Design and manufacture of umanned aerial vehicles (UAV) wing structure using composite materials

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Design and manufacture of umanned aerial vehicles (UAV) wing structure using composite materials

Planung und Bau einer Flügelstruktur für unbemannte Luftfahrzeuge (UAV) unter Verwendung von Kompositwerkstoffen

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This paper presents possibilities of simulating composites materials in SolidWorks environment and selected aspects of the unmanned aerial vehicle (UAV) design and manufacturing process (real airplane model was used in Air Cargo Challenge 2013 competition). First part of article contains informations about types of composite materials, mechanical properties and their structures. UAV wing design process is presented as multistage task. Next part of manuscript includes numerical analysis which deals with two cases of composites structures: laminates and sandwich composites. Differences in values of displacement and principal stresses (P1, P3) were shown in considered structures based on equal boundary conditions. Final part of article deals with development of manufacture process.

Keywords: UAV / composite structure / sandwich structure / CAx / Numerical analysis

Dieser Beitrag stellt Möglichkeiten der Simulation von Verbundwerkstoffen in Solidworks-Umgebung und ausgewählte Aspekte der Konstruktions- und Fertigungsprozesse von unbemannten Luftfahrzeugen (UAV) vor (reales Flugzeugmodell wurde im Air Cargo Ausschreibungswettbewerb 2013 verwendet). Der erste Teil des Artikels enthält Informationen über die Verbundwerkstoffvarianten, die mechanischen Eigenschaften und ihre Strukturen. Der UAV-Flügel-Design-Prozess wird als mehrstufige Aufgabe vorgestellt. Der anschließende Teil befasst sich mit der numerischen Analyse an zwei Fällen von Verbundstrukturen: Laminate und Sandwich-Verbundwerkstoffe. Es werden Unterschiede in den Werten der Verschiebung und der Hauptspannungen (P1, P3) bei den in Betracht gezogen Strukturen bei gleichen Randbedingungen dargestellt. Der letzte Teil des Artikels befasst sich mit der Entwicklung von Herstellungsverfahren.

Schlüsselwörter: UAV / Kompositstruktur / Sandwichstruktur / CAx / Numerische Mathematik

1 Introduction

Popularity of unmanned aerial vehicles (UAV) forces today's engineers to design and manufacture products in a short time and at reasonable price. It is possible due to the wide availability of modern composite materials and development of computer systems. CAD environment gives the possibility to create a virtual model. Next, using CAE tools, it is possible to simulate many behaviors of physical objects. Results of numerical analysis, in many cases, should be compared with experimental data. Using of CAx

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tools designers meaningfully decrease price of the introduction of new product [1, 2].

Due to high requirements of the modern UAV's (high strength-low weight) composite materials especially polymer matrix composites reinforced with continues fibers are most appropriate choice [3]. These materials are characterized by Young module twice as high compared to aluminum alloys while retaining two times lower weight. Difficulties in use of composites materials are related to the anisotropic/orthotropic structure. Use of these materials in numerical analysis requires knowledge of material constants and mechanical properties in principal axes. Only fully defined composite material guarantees real structure behavior and reliability of calculation results [4].

Development of wings for unmanned aerial vehicle is a multistage task which includes: airfoil selection, geometrical calculations, structural design, materials selection, numerical analysis and manufacturing. Main task of wing development process is to design structure characterized by high strength along with low weight.

2 Composite materials

Composite materials consist of two or more phases with significantly different properties (physical, chemical). Combined together they form a material with characteristics different from individual components. Each phase remain separate within the finished structure. Components of composites due to their function can be divided into filler (reinforcement) and matrix. Mostly used matrices materials are polymers, metals, ceramics and carbon while fillers materials are glass, carbon, aramid and boron.

Reinforcement occur in different forms: continues fibers, short fibers and particles [4].

2.1 Polymer matrix composites

Polymer matrix composites are most common due to low costs of fabrication and attractive properties: low density, high strength, high stiffness, corrosion resistance and vibration damping ability. Reinforced with continues fibers obtain mechanical properties similar or higher than conventional materials and present high strength-to-weight and stiffness-to-weight ratio. Therefore polymer matrix composites reinforced with continues fibers are widely used for lightweight structures, for example in aviation as parts of airframes structures. The most common materials used for matrices are vinyl ester, epoxy, phenol, formaldehyde (PF), polyamide (PI) polyimide and polypropylene (PP) resins. Fillers due to the form of occurrence may take the form of mats (made form short fibers) or fabrics (continue fibers) [5]. The most frequent occurring types of fabrics used as reinforcement for polymer composites are: plain weave, twill wave, satin weave and unidirectional fabric. *Table 1* shows the mechanical properties of selected polymer (epoxy resin) matrix composites [6].

