STAT448-Assignment1

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Course: STAT448-20S1

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Q1 (a) Compute ordinary least squares estimates of the coefficients β_0 and β_1 using linear algebra calculations by hand and with explanatory comments.

Let response variable $y = \begin{bmatrix} 5 \\ 9 \\ 13 \end{bmatrix}$, explanatory variables $X = \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}$, parameter $B = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$

Because OLS tries to find B to minimise the SSE(sum of the square of the errors).

We have $SSE = \epsilon^T \epsilon = (y - XB)^T (y - XB)$, which is a quadratic function of vector B.

SSE would be minimised, when the partial derivative of SSE with respect to B is equal to zero.

Now we can use OLS solution to estimate $\hat{B} = (X^T X)^{-1} X^T y$

Then we calculate $X^T X = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1+1+1 & 3+4+5 \\ 3+4+5 & 9+16+25 \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 12 & 50 \end{bmatrix}$

$(X^T X)^{-1} = \frac{1}{50*3-12*12} \begin{bmatrix} 50 & -12 \\ -12 & 3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 50 & -12 \\ -12 & 3 \end{bmatrix}$

$X^T y = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 5 \\ 9 \\ 13 \end{bmatrix} = \begin{bmatrix} 27 \\ 116 \end{bmatrix}$

Then we have $\hat{B} = \frac{1}{6} \begin{bmatrix} 50 & -12 \\ -12 & 3 \end{bmatrix} \begin{bmatrix} 27 \\ 116 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 50*27 - 12*116 \\ -12*27 + 3*116 \end{bmatrix} = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$

$\hat{B} = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$, so intercept $\beta_0 = -7$ and slope $\beta_1 = 4$

(b) Calulate by hand the estimates of the residuals $\hat{\epsilon}$.

We have $\hat{\epsilon} = y - \hat{y} = y - X\hat{B} = \begin{bmatrix} 5 \\ 9 \\ 13 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} -7 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 13 \end{bmatrix} - \begin{bmatrix} -7+12 \\ -7+16 \\ -7+20 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

So, $\hat{\epsilon} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$