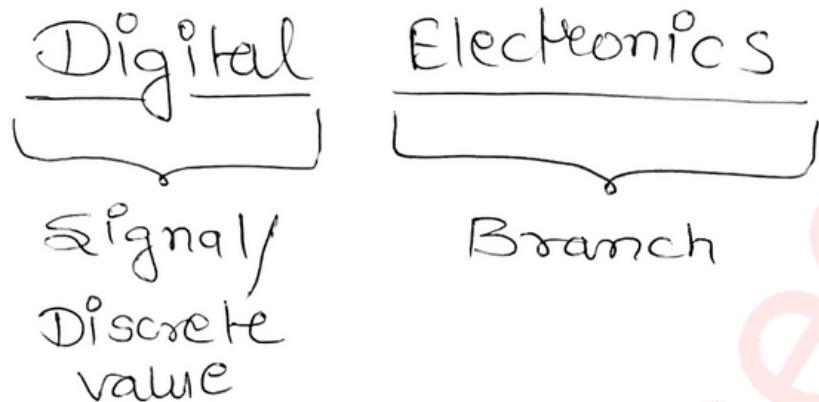


Introduction



→ Analog

◦ Continuous
Nature

◦ Sine wave

◦ Analog circuits
[Resistors, capacitors,
inductors]

◦



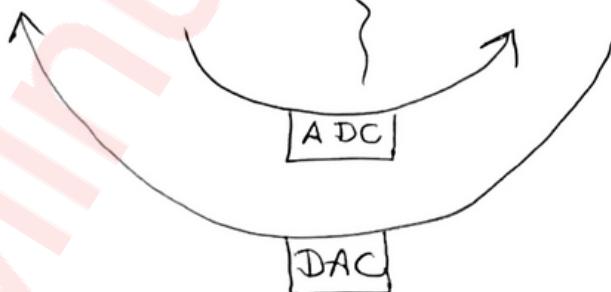
VS

Digital

◦ Discrete nature
[0 & 1]

◦ Square wave

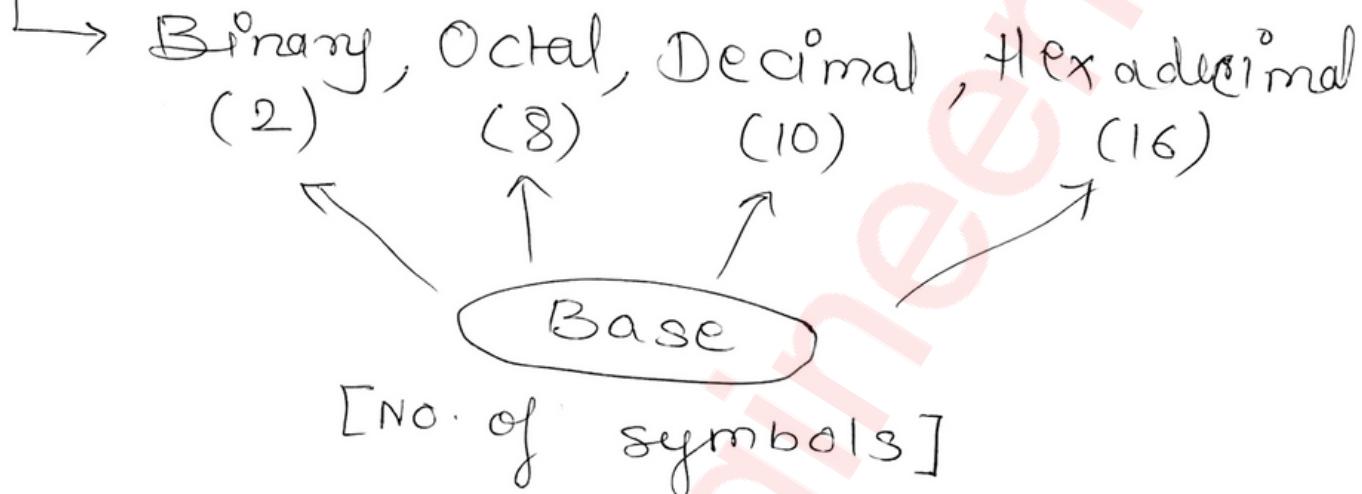
◦ Digital circuits
[Transistors, logic gates,
ICs]



