<u>Assignment 3</u>

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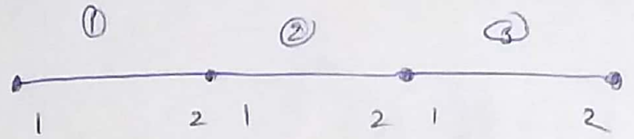
ME16B001

1. $$-\frac{d^2u}{dx^2} - u + x^2 = 0 \qquad 0 < x < 1 \qquad u(0) = u(1)$$

$a = 1 \qquad c = -1 \qquad f = -x^2$

(a) linear elements



$$Ku = f + Q$$

K ↙ coefficient matrix f ↓ source vector

$$K\text{matrix} = \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{bmatrix}$$

$$f\text{ matrix} = \begin{bmatrix} F_1^1 \\ F_1^2 + F_2^1 \\ F_2^2 + F_3^1 \\ F_3^2 \end{bmatrix}$$

$$K_{ij} = \int_{x_a}^{x_b} \left(a \frac{d\psi_i}{dx} \frac{d\psi_j}{dx} + c\,\psi_i \psi_j \right) dx \qquad F_i^e = \int_{x_a}^{x_b} f \psi_i^e \, dx$$

where $\psi_1 = \dfrac{x_2 - x}{x_2 - x_1} = 1 - 3x \qquad \psi_2 = \dfrac{x - x_1}{x_2 - x_1} = 3x$

Using the integral relation get

$$k = \begin{bmatrix} 2.888 & -3.0555 & 0 & 0 \\ -3.0555 & 5.777 & -3.0555 & 0 \\ 0 & -3.0555 & 5.777 & -3.0555 \\ 0 & 0 & -3.0555 & 2.888 \end{bmatrix}$$

$$f_{matrix} = \begin{bmatrix} -0.0030864 \\ -0.0432098 \\ -0.15432 \\ -0.13271 \end{bmatrix}$$

$$\theta \text{ matrix} = \begin{bmatrix} m \\ 0 \\ 0 \\ n \end{bmatrix}$$

$KU = f + \theta$

$$\begin{bmatrix} 2.888 & -3.0555 & 0 & 0 \\ -3.0555 & 5.777 & -3.0555 & 0 \\ 0 & -3.0555 & 5.777 & -3.0555 \\ 0 & 0 & -3.0555 & 2.888 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} -0.0030864 + m \\ -0.0432098 \\ -0.15432 \\ -0.13271 + n \end{bmatrix}$$

Boundary condition $u(0) = u(1)$

$\therefore$ let $U_1 = U_2 = P$ (some known quantity)

$$\left(2.888 + (-3.0555) + 0 + 0\right) U_1 = -0.0030864 + m$$

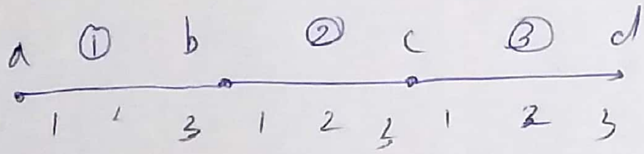
$$m = -\frac{67}{400} P + 0.030864$$

$$\left(0 + 0 + (-3.055) + 2.88\right) U_2 = -0.13271 + n$$

$$n = \frac{-67}{400} P + 0.13271$$

m & n can be substited back in eqn. $KU = f + Q$ and solved.

(b) Qudratic elements.



$$\psi_1 = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}$$

$$\psi_2 = \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}$$

$$\psi_3 = \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}$$

$$
\begin{bmatrix}
K_{11}^1 & K_{12}^1 & K_{13}^1 & & & & \\
K_{21}^1 & K_{22}^1 & K_{23}^1 & & & & \\
K_{31}^1 & K_{32}^1 & K_{33}^1+K_{11}^2 & K_{12}^2 & K_{13}^2 & & \\
 & & K_{21}^2 & K_{22}^2 & K_{23}^2 & & \\
 & & K_{31}^2 & K_{32}^2 & K_{33}^2+K_{11}^3 & K_{12}^3 & K_{13} \\
 & & & & K_{21}^3 & K_{22}^3 & K_{23}^3 \\
 & & & & K_{31}^3 & K_{32}^3 & K_{33}^3
\end{bmatrix}
\begin{bmatrix}
U_a^1 \\ U_a^2 \\ U_b \\ U_b^2 \\ U_c \\ U_c^2 \\ U_d
\end{bmatrix}
=
\begin{bmatrix}
f_1^1 \\ f_1^2 \\ f_1^3 + f_2^1 \\ f_2^2 \\ f_2^3 + f_3^1 \\ f_3^2 \\ f_3^3
\end{bmatrix}
$$