Chemical Engineering Science 61 (2006) 6180–6188

Chemical Engineering Science

www.elsevier.com/locate/ces

Segregation of cohesive powders in a vibrated granular bed

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Received 30 August 2005; received in revised form 18 May 2006; accepted 28 May 2006
Available online 9 June 2006

Abstract

The effects of small amounts of added liquid on the segregation behavior of a granular system under vertical vibration by DEM simulation are investigated in this study. The cohesive forces of grains are incorporated into DEM simulations via a simplified dynamic liquid bridge force model. The simulation results show that capillary forces in addition to viscous forces have an important effect on the segregation phenomenon. The segregation rate of larger intruder rises to the top of the bed is found to depend on the liquid content. The segregation rate is sharply increased when a small amount of liquid is added to granular system. A transition to the reduction of segregation rate occurs at a critical liquid content. It has shown that this transition can be interpreted as the increase of attractive force between grains due to viscous force. The viscous forces make the particles stick more tightly to each other and retard the movement of particles, thus reducing the segregation rate. The segregation rate is also related to the convection motion of the granular system. The presence of convection enhances the segregation rate of wet granular materials.

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Keywords: Segregation; DEM; Vertical vibration; Cohesive; Liquid bridge

1. Introduction

Segregation is a process that has long been used in industry for the processing and handling of bulk solids. When a granular mixture is shaken vertically under gravity, the grains tend to segregate with the larger particles moving to the top of the bed. This is commonly referred as the "Brazil nut Effect" (Rosato et al., 1987). There have been many studies discussing the influence of powder properties such as size, density and friction on the segregation of granular materials in a vibrated bed based on simulation (Rosato et al., 1987; Jullien et al., 1992; Hong et al., 2001; Fierro et al., 2003), and on experimental tools (Knight et al., 1993; Duran et al., 1994; Liffman et al., 2001; Burtally et al., 2002; Huerta and Ruiz-Suárez, 2004). Though universally recognized, the mechanisms of this phenomenon are still poorly understood and so far there is no clear theory for the complex mechanisms. There have been three mechanisms proposed for the segregation in a vibrated granular bed: geometrical reorganization (Jullien et al., 1992), size percolation (Williams, 1963) and convection (Knight et al., 1993; Duran et al., 1994). The interstitial gas also plays an important role in determining the size segregation (Möbius et al., 2001, 2004; Rhodes et al., 2003; Yan et al., 2003). Contra-intuitively, a large and light intruder can also sink to the bottom of a deep granular bed (Shinbrot and Muzzio, 1998; Yan et al., 2003; Breu et al., 2003; Shinbrot, 2004). Several theoretical investigations have examined the rise and descent of the larger particles in a vertically vibrated bed (Hong et al., 2001; Shishodia and Wassgren, 2001; Liffman et al., 2001; Jenkins and Yoon, 2002; Rhodes et al., 2003; Tarzia et al., 2005). Recently, Ciamarra et al. (2006) investigated the species segregation in a granular mixture subject to vertical taps by molecular dynamics simulations. They showed that the larger particles rise to the top or sink to the bottom, which are dependent on the particle properties and the shaking intensity.

The cohesive forces between granular materials may be caused by van der Waals forces, electrostatic forces, magnetic forces or the capillary forces arising from liquid bridges. In a number of industrial processes, liquids are often added to reduce segregation effects of granular systems. The forces induced by liquid bridges with constant physical properties have been extensively studied both theoretically and experimentally

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doi:10.1016/j.ces.2006.05.048

(Fisher, 1926; Mason and Clarke, 1965; Mehrotra and Sastry, 1980; Ennis et al., 1990). Fraysse et al. (1999) experimentally measured the maximum angle of stability of a sandpile with the humidity effects of liquid, demonstrating that the wetting properties of the liquid have a strong influence on the granular medium. Although the flow behaviors of cohesive granular materials have attracted much interest over the past few years, there have been very few systematic studies of segregation in wet granular materials. Li and McCarthy (2003) experimentally observed the mixing and segregation phenomena within a tumbler mixer, finding that the segregation could be increased by adding a liquid such as water. Geromichalos et al. (2003) experimentally studied the segregation–mixing transition of wet granular mixtures shaken in horizontal circles. They demonstrated that segregation can be enhanced by adding a small amount of liquid. Rhodes et al. (2003) experimentally studied the Brazil-nut effect in a vertically vibrated granular bed. They showed that the rise time of the intruder was strongly influenced by humidity of the air, by over an order of magnitude. Scheel et al. (2004) experimentally investigated the influence of small amounts of liquid on the dynamic behavior of a vertically vibrated granular system. They demonstrated that the critical acceleration for fluidization of the system increased with the water content. In contrast to experiment, there are very few systematic studies of segregation in wet vibrated granular materials by numerical simulation. Using DEM simulation, the effects of small amounts of liquid on the self-diffusion and mixing behaviors of vibrated granular systems have been investigated by Hsiau and Yang (2003). In the current study, discrete element method (DEM) simulation taking into account the cohesive force of liquid bridge is employed to analyze the segregation behaviors in a vertically vibrated bed. The rise velocity of the intruder is measured to analyze the segregation of the granular system. The effects of interstitial liquids on the size segregation are examined under different vibration conditions. The segregation mechanism of cohesive vibrated granular materials is also investigated in detail.

2. Simulation model

2.1. Equation of particle motion

The DEM proposed by Cundall and Strack (1979) incorporating the cohesive effect of liquid bridges was used to simulate the segregation behavior of vibrated granular materials in this study. The DEM calculations were performed using Newton's second law of motion and the force-displacement law at the contact point. The governing equation of motion for particle i is

$$m_i \frac{\mathrm{d}\mathbf{V}_i}{\mathrm{d}t} = \mathbf{F}_{gi} + \sum (\mathbf{F}_{cij} + \mathbf{F}_{lij}), \tag{1}$$

$$I_i \frac{\mathrm{d}\boldsymbol{\omega}_i}{\mathrm{d}t} = \sum (\mathbf{R}_i \times \mathbf{F}_{cij}), \tag{2}$$

where m_i and I_i are the particle mass and moment of inertia; $\mathbf{V}_i$ and $\boldsymbol{\omega}_i$ are the particle translational and rotational velocities; $\mathbf{F}_{gi}$ is the gravitational body force of particle; $\mathbf{F}_{cij}$ is the contact force of particle i due to particle j; $\mathbf{F}_{lij}$ is the liquid bridge force between particles i and j; and $\mathbf{R}_i$ is the vector directed from the center of particle i to the contact point.

