Assignment 2 ME5102

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Problem 3: Explicit, implicit and CN time stepping schemes

Given problem for which the solution has been attempted.

$$f = 2te^{-t} - u$$

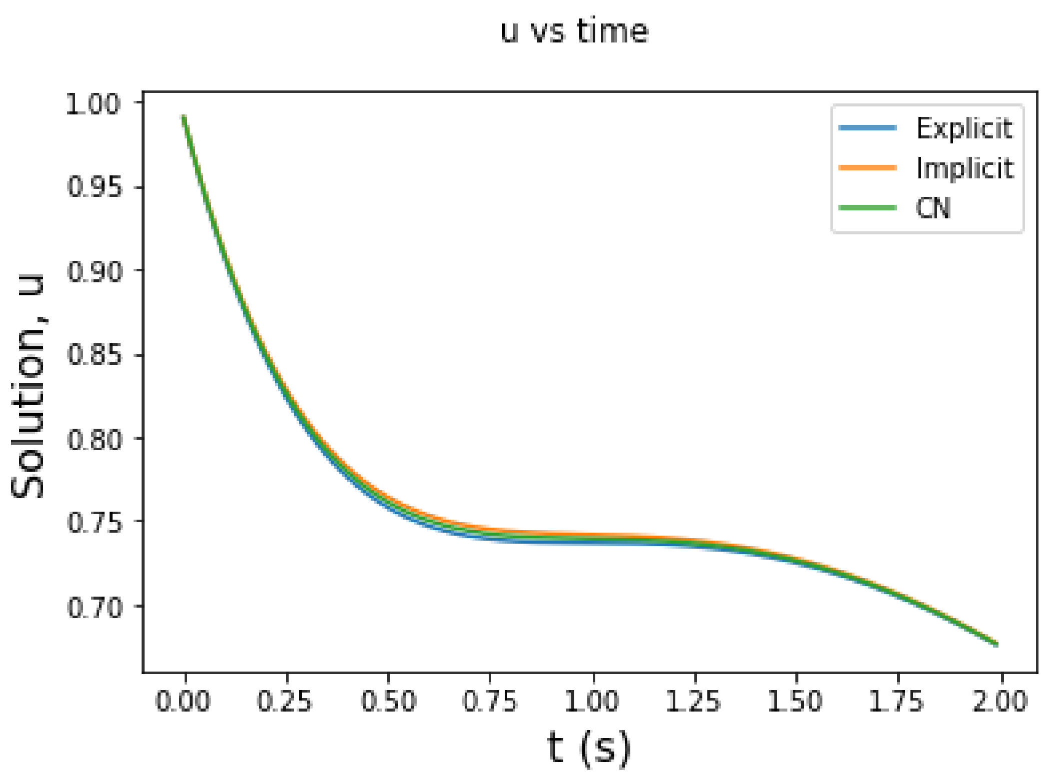
$$u(t = 0) = 1$$

$$0 \leq t \leq 2$$

Note: *this is my third attempt of the same problem as it kept changing every now and then, doing one more time was not possible*

```
In [3]: import numpy as np
        import matplotlib.pyplot as plt
        import math
        # explicit scheme
        u=[]
        t=0.01
        for i in range(200):
            if i==0: # case t=0
                u.append(1-0.01)
            else:
                u.append(u[i-1]+(0.01)*(2*t*np.exp(-t)-u[i-1]))
            t=t+0.01
        # implicit scheme
        ub=[]
        t=0.01
        for i in range(200):
            if i==0: # case t=0
                ub.append((1+ (0.01)*2*(0.01)*np.exp(-0.01))/(1+0.01))
            else:
                ub.append((ub[i-1]+(0.01)*2*(t+0.01)*np.exp(-(t+0.01)))/(1+0.01))
            t=t+0.01
        # CN scheme
```

