DEPARTMENT OF MECHANICAL ENGINEERING AT IIT TIRUPATI
ME 5101 COMPUTATIONAL FLUID DYNAMICS

Homework Assignment 1, Due Mon, 28 Jan 2019, before 11 AM, in class
(Late Submissions will not be accepted)

(Please print out this sheet and attach to the front of your completed Assignment)

Name (in full, upper case, write legibly) ______________________________

Roll Number ______________________________

Instructions

1. Use A4 size paper only with proper margins
2. Answer all questions in detail
3. Graphs should be in pencil, or must be made using software, printed out & attached
4. Leave enough space between lines (Double spacing preferred)
5. Label all plots, sketches and graphs properly. Include correct units.
6. Sloppy work will attract severe penalty. Marks may be deducted from overall score.
7. **Plagiarism** will lead to FAILURE (U GRADE) in this course

1. Complete the following table

	Differential Equation	ODE or PDE	Order	Degree	Independent Variable(s)	Dependent Variable(s)	Linear or Nonlinear
1	$y' = x^2 + 5y$						
2	$xy'' - 4y' - 5y = e^{2\sin(x)}$						
3	$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y}$						
4	$\left(\frac{d^3 s}{dt^3}\right)^2 + \left(\frac{d^2 s}{dt^2}\right)^4 = s - 3t$						
5	$y^2 + y' = \sqrt{t}$						
6	$\frac{7ty' - y''}{\sqrt{t^2 - 1}} = y\ln[\pi + \sin(t)]$						
7	$y' = t\sin(y)$						
8	$u_{xx} = (x^4 + y^4)u_{yy} + \sin(xy)u$						
9	$(u_x)^2 - \left(u_y\right)^2 = u$						
10	$y^2 = y_{xx} + 5y$						

2. Determine if the following differential equations are parabolic, hyperbolic or elliptic.

a	$\frac{\partial u}{\partial y} + 4\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial y \partial x} - 5\frac{\partial u}{\partial x} = \sin(xy)$	**f**	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial y \partial x} = 2$
b	$2x\frac{\partial^2 u}{\partial x^2} - 4\sqrt{x+2}\frac{\partial^2 u}{\partial y \partial x} - 5y\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial y^2} = 0$	**g**	$\frac{\partial T}{\partial y} = 2\frac{\partial^2 T}{\partial x^2}$
c	$e^x\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial y \partial x} + y\frac{\partial^2 u}{\partial y^2} = 0$	**h**	$\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial x^2}$
d	$\frac{\partial u}{\partial y} + 3\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 u}{\partial y^2} - x\frac{\partial u}{\partial y} = 4$	**i**	$\frac{\partial^2 T}{\partial y \partial x} = 0$
e	$u_{xx} = (x^4 + y^4)u_{yy} + \sin(xy)u$	**j**	$\nabla^2 T(x,y) = 0$

3. Consider a standard living hall with doors, windows, fans, bulbs and an air conditioner. It is desired to model the flow of air inside this room.

a. What are the unknowns in this problem?
b. What are the governing equations that you will use to model this flow? (Do not just name them, write down the actual governing equations)
c. Is this equation system closed mathematically?
d. If the equation system is closed, why? If it is not closed, how will you close it?
e. Is the flow 1D/2D/3D? Is it viscous/inviscid? Is it compressible or incompressible? Is it laminar or turbulent?
f. What are the important boundary conditions in this problem?

4. For a Newtonial fluid, $\vec{\vec{\tau}} = 2\mu\vec{\vec{S}}$, where μ is the dynamic viscosity & $\vec{\vec{S}} = \frac{1}{2}\begin{bmatrix} \frac{\partial u}{\partial x}+\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} & \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}+\frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x}+\frac{\partial u}{\partial z} & \frac{\partial w}{\partial y}+\frac{\partial v}{\partial z} & \frac{\partial w}{\partial z}+\frac{\partial w}{\partial z} \end{bmatrix}$

Now, consider the continuity equation (conservation of mass) $\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\vec{\nabla}.\vec{V} = 0$ and the Cauchy equation (conservation of momentum), $\frac{\mathrm{D}\vec{V}}{\mathrm{D}t} = \rho\vec{g} + \vec{\nabla}.\vec{\vec{\sigma}}$ where ρ is the fluid density, $\vec{g}$ is the acceleration due to gravity, $\vec{V} = [u, v, w]$ is the velocity vector and $\vec{\nabla} = \hat{\mathbf{i}}\,\frac{\partial}{\partial x} + \hat{\mathbf{j}}\,\frac{\partial}{\partial y} + \hat{\mathbf{k}}\,\frac{\partial}{\partial z}$ is the del operator.

a. Consider the equation $\vec{\vec{\sigma}} = -p\vec{\vec{I}} + \vec{\vec{\tau}}$. Explain the three terms involved. What is $\vec{\vec{S}}$?
b. Is the term $\vec{\nabla}.\vec{\vec{\sigma}}$ a scalor, vector or a tensor? Evaluate it.
c. After evaluating $\vec{\nabla}.\vec{\vec{\sigma}}$, simplify it for a Newtonian fluid by using the equations for $\vec{\vec{\sigma}}$ and $\vec{\vec{\tau}}$.
d. Simplify the continuity equation for incompressible flow. Write the resulting equation in both scalar form and vector form.
e. You may recognize that the Cauchy's equation is a vector equation. Using your result from (c), write out the x-component of the Cauchy equation.
f. Simplify the result in part (e) for incompressible, isothermal flow of a Newtonian fluid with constant viscosity.
g. Generalize the result in (f) and write down the corresponding equations in y and z directions.
h. Now, gather the simplified equations in x y and z directions, and write down a single vector equation representing all the three.

5. Consider the integral $I = \int_a^b \frac{dr}{1+r^2}$ with $a = 0$ and $b = 2$. (Answer parts (a) to (e) below)

a. Evaluate I analytically and find its exact value.
b. Write the general formula to evaluate I using trapezoidal rule using n trapezoidal strips each of width $h = \Delta r$, so that $nh = b - a$.
c. Now write a program (WAP) to evaluate I numerically using trapezoidal rule for any generic value of h. (Attach your code. It should be neatly formatted and labelled.)
d. Choose 10 different values for h within the range 0.0001 to 0.1 (inclusive of the end points). Using your program, evaluate I for these 10 different values of h. Tabulate your results and complete the table below.

#	h	I_{approx} (calculated using trapezoidal rule)	Absolute error, e = $\lvert I_{\text{approx}} - I_{\text{exact}} \rvert$
1	0.0001		
…			
10	0.1		

e. Plot log(e) vs log(h) and find the <u>**slope**</u> of the graph.

6. WAP to find the root of the equation $e^{0.3x}\ln(x) = 2 + x$ using Newton-Raphson method. Your program should stop when the error drops to less than or equal to 10^{-5}. Choose the initial guess for x to be $x = 6$. Tabulate your results and plot absolute error vs iteration number. (Attach your code. It should be neatly formatted and labelled.)