Table 1. Mechanical properties of selected polymer matrix composites

Tabelle 1. Mechanische Eigenschaften von ausgewählten Polymermatrix-Verbundwerkstoffen

	Symbol	Units	Std CF Fabric	E glass Fabric	Kevlar Fabric	Std CF UD	E glass UD	Kevlar UD
Young's Module 0°	E1	GPa	70	25	30	135	40	75
Young's Module 90°	E2	GPa	70	25	30	10	8	6
In-plane Shear Modules	G12	GPa	5	4	5	5	4	2
Major Poisson's Ratio	ν12		0,1	0,2	0,2	0,3	0,25	0,34
Ult. Tensile Strength 0°	Xt	MPa	600	440	480	1500	1000	1300
Ult. Comp. Strength 0°	Xc	MPa	570	425	190	1200	600	280
Ult. Tensile Strength 90°	Yt	MPa	600	440	480	50	30	30
Ult. Comp. Strength 90°	Yc	MPa	570	425	190	250	110	140
Ult. In-Plane Shear Stren.	S	MPa	90	40	50	70	40	60
Density		kg/m^3	1600	1900	1400	1600	1900	1400

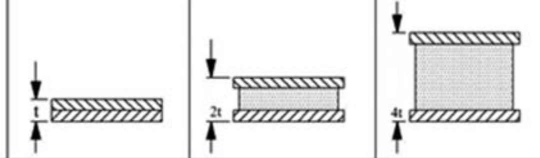
Figure 1. Composite laminate with layer orientation [0/–45/90/+45/0/0/+45/90/–45/0]

Bild 1. Kompositlaminate mit Schichtorientierung [0/–45/90/+45/0/0/+45/90/–45/0]

2.2 Laminates and sandwich structures

Polymer matrix composites used in aviation commonly occurs in form of symmetrical and asymmetrical laminates. Laminates are assemblies of fibrous composites layers, bonded together to provide required engineering properties including strength, stiffness, weight etc. Change in laminate layers orientation significantly affect its properties. In order to create hybrid laminate, layers of different materials may be used [6]. Individual layers of laminate are mostly orthotropic. Example of composite laminate made of unidirectional carbon fiber and polymer matrix is shown in *Figure 1*. Laminates are often described by orientation code, for example [0/–45/90/+45/0/0/+45/90/–45/0].

Another structures increasingly used in aviation are sandwich composites [7]. It is specific type of composite material resulting from bonding two thin, stiff skins to lightweight thick core. Those structures are used in aerospace, aircraft and marine industries



	Solid Material	Core Thickness "t"	Core Thickness "3t"
Stiffness	1,0	7,0	37,0
Flexural Strenght	1,0	3,5	9,2
Weight	1,0	1,03	1,06

Figure 2. Increase of bending stiffness and strength in sandwich structures

Bild 2. Erhöhung der Biegesteifigkeit und Festigkeit in Sandwichstrukturen

due to high bending stiffness and strength. In sandwich composites skins carry most of in plane and bending loads while core transfer shear and normal loads. Laminates of glass or carbon fiber polymer composites are widely used as thin skin material because of their mechanical properties. Most commonly used core materials are: polyurethane, polyethylene, balsa wood, honeycomb cores made of foams, aluminum alloys etc. Bending stiffness and strength of sandwich structure rises along with core thickness [8], *Figure 2*.

3 UAV wing design

All mechanical devices are designed to perform clearly defined tasks. Wings of unmanned aerial vehicles according to its destination vary in airfoil shape, thickness, chords dimensions, span, surface area and geometry [9]. Despite the differences, design concept is similar to all types of structures, *Figure 3*.

