Math Education Qualifying Exam, Fall 2011 **Name:**

Instructions: *You must respond to Question 1. In addition, respond to 2 (two) other questions of your choosing from the remaining list. Provide responses to entire questions, not parts of multiple ones. Write your response to each sub-question below that sub-question. Delete the two questions that you choose not to answer; the file you turn in should contain only the questions you answered and your responses.*

All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. The recommended length limit for each response is roughly 1000 words. If you use diagrams in your responses, it is recommended that you draw them by hand, label each with a title (e.g., "Figure x"), and insert a clear reference to each one in the appropriate place in your narrative. Turn in any diagrams with your responses. Your responses may cite literature that is not explicitly mentioned in a question.

You have 3 hours to complete the exam. Good luck and good writing!

Indicate whether you are taking this exam as a: PhD qualifier ___, Master's qualifier ___, Neither ___.

Question 1 (Mandatory)

a) In *The Teaching Gap*, authors Stigler and Hiebert (1999) frame teaching as a "cultural activity". Characterize and explain what the authors mean by this. In doing so, briefly summarize the educational research context (the problem, collected data, and salient results) and the type of evidence that gave rise to this framing.

b) Discuss the significance and some important implications of viewing teaching as a cultural activity—what is there to be gained from viewing teaching in this way?

c) Thompson (1994) presented this figure to convey his broad perspective of key ideas in the literature on the learning and teaching of functions:

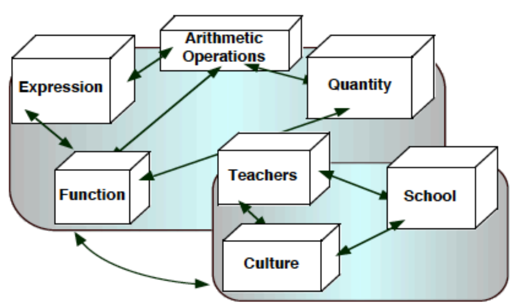

Figure 1. Students' concepts always emerge in relation to other concepts they hold, and in relation to teachers' orientatons and in relation to cultural values expressed by teachers, peers, and families.

(*i*) Briefly describe the perspective embodied in the figure, and (*ii*) discuss at least two important connections you see between this perspective and the idea of *teaching as a cultural activity* (i.e., what do the two have to do with each other?)

Question 2

a) Characterize Carlson et al.'s (2002) *covariation framework*. Your characterization should include a description of: (*i*) what it means to reason covariationally, (*ii*) what the framework is about, and (*iii*) the framework's key components and interconnections among them.

b) Discuss the ostensible usefulness of this framework. Include in your discussion at least one illustrative example drawn from the research literature (be explicit and clear as to what your example illustrates).

Question 3

Konold (1989) proposed an empirically-derived model of informal reasoning under conditions of uncertainty that he calls the *outcome approach*.

a) Characterize the outcome approach: describe what it is about, what its components are and how they work together to comprise the model.

b) Give an interpretation that an outcome-oriented individual would make of this weather forecast: *"There is a 70% chance of rain tomorrow"*.

In what essential ways would such an interpretation differ from a relative frequency interpretation?

c) How is the outcome approach useful as an explanatory construct?—*What* might it explain and *how* might it do so? (Your response should indicate the significance of model.)

Question 4

In the two articles, *Talking About Rates Conceptually, Parts I and II*, authors Thompson and Thompson (1994, 1996), report on a sequence of lessons that were intended to help a middle school student (Ann) develop an understanding of speed as a rate. The authors offer the following description of teaching that is guided by a conceptual orientation:

A teacher with a conceptual orientation is one whose actions are driven by:

- an image of a system of ideas and ways of thinking that he or she intends the student to develop,
- an image of *how these ideas and ways of thinking can develop,*
- ideas about *features of materials, activities, expositions, and students' engagement with them* that can orient students' attention in productive ways, and
- *an expectation and insistence that students be intellectually engaged* in tasks and activities.

a) Interpret each of these components of conceptual teaching (e.g., describe briefly what is entailed).

b) Describe how these components were present in the design and enactment of this teaching experiment, and how they contributed to the teaching experiment's outcomes.

Question 5

a) Characterize the *action* and *process* conceptions of function, as developed by Breidenbach, Dubinsky, Hawks & Nichols (1992).

b) Provide an example of indicators or beliefs that would be consistent with an action versus a process conception of function.

c) Briedenbach et al. (1992) included the following question as part of their effort to assess whether students had developed process conceptions of functions at the end of their instructional study:

> f, g, h are functions whose domain and ranges are the set of all real numbers, and such that $h = f \circ g$.
>
> If only the information in the following table were known, would it be possible to find $f(2)$? If so, find it and if not explain why not.
>
> | x | $h(x)$ | $g(x)$ |
> |---|---|---|
> | -1 | 1 | -3 |
> | 4 | π | 1 |
> | π | 0 | 2 |

Answer the question and explain how the reasoning employed in answering it might entail a process conception of function.

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