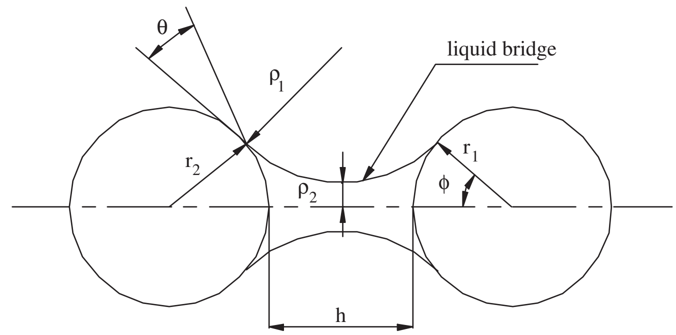

Fig. 1. Schematic of the liquid bridge force model acting between two particles.

2.2. Contact force model

The contact force between particles is calculated when the particles collide with the neighboring particles or with the walls, and the particle contact force can be separated into the normal and tangential components. The normal component of the force acting on particle i from particle j is

$$F^n_{j\to i} = k_n \Delta n - c_n m_e (\mathbf{V}_{\mathrm{rel}} \cdot \mathbf{n}), \tag{3}$$

where Δn is the amount of overlap between the particles i and j; k_n is the normal elastic constant; c_n is the dashpot coefficient; $\mathbf{V}_{\mathrm{rel}}$ is the relative velocity between particles; $\mathbf{n}$ is the unit vector of normal direction; and m_e is the effective mass, $m_i m_j/(m_i + m_j)$. The normal contact force between particles is modeled as a linear spring in parallel with a dashpot element. The spring force provides an elastic restoration and the damping force dissipates the energy as the particles collide with neighboring particles or with the walls. The tangential component of the force is given by

$$F^s_{j\to i} = -c_s m_e (\mathbf{V}_{\mathrm{rel}} \cdot \mathbf{t}) - \mathrm{sign}(\delta s) \min(k_s|\delta s|, f_s|F^n_{j\to i}|), \tag{4}$$

where k_s is the tangential elastic constant, c_s is the dashpot coefficient, $\mathbf{t}$ is the unit vector of tangential direction, δs is the tangential displacement during a contact, and f_s is the frictional coefficient of particles. When the magnitude of the tangential elastic force, $k_s|\delta s|$, exceeds the magnitude of the frictional sliding force, $f_s|F^n_{j\to i}|$, then the frictional sliding force becomes active and replaces the tangential elastic force.

2.3. The cohesive force model

The cohesive forces of liquid bridge arise from the dynamic bridge strength based on the superposition of lubrication and circular approximation is considered in the simulation model. The liquid configuration between two particles is shown in Fig. 1. The capillary forces include the liquid surface tension force F_s and the reduced hydrostatic force F_h, which is given by

$$F_c = F_s + F_h = \pi\gamma\rho_2\left(\frac{\rho_1+\rho_2}{\rho_1}\right), \tag{5}$$

where γ is the liquid surface tension, ρ_1 and ρ_2 are the principal radii of curvatures of the bridge which are given by

$$\rho_1 = \frac{s_i + r_i(1-\cos\phi_i)}{\cos(\phi_i+\theta)}, \tag{6}$$

$$\rho_2 = r_i \sin\phi_i - \rho_1[1-\sin(\phi_i+\theta)], \tag{7}$$

where r_i is the radius of particle i, s_i is the half-separation distance of particle i, ϕ_i is the half-filling angle and θ is the solid–liquid contact angle in radians. To maintain a constant volume between two particles, the half-filling angle between particles is used to calculate the different distances between particles. An interval-halving iteration method is used to obtain the corresponding value of the half-filling angle. Another important parameter is the rupture distance of the liquid bridge. Lian et al. (1993) showed that the critical separation distance for rupture between two spheres, S_c, was proportional to the cube root of the liquid volume:

$$S_c = (1+0.5\theta)V^{1/3}. \tag{8}$$

In addition to the capillary force, the dynamic bridge force due to viscosity of liquid is also considered in this study. Adams and Perchard (1985) employed the lubrication approximation and the elastohydrodynamic collision model between two elastic spheres to analyze the viscous force of the pendular liquid bridge between two rigid spheres. The viscous force in the normal direction

$$F_{v_n} = 6\pi\mu v_n \frac{(R^*)^2}{h}, \tag{9}$$

where μ is the viscosity of the liquid, v_n is the relative normal velocity between particles, h is the distance between particles, and R^* is the reduced radius of two particles, $R^*=r_i r_j/(r_i+r_j)$. In the tangential direction, the viscous force between a sphere and a rigid wall derived by Goldman et al. (1967) is used in this study. The analytical solution of the tangential viscous force is

$$F_{v_t} = \left(\frac{8}{15}\ln\frac{R^*}{h} + 0.9588\right)6\pi\mu R^* v_t, \tag{10}$$

where v_t is the relative tangential velocity. The total dynamic liquid bridge force is obtained by the superposition of the capillary force and the viscous force. The interaction between a particle and a wall is similar to that described above.

3. Results and discussion

To observe the Brazil-nut effect of cohesive powders in a vertically vibrated container, glass lime beads (particle density $\rho_p = 2500\,\text{kg/m}^3$) with an average diameter of 1 mm (standard deviation of 0.05 mm) were used as the granular materials in this study. The large disks (or intruders) with various diameters and similar density of glass beads were used as the intruders in the simulation tests. Water with surface tension of 0.0725 N/m and viscosity of 0.0012 Pa/s was chosen as the interstitial liquid between particles and with the walls. The height, width, and depth of the inside space of the tank used in the simulation were 30, 40 and 1.2 mm, respectively. The related simulation parameters are listed in Table 1. In the beginning, the intruder was placed at the bottom of a monodispersed glass sphere bed. The system was then vibrated sinusoidally in the vertical direction. The rise time of the intruder is recorded to calculate the rise velocity, which is denoted as $u_{\text{rise}} = y_0/t$, where y_0 is the distance between the top of the intruder and the bed surface in unit of mm, and t is the total rise time of the intruder in unit of s.