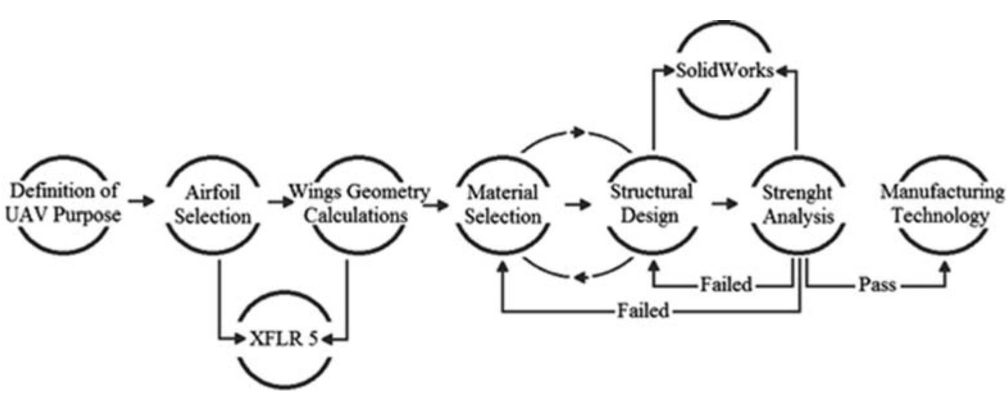


Figure 3. UAV wing workflow

Bild 3. UAV Flügel Workflow

3.1 Airfoil selection

One of the most important stages in wing design process is proper airfoil selection. It involves of specifying the destination and mission of given aircraft. In fact, it's impossible to find universal airfoil, working in all flight conditions and in all configurations of airplane.

XFLR 5 is a free software that allows numerical analysis of airfoils, wings and whole airframes. In calculation this software uses a non-linear lifting line method considering standalone wing and Vortex Lattice Method (VLM) along with 3D Panel Method for aerodynamic analysis of wings and airframe. XFLR5 operates at Reynolds numbers. Results of the analysis are lift and drag coefficients in case of airfoils calculations and lift and drag forces in case of wing and airframe simulation.

After defining specific group of profiles corresponding to assumed requirements compare of their aerodynamic characteristics (lift/drag coefficient) should be done.

3.2 Wing geometrical calculations

Further step (after airfoil selection) of UAV design involves main wing geometrical calculations. This type of studies may be performed in "Wing and Plane Design" (one of XFLR 5 modules). The abovementioned phase concerns calculation of wing main dimensions based on the project assumptions and selected airfoil. During this phase geometrical dimensions of main wing as: span, root and tip chords, twists, dihedrals and airfoil distribution are established [10].

3.3 Materials selection and structural design

Knowledge about types of loads acting on designed element is essential during creation process of new construction. In case of UAV wing structures most important are bending loads and torsion loads derived from acting lift force. Due to high requirements of the modern UAV's (high strength-low weight) composite materials especially polymer matrix composites reinforced with continues fibers are most appropriate choice. In order to transfer mentioned loads composite structures has been designed. First proposed structure was laminate composite consisting of glass fabrics (outside) and unidirectional carbon fabrics (inside) reinforced with balsa wood ribs for proper shape. Created construction represent type of sandwich structure, since upper and lower layers of fabrics are separated with balsa wood ribs. Second analyzed structure characterized by additional foam core separating layer of glass-carbon fabrics.

First step towards the creation of a structural design of calculated wing geometry (dimensions designed in XFLR 5 software) was to import airfoil coordinates into the SolidWorks environment. Airfoil was downloaded from an online database containing description of shape using X, Y, Z coordinates.

The next step was to create a sketch of the imported curve, setting appropriate dimensions and position relative to the global coordinate system using intelligent dimensioning tools and relationships. Last step relied on creation of virtual 3D model from an existing sketch [11].

Complete structural design of aircraft wing allows setting different materials for various elements of

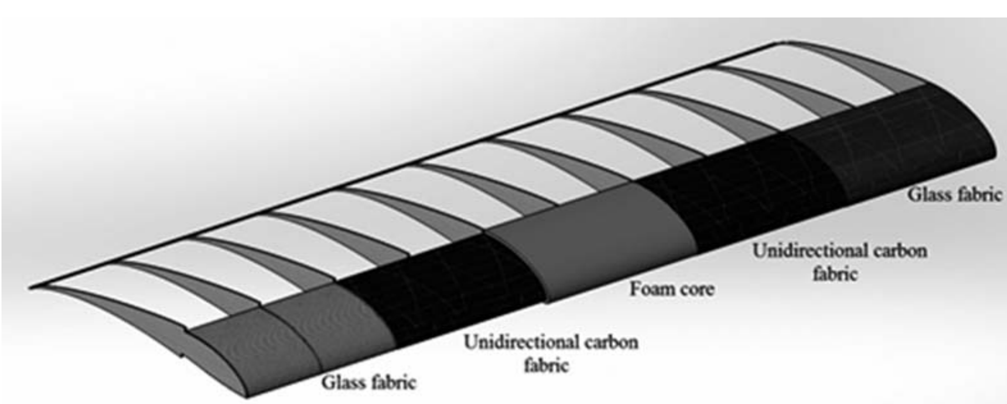


Figure 4. Structural design of aircraft wing
Bild 4. Strukturelle Gestaltung des Flugzeugflügels

construction. SolidWorks environment gave possibility to control weight of created structure at early design stage, *Figure 4*.

