

DEPARTMENT OF MECHANICAL ENGINEERING AT IIT TIRUPATI

ME 5101 COMPUTATIONAL FLUID DYNAMICS

Homework Assignment 3, Due before 9 AM on Mon, 18 Feb 2019 (Submit to Jivrakh)
(Late Submissions will not be accepted)

(Please print out this sheet and attach to the front of your completed Assignment)

Name (in full, upper case, write legibly) _____

Roll Number _____

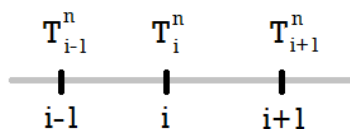
Instructions

1. Use A4 size paper only with proper margins
2. Answer all questions in detail
3. Graphs should be in pencil, or must be made using software, printed out & attached
4. Leave enough space between lines (Double spacing preferred)
5. Label all plots, sketches and graphs properly. Include correct units.
6. Sloppy work will attract severe penalty. Marks may be deducted from overall score.
7. **Plagiarism** will lead to FAILURE (U GRADE) in this course

(Do not get disappointed on seeing the size of this assignment. Once you are ready with your program, it is easy to get the graphs!)

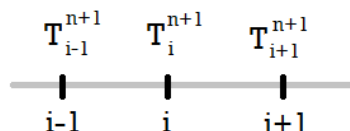
Consider a large planar wall of width (or thickness) $L = 0.24$ m and uniform cross sectional area $A = 8$ m² (this area is perpendicular to the x -axis). The conductivity, density and specific heat of the material of the slab are $k = 0.6$ W/m-K, $\rho = 3000$ kg/m³ and $C = 0.2$ J/kg-K respectively. There is no internal heat generation in the wall. Initially at $t = 0$, the entire wall is at a uniform temperature of 25°C. Immediately after $t = 0$, the Temperatures of the left ($x = 0$) and right ($x = L$) ends of the wall are increased to 100°C and 75°C respectively. You are required to analyze the heat transfer in the wall and to obtain the time-evolution of the temperature profile in the wall using numerical techniques. Temperature profile means $T(x)$; i.e, Temperature as a function of x .

time step n

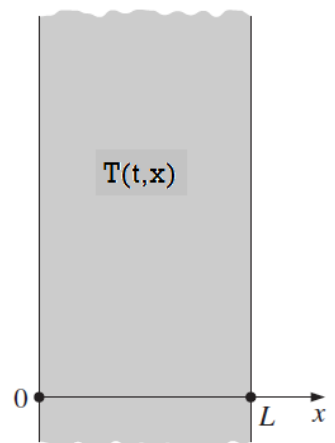


$i = 0, 1, 2, \dots, P-2, P-1, P$

time step $n+1$



$n = 0, 1, 2, \dots, Q-2, Q-1, Q$



1. What is the governing equation for this problem? List the boundary and/or initial conditions.
2. Derive the governing equation (you may refer to some standard text)
3. Consider the stencil shown above for three nodes $i-1$, i and $i+1$ at two different time steps n and $n+1$. Discretize the governing equation using central differencing for space, and (a) explicit differencing for time (b) implicit differencing for time and (c) Crank Nicolson technique for time. For all three cases, use a uniform grid with $x_{i+1} - x_i = h$ for all values of i and $t^{n+1} - t^n = s$ for all values of n . Use $r = \alpha s / h^2$.

4. WAP to find the temperature profile in the wall at any given instant of time, $t = t_{\text{interim}}$. Your program should accept some input and produce the necessary output. **Print out your code and attach it!!**
 Input: t_{interim} ; method; h; s {Here the variable method is an integer which can take values 1, 2 or 3; method = 1 implies explicit; method = 2 implies implicit and method = 3 implies CN}.
 Output: A vector $[T_0, T_1, T_2, \dots, T_{P-2}, T_{P-1}, T_P]$ at $t = t_{\text{interim}}$ using the value of “method” which was given as input.
5. Find $T(x)$ for the following values using your code. Plot the resulting curve, T vs x for all the cases in 18 different graphs. (Label axes properly with units). Please arrange your 18 graphs in 3 pages. Each page should contain 6 graphs. Page 1 should have graphs for cases (a-f); page 2 for (g-l) and page 3 for (m-r). In each page, arrange graphs in a matrix of 2 columns and 3 rows. Calculate the value of r for each case and list it on that respective graph. The label for each graph should be $s = \underline{\hspace{2cm}}$, method = $\underline{\hspace{2cm}}$, and $r = \underline{\hspace{2cm}}$. Comment on your results. See if you are obtaining realistic values for temperature in all the 18 cases listed below.
- $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.010 \text{ s}$, method = 1 (explicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.025 \text{ s}$, method = 1 (explicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.040 \text{ s}$, method = 1 (explicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.060 \text{ s}$, method = 1 (explicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.075 \text{ s}$, method = 1 (explicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.120 \text{ s}$, method = 1 (explicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.010 \text{ s}$, method = 2 (implicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.025 \text{ s}$, method = 2 (implicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.040 \text{ s}$, method = 2 (implicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.060 \text{ s}$, method = 2 (implicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.075 \text{ s}$, method = 2 (implicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.120 \text{ s}$, method = 2 (implicit)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.010 \text{ s}$, method = 3 (CN)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.025 \text{ s}$, method = 3 (CN)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.040 \text{ s}$, method = 3 (CN)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.060 \text{ s}$, method = 3 (CN)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.075 \text{ s}$, method = 3 (CN)
 - $t_{\text{interim}} = 6 \text{ s}$, $h = 0.01 \text{ m}$, $s = 0.120 \text{ s}$, method = 3 (CN)
6. Now modify your program to find the time-evolution of temperature in the wall. For this purpose, the input will be $\{t_m\}_{m=0}^{10}$; method; h; s. Output should be T vs x for all values of $t = t_1, t_2, t_3, \dots, t_9, t_{10}$. Plot all the curves in a single graph. (Label axes properly with units). Also derive the steady-state solution and plot it in the same graph for reference. Does the wall reach steady state? If so, can you find out? For this problem, use $h = 0.012 \text{ m}$, $s = 0.05 \text{ s}$, method = 1 (explicit) and $t_0 = 0$; $t_1 = 1 \text{ s}$; $t_2 = 2 \text{ s}$; $t_3 = 5 \text{ s}$; $t_4 = 10 \text{ s}$; $t_5 = 20 \text{ s}$; $t_6 = 40 \text{ s}$; $t_7 = 50 \text{ s}$; $t_8 = 65 \text{ s}$; $t_9 = 80 \text{ s}$; $t_{10} = 100 \text{ s}$.