Mathematics Education Qualifying Exam
Spring 2011

Instructions: *You <u>must</u> respond to Questions 1 and 2. In addition, respond to two (2) other question of your choosing from the remaining list. Provide responses to entire questions, not parts of multiple ones. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. The recommended length limit for each response is roughly 1000 words. If you use diagrams in your responses, draw them by hand, label each with a title (e.g., "Figure x"), and insert a clear reference to each one in the appropriate place in your narrative. Turn in any diagrams with your responses. Your responses may cite literature that is not explicitly referenced in a question. Good luck and good writing!*

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Question 1 (Mandatory)

Cobb (2007) compares and contrasts four theoretical perspectives that have significantly influenced research in mathematics education: *experimental psychology*, *cognitive psychology*, *sociocultural theory*, and *distributed cognition*.

(a) Given that a central purpose of mathematics education research is to develop insight into how individuals learn, summarize the characterization of the individual and what counts as learning from each of these perspectives.

(b) Historically, the concerns and interests that motivated these perspectives differ from those of mathematics educators. How might the various perspectives be sources of ideas to be appropriated and adapted in the service of mathematics education?

Question 2 (Mandatory)

Roh conducted a study that explored calculus students' understanding of aspects of the logical structure of the ε-N definition of the limit of a sequence:

A sequence $\{a_n\}_{n=1}^{\infty}$ is convergent to L if for any positive real number ε, there is a natural number N such that for all $n>N$, $|a_n - L| < \varepsilon$.

i) Describe the targeted understanding of limit of a sequence that Roh devised and employed in her study.

ii) Describe Roh's epsilon strip activity. How was this activity employed both as an instructional intervention and, together with her model targeted understanding, as a basis for interpreting the data (i.e., as a research tool) generated in her study?

iii) Summarize Roh's results describing students' ways of using the counting process. What were the implications of her study regarding "essential components that students must conceptualize in order to understand the relationship between ε and N in defining the limit of a sequence" (p. 277)?

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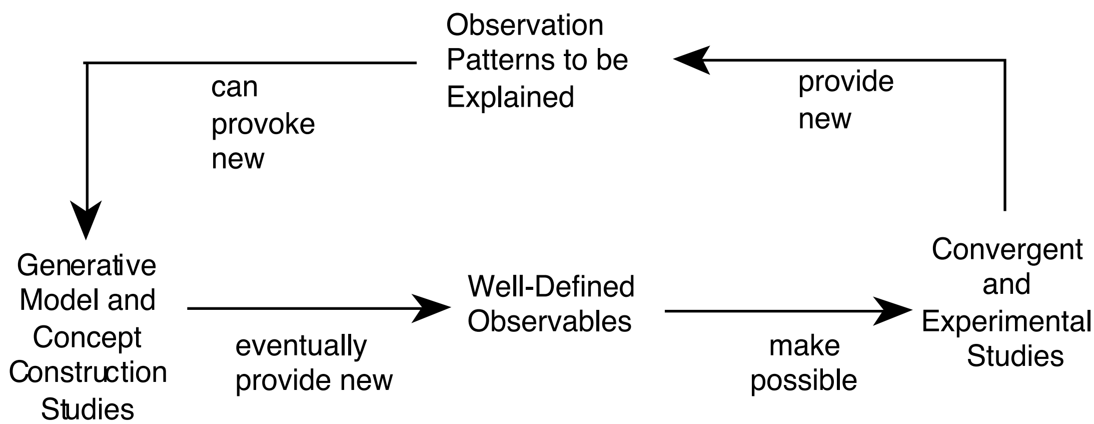
Respond to any *two (2)* *of the following questions*

Question 3

Clement (2000) presented the following figure to convey "How Work at a Convergent Level Can Initiate Work at a Generative Level" in his discussion of convergent and generative approaches to the analysis of interview data.



Discuss the cycle of research activity that Clement aims to convey in this figure, and how (in his view) it contributes to the generation of scientifically-based insights and models of cognition in educational research. Your discussion should:

i) Explain what Clement means by both *generative* and *convergent* approaches in educational research, and summarize his view of the relative advantages of each of them.

ii) Provide one or two example(s) that illustrate the two approaches

Question 4

Your recent work in RUME II involved designing and conducting a "teaching interview" with an individual learner around a set of mathematical tasks. Summarize your method of creating the teaching-interview protocol and analyzing the data generated in the interview you conducted. Your summary must include a description of:

a) The mathematical task(s) employed, and the underlying mathematical concepts that those tasks were designed to address.

b) The theoretical perspective(s) that you employed in the design and analysis of the interview, or that you *would* employ were you to do it again. In particular, describe the role that conceptual analysis of the mathematical learning objectives (Thompson & Saldanha, 2003) played in your actual design and analysis of the interview, and in any envisioned future revisions of your interview protocol.