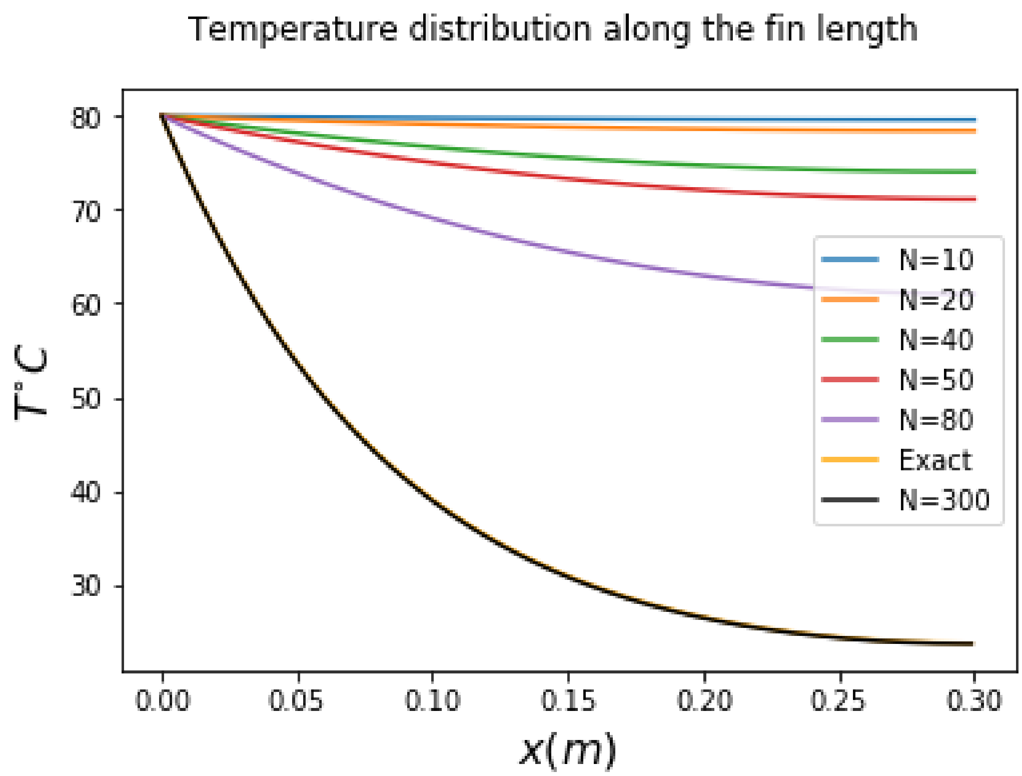
```
uc=[]
t=0.01
for i in range(200):
    if i==0:
        uc.append((1+(0.5)*(0.01)*(2*t*np.exp(-t)-1+2*(t+0.01)*np.exp(-(t+0.01))))/(1+0.
    else:
        uc.append((uc[i-1]+(0.5)*(0.01)*(2*t*np.exp(-t)-uc[i-1]+2*(t+0.01)*np.exp(-(t+0.
    t=t+0.01
# CN scheme end

t = np.arange(0, 2, 0.01)
plt.plot(t, u, label="Explicit")
plt.plot(t, ub, label="Implicit")
plt.plot(t, uc, label="CN")
plt.legend()
plt.xlabel('t (s)', fontsize=16)
plt.ylabel('Solution, u', fontsize=16)
plt.suptitle('u vs time')
plt.show()
```



Problem 4: 1-D Heat loss through circular fin

```
In [4]: from sympy import *
        import matplotlib.pyplot as plt
        import numpy as np
        from numpy import *
        from array import *
        from scipy.sparse import *

        def TDMA(T,d, n):
                a=[]
                b=[]
                c=[]
                cs=[]
                ds=[]
                u=[]

                for i in range(len(T)):
                        for j in range(len(T[0])):
                                if (j-i)==1:
                                        c.append(T[i][j])
                                if (i-j==1):
                                        a.append(T[i][j])
                                if i==j:
                                        b.append(T[i][j])
                # acccomodating n nodes
                tb=b[len(b)-1]
                ta=a[len(a)-1]
                a.pop()
                b.pop()
                for i in range(n-4):
                        a.append(a[len(a)-1])
                        b.append(b[len(b)-1])
                        c.append(c[len(c)-1])
                        d.append(d[len(d)-1])
                a.append(ta)
                b.append(tb)
                T = diags([b,a,c], [0,-1, 1]).todense()
                #print(T)
                #print("T",len(T))

                for i in range(len(d)):
                        u.append(0)

                for i in range(len(c)):
                        if i==0:
                                cs.append(c[i]/b[i])
                        elif i!=0:
```

```
                        cs.append(c[i]/(b[i]-a[i-1]*cs[i-1]))

        for i in range(len(d)):
                if i==0:
                        ds.append(d[i]/b[i])
                elif i!=0:
                        ds.append((d[i]-a[i-1]*ds[i-1])/(b[i]-a[i-1]*cs[i-1]))

        for i in range(len(d)-1,-1,-1):
                if i == len(d)-1:
                        u[i]=ds[i]
                elif i!= len(d)-1:
                        u[i]=ds[i]-cs[i]*u[i+1]
        return(u)

n=299
m=0.3/(n-1)
c=(400/3)*m*m
# Matrix equation AX=B, n is the number of nodes
A = [[1, 0, 0,0,0],[1,-(c+2),1,0,0],[0,1,-(c+2),1,0],[0,0,1,-(c+2),1],[0,0,0,2,-(c+2)]]
B=[80,-c*20,-c*20,-c*20,-c*20]

# NN=[10]
NN=[10,20,40,50,80]
for N in NN:
        l=np.arange(0,0.30005,0.3/N)
        # print(len(TDMA(list(A),list(B),N-1)))
        # print(TDMA(list(A),list(B),N-1))
        # print(l)
        plt.plot(l, TDMA(list(A),list(B),N),label="N="+str(N))

x = np.arange(0, 0.3, 0.001)
T_exact = 0.058728*np.exp((20*x)/(np.sqrt(3)))+59.9412719*np.exp((-20*x)/(np.sqrt(3)))+2

plt.plot(x, T_exact, label="Exact",color='orange')
plt.plot(x, TDMA(list(A),list(B),int(n)),label="N=300", color='black')
plt.legend()
plt.xlabel(r'$x (m)$', fontsize=16)
plt.ylabel(r'$ T^{\circ}C $', fontsize=16)
plt.suptitle('Temperature distribution along the fin length')
plt.show()
```



Please note that the exact solution and solution with N = 300 are almost coinciding.

```
In [ ]:
```