3.4 Numerical analysis

Strength analysis is closely related to the structural design (in our case UAV wing) verifying given geometry and used materials under posed boundary conditions. In order to improve the accuracy of numerical analysis solid model has been replaced by a surface model preserving geometrical dimensions.

Defining materials in SolidWorks environment requires describing properties such as Elastic modulus E, Poisson ratio ν, shear modulus G, tensile strength, compressive strength, density. For complete characterization of orthotropic materials (composites used in analysis) six materials constants are required (properties in two axes are the same) and following mechanical properties: tensile strength and compressive strength in two directions, shear strength in xy, density [12]. Thermal properties have been omitted due to nature of the analysis. In course of analysis Rohacel IG 71 and Balsa wood were considered as isotropic structures. Mechanical properties of materials used in analysis are presented in *Table 2* [13, 14].

SolidWorks Simulation gave possibility to analyze composites up to 50 individual layers [15]. Software assumes perfect bond between them. Most common types of composites available in SolidWorks Simulation are:

- Symmetric laminate – defined by symmetrical distributed layer thicknesses, fiber orientations and properties of the layers.
- Asymmetric laminate – characterized by no symmetry of used layers, orientations and properties in reference to central plane.
- Sandwich materials – special case of symmetric laminates with central layer characterized by greater thickness and lower mechanical properties compared to other layers. Used when higher resistance to bending load is required.

Composite property manager allows to set required number of layers, thicknesses, orientations and layer materials. This tool shows laminate coordinate system and direction of each individual layers. It is important to point that coordinate system of composite is not the same as model or assembly. In considered case numbers of layers was set to 4/5 depending on type of analyzed structure. Thickness of composites layers amounted 0.3 mm and foam core 3 mm. Or-

Table 2. Mechanical properties of materials used in analysis
Tabelle 2. Mechanische Eigenschaften von in der Analyse verwendeten Materialien

Material constant	Carbon/Epoxy UD fabric	S2-Glass/Epoxy Plain wave fabric	Rohacel IG 71 (as isotropic)	Balsa wood (as isotropic)
Elastic modulus in x	147e9 N/m^2	27.1e9 N/m^2	92e6 N/m^2	3.0e9 N/m^2
Elastic modulus in y	10.3e9 N/m^2	27.1e9 N/m^2		
Elastic modulus in z	10.3e9 N/m^2	12.0e9 N/m^2		
Poisson's ratio in xy	0.27	0.11	0.2	0.25
Poisson's ratio in yz	0.54	0.18		
Poisson's ratio in xz	0.27	0.18		
Shear modulus in xy	7e9 N/m^2	2.9e9 N/m^2	29e6 N/m^2	2.0e8 N/m^2
Shear modulus in yz	3.7e9 N/m^2	2.14e9 N/m^2		
Shear modulus in xz	7e9 N/m^2	2.14e9 N/m^2		
Mass density	1600 kg/m^3	1850 kg/m^3	75 kg/m^3	75 kg/m^3
Tensile strength in x	2280e6 N/m^2	604e6 N/m^2	2.8e6 N/m^2	20e6 N/m^2
Tensile strength in y	57e6 N/m^2	604e6 N/m^2		
Comp. strength in x	1725e6 N/m^2	291e6 N/m^2	1.5e6 N/m^2	12.5e6 N/m^2
Comp. strength in y	228e6 N/m^2	291e6 N/m^2		
Shear strength in xy	76e6 N/m^2	58e6 N/m^2	1.3e6 N/m^2	

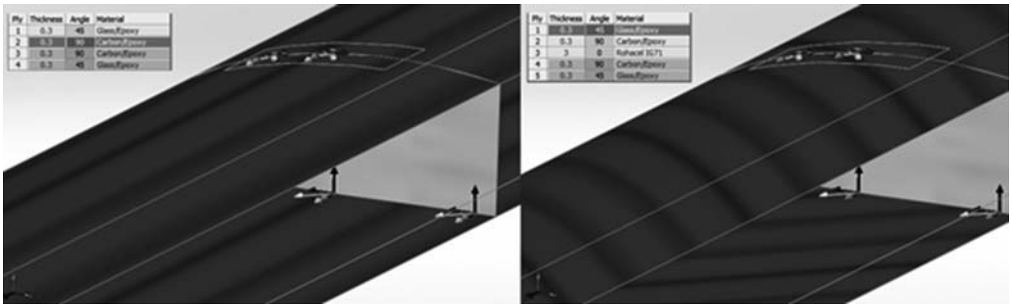


Figure 5. Orientation, thickness, layer material and coordinate system
Bild 5. Orientierung, Dicke, Schichtmaterial und Koordinatensystem