Table 1
Simulation parameters

Parameter	Value	Units
Particle diameter, d	0.001	m
Particle density, ρ_p	2500	kg/m^3
Coefficient of restitution, e_p		
Particle/particle	0.9	
Particle/wall	0.9	
Friction coefficient, f_s		
Particle/particle	0.5213	
Particle/wall	0.5027	
Normal damping coefficient, c_n		
Particle/particle	8.337×10^{-2}	N s/m
Particle/wall	1.667×10^{-3}	N s/m
Tangential damping coefficient, c_s		
Particle/particle	5.294×10^{-2}	N s/m
Particle/wall	1.058×10^{-3}	N s/m
Normal spring constant, k_n		
Particle/particle	5280	N/m
Particle/wall	10580	N/m
Tangential spring constant, k_s		
Particle/particle	5280	N/m
Particle/wall	10580	N/m
Time step, Δt	5.0×10^{-6}	s

When the granular matters are subjected to vertical vibration, the size segregation could be induced by the convective motion of the smaller particles, which drag the intruder to the top (Knight et al., 1993) or by the void-filling of small particles below large intruder, which lift the large particle (Rosato et al., 1987; Duran et al., 1994). Several studies have demonstrated that the presence of convection can enhance size segregation in a vibrated system (Taguchi, 1992; Gallas et al., 1992; Knight et al., 1993). To investigate the effect of convection on Brazil-nut behaviors in a cohesive granular system, the convection motion of small particles subjected to a vertical vibration is first determined with different humidity contents. Fig. 2 shows the time sequences of the upward motions of intruders in a bed of small glass beads with different volume fraction of the added liquid V^* for the size ratio D/d of 12 under the dimensionless vibration acceleration of $\Gamma=2.4$ and vibration frequency of $f=20$ Hz. The dimensionless vibration acceleration Γ is defined as $\Gamma = a(2\pi f)/g$, where a is the amplitude of vibration, f is the vibration frequency and g is the gravity acceleration. The volume fraction of the added liquid V^* is defined as the volume

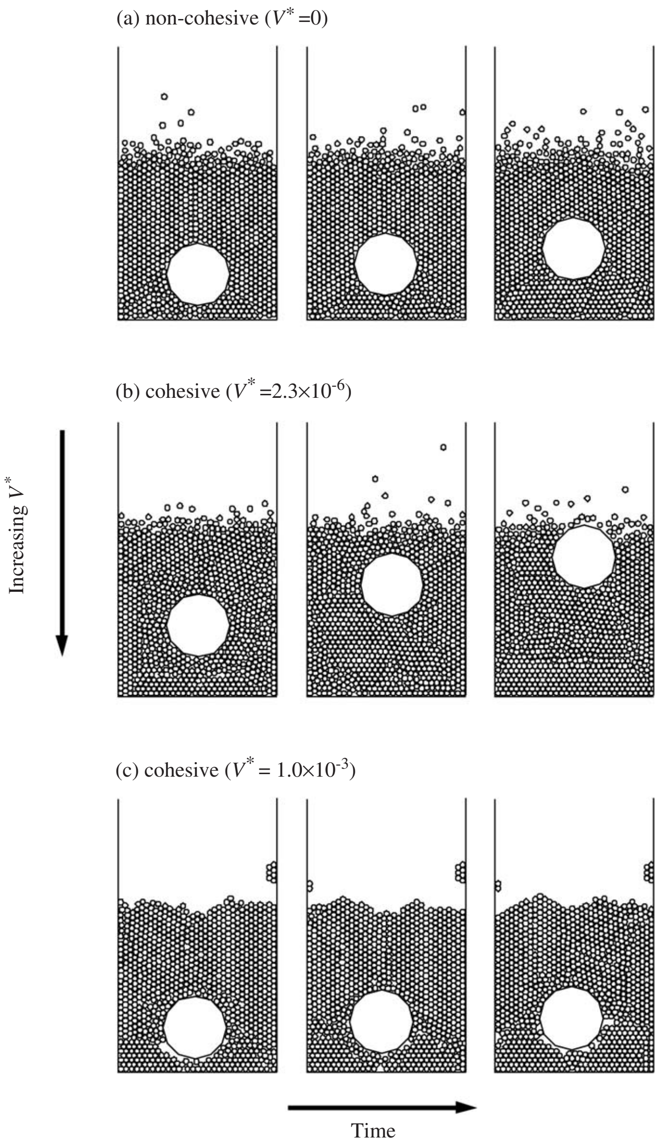

Fig. 2. Comparison of size segregation for different volume fraction V^* of 0.0, 2.3×10^{-6} and 1.0×10^{-3} (from top to bottom) with $\Gamma = 2.4$ and $D/d = 12.0$. The arrows indicate the direction of increasing time (horizontal) and increasing volume fraction of interstitial liquid (vertical).

of the liquid divided by the total volume of the small beads. From the images it is clear that the large intruder essentially moves through the small particles as it rises up to the surface. It appears that the cohesive force due to liquid bridge may play a dominant role in determining the upward motion of the intruder. The intruder rises quickly to the top at an intermediate volume fraction of $V^* = 2.3 \times 10^{-6}$ and rises slowly at a large volume fraction of $V^* = 1.0 \times 10^{-3}$ in comparison to the rates of noncohesive materials ($V^* = 0.0$).

Fig. 3 shows the mean velocity fields of small glass beads with different humidity contents for the size ratio of $D/d = 12$ with $\Gamma = 2.4$ and $f = 20$ Hz. From the figures off velocity fields, two apparent convection cells are found for the cases of $V^* = 0$ and $V^* = 2.3 \times 10^{-6}$. In these approaches, the particles move upward in the central part and move downward along the side

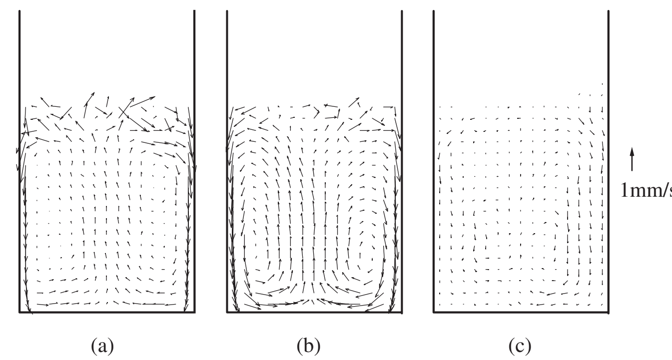

Fig. 3. Mean velocity fields of particles in a vertical vibrated bed with different volume fraction of the added liquid V^* for the case of $D/d = 12$ and $\Gamma = 2.4$. The dimensionless liquid volumes V^* from left to right are: (a) $V^* = 1.0 \times 10^{-3}$; (b) $V^* = 2.3 \times 10^{-6}$; (c) $V^* = 0.0$.

walls. In the contrast, the convection cells seem to have disappeared for the case of $V^* = 1.0 \times 10^{-3}$. It is because that the cohesive force causes the particles to cluster together and retard the particles' relative motions, resulting in the decrease of the velocity fields. Molecular dynamics simulations in a vibrated granular system previously showed that the shear friction between particles during collisions can lead to convection (Gallas et al., 1992; Taguchi, 1992). For wet granular beds ($V^* > 0$), the interaction between the cohesive force due to liquid bridge and the frictional force during collision may play dominant roles in determining the convection. Nase et al. (2001) demonstrated that the cohesive force is significantly larger than the shearing forces in wet dense granular flows. When a small amount of liquid is added to the system, the liquid bridges between particles develop, which introduce cohesive forces between particles. If the humidity content is relatively small which corresponds to a small value of V^*, the liquid bridge between particles may be ruptured during vertical vibration. Thus the small amounts of liquid may produce a lubrication effect and reduce the frictional interaction between particles, which results in the enhancement of the convection, as shown in Fig. 3(b). However, if the humidity content increases, which corresponds to a large value of V^*, the effects of dynamic viscous forces between particles become important. This is because the liquid bridge completely develops, and this always acts opposite to the particle motion and suppresses the relative motion between particles, leading to the reduction of convection as shown in Fig. 3(a).

