Instructions: *You* <u>*must*</u> *respond to either Question 1 or Question 2. In addition, respond to 2 (two) other questions of your choosing from the remaining list. Provide responses to entire questions, not parts of multiple ones.*

All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. The recommended length limit for each response is roughly 1000 words. If you use diagrams in your responses, it is recommended that you draw them by hand, label each with a title (e.g., "Figure x"), and insert a clear reference to each one in the appropriate place in your narrative. Turn in any diagrams with your responses. Your responses may cite literature that is not explicitly mentioned in a question. Good luck and good writing!

<u>Question 1</u>

In *The Teaching Gap*, authors Stigler and Hiebert (1999) frame teaching as a "cultural activity". Characterize and explain what the authors mean by this. In doing so, summarize the type of evidence that formed the basis of this framing, and summarize the educational research context (the problem, collected data, and salient results) from which this framing emerged. Elaborate some of the main arguments for the significance of viewing teaching as a cultural activity.

<u>Question 2</u>

Imagine that you have been hired by a university mathematics department as a faculty member specializing in mathematics education research. In your daily interactions with your mathematician colleagues you find yourself having to explain the raison d'être for mathematics education research as a scholarly field.

Write a narrative that captures what you would say to your colleagues who want to better understand the following: what the field is about, some key problems that pre-occupy the field, approaches and methods for addressing such problems, standards of evidence for claims and conclusions in the field, and how these differ from parallel aspects in research in mathematics. You wish to present a viewpoint that strikes a balance between arguing for the scholarly character and merits of mathematics education research and the reality that this is still a relatively young academic discipline. Bolster your arguments and viewpoints by drawing on the articles by Schoenfeld (2000) and Sierpinska et al. (1993), as well as other scholars/works of your choosing.

Question 3

a) Characterize Carlson et al.'s (2002) *covariation framework*. Your discussion should include a description of what it means to reason covariationally, what the framework is about, and it should describe the framework's various components and interconnections among them.

b) What is the ostensible usefulness of this framework? Illustrate and bolster your argument with at least one example of its usefulness drawn from the research literature (be explicit and clear as to what your example illustrates).

Question 4

In Chapter 2 of *Understanding in Mathematics*, Sierpinska (1992) elaborates the following components of an act of understanding:

- The understanding subject
- The object of understanding
- The basis of understanding
- Mental operations involved in understanding: identification, discrimination, generalization, and synthesis.

a) Characterize what Sierpinska means by each of these components.

b) Discuss how these components and interrelations among them, when taken as a whole, might comprise part of a useful framework for analyzing and interpreting a student's mathematical sense making activities. Situate your discussion within a concrete example (it could be a hypothetical example) in order to help communicate your arguments and points.

Question 5

a) Compare and contrast the ideas of *cognitive obstacle* (Herscovics, 1989) and *epistemological obstacle* (Sierpinska, 1992): characterize each one, as elaborated by these authors, and discuss similarities and distinctions between them.

b) For each of these ideas, provide an example that illustrates and clarifies how the idea has been used as an explanatory construct in the learning of mathematics. You may draw on Herscovics' (1989) and Sierpinska's (1992) chapters for examples.

Question 6

In a sequence of articles—*Talking About Rates Conceptually, Parts I and II*—authors Thompson and Thompson (1994, 1996) report on a sequence of lessons involving interactions between a teacher (Bill) and a middle school student (Ann). These interactions occurred in the context of a protracted effort to help Ann develop an understanding of speed as a rate. The authors made the following statement in both articles:

> "The image of speed we intended students construct through this unit is composed of these items, which themselves are constructions:
>
> 1. Speed is a quantification of motion;
> 2. completed motion involves two completed quantities—distance traveled and amount of time required to travel that distance (this must be available to students both in retrospect and in anticipation);
> 3. speed as a quantification of completed motion is made by multiplicatively comparing distance traveled and amount of time required to go that distance;
> 4. there is a direct proportional relationship between distance traveled and amount of time required to travel that distance. That is, if you go *m* distance units in *s* time units at a constant speed, then at this speed you will go *a/b x m* distance units in *a/b x s*." (Thompson & Thompson, 1994, p. 283)

a) Briefly interpret each of the items above (i.e., describe what they mean).

b) Describe two arguably distinct ways in which the authors used the above scheme of ideas in their research study.

c) Discuss how the authors' use of this scheme is related to what Thompson (2008) writes about in his later article *Conceptual analysis of mathematical ideas: Some spadework at the foundation of mathematics education*. In particular, discuss this relationship with regard to the ideas of coherence and meaning elaborated in the later article.

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