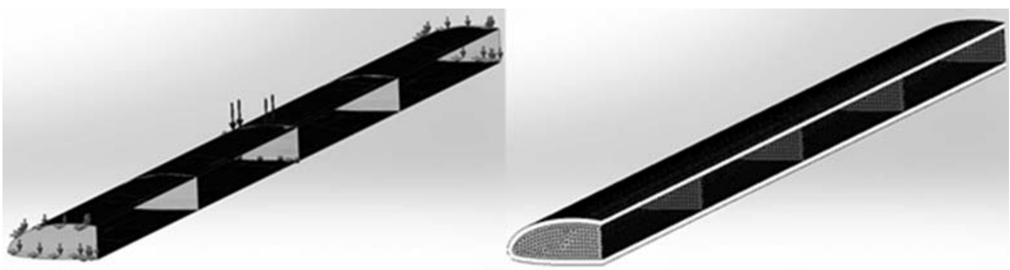


Figure 6. Boundary conditions for analysis, view of created mesh
Bild 6. Randbedingungen für die Analyse, Ansicht des erstellten Netzes

ientations of layers were as follows [45/90/90/45], [45/90/0/90/45], *Figure 5*.

Boundary conditions based on wing static test used in analysis involved attaching wings at the ends, applying external load in middle of the structure. In flight conditions such test simulate overload of 2.5 g. Fixation of tested element was carried out by taking off translations at the ends of the wing in X and Y axis while translation in the Z-axis was taken away on rib in the middle of the structure (such fixing allows displacement of wing tips in Z-axis reflecting exact terms of static test). For purpose of analysis it was necessary to create finite element mesh. Applied mesh density was equally distributed for the whole model (global size of element 2.4 mm, tolerance 0.12). *Figure 6* presents discussed boundary conditions with external load of 300 N (wing span 1000 mm, length of composite structure 88.5 mm, height of structure 32 mm)

For the analysis, the iterative solver FFE Plus were applied. This method uses approximate techniques to calculate the solution and repeat the process until the difference between two consecutive solutions is significantly small or does not exceed set er-

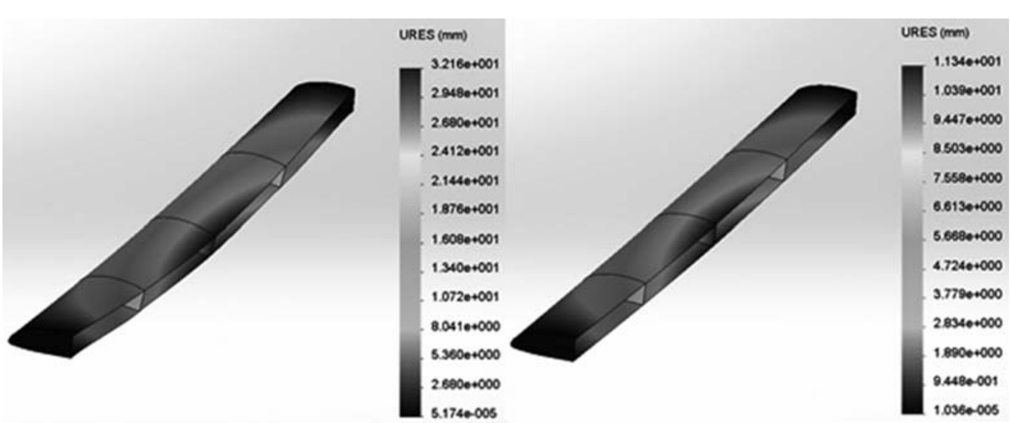


Figure 7. Displacement of carbon-glass fiber lamina (left side), sandwich structure (right side)
Bild 7. Die Verschiebung der Sandwichstruktur (rechts), Kohlenstoff-Glasfaser-Schichten (links)

