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Density effect on mixing and segregation processes in a vibrated binary granular mixture

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Abstract

The mixing and segregation processes of binary granular mixture with identical sizes but different densities particles subjected to vertical oscillatory excitation are investigated in this study. The spatial distributions of vibrated binary steel–glass beads mixture are visualized by simulation and the results are similar to the experimental data. The time evolution of pattern formations show that the heavy particles first move toward the center of the bed and then concentrate near the centers of the two convection cells of the vibrated system. The mechanism causing the mixing and segregation is strongly dependent on the momentum exchange of each species which is related to the granular temperature gradients of mixture components. The influences of solid fraction and granular temperature profiles on the mixing of the mixture are examined under different operating conditions. The simulation results show that the granular temperatures of heavy particles are higher than those of light particles, indicating that the granular temperatures do not equilibrate for the mixture system. The convection motion also plays an important role in determining the mixing of the system. For understanding the extent of granular mixing, the segregation intensity was determined to quantify the mixing rate of binary mixtures. The segregation intensity shows that the mixing rate increases with the vibration strength, but decreases with the initial heights of mixture.

Keywords: Mixing; Segregation; Granular mixture; Density; DEM

1. Introduction

Granular mixing is achieved through the operation of two or more components of particulates, which are scattered in a random motion in the granular flow system. The mixing of granular materials is unquestionably important in several industrial processes such as pharmaceuticals, metallurgy and ceramics. An important and fundamental problem for mixing of granular materials is the tendency for mixtures to demix or segregate. Small differences in either size or density of granular mixture system can lead to segregation [1,2]. The fundamental process behind the mixing of granular materials was not well understood as compared with the mixing of regular fluids [3]. The mechanisms contributing to the mixing of granular materials were usually described by the kinematics of the bulk flow, the diffusive motions resulted from the fluctuation velocities and the segregation resulted from the difference in material properties. Although the mixing and segregation phenomena play an important role in handling particle solids, the design of the mixer was based on the extrapolation from the existing empirical knowledge. So, the understanding of the mixing processes for granular flow systems was imperative.

The shaker is one of the typical devices for mixing and drying granular materials. The mixing and segregation problems of vibrated granular mixture had been the subjects of a number of studies for many years. Zik and Stavans [4] and Barker and Mehta [5] analyzed the mixing of cohesionless granular materials under vertical vibration using both experiment and simulation. They used self-diffusion coefficients to quantify the mixing of vibrated granular bed. Akiyama et al. [6] experimentally characterized the convective motions and mixing of cohesionless particles by means of fractal properties. Henrique et al. [7] performed MD simulation to investigate the mixing process of granular mixture. They examined the efficiency of diffusion as the mixing mechanism of granular system with

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Table 1
Related material properties of the binary granular mixture

Material	ρ (kg/m^3)	E (GPa)	v
Steel	9195	190	0.305
Glass	2475	46.2	0.245

external random acceleration. Molina-Boisseau and Bolay [8] experimentally studied the mixing of polymeric powders and the glass beads in a shaker. They demonstrated that the acceleration of vibration was the most important parameter that governs mixing. Using DEM simulation, Hsiau and Yang [9] studied the dependence of self-diffusion motions and mixing of cohesive powders in a two-dimensional vibrated granular bed.

Because of the differences in size and density, the mixing was usually accompanied by segregation. The size segregation for a mixture with different size but equal density was observed for large particles to rise to the top of the bed, which is commonly recognized as the "Brazil nut effect" [10]. The large and light particles could sink to the bottom of a deep granular bed [11–13]. Three mechanisms have been proposed for the segregation in a vibrated granular mixture: geometrical reorganization [14], size percolation [15] and convection [16,17]. Jenkins and Yoon [18] employed kinetic theory to study the segregation of a binary mixture due to the difference in size and mass in a gravitational field. They derived the criteria for spheres or disks that will rise as a function of density and size ratio. Although the size difference is generally the most important factor causing segregation in a vibrated granular mixture, the difference in density of mixture is also an important issue and is less studied previously. The main objective of the present work is to examine the mixing and segregation process of a binary solids mixture with equal sizes but different densities subjected to vertical vibration. Using DEM simulation, the momentum exchanges due to the temperature gradient of each species of the mixture are investigated with different vibration conditions. The influences of convection motion and the initial mixture height of the system on the mixing and segregation are also examined carefully.

2. Simulation model

The DEM proposed by Cundall and Strack [19] was used to simulate the mixing of binary granular mixture under external vibration in this study. The governing equations of motion for particle i are

$$m_i \frac{\mathrm{d}\mathbf{V}_i}{\mathrm{d}t} = \mathbf{F}_{gi} + \Sigma(\mathbf{F}_{cij} + \mathbf{F}_{lij}) \tag{1}$$

$$I_i \frac{\mathrm{d}\boldsymbol{\omega}_i}{\mathrm{d}t} = \Sigma(\mathbf{R}_i \times \mathbf{F}_{cij}) \tag{2}$$

where m_i and I_i are the particle mass and moment of inertia; $\mathbf{V}_i$ and $\boldsymbol{\omega}_i$ are the particle translational and rotational velocities; $\mathbf{F}_{gi}$ is the gravitational body force of particle; $\mathbf{F}_{cij}$ is the contact force of particle i due to particle j; $\mathbf{F}_{lij}$ is the liquid bridge force between particles i and j; $\mathbf{R}_i$ is the vector directed from the center of particle i to the contact point. The normal contact force between particles is modeled as elastic with a viscous dissipation element. The normal contact force acting on particle from particle j is given by

$$F^n_{j\to i} = k_\mathrm{n}\Delta n^{3/2} - c_\mathrm{n} m_\mathrm{e}(\mathbf{u}_\mathrm{rel}\cdot\mathbf{n}) \tag{3}$$

where Δn is the overlap between the particles i and j; k_n is the normal elastic constant from the Hertz theory [20]; c_n is the dashpot coefficient; $\mathbf{u}_\mathrm{rel}$ is the relative velocity between particles; $\mathbf{n}$ is the unit vector of normal direction; m_e is the effective mass, $m_i m_j/(m_i+m_j)$. The normal elastic constant k_n is related to the particle radii and elastic properties of particles which is given by