In a vertically vibrated granular system, the motions of particles are driven by the kinetic energy input from the base of the container. The granular temperature and the solid fraction profiles along the bed depth have previously been used to quantify the dynamic behaviors and the expansion of noncohesive granular bed, respectively (Lan and Rosato, 1995; Yang and Hsiau, 2000). The granular temperature is proportional to the kinetic energy of the fluctuation velocity. Fig. 4 shows the comparison of solid fraction profile (or density profiles) $\nu(y/d)$ of the small particles with different humidity contents for the size ratio of $D/d = 12$ with $\Gamma = 2.4$ and $f = 20$ Hz. The corresponding dimensionless granular temperature profiles $T_d(y/d)$ are shown in Fig. 5. The dimensionless granular temperature $T_d(y/d)$ is

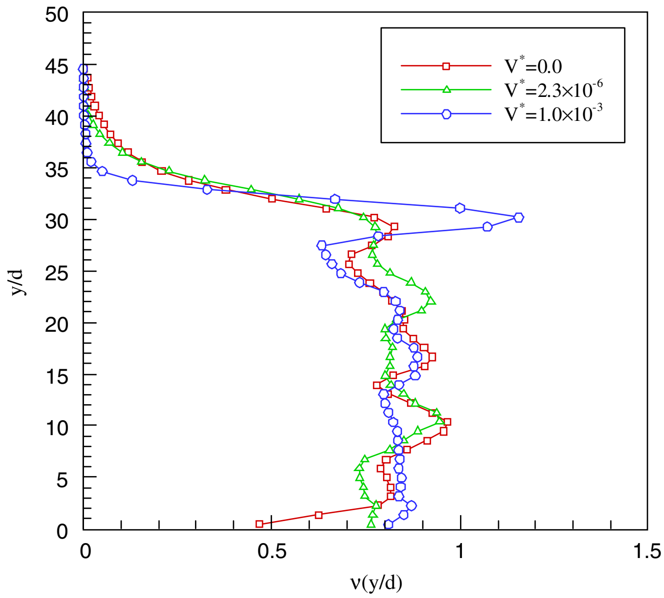

Fig. 4. Solid fraction profiles $v(y/d)$ with different volume fraction of the added liquid V^* for the case of $D/d = 12$ and $\Gamma = 2.4$.

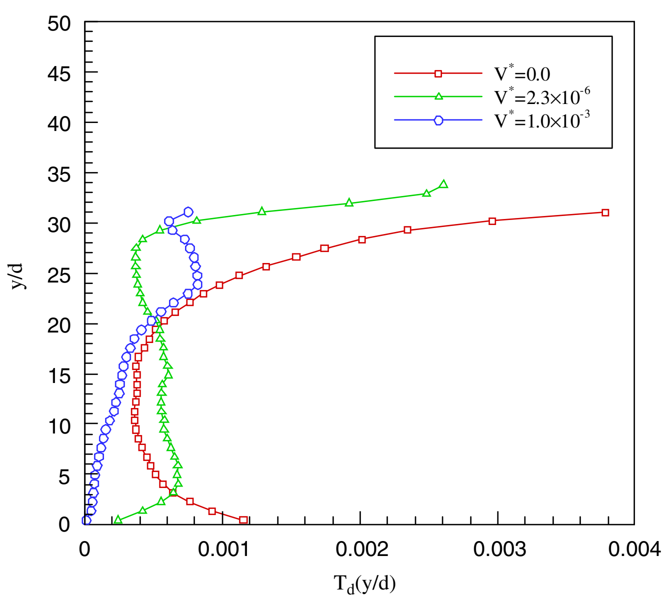

Fig. 5. Dimensionless granular temperature profiles $T(y/d)$ with different volume fraction of the added liquid V^* for the case of $D/d = 12$ and $\Gamma = 2.4$.

defined as $T_d(y/d) = T(y/d)/gd$, where $T(y/d)$ is the granular temperature profile of the small particles along bed depth, which is calculated from

$$T(y) = \frac{1}{3}(\langle C(y,t)\rangle_L)^2, \tag{11}$$

where $\langle C(y,t)\rangle_L$ is the long-term averaged cumulative mass-weighted fluctuation velocity in layer y. It appears that the solid fraction profiles have a peak near the bottom of the bed and they oscillate gradually along the depth to the top surface for the cases of $V^* = 0.0$ and 2.3×10^{-6}. A larger value of solid fraction indicates a higher probability of collisions and

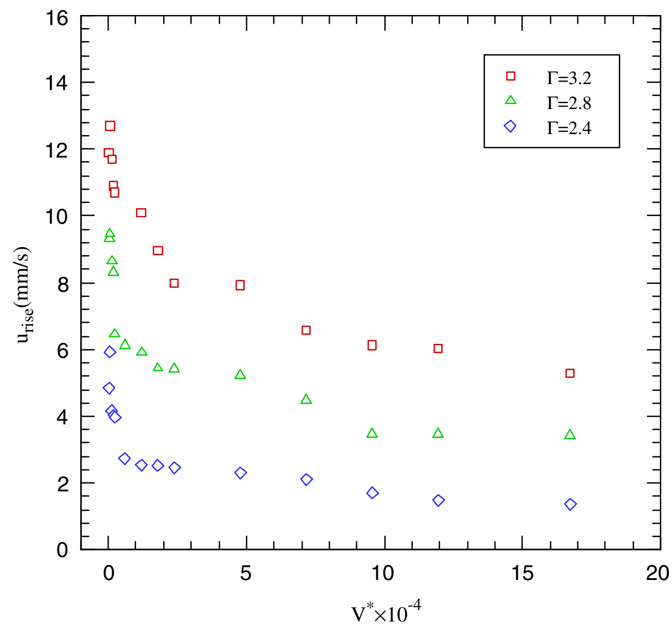

Fig. 6. Rise velocity of the intruder u_{rise} as a function of V^* with the size ratio D/d of 12 under three vibration amplitudes of $\Gamma = 2.4, 2.8$ and 3.2.