Number System



Representation

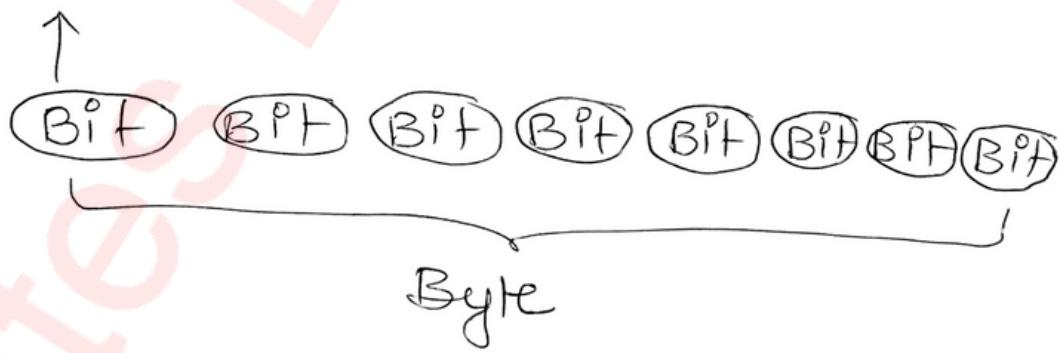


→ MSB

& LSB

e.g.

0 1 1 0



Binary

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Octal

0
1
2
3
4
5
6
7

Decimal

0
1
2
3
4
5
6
7

Hexadecimal

0,	1,	2,	3,	4,	5,	6,	7,	8,	9,
A,	B,	C,	D,	E,	F				
↓	↓	↓	↓	↓	↓				
(10)	(11)	(12)	(13)	(14)	(15)				
						Total	=	(16)	

Binary Arithmetic

↳ addition, subtraction, division, multiplication.

⇒ Addition:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 0 \quad [\text{Carry} = 1]$$

⇒ Subtraction:

$$0 - 0 = 0$$

$$0 - 1 = 1 \quad [\text{Borrow} = 1]$$

$$1 - 0 = 1$$

$$1 - 1 = 0$$

⇒ Multiplication

$$0 \times 0 = 0 \times 1 = 1 \times 0 = 0$$

$$1 \times 1 = 1$$

• Division

$$0 \div 0 = ND$$

$$0 \div 1 = 0$$

$$1 \div 0 = ND$$

$$1 \div 1 = 1$$

• 1's complement

$$\text{eg: } 1010101 \Rightarrow 0101010$$

$$\text{• } \underline{2\text{'s complement}} = 1 + \underline{1\text{'s complement}}$$

$$\begin{array}{r} 1010101 \\ \downarrow 1\text{'s complement} \end{array}$$

$$\begin{array}{r} 0101010 \\ + 1 \\ \hline 0101011 \leftarrow 2\text{'s complement.} \end{array}$$

Shortcut : from LSB complement bits
as we see first '1'

$$\Rightarrow \begin{array}{r} 1010101 \\ \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \end{array} \leftarrow \text{first } \underline{1}'$$

$$0101011$$

$$\Rightarrow \begin{array}{r} 1010000 \\ \downarrow \downarrow \downarrow \end{array} \leftarrow \text{first } \underline{1}'$$

$$\Rightarrow 011$$

Eg's. ^o Decimal Number System

$$555.55 = 5 \times 10^2 + 5 \times 10^1 + 5 \times 10^0 + 5 \times 10^{-1} + 5 \times 10^{-2}$$

^o Binary Number System

$$\begin{aligned} 10101.01 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} \\ &= 16 + 0 + 4 + 0 + 1 + 0 + \frac{1}{4} = 0.25 \\ \Rightarrow 21.25 \end{aligned}$$

^o Octal Number System

$$\begin{aligned} 525.5 &= 5 \times 8^2 + 2 \times 8^1 + 5 \times 8^0 + 5 \times 8^{-1} \\ &= 320 + 40 + 5 + 0.625 \\ &= 365.625 \end{aligned}$$

$$(10101)_2$$

↓ ↓ ↓ ↓ ↓
 $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
 ↓ ↓ ↓ ↓ ↓
 $16 + 0 + 4 + 0 + 1$

$$(21)_{10}$$

↓ Convert to Octal.
[Divide by 8]

$$\begin{array}{r} 2 \\ 8) \overline{21} \\ 16 \\ \hline 5 \end{array} \rightarrow (25)_8$$

$$\begin{aligned}
 &= 2 \times 8^1 + 5 \times 8^0 \\
 &= 16 + 5 = (21)_{10} \quad \checkmark
 \end{aligned}$$

$$\Rightarrow (555.55)_{10}$$

↓ ↓
convert to Octal.