Question 5

Some mathematics education researchers have been involved in an effort to flesh out "Advanced Mathematical Thinking" (AMT). This effort has involved formulating and adopting different definitions of AMT. Choose 2 (two) of the following articles, and summarize, compare, and contrast the authors' definition of AMT. Include a discussion of the relationship between theory and research, more explicitly how the various definitions impact the types of questions asked, the methods employed, and the contributions made to the field of mathematics education research.

- Edwards, B. S., Dubinsky, E., & McDonald, M. A. (2005). Advanced mathematical thinking. *Mathematical Thinking and Learning, 7*(1), 15-25.

- Harel, G., & Sowder, L. (2005). Advanced mathematical-thinking at any age: Its nature and its development. *Mathematical Thinking and Learning, 7*(1), 27-50.

- Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing mathematical activity: A practice-oriented view of advanced mathematical thinking. *Mathematical Thinking and Learning, 7*(1), 51-73.

Question 6

a) Discuss Harel & Sowder's (2005) constructs *Ways of Understanding* and *Ways of Thinking*. In addition to explicating the meaning of these constructs, your discussion should describe key distinctions among them, and the constructs' potential usefulness—i.e., what work do these constructs do for these researchers' efforts to understand and characterize individuals' mathematical reasoning and learning?

b) Harel & Sowder (2005) presented the following student argument "justifying" the log law:

$$log(a_1 \cdot a_2 \ldots \cdot a_n) = log\ a_1 + log\ a_2 + \ldots + log\ a_n\text{, for all positive integers } n.$$

<u>Student argument</u>

$\log(4 \cdot 3 \cdot 7) = \log 84 = 1.924$
$\log 4 + \log 3 + \log 7 = 1.924$
$\log(4 \cdot 3 \cdot 6) = \log 72 = 1.857$
$\log 4 + \log 3 + \log 6 = 1.857$
Because these work, then $\log(a_1 \cdot a_2 \ldots a_n) = \log a_1 + \log a_2 + \ldots + \log a_n$.

Identify *ways of understanding* entailed in this argument (i.e., what parts of it are instances of them and how are they so?), and identify inferred *ways of thinking* that might have been at play in this student's reasoning.

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Question 7

Cobb and Yackel (1996) developed the so-called *emergent* interpretive framework as part of their ongoing efforts to "account for students' mathematical development as it occurs in the social context of the classroom". The central constructs of this framework are listed in the figure below.

SOCIAL PERSPECTIVE	PSYCHOLOGICAL PERSPECTIVE
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Characterize Cobb and Yackel's emergent framework with an eye to communicating its overarching perspectives, its components, and the work it arguably does for the enterprise of mathematics education research as seen from the authors' viewpoint. Your characterization should describe and explain the meaning of each of the following:

- The two broad perspectives that comprise the framework
- The individual constructs that make up the core components of each perspective
- The interplay between the two broad perspectives and between individual constructs as envisioned by the authors

Readings

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics, 58*(1), 45-75.

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In R. Lesh & A. Kelly (Eds.), *Handbook of research methodologies for science and mathematics education* (pp. 547-589). Hillsdale, NJ: Lawrence Erlbaum.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Greenwich, CT: Information Age Publishing.

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Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22*(3), 170-218.

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Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education* (pp. 267-307). Hillsdale, NJ: Erlbaum.

Tall, D. (2002). The psychology of advanced mathematical thinking. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 3-21). New York, New York: Springer.

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Thompson, P., & Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, G. Martin & D. Schifter (Eds.), *Research companion to the Principles and Standards for School Mathematics* (pp. 95-114). Reston, VA: National Council for Teachers of Mathematics.

Saldanha, L. & Thompson, P. (2007). Exploring Connections between Sampling Distributions and Statistical Inference: an Analysis of Students' Engagement and Thinking in the Context of Instruction Involving Repeated Sampling, *International Electronic Journal of Mathematics Education,* 2(3). http://www.iejme.com

von Glasersfeld, E. (1995). Constructing agents: The self and others. In L. P. Steffe & P. W. Thompson (Eds.), *Radical Constructivism: A Way of Knowing and Learning* (pp. 113-128). New York, NY: RoutledgeFalmer.