more energy dissipation between particles. From the figures of granular temperature profile, the granular temperature profiles present different shapes as the volume fraction V^* varies. For the case of dry materials ($V^* = 0.0$), the granular temperature profile has a maximum value near the surface and a minimum value at the intermediate layer, which corresponds to the location of minimum solid fraction as shown in Fig. 4. For wet materials, the kinetic energies during vibration are dissipated by the inelastic collisions, the frictional interactions between particles and the viscous dissipation of added liquid. As a small amount of liquid is added to the system ($V^* = 2.3 \times 10^{-6}$), the granular temperature profile becomes more uniform along the depth of the bed. This is because the small amount of liquid reduces the frictional force and produces a lubricant effect between particles, resulting in less energy dissipation between particles. Thus, the granular temperature increases along the depth of the bed. In comparison, at the higher volume fraction of $V^* = 1.0 \times 10^{-3}$, the viscous forces due to liquid bridge dissipate most of the kinetic energies of the particles, causing the lower region of the granular bed to behave like a solid state. So the granular temperatures decrease along the depth as compared to those of dry materials. Fig. 6 shows the rise velocity of the intruder as a function of V^* for the size ratio of $D/d = 12$ under different vibration conditions of $\Gamma = 2.4, 2.8$ and 3.2. As shown in Fig. 6, it appears that the rise velocity of the intruder increases with the vibration acceleration Γ. For a given vibration acceleration Γ, the rise velocities of the intruders increase to a maximum value by adding a small amount of liquid to granular systems. As the liquid content increases continuously, the particles may cluster around each other and retard the movements of the particles, reducing the rise velocity of the intruder. Hsiau et al. (2004) experimentally investigated the influence of moisture content on the rising velocity of a

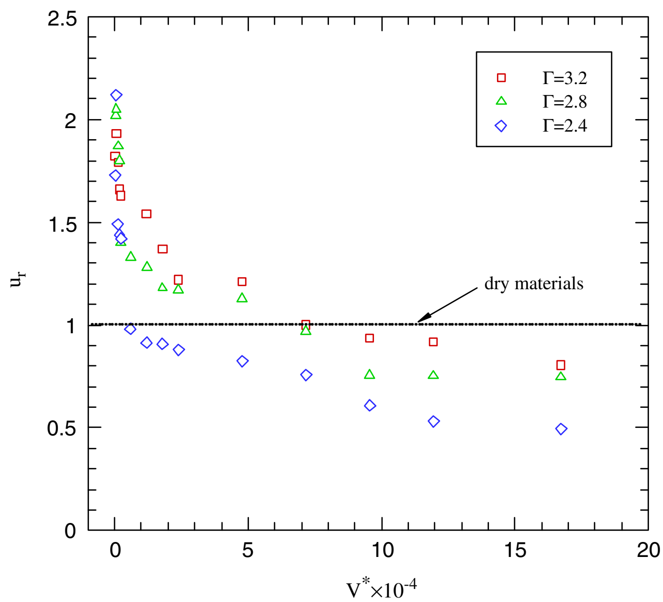

Fig. 7. Normalized rise velocity of the intruder u_r as a function of V^* for the case of size ratio $D/d = 12$ under three vibration amplitudes of $\Gamma = 2.4, 2.8$ and 3.2. The horizontal dotted line represents $u_r = 1$ which is corresponding to the case of dry materials.

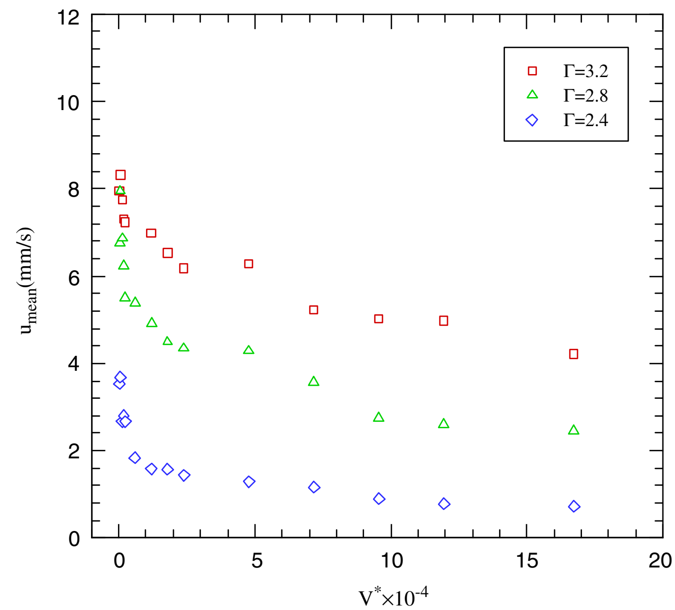

Fig. 8. Mean rise velocity in the central part of the bed u_{mean} as a function of V^* with the size ratio D/d of 12 under three vibration amplitudes of $\Gamma = 2.4, 2.8$ and 3.2.

larger intruder in the background of smaller beads of vibrated granular systems. They demonstrated that the rise velocity of the intruder increased by adding a small amount of water and decreased with greater amount of moisture content. Since the size scales of the granular system used in this study are different from the ones in the experiments, it is difficult to compare the simulation data with those in experiment. However, the effects of liquid contents on the rise velocity of the intruder are consistent in the simulation and in the experiments by Hsiau et al. (2004).

In order to compare the segregation rate between the wet granular system and the dry materials, the rise velocity of the intruder for wet powders is normalized to that of dry materials, which is given by

$$u_r = \frac{u_{\text{rise}}}{u_{\text{rise},nc}}, \tag{12}$$

where $u_{\text{rise},nc}$ is the rise velocities of the intruder for dry materials. Fig. 7 shows the normalized rise velocity of the intruder u_r as a function of V^* for the size ratio of $D/d = 12$ under different vibration conditions of $\Gamma = 2.4, 2.8$ and 3.2. The horizontal dotted line represents $u_r = 1$ which corresponds to the case of dry materials. A nonmonotonic dependence is observed for the rise velocity of the intruder and the added liquid contents. For relatively lower vibration acceleration of $\Gamma = 2.4$, lubricant effects between particles are induced by adding a small amount of liquid during the vibration cycle, leading to an increase of u_r. However, the effect of added liquid becomes increasingly important as V^* is approximately greater than 1.0×10^{-4}. Beyond 1.0×10^{-4}, most of the particles in the granular bed are more condensed, leading to a reduction of convection, and thus $u_r < 1$. In contrast, for relatively strong vibration accelerations of $\Gamma = 2.8$ and 3.2, the rise velocity of the intruder for wet particles is smaller than dry particles as V^* is greater than a critical value of 7.0×10^{-4}. This is because with strong agitation inputs from the base of the container, most particles of the system are in continuous motion. Thus, the liquid bridge between particles is more difficult to develop during the vibrated motion until more liquids are added to the system ($V^* > 7.0 \times 10^{-4}$).

