Math Education Qualifying Exam, Fall 2012 Name:

Instructions: *You* <u>*must*</u> *respond to Question 1. In addition, respond to 2 (two) other questions of your choosing from the remaining list. Provide responses to entire questions, not parts of multiple ones. Write your response to each sub-question below that sub-question. Delete the two questions that you choose* <u>*not*</u> *to answer; the file you turn in should contain only the questions you answered and your responses.*

All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. The recommended length limit for each response is roughly1000 words. If you use diagrams in your responses, it is recommended that you draw them by hand, label each with a title (e.g., "Figure x"), and insert a clear reference to each one in the appropriate place in your narrative. Turn in any diagrams with your responses. Your responses may cite literature that is not explicitly mentioned in a question.

You have 3 hours to complete the exam. Good luck and good writing!

Indicate whether you are taking this exam as a: PhD qualifier ___, Master's qualifier ___, Neither ___.

<u>Question 1 (Mandatory)</u>

Imagine that you are part of a conversation in which someone asserts that mathematics education research is relatively straight-forward because it deals with the production of mathematical solutions that can be judged to be either correct or incorrect. Frame a response to this assertion that touches on the following questions:

a) What does it mean to understand mathematics?
b) What does mathematics education research have to say about the importance of incorporating students' understandings in curricular design?
c) How does mathematics education research inform us regarding the relationship between student performance and student understanding?
d) What does mathematics education research have to say about teaching to support student understanding of mathematics?

Support your argument by drawing on notions from research literature we have read during this semester.

<u>Question 2</u>

In *The Teaching Gap*, authors Stigler and Hiebert (1999) frame teaching as a "cultural activity" and compare mathematics teaching practices in Japan and Germany with those in the United States.

a) Characterize and explain what the authors mean by teaching as a cultural activity. Briefly summarize the educational research context (the problem, collected data, and salient results) and the type of evidence that gave rise to this framing.

b) Relate the idea of teaching as a cultural activity with what you understand of the difficulty American mathematics teachers have in changing the mathematics they teach and how they teach it.

c) Discuss the significance and some important implications of viewing teaching as a culture activity – what is there to be gained from viewing teaching in this way? And how can we improve mathematics teaching if we consider teaching as a cultural activity?

Question 3

Carlson et al. (2002) proposed and used “covariation framework” to assess students’ covariational reasoning in dynamic functional relationship.

a) Characterize the covariation framework. Your discussion should include a description of (*i*) what the authors mean by reasoning covariationally, (*ii*) what the covariation framework is about, and (*iii*) what constitutes the framework and how the components are interconnected.

b) Explain how student reasoning employed in answering the following problem (Figure 1.) might entail various conceptions of a function as covariation of quantities.

Imagine this bottle filling with water. Sketch a graph of the height as a function of the amount of water that’s in the bottle.

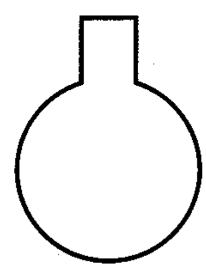

Figure 1. **The Bottle Problem.**

c) What are your opinions on the ostensible usefulness of Carlson et al.’s (2002) covariation framework in designing tasks and instructional prompts to promote coherence in students’ understanding of function? Support your argument by drawing from the research literature.

Question 4

Tall and Vinner (1981) use the notions of *concept image*, *concept definition*, and *concept definition image* to highlight various roles that images and definitions may play in students' mathematical reasoning.

a) Characterize what Tall and Vinner mean by these constructs. Include a description of (*i*) how individuals develop concept images and definitions, (*ii*) what it means to experience cognitive conflict, and (*iii*) how inappropriate concept images can emerge.
b) Compare and/or contrast Tall and Vinner's construct of concept image with the word *image* of Carlson et al. (2002) in describing the images of covariation and that of Thompson and Thompson (1994, 1996) in describing the images of speed.
c) Discuss some implications of these constructs for the curricular development of mathematical ideas.

<u>Question 5</u>

Thompson (1993, 1994) highlight the importance of quantitative reasoning in the development of the concept of speed and the concept of difference.

a) Characterize what these researchers mean by *quantities*, *quantitative reasoning*, and *quantitative operations*. Include (*i*) a description of what each construct means; and (*ii*) a discussion of how quantitative reasoning are tied to assimilation, generalization, and transfer of mathematical knowledge.
b) Give an example from one of our readings that illustrates what role the quantitative reasoning plays in the development of student understanding of a mathematical concept.
c) Discuss the implications of these constructs for the design of curriculum and instructional sequences.

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