSphere$`Mauchly's Test for Sphericity` %>%
  dplyr::select(Effect, W, p) %>%
  knitr::kable(
    digits = 4,
  col.names = c("Effect", "Mauchly's W", "p"),
  caption = "Mauchly's Sphericity Test",
  align = c('l', "c", "c"),
  booktab = TRUE
) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
  )
beerTable1 <- parameters::model_parameters(</pre>
    model = beerOmni, # the RCBD with all fixed effects
    omega_squared = "partial",
    eta_squared = "partial",
    epsilon_squared = "partial"
)
beerTable1$p <- lapply(</pre>
 X = beerTable1$p,
  FUN = pvalRound
knitr::kable(
 x = beerTable1,
  digits = 4,
  col.names = c("Source", "SS", "df", "MS", "F", "p-value",
                "Partial Omega Sq.", "Partial Eta Sq.", "Partial Epsilon Sq."),
  caption = "ANOVA Table for Beer Judging Study",
  align = c('l', rep('c', 8)),
  booktab = TRUE
) %>%
```

```
kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("scale_down", "HOLD_position")
  )
# Use the block relative efficiency function
block.RelEff(
  aov.obj = beerOmni,
 blockName = "judge",
  trtName = "beer"
)
correctedTable <- beerSphere$`Sphericity Corrections` %>%
  dplyr::select(GGe, `p[GG]`, HFe, `p[HF]`)
correctedTable$`p[GG]` <- lapply(</pre>
 X = correctedTable$`p[GG]`,
 FUN = pvalRound
)
correctedTable$`p[HF]` <- lapply(</pre>
 X = correctedTable$`p[HF]`,
 FUN = pvalRound
)
knitr::kable(
 x = correctedTable,
 digits = 4,
  col.names = c("Greenhouse-Geisser", "p-value", "Huynh-Feldt", "p-value"),
  caption = "Sphericity Corrections",
  align = "c",
  booktab = TRUE
) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
  )
# Using emmeans
beerPH <- emmeans::emmeans(</pre>
  object = beerMixed, # Notice the use of the mixed model here
 specs = pairwise ~ beer,
 adjust = "tukey",
  level = 0.92
)
## Point Estimates
as.data.frame(beerPH$emmeans) %>%
  knitr::kable(
    digits = 4,
    col.names = c("Type of Beer", "Marginal Mean", "SE", "DF",
                  "Lower Bound", "Upper Bound"),
    caption = "Marginal Means-Tukey 92\\% Adjustment",
    align = c("l", rep("c", 5)),
    booktabs = TRUE
  ) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
```

```
# Pairwise Comparisons
as.data.frame(beerPH$contrasts) %>%
  knitr::kable(
    digits = 4,
    col.names = c("Comparison", "Difference", "SE", "DF",
                  "t Statistic", "p-value"),
    caption = "Marginal Means-Tukey 92\\% Adjustment",
    align = c("l", rep("c", 5)),
    booktabs = TRUE
  ) %>%
  kableExtra::kable styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
tempEMM <- emmeans::emmeans(</pre>
  object = beerMixed,
  specs = "beer"
)
# Pass the stored marginals into the effect size function
cohenTemp <- emmeans::eff_size(</pre>
 object = tempEMM,
 sigma = sigma(beerMixed),
  edf = df.residual(beerMixed)
)
# Create a data frame, add on the probability of superiority
# Send that data frame into a nice table
as.data.frame(cohenTemp) %>%
  dplyr::mutate(
    ps = probSup(effect.size),
    .after = effect.size
  dplyr::select(contrast, effect.size, ps) %>%
  knitr::kable(
   digits = 3,
    col.names = c("Comparison", "Cohen's d", "Probability of Superiority"),
    align = "lcc",
    caption = "Effect Sizes for Beer",
    booktab = TRUE
  ) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = "HOLD_position"
  )
# Hasse Diagram for Shoe Advertisement Study
modelLabels <- c("1 Sell Shoes 1", "2 Campaign 1", "10 (Market) 8", "3 Time Point 2",
                 "6 Campaign × Time Point 2", "30 (Markets X Time) 16")
modelMatrix <- matrix(</pre>
  data = c(FALSE, FALSE, FALSE, FALSE, FALSE, TRUE, FALSE, FALSE, FALSE, FALSE,
           FALSE, TRUE, TRUE, FALSE, FALSE, FALSE, FALSE, TRUE, FALSE, FALSE, FALSE,
           FALSE, FALSE, TRUE, TRUE, FALSE, TRUE, FALSE, FALSE, TRUE, TRUE, TRUE,
           TRUE, TRUE, FALSE),
```