To distinguish whether the convection or the cohesion is more important in determining the segregation of wet materials, it is necessary to subtract the mean rise velocity in the central part of the bed, u_{mean}, from the rise velocity of the intruder, u_{rise}. The remaining component is referred to the “net rise velocity” of the intruder, $u_{\text{rise,net}}$. Figs. 8 and 9 show that u_{mean} and $u_{\text{rise,net}}$ varied with V^* under different vibration accelerations. As shown in Figs. 8 and 9, both u_{mean} and $u_{\text{rise,net}}$ are observed to decrease with increasing the volume fraction V^*. Fig. 10 shows the ratio of $u_{\text{rise,net}}$ and u_{mean} against V^* under different vibration accelerations. It appears that the ratio of $u_{\text{rise,net}}/u_{\text{mean}}$ is strongly dependent on Γ and V^*. For relatively low vibration strength of $\Gamma = 2.4$, $u_{\text{rise,net}}/u_{\text{mean}}$ increases with V^*, indicating that the convection is less important by increasing V^*. The reason for this is that the liquid bridges between particles always suppress the particles’ motion by adding more liquids to system, leading to the reduction of u_{mean} and the increase of $u_{\text{rise,net}}/u_{\text{mean}}$. In comparison, for relatively high vibration strength of $\Gamma = 3.2$, the ratio of $u_{\text{rise,net}}/u_{\text{mean}}$ monotonously decreases with V^*, indicating that the effect of convection on segregation is increasingly important as the vibration strength increases. As a large driving energy is input from the base of the container, most particles of the system are in fluidized conditions. Then the lubricant effect is induced by adding a small amount of liquid to system, which

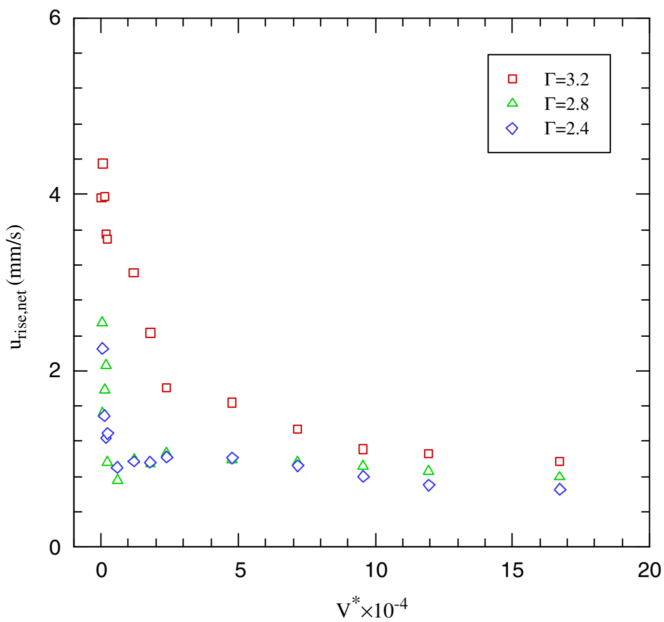

Fig. 9. Net rise velocity of the intruder $u_{\mathrm{rise,net}}$ as a function of V^* with the size ratio D/d of 12 under three vibration amplitudes of $\Gamma = 2.4$, 2.8 and 3.2.

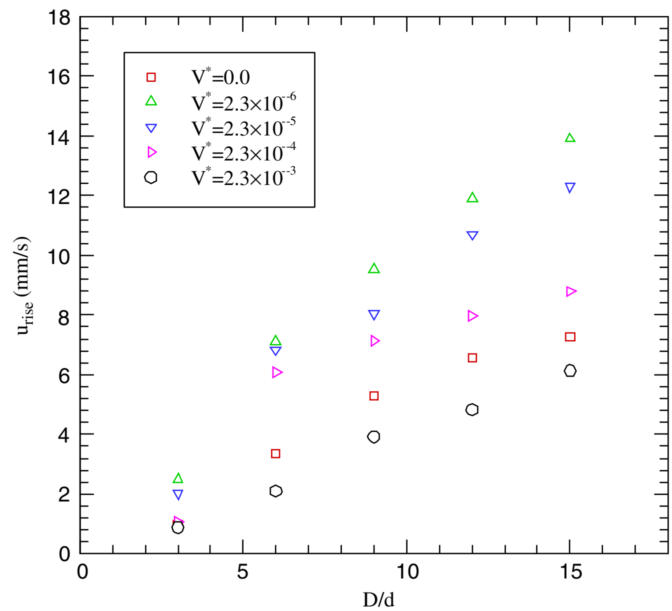

Fig. 11. Rise velocity of the intruder as a function of size ratio D/d and volume fraction of the added liquid V^* under the vibration condition of $\Gamma = 2.4$.

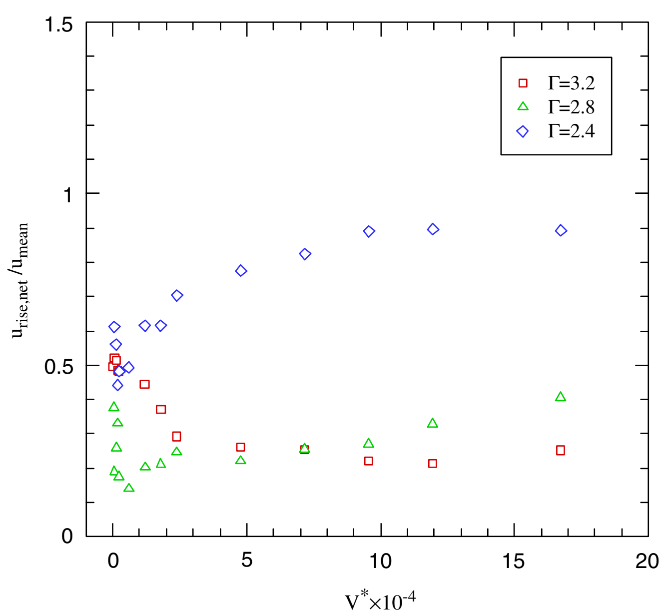

Fig. 10. Ratio of $u_{\mathrm{rise,net}}$ and u_{mean} against V^* with the size ratio D/d of 12 under the three vibration amplitudes of $\Gamma = 2.4$, 2.8 and 3.2.

