RUME 2 Qualifying Examination/Final Examination

May 2, 2012

Instructions:

You may not open any file other than this exam on your computer. Turn off all electronic devices except your computer. You may not use any text prepared ahead of this exam.

Before anything else, use File/Save As to save this file on your desktop with your initials at the end.

You <u>must</u> respond to Questions 1 through 3. In addition, respond to 1 (one) other question of your choosing from the remainder. Type your responses within this document below the part of the question to which you are responding.

If you use a diagram in a response, please draw it by hand on a separate sheet of paper. Label the figure with a title (e.g., "Figure x"), and insert a paragraph like this

INSERT FIGURE X ABOUT HERE.

Make sure that your name is written on each hand-drawn sheet.

All responses should be in essay form. Aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. A lengthy response is not necessarily the same as a good response. The recommended limit for each question is roughly 1000 words total (i.e., all parts inclusive).

Your responses may cite literature that is not explicitly mentioned in a question. Indeed, you are <u>encouraged</u> to cite literature not mentioned in a question.

Save your file frequently! When finished, email your document to <u>pat@pat-thompson.net</u> and copy your email to <u>luis.saldanha@asu.edu</u>. Give your hand-drawn figures to Luis Saldanha. He will have Caleb Street scan your figures and email them to me.

You have 3 hours to complete the exam. Good luck, and good writing!

Indicate whether you are taking this exam as: PhD qualifier, Master's qualifier, Neither
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Your name:

Question 1 (Mandatory)

Steffe and Thompson (2000) went to great pains to explain how teaching, conceived and implemented appropriately, can be a scientific method. Explain their argument. Your explanation should address the nature of a scientific method and the ways that proper uses of teaching fit within the nature of a scientific method.

Question 2 (Mandatory)

Use the Chuck interview as a context to address parts (a) and (b).

a. Cobb and Yackel's (1996) described their emergent perspective as having two functions: As guiding the construction of a theory of subjects' mathematical learning, and as guiding researchers' theorizing and analyzing. Cobb (2007) went on to describe four generic perspectives within which math education research is done, and urged researchers to embrace the notion of *bricolage* with respect to doing the work of math education research. Explain how these perspectives might have played out in the design, conduct and analysis of the Chuck interview.

Respond here

b. Explain how Glasersfeld's (1995) explication of the construction of self and others, and Steffe & Thompson's distinction between *students' mathematics* and *mathematics of students*, can inform a researcher's analyses of Chuck's mathematical behaviors during the interview. Are these issues subsumed among those addressed in the perspectives described in 2.a?

Question 3 (Mandatory)

a. In his introduction to the volume *Advanced Mathematical Thinking*, Tall examines specific concept areas in mathematics (e.g., vector spaces, functions and limits) and points to chapters that address what constitutes knowing them in an advanced form. He also discusses specific mathematical processes -- like abstraction and generalization. The MTL articles by Selden and Selden, Harel and Sowder, and Edwards did not focus on specific mathematical ideas and touched only briefly on specific mathematical processes. Write what you anticipate Tall would say about the MTL articles on advanced mathematical thinking.

Respond here

b. Carlson & Bloom (problem solving) and Roh (ESM-limits) are generally not thought of in terms of studying advanced mathematical thinking. But they could be seen as such. Comment on each from the perspective of what we might learn from them about advanced mathematical thinking.

Optional Question – Respond to Question 4 or Question 5

Question 4

a. Briefly stated, what were the Kaminski's major conclusions in her *Science* article and what were her justifications for them?

Respond here

b. Briefly stated, what were De Bock *et al.*'s qualifications of Kaminski's conclusions and what were their justifications for them?

Respond here

c. McCallum (2010) made this observation about Kaminski's tasks and analyses of them.

My fundamental objection to the experiment is that I believe that the two treatment groups were not given the same mathematical structure to work with.

Consider the following two groups (as in the abstract mathematical structure known as a group):

- Group A has 3 elements. Two of them behave in the following symmetric way: adding each to itself yields the other, and adding the two of them in any order yields the third element of the group. The third element behaves differently: adding it to any element of the group, in any order, yields that element back again.
- Group B has 3 elements, 1, 2, and 3. You add them just the way you normally add these numbers, except that if the ordinary sum is bigger than 3 you subtract 3 from it to get the sum in the group. Of course, the two groups are isomorphic. However, let me propose three hypotheses:
 - 1. Some students in each of the treatment groups tried to learn the operations by discerning a structure, rather than just memorizing the operations
 - 2. Among students who tried to discern structure, those asked to work with the symbols oval, diamond, and flag discerned something like structure A, whereas students given the concrete examples discerned something like structure B
 - 3. Seeing the isomorphism between the two structures A and B is more cognitively demanding than grasping the structures separately.

Do De Bock *et al.* address McCallum's criticism? Explain. If De Bock *et al.* did not address this criticism, then comment upon its validity.

Question 5 (Optional)

You designed an interview to investigate students' understandings of derivatives.

a. Describe the tasks you used in it and justify each task's inclusion.

Respond here

b. Describe the essential features of your interview protocol and interview method. Point to important literature, or give your own reasoning, that warrants each feature.