```
nrow = 6,
  ncol = 6,
  byrow = FALSE
)
hasseDiagram::hasse(
data = modelMatrix,
labels = modelLabels
shoesWide <- read.table(</pre>
  file = "https://raw.github.com/neilhatfield/STAT461/master/dataFiles/shoes.csv",
 header = TRUE,
  sep = ","
)
shoesWide$campaign <- as.factor(shoesWide$campaign)</pre>
shoesWide$market <- as.factor(shoesWide$market)</pre>
# Make a long version of the data
shoesLong <- tidyr::pivot_longer(</pre>
  data = shoesWide,
  cols = dplyr::starts_with("t"),
  names_to = "time",
  names_ptypes = list("time" = factor()),
  values_to = "sales"
)
# Recode times to before, during, after and puts an ordering
shoesLong$time <- dplyr::recode_factor(</pre>
  shoesLong$time,
  "t1" = "before",
  "t2" = "during",
  "t3" = "after",
  .ordered = TRUE
)
# Omnibus Model
shoesModel <- aov(</pre>
  formula = sales ~ campaign * time + Error(market %in% campaign),
  data = shoesLong
)
# Use the nlme package to fit a model that we can use for assumption checking
shoesAssumptions <- nlme::lme(</pre>
 data = shoesLong,
  fixed = sales ~ campaign * time,
  random = ~ 1|market
)
# QQ plot for residuals
car::qqPlot(
 x = residuals(shoesAssumptions),
  distribution = "norm",
  envelope = 0.90,
  id = FALSE,
  pch = 20,
  ylab = "Residuals (sales)"
)
```

```
# Market Effects
car::qqPlot(
 x = unlist(
    lme4::ranef(
      object = shoesAssumptions,
      whichel = c("market")
    )
 ),
 distribution = "norm",
 envelope = 0.90,
 id = FALSE,
 pch = 20,
 ylab = "Market"
# Tukey-Anscombe Plot
ggplot(
 data = data.frame(
   residuals = residuals(shoesAssumptions),
   fitted = fitted.values(shoesAssumptions)
 mapping = aes(x = fitted, y = residuals)
  geom_point(size = 2) +
 geom_hline(
   yintercept = 0,
    linetype = "dashed",
    color = "grey50"
  ) +
  geom_smooth(
    formula = y \sim x,
    method = stats::loess,
   method.args = list(degree = 1),
    se = FALSE,
   size = 0.5
 ) +
 theme_bw() +
 xlab("Fitted values (sales)") +
 ylab("Residuals (sales)")
ggplot(
 data = shoesLong,
 mapping = aes(
    x = time,
   y = sales,
    color = paste(campaign, market, sep = ":"),
    group = paste(campaign, market, sep = ":")
) +
 geom_point() +
 geom_line() +
 theme_bw() +
 xlab("Time Point (relative to campaign)") +
 ylab("Sales (coded)") +
 labs(
    color = "Campagin:Market"
# Growth Curves
```