$$k_\mathrm{n} = \frac{4}{3}\sqrt{\frac{r_i r_j}{r_i + r_j}}\left[\frac{E_i E_j}{E_j(1-v_i^2) + E_i(1-v_j^2)}\right] \tag{4}$$

where r_i and r_j are radii of particles i and j; E_i and E_j are elasticity moduli of particles i and j; and v_i and v_j are Poisson's ratios of particles i and j. The related material properties of the binary mixture are given in Table 1. The tangential component of the force is given by

$$F^\mathrm{s}_{j\to i} = -c_\mathrm{s} m_\mathrm{e}(\mathbf{u}_\mathrm{rel}\cdot\mathbf{t}) - sign(\delta s)\min(k_\mathrm{s}|\delta s|, f_\mathrm{s}|F^\mathrm{n}_{j\to i}|) \tag{5}$$

where k_s is the tangential elastic constant, c_s is the dashpot coefficient, $\mathbf{t}$ is the unit vector of tangential direction, δs is the tangential displacement during a contact, and f_s is the frictional coefficient of particles. When the magnitude of the tangential elastic force, $k_\mathrm{s}|\delta s|$, exceeds the magnitude of the frictional sliding force, $f_\mathrm{s}|F^\mathrm{n}_{j\to i}|$, then the frictional sliding force becomes active and replaces the tangential elastic force.

3. Experimental apparatus

The experimental apparatus used in this study is illustrated in Fig. 1. An electromagnetic vibration system (Gearing and Watson, V-20) was served as the vertical shaker, and the shaker was vertically driven by sinusoidal signals. The

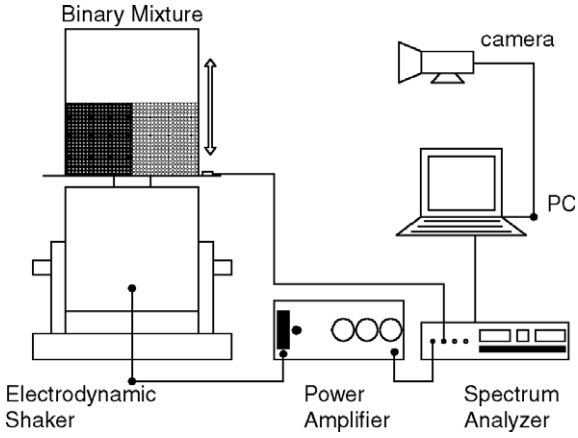

Fig. 1. Schematic drawing of the experimental apparatus.

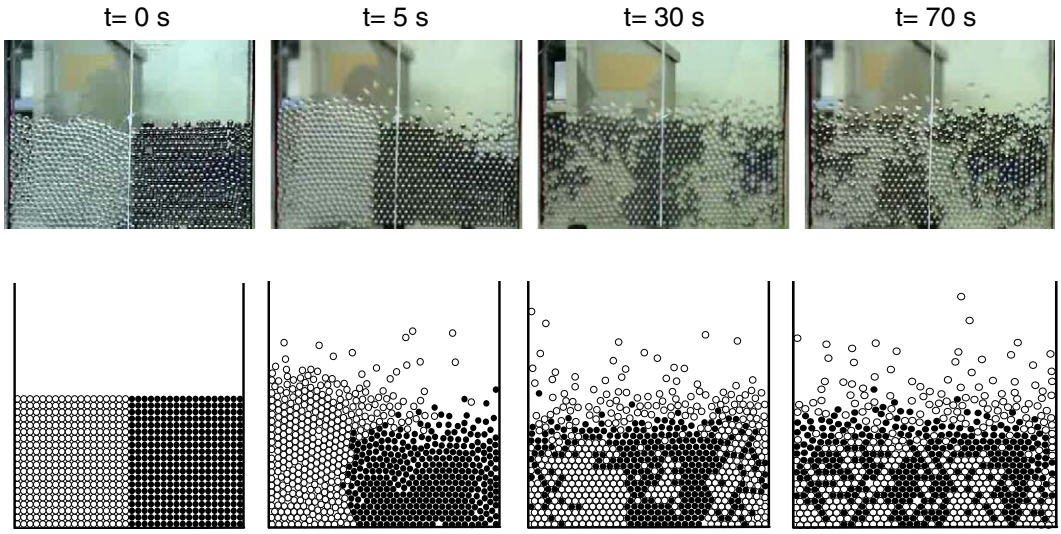

Fig. 2. Time evolutions of the segregated state to an equilibrium spatial distribution for the binary mixture of steel–glass beads under Γ=3.0 and f=15 Hz.

vibrational acceleration was measured by an accelerometer fixed at the shaker. The dimensionless acceleration amplitude Γ was defined as $\Gamma = a(2\pi f)^2/g$, where a and f are the vibration amplitude and vibration frequency, respectively. The binary granular mixture consisted of solid spheres of steel and soda lime glass of diameter d=3 mm. A tank with glass plates as the front and back walls and the side and bottom wall were glued by a layer of # 400 sandpapers, was driven by the shaker. The height, width and depth of the inside space of the tanks were 21.6, 10.8 and 0.35 cm, respectively. The geometry of the tank and the properties of binary mixture used in the experiment were the same with simulation tests. A digital video camera was used to record the particles' motion of different species. The digital images were transported to a personal computer for mixing analysis.

4. Results and discussion

In many industrial processes, it is necessary to homogenize an initially segregated binary granular mixture of different particle properties (size, shape, density or surface roughness) in a vertical shaker. In these cases, the competition between mixing and segregation determines the equilibrium concentration distributions of the mixing state.