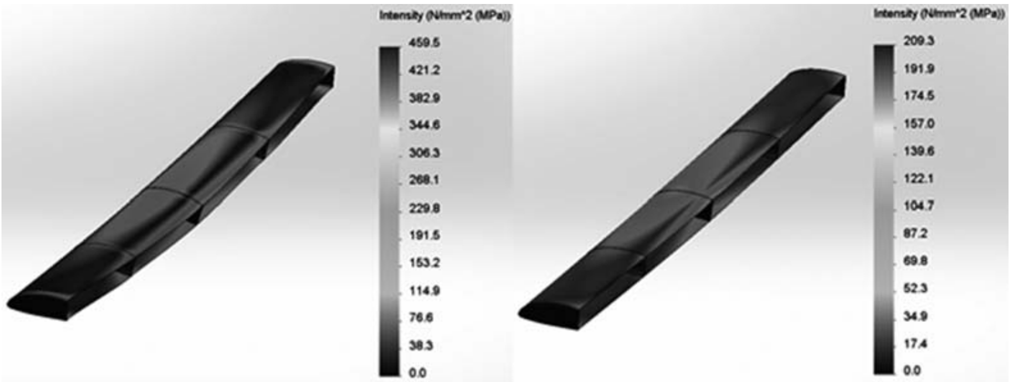


Figure 8. Stress Intensity (P1-P3) of carbon-glass fiber lamina (left side), sandwich structure (right side)
Bild 8. Spannungsintensität (P1-P3) der Sandwichstruktur (rechts), Kohlenstoff-Glasfaser-Schichten (links)

ror limits [16]. Studies have also been carried out using the Direct Sparse solver (appropriate when solving assemblies of parts with widely different material properties). Calculation results of both solvers overlapped (FEE plus perform calculations faster).

During the analysis two cases of composite structure were considered:

- sandwich construction based on carbon/glass fabric/epoxy resin laminate filled with balsa ribs
- sandwich construction based on carbon/glass fabric/foam core/epoxy resin sandwich composite filled with balsa wood ribs for proper shape.

Evaluation of the test results will begin with displacement plots analysis, *Figure 7*. It represents the resultant displacement (it is possible to check displacement in requested axis) of the object under the influence of applied forces and restraints. Notable is three times smaller deflection in the structure consisting of foam core. Difference in weight between two analyzed composites is negligible. Stiffness of the structure is very important aspect in the design of airframe because change in geometry resulting from deflection affects significantly aerodynamics of created aircraft.

Next analyzed plot presents distribution of highest values of "stress intensity", which is defined as P1-P3 = $2 \cdot \tau_{max}$, occurring in researched structure (P1-1st and P3-3rd principal stresses), *Figure 8*. "Stress intensity" value is based on Tresca yield criterion and in complex stress state is more conservative then HMH criterion. Stresses intensities shown at figure below are expressed in absolute value which means types of load (tension, compression) are not taken into account only their equivalent maximum value. It can be noted that in case of sandwich structure occurring stress is two and half times lower than in laminate composite. Assuming that both structures satisfy project assumptions, sandwich structure due to lower stress intensity, could be manufactured with thinner layer of carbon fiber thus reducing weight of the designed element.

For complete description of design structure it is necessary to perform analysis allowing to present types of loads with their occurrence. SolidWorks environment gives possibility to analyze state and the type of stress on each individual layer of tested composite. Figure below shows values and distribution of tensile (P1-1st Principal Stress) and compressive stresses (P3-Third Principal Stress) appearing on carbon fiber layers of examined structure, *Figure 9*.

As predicted tensile stress appeared mostly on the lower part of construction while compressive stress occurred on the upper part. Difference of stresses values varies between two and four times lower in case of sandwich composite. Such large divergences are the result of different tendencies in the occurrence of stress fields in considered structures. SolidWorks Simulation allows to analyze not only differences of stresses on layers but also on sides (top, bottom) of individual layer. On the basis of analysis noticeable is existence of tensile and compressive stresses on both sides of researched composites. It is caused by geometry of examined structure (open composite profile filled with balsa ribs) and type of applied load. Analyzing the points of occurrence of the largest tensile and compressive stress on the lower and upper surface with respect to the layers of the composite and side of the layer, provides information about the tendencies of stress occurrence across considered element.