$$\begin{array}{r} 69 \\ 8) \overline{555} \\ 48 \\ \hline 75 \\ 72 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 8 \\ 8) \overline{69} \\ 64 \\ \hline 5 \end{array}$$

$$\begin{array}{r}
 853.341 \\
 0.44 \times 8 = 3.52 \\
 \hline
 0.52 \times 8 = 4.16 \\
 \hline
 0.16 \times 8 = 1.28
 \end{array}$$

$$\bullet (101010.10)_2$$

$$\begin{array}{l} \xrightarrow{\quad} (222.2)_4 \\ \xrightarrow{\quad} (52.4)_8 \\ \xrightarrow{\quad} (2A.8)_{16} \end{array}$$

[\leftarrow \rightarrow] direction.
apply o's apply o's

'But' to go from $B_4 \rightarrow B_8 \quad \{$
 $B_8 \rightarrow B_{16} \}$

\Rightarrow Try to first get it in
 B_2 then next.

$$B_4 \rightarrow B_2 \rightarrow B_8 / B_{16}$$

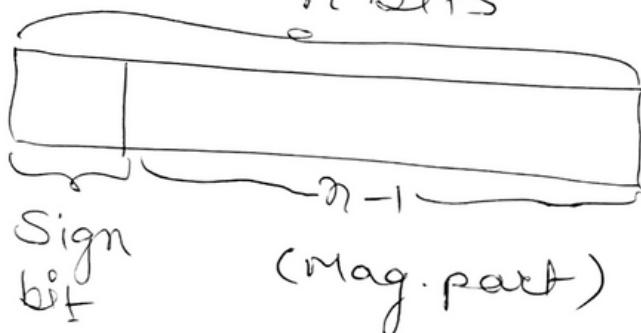
Signed Numbers



Magnitude
+
Sign

(+5, -5)

n bits



Unsigned Numbers



Only magnitude
No sign (5, 55)

→ 5 bit

$$\rightarrow 2^n - 1$$

$$= 2^5 - 1$$

$$= 31 \quad \boxed{\text{total/possible.}}$$

0 → +ve $[-(2^{n-1}) \text{ to } +(2^{n-1}-1)]$ (SM)



Decimal values

2's C

+5

0101

+3

0011

+1

0001

+0

0000

-0

0000

-1

1111

-3

1101

-5

1011

Signed magnitude

↑
SB

↓

Same

0011

0001

0000

0000

1001

1011

1101

\Rightarrow Eg Given

10110

SB mag.

$1 \rightarrow -ve$
 $0 \rightarrow +ve$

$1 \times 2^4 \quad 0 \times 2^3 \quad 1 \times 2^2 \quad 1 \times 2^1 \quad 0 \times 2^0$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$(-16) + 0 + 4 + 2 + 0$

$-16 + 6 = \underline{-10}$

$(2'sC)$ $\begin{array}{r} 10110 \\ (-ve) \quad (10) \end{array}$

\Rightarrow Subtraction by Addition ($2'sC$)

eg: $10 - 5$

\downarrow
 $10 + (-5)$

$(+ve)$ $\begin{array}{r} 1010 \\ + 11011 \\ \hline 10 \end{array}$

$\frac{-ve}{-16+8+0+2+1}$

$= -5$

0101
 $\downarrow 2'sC$

1011

$\begin{array}{r} 101010 \\ + 11011 \\ \hline 00101 \end{array}$

\downarrow
+ve

$\underline{\underline{5}}$

$$\Rightarrow 10 - 11$$

↓

$$10 + (-11)$$

↓

↓ 2's com

$$\begin{array}{r} 01010 \\ \text{the } \frac{1}{10} \\ \hline 10 \end{array}$$

$$\begin{array}{r} 1 \\ -\text{ve } \underline{0101} \\ \hline \end{array} \rightarrow$$

$$\begin{array}{r} 1011 \\ 0101 \\ \hline 1111 \\ \text{---} \\ \text{-ve no.} \end{array}$$

$$\begin{array}{ccccccc} 1 \times 2^5 & 1 \times 2^4 & 1 \times 2^3 & 1 \times 2^2 & 1 \times 2^1 & 1 \times 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ -32 & +16 & +8 & +4 & +2 & +1 \\ -32 & +31 & = & \underline{\underline{-1}} & \text{ans.} \end{array}$$

\Rightarrow BCD code

↴ 8 4 2 1
 ↴ 2 4 2 1

$\left. \begin{array}{c} \\ \\ \end{array} \right\}$ weighted code.