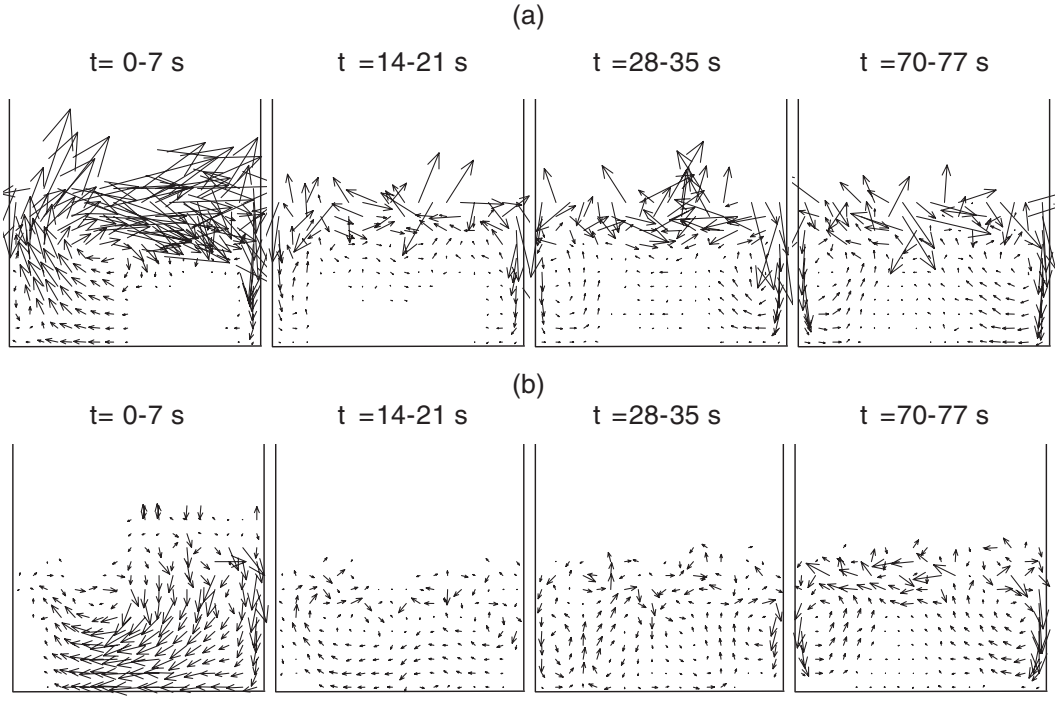

Fig. 3. Average velocity fields of binary mixture during different time ranges for Γ=3.0 and f=15 Hz (a) glass beads (b) steel beads.

To investigate the mixing and segregation process of initially segregated binary mixture, a series of simulations and experiments are performed with different vibration conditions. Two identical sizes but different densities of steel–glass beads with the arrangement of right-hand and left-hand are used as the binary mixture in the current study. The number fraction of steel beads and glass beads in all the cases is fixed at 0.5, respectively. The total numbers of the particles are varied by changing the initial mixture height of the bed. Fig. 2 shows the time evolution of the segregated state to an equilibrium distribution for the binary mixture of steel–glass beads under Γ=3.0 and f=15 Hz. Both the experiment and the simulation are tested and a similar result between the two has been obtained. From the visual observation of the spatial distributions of images, an intricate evolution of mixing process is found for the system. At the beginning, the glass beads quickly expand into the upper region and then move downward along the side walls, while the steel beads tend to move toward the central regime of the bed. After a few seconds, the steel beads start to mix with the glass beads in the interior of the bed. Finally, a segregated state of the mixture evolves to a well mixed state within 70 s.

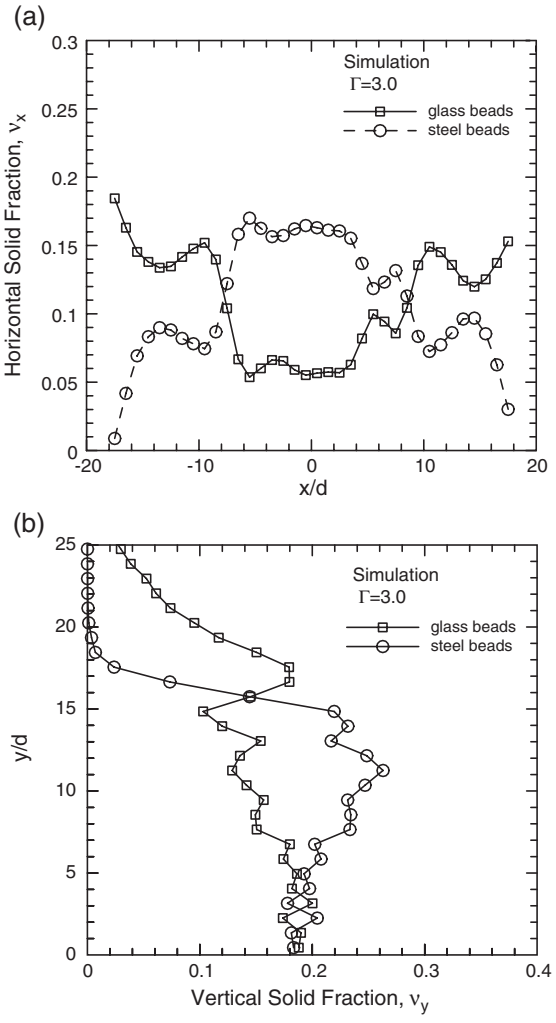

Fig. 4. Average solid fraction profiles as a function of the height of granular bed for steel–glass mixture under Γ=3.0 and f=15 Hz: (a) horizontal direction and (b) vertical direction.

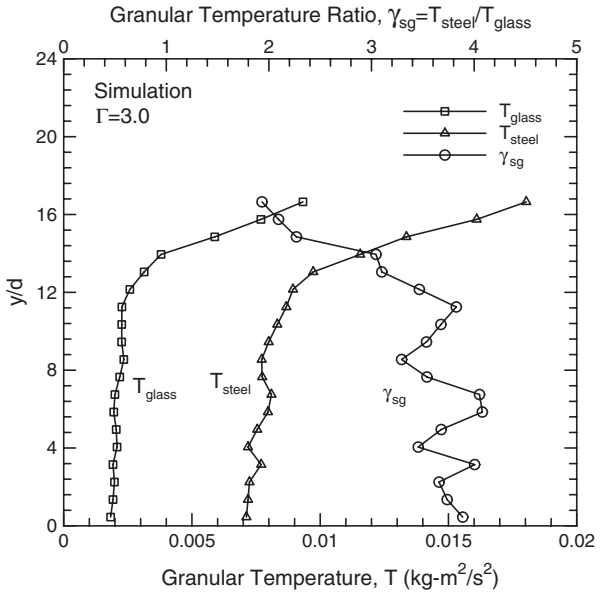

Fig. 5. Granular temperature profiles as a function of the height of granular bed for Γ=3.0 and f=15 Hz.

