(Part I, Fall 09)

Instructions: *You* <u>*must*</u> *respond to either Question 1 or Question 2. In addition, respond to 2 (two) other questions of your choosing from the remaining list. Provide responses to entire questions, not parts of multiple ones.*

All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. The recommended length limit for each response is roughly 1000 words. If you use diagrams in your responses, it is recommended that you draw them by hand, label each with a title (e.g., "Figure x"), and insert a clear reference to each one in the appropriate place in your narrative. Turn in any diagrams with your responses. Your responses may cite literature that is not explicitly mentioned in a question. Good luck and good writing!

<u>Question 1</u>

Sierpinska (1992) paints the following picture of what is involved in learning:

> Acts of understanding or acts of overcoming an obstacle are very demanding on both intellectual concentration and emotional tension. In fact, the second always accompanies the first. Tension cannot be eliminated from learning, it is the very essence of it: *there is no royal road to geometry*, or, as Byers paraphrases it: "there is no painless road to learning."

Elaborate on this characterization of learning and understanding. Summarize evidence from research that supports this claim, and discuss the significance of viewing learning in this manner. Use the findings that are presented in *The Teaching Gap* (Stigler & Hiebert, 1999) to describe how this view of learning is realized in various cultures around the world.

<u>Question 2</u>

Imagine that you are part of a conversation in which someone asserts that mathematics education research is relatively straight-forward because it deals with the production of mathematical solutions that can be judged to be either correct or incorrect.

Frame a response to this assertion that touches on the types of questions that mathematics education research addresses, and that provides evidence supporting a richer picture of what it means to understand mathematics. Include a discussion of the complex and nuanced relationship that exists between performance and understanding. Support your argument by drawing on notions such as pseudo-analytical thought processes and behavior as discussed in Carlson, Jacobs, Coe, Larsen, & Hsu (2002).

Question 3

Oehrtman (2009) uses metaphor as a construct to describe student understanding of the limit concept.

a) Describe what is meant by metaphor in this context, the distinction between strong and weak metaphors, and the metaphors for limit that emerged in the research.
b) What are some implications for using metaphor as a construct to describe student understanding and the development of conceptual understanding?

Question 4

In the two articles, *Talking About Rates Conceptually, Parts I and II*, authors Thompson and Thompson (1994, 1996), report on a sequence of lessons that were intended to help a middle school student (Ann) develop an understanding of speed as a rate. The authors offer the following description of teaching that is guided by a conceptual orientation:

A teacher with a conceptual orientation is one whose actions are driven by:

- an image of a system of ideas and ways of thinking that he or she intends the student to develop,
- an image of *how these ideas and ways of thinking can develop,*
- ideas about *features of materials, activities, expositions, and students' engagement with them* that can orient students' attention in productive ways, and
- *an expectation and insistence that students be intellectually engaged* in tasks and activities.

a) Interpret each of these components of conceptual teaching (*e.g.,* describe briefly what is entailed).
b) Describe how these components were present in the design and enactment of this teaching experiment, and how they contributed to its eventual success.

Question 5

a) Characterize the *action* and *process* conceptions of function, as developed by Breidenbach, Dubinsky, & Nichols (1992). Include a discussion of the *operational* and *structural* conceptions and the process of *reification* that are discussed by Sfard (1992).
b) Provide an example of indicators or beliefs that would be consistent with an action versus a process conception of function.

Question 6

Tall & Vinner (1981) present the constructs of *concept image*, *concept definition,* and *concept definition image*.

a) Characterize what Tall & Vinner mean by these constructs. Include a description of how individuals develop concept images and definitions, what it means to experience cognitive conflict, and how inappropriate concept images can emerge.
b) What are some of the implications of these constructs for the curricular development of mathematical ideas?

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the Process Conception of Function. *Educational Studies in Mathematics, 23*(3), 247-285.

Carlson, M. P., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education, 33*(5), 352-378.

Oehrtman, M. (2009). Collapsing dimensions, physical limitation, and other student metaphors for limit concepts. *Journal for Research in Mathematics Education, 40*(4), 396-426.

Sfard, A. (1992). Operational origins of mathematical notions and the quandary of reification - the case of function. In E. Dubinsky & G. Harel (Eds.), *The concept of function: Aspects of epistemology and pedagogy*. Washington, D. C. : Mathematical Association of America.

Sierpinska, A. (1992). On understanding the notion of function. In E. Dubinsky & G. Harel (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 25-58). Washington, D. C.: Mathematical Association of America.

Stigler, J. W., & Hiebert, J. (1999). *The Teaching Gap*. New York, NY: Free Press.

Tall, D., & Vinner, S. (1981). Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity. *Educational Studies in Mathematics, 12*(2), 151-169.

Thompson, A. G., & Thompson, P. W. (1996). Talking about rates conceptually, Part II: Mathematical knowledge for teaching. *Journal for Research in Mathematics Education, 27*(1), 2-24.

Thompson, P. W., & Thompson, A. G. (1994). Talking about Rates Conceptually, Part I: A Teacher's Struggle. *Journal for Research in Mathematics Education, 25*(3), 279-303.