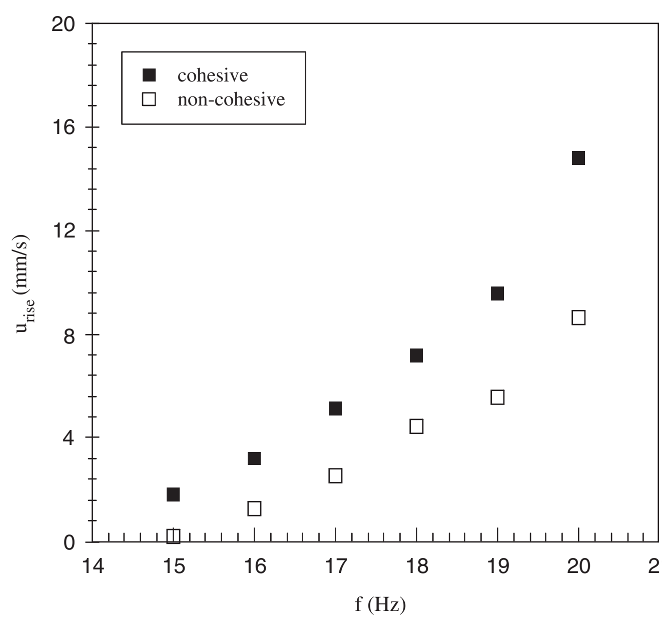

Fig. 12. Rise velocity of the intruder against the vibration frequency f with the vibration amplitude $a = 2\,\mathrm{mm}$ and $V^* = 2.3 \times 10^{-5}$.

results in the enhancement of the convection. In this situation the effect ofconvection becomes more important, leading to the reduction of $u_{\mathrm{rise,net}}/u_{\mathrm{mean}}$. The effects of the variations of the size ratio D/d on the segregation for wet materials are investigated below. Fig. 11 shows that the rise velocity of the intruder varies with the size ratio D/d for wet materials under the vibration condition of $\Gamma = 2.4$. As shown in Fig. 11, the rise velocity of the intruder increases with D/d for both wet ($V^* > 0$) and dry ($V^* = 0$) cases. As the size ratio D/d is small, the intruder rises slowly to the surface almost preserving its position relative to the surrounding small particles. With increasing the size ratio, the rise velocity of the intruder also increases due to the void-filling effect of small particles beneath the large intruder. Furthermore, for a given size ratio D/d, the rise velocity of the intruder can be enhanced by adding a small amount of liquid to granular system and the rise velocity decreases as more liquid is added to the system. Hsiau et al. (2004) experimentally investigated the rise velocity of an intruder in a wet vibrated granular bed. They also found that the rise rate of the intruder for wet powders was slightly faster than that of dry powders when a small amount of liquid was added to the system. Fig. 12 shows the rise velocity of the intruder against the vibration frequency f with the vibration amplitude $a=2$ mm. As shown in Fig. 12, the rise velocity of the intruder increases with f for both wet and dry materials. As the vibration amplitude stays constant, a higher vibration frequency indicates a higher kinetic energy input from the base of the vibrated bed, resulting in the increase of convection strength. Therefore the large intruder rises to the surface of the bed quickly due to the enhancement of convection.

4. Conclusion

In this study, DEM simulation incorporating a simplified model of liquid bridge is employed to investigate the Brazil-nut effect in a cohesive granular bed subjected to vertical vibration. The simulation results show that the segregation behaviors for wet powders are strongly dependent on both the convection motion of small particles and the amount of liquid added to system. The time evolutions of spatial distributions of cohesive powders with different liquid contents show that the segregation is enhanced at an intermediate amount of liquid content and slows down at a large liquid content as compared to those of dry materials. The granular temperature and solid fraction profiles of the bed are determined to interpret the segregation of cohesive materials in a vertically vibrated bed. For relatively strong vibration strength and a small amount of liquid content, the interstitial liquids reduce the frictional force and produce a lubricant effect between particles. In this situation the agitation energy input from the base of the container is less dissipated, enhances the convection and segregation rate. However, for relatively weak vibration strength and more liquid content, the convection and segregation rate decreases due to the dynamic viscous force and capillary force of liquid bridge.

Notation

a	vibration amplitude
c_n	normal dashpot coefficient
c_s	tangential dashpot coefficient
$C(y,t)$	fluctuation velocity in layer y
d	mean small particle diameter
D	intruder diameter
f	vibration frequency
f_s	frictional coefficient
F_c	capillary force of liquid bridge
$\mathbf{F}_{cij}$	contact force between particles i and j
$\mathbf{F}_g$	gravitational body force
$\mathbf{F}_{lij}$	liquid bridge force between particles i and j
F_v	viscous force of liquid bridge
F^n	normal contact force
F^s	tangential contact force
$\mathbf{g}$	gravitational accelaration
h	distance between particles
I	particle moment of inertia
k_n	normal elastic constant
k_s	tangential elastic constant
m	particle mass
m_e	effect particle mass
Δn	amount of overlap between the particles
$\mathbf{n}$	normal unit vector
R^*	reduced radius of two particle
$\mathbf{R}$	vector directed from the center of particle to contact point
S_c	critical rupture distance
Δt	time step
$\mathbf{t}$	tangential unit vector
T_d	dimensionless granular temperature
u_{rise}	rise velocity of the intruder for wet particle
$u_{\mathrm{rise},nc}$	rise velocity of the intruder for dry particle
$u_{\mathrm{rise,net}}$	net rise velocity of the intruder
$\mathbf{V}_{\mathrm{rel}}$	relative velocity between particles
V^*	dimensionless liquid bridge volume
y_0	distance between the top of the intruder and the bed surface

Greek letters

Γ	dimensionless vibration acceleration amplitude
δs	tangential displacement
θ	solid–liquid contact angle
μ	liquid viscosity
ρ	principle radii of curvature of liquid bridge
ρ_p	particle density
ϕ	half-filling angle

Acknowledgments

The author gratefully acknowledges the financial support from the National Science Council of the ROC for this work through project NSC 92-2212-E-270-005.

References

Adams, M.J., Perchard, V., 1985. The cohesive forces between particles with interstitial liquid. Institute of Chemical Engineers Symposium 91, 147–160.

Breu, A.P.J., Ensner, H.M., Kruelle, C.A., Rehberg, I., 2003. Reversing the Brazil-nut effect: competition between percolation and condensation. Physical Review Letters 90, 014302.