Fig. 3(a) shows the average velocity fields of glass beads during different time ranges. At the beginning, the upward motions of the light particles are apparent, especially in the first 7 s. Then the glass beads are tapped randomly on the free surface and then move downwards at both side walls during the 14–21 s time interval. There is a region along the bed floor where there is no velocity vector. It means that the steel beads occupy the space and the segregation phenomenon is developed during those time periods. Finally, two clockwise convection cells form at both sides of the bed during the 70–77 s time interval. Fig. 3(b) shows the average velocity fields of the steel beads during different time ranges. It is observed that the steel beads diffusively move toward the central regime in the first 7 s. After a few seconds, four weak convection cells have been formed in the interior of the bed during the 28–35 s time intervals. The heavy particles move downwards along the side walls and move upwards with smaller velocities in the interior of the bed. Finally, the two apparent symmetric convection cells are found during the 70–77 s time intervals.

When a binary granular mixture is subjected to a vertical vibration, the flow behaviors and mixing phenomenon are contributed significantly to the momentum exchange of each species causing the granular temperature gradient of components. The granular temperature is associated with the fluctuation velocity by averaging the deviations between the mean velocity and particle velocities in a strip or a cell. In this study, the computational domain is partitioned into horizontal layers in the depth direction. Then the granular temperature $T(y)$ is calculated from

$$T(y) = \frac{1}{3} m(\langle C(y,t)\rangle)^2 \tag{6}$$

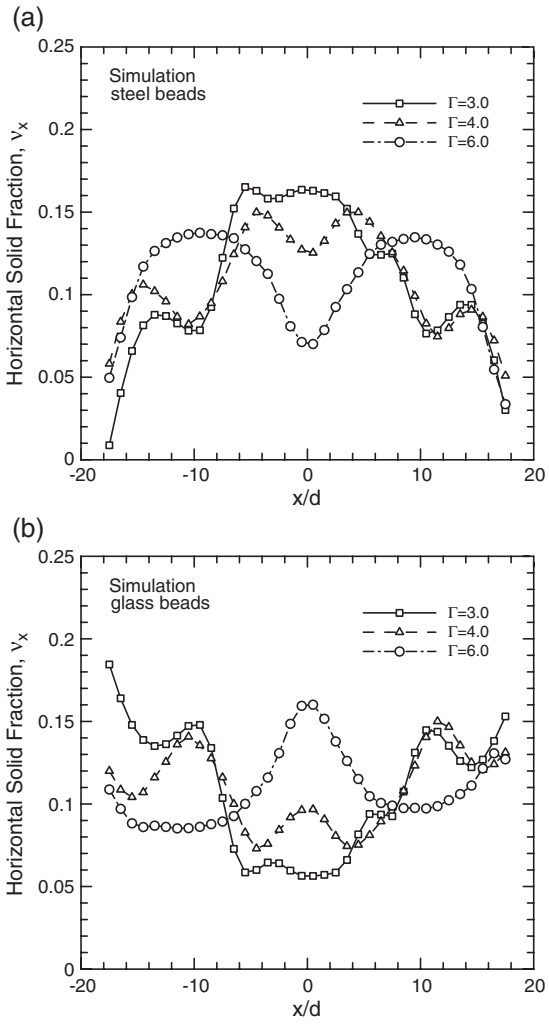

Fig. 6. Effect of vibration intensity Γ on the solid fraction profiles in the horizontal direction of (a) glass beads and (b) steel beads.

where m is the mass of a grain and $\langle C(y,t)\rangle$ is the long-term averaged fluctuation velocity in layer y along the depth direction, which is given by

$$\langle C(y,t)\rangle = \left(\frac{\sum_{t=t_1}^{t=t_2}\{\sum_{i\in y}[u_i(t)-u(y,t)]^2\}}{\sum_{t=t_1}^{t=t_2}N(y,t)}\right)^{1/2} \tag{7}$$

where $u_i(t)$ is the velocity of particle i, $N(y,t)$ and $u(y,t)$ are the total number and the instantaneous mean velocity of particles in layer y, respectively. Similarly, the long-term averaged solid fraction profiles along the horizontal and vertical layers are defined as

$$v(x) = \frac{\sum_{t=t_1}^{t=t_2}\left(\sum_{i\in x}V_x(i)\right)}{\sum_{t=t_1}^{t=t_2}V(x)} \tag{8}$$

$$v(y) = \frac{\sum_{t=t_1}^{t=t_2}\left(\sum_{i\in y}V_y(i)\right)}{\sum_{t=t_1}^{t=t_2}V(y)} \tag{9}$$

where $V_x(i)$ and $V_y(i)$ are the volume of particle i in the horizontal layer x and vertical layer y and $V(x)$ and $V(y)$ are the volume of horizontal layer x and vertical layer y, respectively.

Fig. 4(a) shows the average solid fraction profiles in the horizontal direction for steel–glass mixture for Γ=3.0 and f=15 Hz. As shown in the solid fraction profiles, there are different solid fraction profiles for each species due to the difference in density. The maximum densities for heavy particles are concentrated around the center region, while the concentrations of light particles in this region are small. In addition, the density profiles of heavy particles have two smaller peaks located at the positions of x=±3 cm, which are close to the locations of the centers of the two convection cells of the vibrated system as shown in the velocity fields of Fig. 3 (a). This indicates that the heavy particles tend to gather around the central convection cells. Hsiau and Chen [22] experimentally studied the segregation process of binary mixture with different density ratios in a granular bed. They also found the