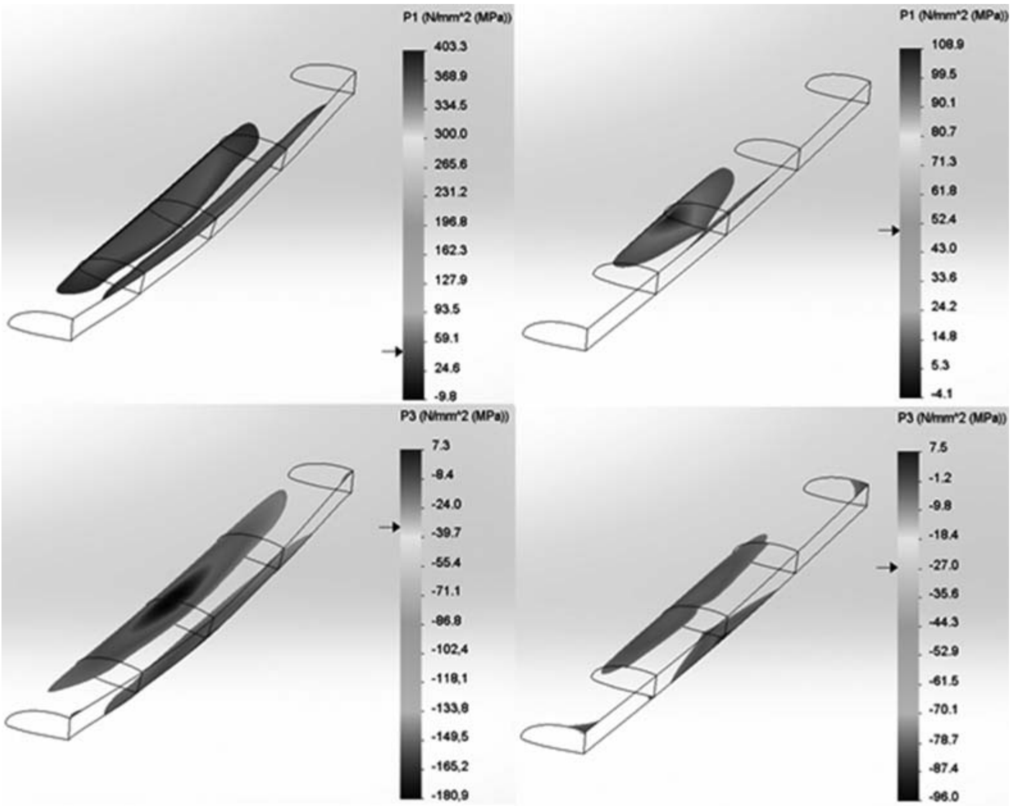


Figure 9. Tensile (P1) and compressive stress (P3) of carbon-glass fiber lamina (left side), sandwich structure (right side)

Bild 9. Zug (P1) und Druckspannung (P3) der Sandwichstruktur (rechts), Kohlenstoff-Glasfaser-Schichten (links)

Above analysis allows to determine structure behavior and stress appearance in respect to given geometry and used materials. Gathered information enables designer to create structure satisfying the requirements set out in the project, *Table 3*.

4 Manufacture

Development of manufacturing technology is an important step in creation process of new construction due to affect on final product quality. In aviation compatibility of design structure geometry and manufactured one is crucial. Small change arising from inaccuracies of execution have significant impact on the aerodynamic properties of considered airframe. Materials used for wing construction (based on above design) are mostly fibrous polymer composites (glass fiber, carbon fiber) with foam (based on polymath-acrylimide) and balsa wood.

Sandwich structure used in abovementioned airframe is main bearing element, therefore manufacturing this part without defect is very important issue. Vacuum bag technique excludes occurrence of

Table 3. Results of numerical analyzes

Tabelle 3. Ergebnisse der numerischen Analyse

	Carbon-Glass fiber laminate	Carbon-Glass-Foam Sandwich composite
Displacement	32.16 mm	11.34 mm
Stress Intensity	459.5 MPa	209.3 MPa
1st Principal Stress	452.3 MPa	205.6 MPa
3rd Principal Stress	–239.3 MPa	–129.3 MPa

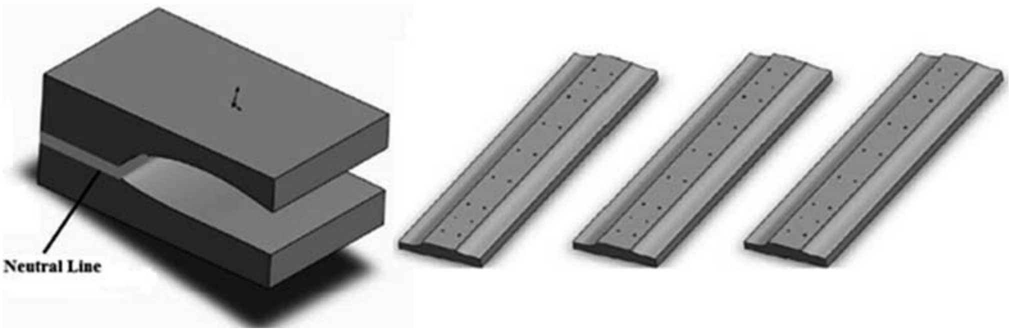


Figure 10. Composite wing structure mold designed in SolidWorks
Bild 10. Form der Verbundflügelstruktur, in Solidworks konstruiert

void (bubbles and wrinkles) in created composite which is main advantage of this method. Vacuum bagging use atmospheric pressure to hold laminate layers together. By this method it is possible to manufacture composites with high fiber content which translate into higher strength to weight ratios [17]. In order to create structure based on given geometry necessary is, to make mold with exact shape, *Figure 10*.