Burtally, N., King, P.J., Swift, M.R., 2002. Spontaneous air-driven separation in vertically vibrated fine granular mixtures. Science 295, 1877–1879.

Ciamarra, M.P., De Vizia, M.D., Fierro, A., Tarzia, M., Coniglio, A., Nicodemi, M., 2006. Granular species segregation under vertical tapping: effects of size, density, friction, and shaking amplitude. Physical Review Letters 96 (5), 058001.

Cundall, P.A., Strack, O.D.L., 1979. A discrete numerical model for granular assemblies. Geotechnique 29, 47–65.

Duran, J., Mazozi, T., Clément, E., Rajchenbach, J., 1994. Size segregation in a two-dimensional sandpile: convection and arching effect. Physical Review E 50, 5138–5141.

Ennis, B.J., Li, J., Tardos, G.I., Pfeffer, R., 1990. The influence of viscosity on the strength of an axially strained pendular liquid bridge. Chemical Engineering Science 45, 3071–3088.

Fierro, A., Nicodemi, M., Coniglio, A., 2003. Edward's approach to horizontal and vertical segregation in a mixture of hard spheres under gravity. Journal of Physics—Condensed Matter 15 (11), S1095–S1105.

Fisher, R.A., 1926. On the capillary forces in an ideal soil; correction of formulae given by W. B. Haines. Journal of Agricultural Science 16, 492–505.

Fraysse, N., Thomé, H., Petit, L., 1999. Humidity effects on the stability of a sandpile. European Physical Journal B 11, 615–619.

Gallas, J.A.C., Herrmann, H.J., Sokolowski, S., 1992. Convection cells in vibrating granular media. Physical Review Letters 69, 1371–1374.

Geromichalos, D., Kohonen, M.M., Mugele, F., Herminghaus, S., 2003. Mixing and condensation in a wet granular medium. Physical Review Letters 90, 168702.

Goldman, A.J., Cox, R.G., Brenner, H., 1967. Slow viscous motion of a sphere parallel to a plane wall. Chemical Engineering Science 22, 637–651.

Hong, D.C., Quinn, P.V., Luding, S., 2001. Reverse Brazil nut problem: competition between percolation and condensation. Physical Review Letters 86, 3423–3426.

Hsiau, S.S., Yang, S.C., 2003. Numerical simulation of self-diffusion and mixing in a vibrated granular bed with the cohesive effect of liquid bridges. Chemical Engineering Science 58, 339–351.

Hsiau, S.S., Tai, C.H., Chiang, M.C., 2004. Effect of moisture content on the motion of powders in a vibrated bed. Advanced Powder Technology 15, 673–686.

Huerta, D.A., Ruiz-Suárez, J.C., 2004. Vibration-induced granular segregation: a phenomenon driven by three mechanisms. Physical Review Letters 92, 114301.

Jenkins, J.T., Yoon, D.K., 2002. Segregation in binary mixtures under gravity. Physical Review Letters 88, 1943011–1943014.

Jullien, R., Meakin, P., Pavlovitch, A., 1992. Three-dimensional model for particle-size segregation by shaking. Physical Review Letters 69, 640–643.

Knight, J.B., Jaeger, H.M., Nagel, S.R., 1993. Vibrated-induced size separation in granular media: the convection connection. Physical Review Letters 70, 3728–3731.

Lan, Y., Rosato, A.D., 1995. Macroscopic behavior of vibrating beds of smooth inelastic spheres. Physics Fluids 7, 1818–1831.

Li, H., McCarthy, J.J., 2003. Controlling cohesive particle mixing and segregation. Physical Review Letters 90, 184301–184304.

Lian, G., Thornton, C., Adams, M.J., 1993. A theoretical study of the liquid bridge forces between two rigid spherical bodies. Journal of Colloid Interface Science 161, 138–147.

Liffman, K., Muniancy, K., Rhodes, M., Gutteridge, D., Metcalfe, G., 2001. A general segregation mechanism in a vertically shaken bed. Granular Matter 3, 205–214.

Mason, G., Clarke, W.C., 1965. Liquid bridges between spheres. Chemical Engineering Science 20, 859–866.

Mehrotra, V.P., Sastry, K.V.S., 1980. Pendular bond strength between unequal sized spherical particles. Powder Technology 25, 203–214.

Möbius, M.E., Lauderdale, B.E., Nagel, S.R., Jaeger, H.M., 2001. Size separation of granular particles. Nature 414, 270.

Möbius, M.E., Cheng, X., Karczmar, G.S., Nagel, S.R., Jaeger, H.M., 2004. Intruders in the dust: air-driven granular size separation. Physical Review Letters 93, 198001.

Nase, S.T., Vargas, W.L., Abatan, A.A., McCarthy, J.J., 2001. Discrete characterization tools for wet granular media. Powder Technology 16, 214–223.

Rhodes, M., Takeuchi, S., Liffman, K., Muniancy, K., 2003. A general segregation mechanism in a vertically shaken bed. Granular Matter 5, 107–114.

Rosato, A., Strandburg, K.J., Prinz, F., Swendsen, R.H., 1987. Why the brazil nuts are on top: size segregation of particulate matter by shaking. Physical Review Letters 58, 1038–1040.

Scheel, M., Geromichalos, D., Herminghaus, S., 2004. Wet granular matter under vertical agitation. Journal of Physics—Condensed Matter 16, S4213–S4218.

Shinbrot, T., 2004. The brazil nut effect—in reverse. Nature 429, 352–353.

Shinbrot, T., Muzzio, F.J., 1998. Reverse buoyancy in shaken granular beds. Physical Review Letters 81, 4365–4368.

Shishodia, N., Wassgren, C.R., 2001. Particle segregation in vibrouidized beds due to buoyant forces. Physical Review Letters 87, 084302.

Taguchi, Y.H., 1992. New origin of convective motion: elastically induced convection in granular materials. Physical Review Letters 69, 1367–1370.

Tarzia, M., Fierro, A., Nicodemi, M., Ciamarra, M.P., Coniglio, A., 2005. Size segregation in granular media induced by phase transition. Physical Review Letters 95, 078001.

Williams, J.C., 1963. The segregation of powders and granular materials. Fuel Society Journal 14, 29–34.

Yan, X., Shi, Q., Hou, M., Lu, K., Chan, C.K., 2003. Effects of air on the segregation of particles in a shaken granular bed. Physical Review Letters 91, 014302.

Yang, S.C., Hsiau, S.S., 2000. Simulation study of the convection cells in a vibrated granular bed. Chemical Engineering Science 55, 3627–3637.