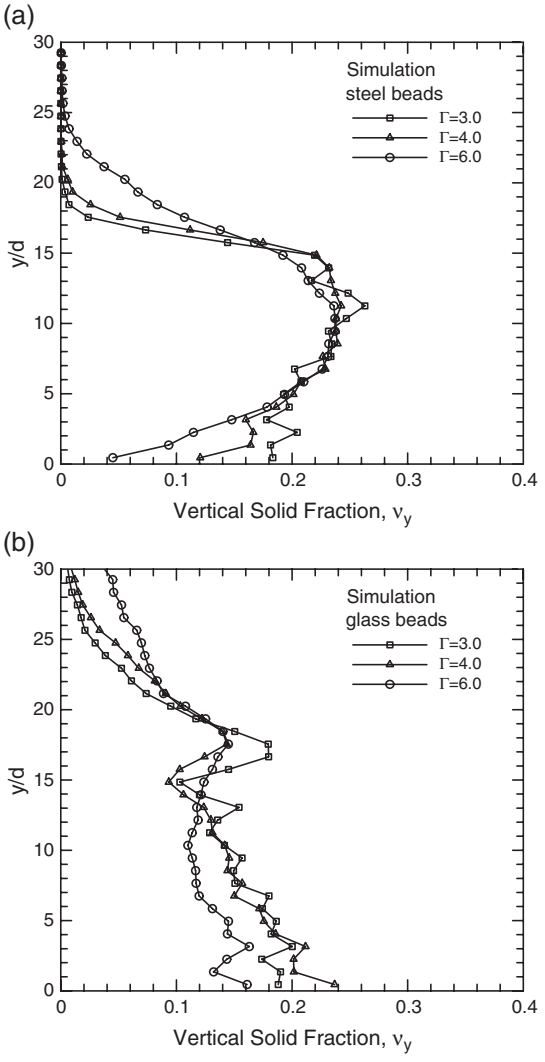

Fig. 7. Effect of vibration intensity Γ on the solid fraction profiles in the vertical direction of (a) glass beads and (b) steel beads.

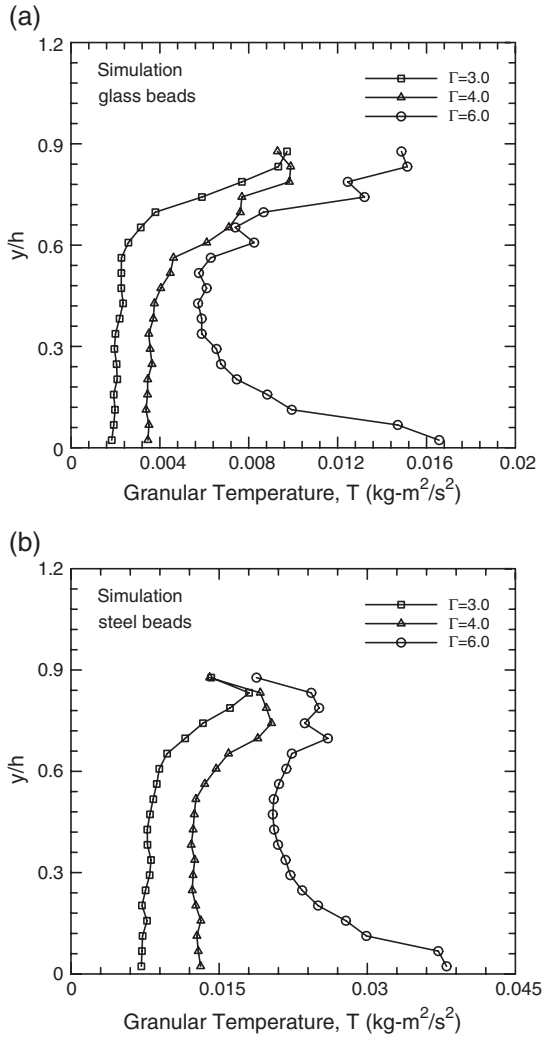

Fig. 8. Granular temperature profiles of binary mixture of steel–glass beads as a function of the height with different vibration intensity Γ of (a) glass beads and (b) steel beads.

heavier particles tended to move towards the centers of the two convection cells of the lighter particles.

Fig. 4(b) shows the average solid fraction profiles in the vertical direction for steel–glass mixture. It shows that the heavy particles have higher concentrations at the intermediate levels, while the light particles have lower values in this region. It is also observed that the density in the upper region for light particle is larger than those of heavy particles. This is because the light particles obtain more energy from the heavy particles due to the collision between particles, resulting in an increase of solid fraction. The maximum solid fraction is located at $y/d=10$ and 18 from the bottom of the bed for the heavy and light particles, respectively. Moreover, the solid fraction profiles have uniform and identical values near the bottom for both species.

The granular temperature profiles as a function of height are shown in Fig. 5. The temperature ratio between the steel beads and glass beads is also plotted. It can be seen that the granular temperature profile has a maximum value near the free surface and a uniform distribution at smaller heights for both species. In general, the temperatures of steel beads are greater than those of

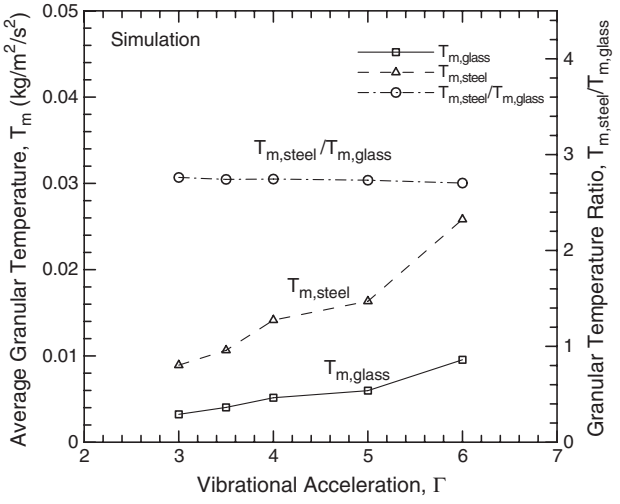

Fig. 9. Effect of vibration intensity Γ on the average granular temperatures of the binary mixture of steel–glass beads.