CAD model of wing was used to obtain mold model [18]. Next, using CAM software, G-code was generated for CNC milling machine. Bottom and top parts of mold was made from MDF panel. Having machined molds, it was necessary to prepare surfaces by painting and polishing them. (obtained smoothness has a large impact on composite quality). Owning prepared molds it is possible to perform lamination using vacuum bag technique, *Figure 11*.

Having manufactured composite structure final step of creating wing construction was attaching balsa wood ribs in order to obtain desired shape and geometry. It is useful to reinforce balsa elements with carbon fiber and apply carbon bar for better stiffness, *Figure 12*.

Described manufacture and design methodology of UAV wing structure was used to build aircraft for Air Cargo Challenge 2013 competitions, *Figure 13*. Among twenty one teams that arrived from all over the world our "Podlasie Tigers" team took 10th place [19]. Largest payload of our classified flight amounted 5 kg (airplane weight was 2.2 kg). Heaviest payload we have managed to pick up during whole competition was 7 kg (over three times the plane weight) however conditions (strong gusts of wind) did not allow to continue the flight.

5 Conclusions

Development of wing for unmanned aerial vehicle is a multistage task which includes: airfoil selection, geometrical calculations, structural design, materials selection, numerical analysis and elaboration of technology. Polymer matrix composites widely used in aviation industry characterize by high strength-to-weight and stiffness-to-weight ratio. Due to high requirements of the modern UAV's (high strength-low weight) composite materials are only appropriate choice.

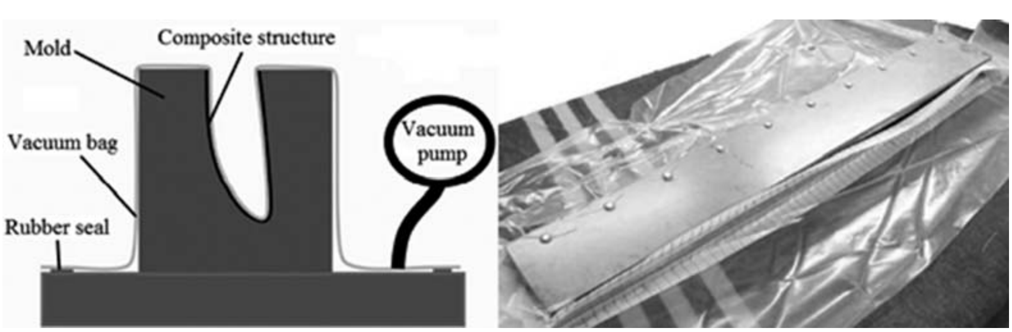


Figure 11. Vacuum bag stand
Bild 11. Vakuumbeutel Stand

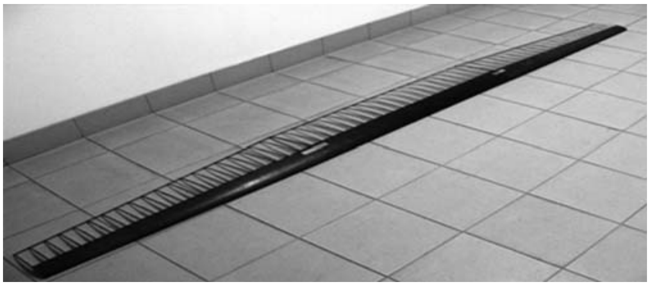


Figure 12. Manufactured wing ready to covering
Bild 12. Hergestellter Flügel vor der Abdeckung

Numerical analysis of composite structures based on equal boundary conditions shows that sandwich structure characterized by foam core layer deflect under load 11.34 mm – it is 3 times smaller than carbon fiber laminate which deflect 32.16 mm considering similar weight. Principal stresses (P1, P3) varies between two and four times lower in case of sandwich composite. CAx system gives possibility to create variety of composite structures, analyze them and compare their strength properties providing valuable information in the early design stage lowering cost of final product.

Lightweight composites structures based on carbon, glass fiber manufactured by vacuum bag technique minimize creation of voids (bubbles and wrinkles) and allows to obtain high fiber content which translate into higher strength of created structure.

Acknowledgement

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Figure 13. Cargo UAV ready to fly
Bild 13. Fracht UAV bereit zum Fliegen

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