Instructions: *You must respond to Question 1, Question 2, and Question 3. In addition, respond to 1 (one) other question of your choosing from the remaining list. Provide responses to entire questions, not parts of multiple ones.*

All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Your responses must be original; you may not insert text that you have prepared prior to this exam. Nor may you consult abstracts or articles that you might have on your computer or in paper form.

You may use diagrams in your responses; label each with a title (e.g., "Figure x"), and insert a clear reference to each figure in the appropriate place in your narrative. Turn in any diagrams with your responses.

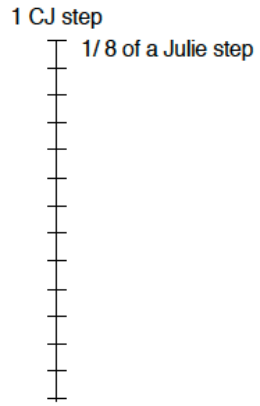
Your responses may cite literature that is not explicitly mentioned in a question. You may use Endnote to insert references. Good luck and good writing!

Qualifying Exam

Question 1

This question refers to the interview you conducted using the step task below:

The figure to the right shows the relationship between the size of CJ's step and the size of Julie's step. Suppose CJ and Julie walk the same distance. How many steps did CJ take in relation to the number of steps Julie took?



Summarize your method of creating the interview protocol and analyzing the interview data of the interview you conducted. You must address:

a) The theoretical perspective(s) that you employed in the design and analysis of the interview, or that you *would* employ were you to do it again.
b) State the mathematical understandings and ways of thinking that you see being involved in responding appropriately to this task. Describe how your insights into these understandings shaped your analysis of the data and the revision of your interview protocol.

Question 2

Cobb (2007) compares and contrasts four theoretical perspectives that have made a considerable mark on mathematics education: experimental psychology, cognitive psychology, sociocultural theory, and distributed cognition.

a) Given that one purpose of mathematics education research is to come to terms with how individuals learn, summarize the characterization of the individual and what counts as learning from each of these perspectives.

I like this question.

b) Historically, the concerns and interests that motivated these perspectives differ from those of mathematics educators. How might the various perspectives be sources of ideas to be appropriated and adapted in the service of mathematics education?

Question 3

Researchers have formulated and adopted different definitions of advanced mathematical thinking (AMT). Select 2 (two) of the following articles, and summarize, compare, and contrast the authors' definition of AMT. Include a discussion of the relationship between theory and research, more explicitly how the varying definitions impact the types of questions asked, the methods employed, and the contributions that are made to the field of mathematics education research.

This is a good question. But see my comments to #6. I suggest you switch them.

* Edwards, B. S., Dubinsky, E., & McDonald, M. A. (2005). Advanced mathematical thinking. *Mathematical Thinking and Learning, 7*(1), 15-25.

* Harel, G., & Sowder, L. (2005). Advanced mathematical-thinking at any age: Its nature and its development. *Mathematical Thinking and Learning, 7*(1), 27-50.

* Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing mathematical activity: A practice-oriented view of advanced mathematical thinking. *Mathematical Thinking and Learning, 7*(1), 51-73.

Question 4

Below is an excerpt from a fifth-grade mathematics lesson on exponents in a public school, quoted from Lampert (1990). Summarize important issues that would arise from the perspectives of a cognitive psychologist and a sociocultural theorist. Include in your discussion, what questions each theorist might have and what methods each might employ to paint more of a picture of the mathematical learning that is taking place.

> The lesson began when I wrote on the blackboard at the beginning of class, "What is the last digit in: 5^4, 6^4, 7^4?" and I challenged the class to tell me if they could prove that their conjectures about what these last digits would be were true without doing the full multiplications…

> Sam asserted, about the last digit in 5^4, "It *has* to end in a 5." I invited everyone in the class to consider the validity of Sam's decisive assertion and to see if they could explain why he seemed to be so sure. The question I was asking was, How does he know that is true? Harriet said, "Well, anything multiplied by 5 has to end in a 5 or a zero," and Theresa quickly added, "but you get a 5 [for a last digit]." Martha observed, "You times the square number, you square it again and get 625. And Carl responded, moving to the level of a mathematical generalization, "You don't have to do that. It's easy, the last digit *is always going to be 5* because you are always multiplying last digits of 5, and 5 times 5 ends in a 5."

Question 5

Glasersfeld (1995) referred to *the construction of others* in his account of the socialization of knowledge. Interview methodology can be viewed as a means for "constructing the interviewee."

a) Summarize Glasersfeld's key ideas relating to *the construction of others* (from a radical constructivist perspective).

b) Relate the issues Glasersfeld addresses to issues of interview methodology.

Question 6

a) Select two of the following perspectives. Discuss how advanced mathematical thinking (AMT) would be defined from these theoretical perspectives:
   - Cognitive psychology
   - Distributed cognition
   - Sociocultural theory

b) What evidence would each seek to support a claim that someone has engaged in advanced mathematical thinking?

Question 6 seems to have a large overlap with #3. I actually prefer #6 to #3 on an exam like this one. Asking about specific articles almost inevitably privileges students who happened, by chance, to review those articles most recently.

REFERENCES

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Greenwich, CT: Information Age Publishing.

Edwards, B. S., Dubinsky, E., & McDonald, M. A. (2005). Advanced mathematical thinking. *Mathematical Thinking and Learning, 7*(1), 15-25.

von Glasersfeld, E. (1995). Constructing agents: The self and others. In L. P. Steffe & P. W. Thompson (Eds.), *Radical Constructivism: A Way of Knowing and Learning* (pp. 113-128). New York, NY: RoutledgeFalmer.