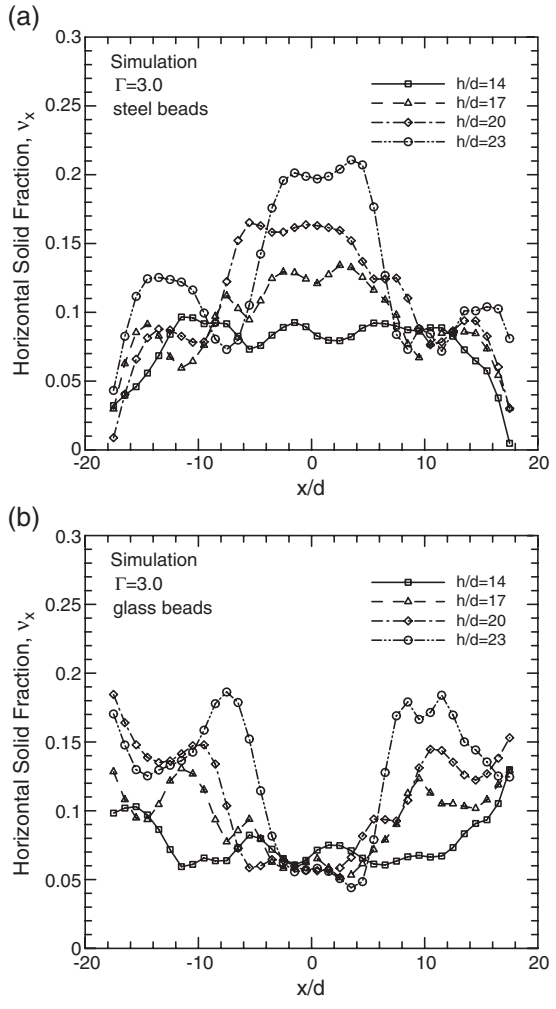

Fig. 10. Solid fraction profiles in the horizontal direction for steel–glass mixture with different mixture heights h/d for $\Gamma=3.0$ and $f=15$ Hz: (a) glass beads and (b) steel beads.

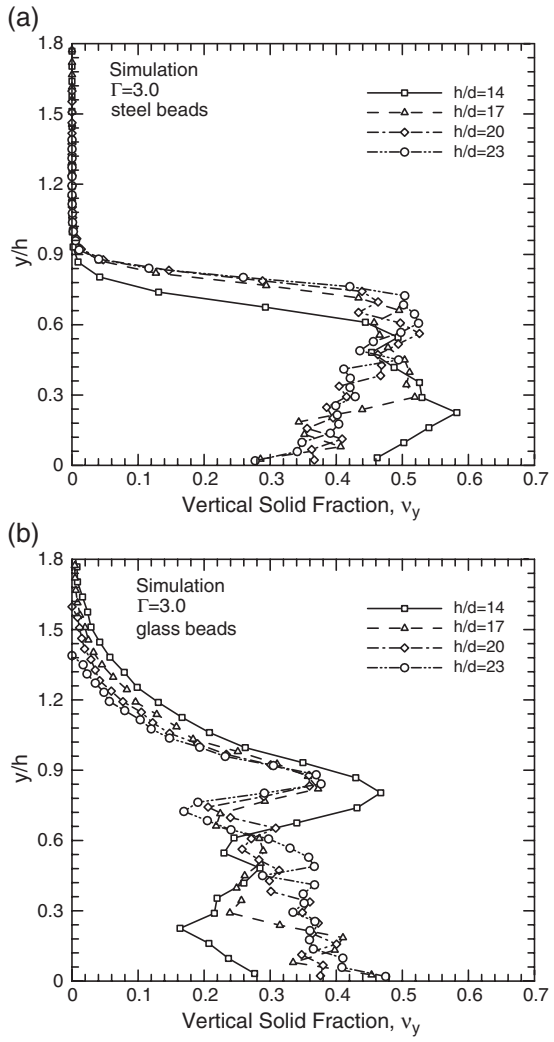

Fig. 11. Solid fraction profiles in the vertical direction for steel–glass mixture with different mixture heights h/d for Γ=3.0 and f=15 Hz: (a) glass beads and (b) steel beads.

glass beads, indicating that the two components of binary mixture do not equilibrate the same granular temperature. This is because the granular temperature is directly related to the mass of the particle. Thus the heavier particles have a higher temperature than the lighter particles. Feitosa and Menon [22] experimentally investigated the granular temperature distributions of a binary mixture of glass balls with aluminum, steel and brass balls. They also found that the granular temperature of the heavy particles was higher than that of light particles in a binary vibrofluidized granular bed.

The effect of vibration strength Γ on the solid fraction profiles in the horizontal direction is shown in Fig. 6. For the case of Γ=3.0, the solid fraction profile for heavy particles have higher concentrations near the center and smaller values close to the side walls as shown in Fig. 6(a). As Γ increases, the densities at the center decreases and more heavy particles shift toward the side walls of the bed resulting in an increase in density in this region. This is because more kinetic energy input from the vibrating wall causes an enhancement in the fluctuation velocities of particles. Then the heavy particles will tend to move toward both the sides of the bed due to the convection motion. Moreover, the trends of the solid fraction profiles for light particles are opposite to those of heavy particles in all cases as shown in Fig. 6(b).

Fig. 7 shows the solid fraction profiles in the vertical direction for steel–glass mixture as a function of Γ. As shown in Fig. 7(a), the variations of solid fraction profiles for heavy particles are not apparent indicating that the solid fraction profiles are not dependent strongly on Γ. The maxima of solid fraction profiles appear at the intermediate layers and the profiles decrease gradually along the altitude to the surface in all cases. However, the solid fraction profiles for light particles are different from those of heavy particles as shown in Fig. 7(b). The light particles have maximum density near the surface and more uniform distributions at a smaller height. An increase of Γ indicates a higher energy entering into the system and the powders easily expand towards higher regions and so the concentrations increase at large heights for both species.

The granular temperature profiles as a function of the height with different vibration intensities Γ are shown in Fig. 8. For the cases of Γ=3.0 and Γ=4.0, the temperature

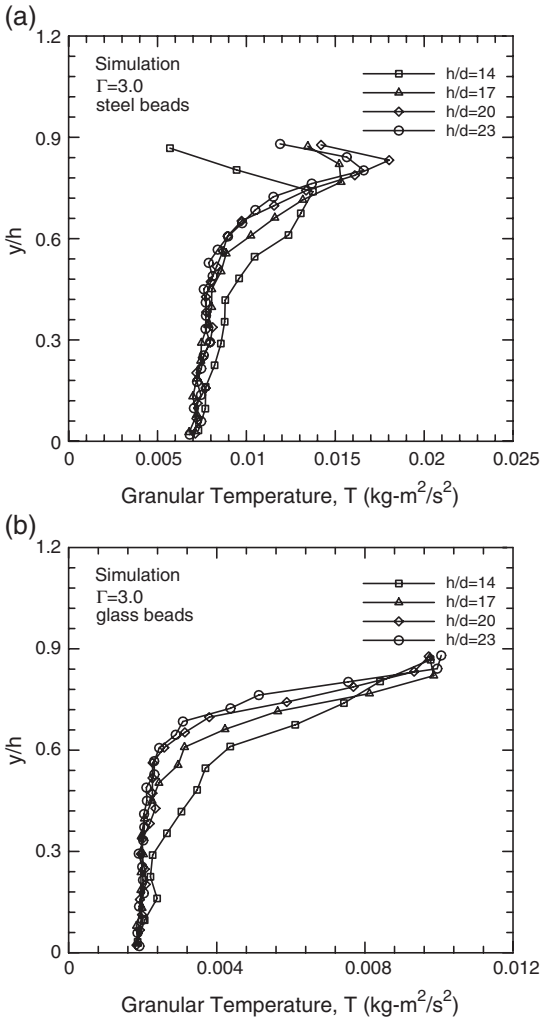

Fig. 12. Granular temperature profiles as a function of the height for steel–glass mixture with different mixture heights h/d for Γ=3.0 and f=15 Hz: (a) glass beads and (b) steel beads.