```
ggplot(
  data = shoesLong,
  mapping = aes(
    x = time,
    y = sales,
    color = market,
    group = market
    )
) +
  geom_point(size = 2) +
  geom_line() +
  facet_wrap(facets = ~campaign) +
  ggplot2::theme_bw() +
  xlab("Time Period") +
  ylab("Sales (coded)") +
  labs(
    color = "Market"
# Sphericity Plot
sphericityPlot(
  dataWide = shoesWide,
  subjectID = c("market", "campaign"),
  colsIgnore = NULL
)
## Look for any groups that have very different amounts of variation
shoesTemp <- summary(shoesModel)</pre>
shoesOmni <- rbind(</pre>
  shoesTemp$`Error: market:campaign`[[1]],
  shoesTemp$`Error: Within`[[1]]
row.names(shoesOmni) <- c("campaign", "market", "time", "campaign:time", "market:time")</pre>
shoesOmni["market", "F value"] <- shoesOmni["market", "Mean Sq"] /</pre>
  shoesOmni["market:time", "Mean Sq"]
shoesOmni["market", "Pr(>F)"] <- pf(</pre>
  q = shoesOmni["market", "F value"],
 df1 = shoesOmni["market", "Df"],
 df2 = shoesOmni["market:time", "Df"],
  lower.tail = FALSE
)
shoesOmni %>%
  tibble::rownames_to_column(
    var = "Source"
  ) %>%
  knitr::kable(
    digits = 4,
    col.names = c("Source", "df", "SS", "MS", "F", "p-value"),
    caption = "ANOVA Table for Athletic Shoes Study",
    align = c('l',rep('c',5)),
    booktab = TRUE
  ) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
```

```
# Using emmeans
shoesCampaignPH <- emmeans::emmeans(</pre>
 object = shoesModel,
 specs = pairwise ~ campaign,
 adjust = "tukey",
 level = 0.99
)
shoesTimePH <- emmeans::emmeans(</pre>
 object = shoesModel,
 specs = pairwise ~ time,
 adjust = "tukey",
 level = 0.99
)
## Point Estimates
as.data.frame(shoesCampaignPH$emmeans) %>%
 knitr::kable(
    digits = 4,
    col.names = c("Campaign", "Marginal Mean", "SE", "DF",
                  "Lower Bound", "Upper Bound"),
    caption = "Marginal Means-Tukey 99\\% Adjustment",
    align = c("l", rep("c", 5)),
    booktabs = TRUE
 ) %>%
 kableExtra::kable styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
as.data.frame(shoesTimePH$emmeans) %>%
 knitr::kable(
    digits = 4,
    col.names = c("Time Point", "Marginal Mean", "SE", "DF",
                  "Lower Bound", "Upper Bound"),
    caption = "Marginal Means-Tukey 99\\% Adjustment",
    align = c("l", rep("c", 5)),
    booktabs = TRUE
 ) %>%
 kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
    font_size = 12,
    latex_options = c("HOLD_position")
# Pairwise Comparisons
as.data.frame(shoesCampaignPH$contrasts) %>%
 knitr::kable(
    digits = 4,
    col.names = c("Comparison", "Difference", "SE", "DF",
                  "t Statistic", "p-value"),
    caption = "Campaign Comparison-Tukey 99\\% Adjustment",
    align = c("l", rep("c", 5)),
    booktabs = TRUE
  ) %>%
 kableExtra::kable_styling(
```

```
bootstrap_options = c("striped", "condensed"),
   font_size = 12,
   latex_options = c("HOLD_position")
as.data.frame(shoesTimePH$contrasts) %>%
  knitr::kable(
   digits = 4,
   col.names = c("Comparison", "Difference", "SE", "DF",
                  "t Statistic", "p-value"),
   caption = "Time Point-Tukey 99\\% Adjustment",
   align = c("1", rep("c", 5)),
   booktabs = TRUE
  ) %>%
  kableExtra::kable_styling(
    bootstrap_options = c("striped", "condensed"),
   font_size = 12,
   latex_options = c("HOLD_position")
```