profiles of heavy particles have uniform distributions at smaller heights and maximum values near the surface as shown in Fig. 8(a). The profiles in these cases for light particles are similar to those of heavy particles as shown in Fig. 8(b). However, for Γ=6.0, the granular temperature profile varies. The temperature profile has a maximum near the bottom and a minimum value at the intermediate layer for both species, which corresponds to the position of minimum solid fraction as shown in Fig. 7.

The effect of vibration intensity Γ on the average granular temperatures of the system is shown in Fig. 9. It is observed that the temperature of each components, $T_{m,glass}$ and $T_{m,steel}$, increase with Γ. Again, the granular temperatures for steel beads are always larger than those of glass beads in all cases. Although the temperature of the two components do not equilibrate, the granular temperature ratio $\gamma_{m,sg}$ remains approximately constant for all cases, indicating $\gamma_{m,sg}$ is independent on Γ.

Fig. 10 shows the solid fraction profiles in the horizontal direction for steel–glass mixture with the normalized initial mixture heights h/d for Γ=3.0 and f=15 Hz. For the case of h/d=14, the solid fraction profiles are uniform throughout the horizontal layers for steel and glass beads. As h/d increases, the solid fraction profiles exhibit oscillation along the horizontal layers. For heavy particles, the density near the center region increases, while the density for light particles in this region decreases. For small value of h/d, the particles of the mixture system are fully fluidized, leading to a uniform distribution of solid fraction for both species. However, as h/d increases, the fluidization process is localized in the upper region. Thus the light particles easily expand into the upper region and the heavy particles are concentrated at the lower region.

Fig. 11 shows the solid fraction profiles in the vertical direction for steel–glass mixture as a function of the normalized initial mixture height y/h for different initial mixture height h/d with Γ=3.0 and f=15 Hz. As h/d increases, the maximum of the density profile for heavy particles moves towards higher regions and the profile becomes wider. For light particles, the solid fraction profiles are similar in all cases. The maximum density appears at the free surface and the solid fraction decays approximately exponentially at large heights.

The granular temperature profiles as a function of y/h with different initial normalized mixture heights h/d for Γ=3.0 and f=15 Hz are shown in Fig. 12. It is clear that the temperatures of heavy particles are greater than those of light particles. In all cases, the granular temperature profiles are uniform at small heights and reach a maximum and decays at large heights for heavy particles. The temperature profiles of light particles also showed similar trend as heavy particles. In addition, as the mixture height h/d increases, the collision rate between particles increases, causing a decrease in temperature for both the components of the mixture as shown in Fig. 12(a) and (b).

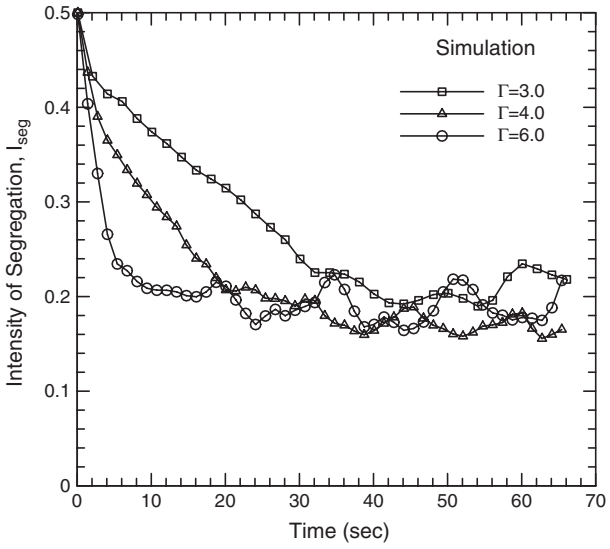

Fig. 14. Time evolution of the intensity of segregation of an initially segregated mixture under different vibration acceleration.

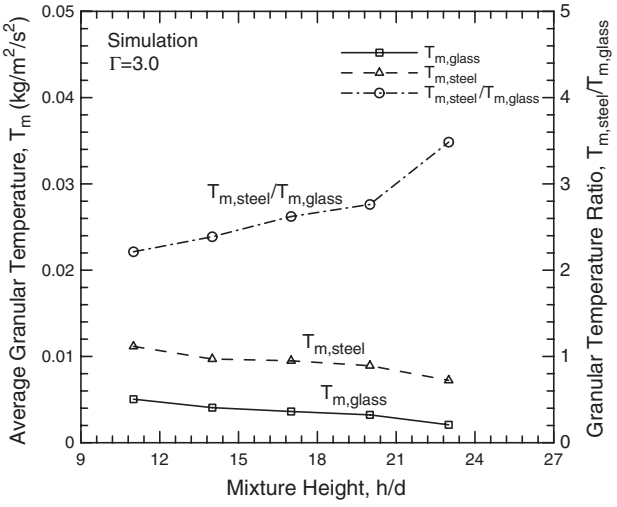

Fig. 13. Effect of granular mixture height h/d on the average granular temperatures of steel–glass mixture system.

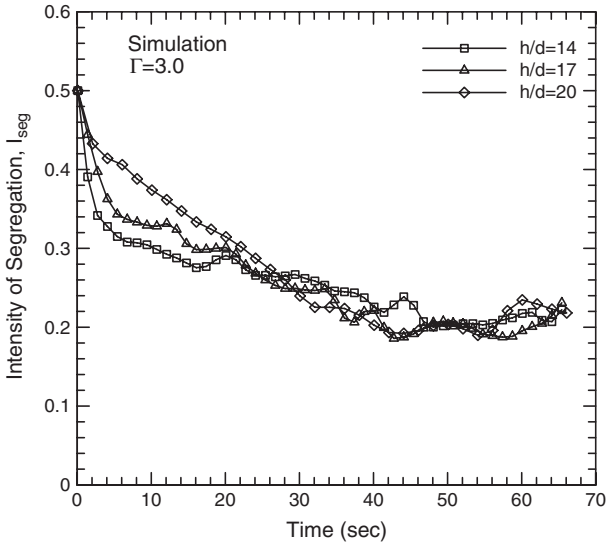

Fig. 15. Time evolution of the intensity of segregation of an initially segregated mixture under different mixture height h/d.

Table 2
Table of mixing rate constants

Vibration intensity	Initial mixture height h/d	mixing rate constant $k_m \times 10^{-2}$
Γ=2.4	20	0.62
Γ=2.6	20	0.96
Γ=3.0	11	2.25
Γ=3.0	14	2.19
Γ=3.0	17	2.11
Γ=3.0	20	2.06
Γ=3.0	23	1.08
Γ=3.5	20	1.82
Γ=4.0	20	2.48
Γ=5.0	20	4.02
Γ=6.0	20	6.95

The effect of normalized initial mixture height h/d on the average granular temperatures of mixture system is shown in Fig. 13. It is observed that the granular temperatures of two species, $T_{m,glass}$ and $T_{m,steel}$, are slightly decreased as h/d increases. The granular temperature is determined by the balance of the kinetic energy entering into the system and the dissipated energy due to the inelastic collisions between particles. A higher h/d indicates a higher probability of collisions and a larger dissipation of energy in the region result in the granular temperature decreases. In addition, the granular temperature ratio $\gamma_{m,sg}$ increases with h/d. This indicates that the heavy particles obtain more energy from the bottom of the bed than light particles as h/d increases.

For a quantitative comparison of the mixing rate, the intensity of segregation is used as the index of mixing. The intensity of segregation is defined as the standard deviation of the compositions of the samples and is calculated from

$$I_{seg} = \left[\frac{\sum_{i=1}^{i=N_r}(\varphi_i-\varphi_m)^2}{N_r-1}\right]^{1/2} \quad (10)$$

where N_r is the number of uniformly distributed cells within the bed, φ_i is the concentration at a specified cell, φ_m is the mean value of the concentration of the mixture system [23]. The concentration φ_i at a specified cell is calculated as the fraction of the heavy (or light) particles to the total particles for which the centers are located in the specified cell.

The variations of the intensity of segregation I_{seg} with different vibration intensities Γ are shown in Fig. 14. From the time evolution of the intensity of segregation, it is clear that I_{seg} decays with time and then fluctuates to an equilibrium value. This indicates that the mixing dominates over segregation during the mixing process. At longer times, the equilibrium values of I_{seg} are located around the values of 0.206, 0.217 and 0.22 for Γ=3.0, 4.0 and 6.0, respectively. In addition, as Γ increases, the intensity of segregation quickly decreases with time, implying a fast rate of mixing at the initial stages of the process. Fig. 15 shows the intensity of segregation I_{seg} as a function of h/d. Again, it is observed that I_{seg} decays with time and then fluctuates to an equilibrium value. At longer times, the equilibrium values of I_{seg} are equal to each other for all the cases, indicating that the equilibrium value of I_{seg} is not strongly dependent on h/d. However, the mixing rate decreases with h/d increase as shown in Fig. 15.

To quantify the mixing rate of the mixture, the simulation data of I_{seg} in all cases are correlated to the exponential function:

$$I_{seg} = I_{seg,0}\exp(-k_m t) \quad (11)$$

where $I_{seg,0}$ is the initial value of the intensity of segregation and k_m is the mixing rate constant. The initial values of the intensity of segregation $I_{seg,0}$ for all cases are equal to 0.5 indicating a completely segregated mixture before vibration. The values of the mixing rate k_m for different vibration conditions and initial mixture heights are listed in Table 2.

5. Conclusion

The mixing and segregation processes of a binary mixture containing glass beads and steel particles subjected to a vertical vibration are investigated in this study. The time evolutions of spatial distributions for glass–steel beads mixture show that the mixing process is an intricate process. For equal-sized particle but different densities, the driving force for mixing or segregation is contributed to the momentum exchange of particles having different granular temperatures. The simulation results showed that the heavier particles have a higher temperature than the lighter particles. The temperature gradient between the mixture components may lead to the mixing of the system. The mechanism causing the mixing and segregation of vibrated granular mixture is also strongly dependent on the convection motion of the mixture components. The presence of convection can enhance the mixing of steel–glass mixture.

The vibration intensity and the initial mixture height have significant influences on the mixing process. From the intensity of segregation, the mixing rate increases with the vibration strength. Meanwhile, the equilibrium values of intensity of segregation showed that the granular mixing is not strongly dependent on the vibration strength. In addition, as the initial mixture height h/d increases, the granular temperatures of each species are slightly decreased. However, the temperature ratio and the mixing rate increase with h/d, indicating that the initial height of the mixture has a significant effect on mixing rate in a vibrated bed.

Notations

a	vibration amplitude
C	fluctuation velocity
c_n	normal dashpot coefficient
c_s	tangential dashpot coefficient
d	mean particle diameter
E	particle elasticity modulus
$\mathbf{F}_{cij}$	contact force between particles i and j
$\mathbf{F}_g$	gravitational body force
F^n	normal contact force
F^s	tangential contact force
f	vibration frequency
f_s	frictional coefficient
h	initial mixture height
I	particle moment of inertia
I_{seg}	the intensity of segregation
k_n	normal elastic constant

k_s tangential elastic constant
m particle mass
m_e effect particle mass
N_r the number of uniformly distributed cells
n normal unit vector
R vector directed from the center of particle to contact point
r particle radius
T granular temperature
t tangential unit vector
$\mathbf{u}_{rel}$ relative velocity
V particle volume

Greek letters
Δn overlap between particles
ΔP reduced hydrostatic pressure
Δt time step
Γ dimensionless vibration acceleration amplitude
δs tangential displacement
v Poisson's ratios
φ concentration
φ_m mean concentration

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