Decimal	8 4 2 1	2 4 2 1
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 0
3	0 0 1 1	0 0 1 1
4	0 1 0 0	0 1 0 0
5	0 1 0 1	1 0 1 1
6	0 1 1 0	1 1 0 0
7	0 1 1 1	1 1 0 1
8	1 0 0 0	1 1 1 0

◦ Non weighted codes

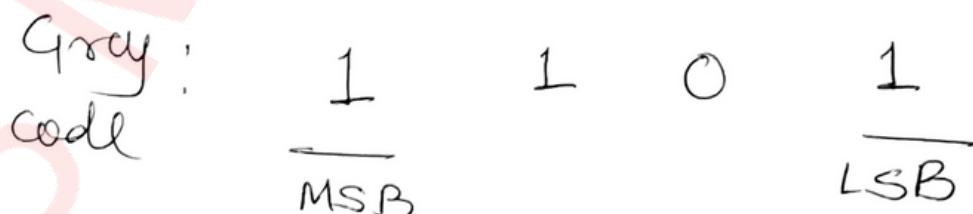
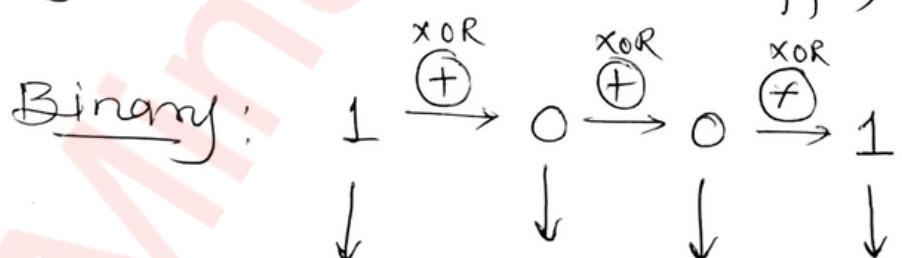
- ↳ Excess -3
- ↳ Gray code

Decimal

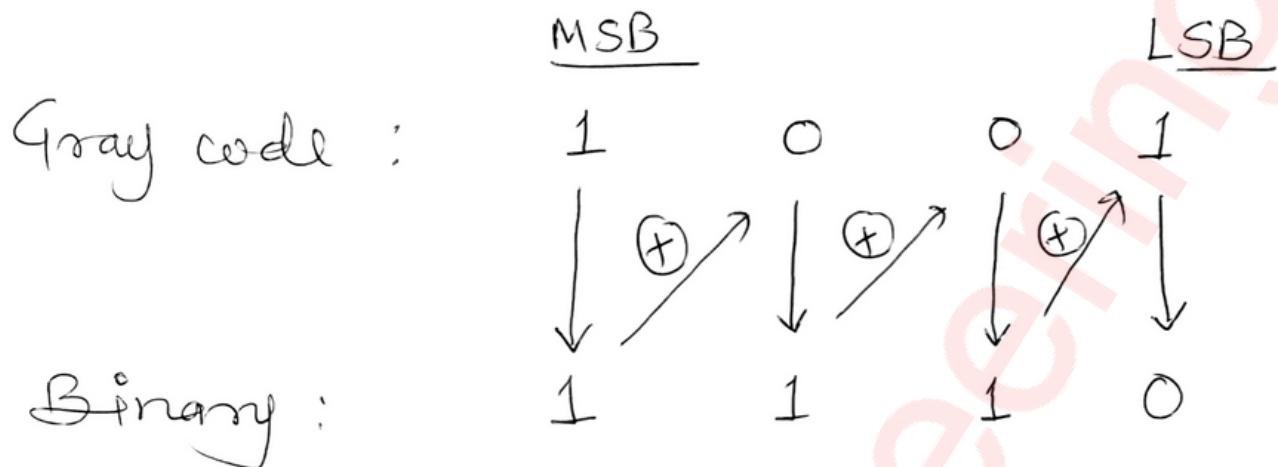
	<u>BCD</u>	<u>Excess-3</u>
↓	<u>8421</u>	<u>(+0011)</u>
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

Complement

◦ Gray Code : (A bit diff)



◦ Gray code → Binary



◦ Boolean Algebra:

↓ Branch of
→ True/false Algebra
→ 1 / 0

: Laws

① Idempotent law:

$$x \cdot x = x \quad \& \quad x + x = x$$

② Associative law

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$x + (y + z) = (x + y) + z$$

③ Distributive law

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

$$x + (y \cdot z) = (x + y) + (x + z)$$

④ Commutative law

$$x \cdot y = y \cdot x$$

$$x + y = y + x$$

⑤ Dc-Morgan law

$$\overline{(x+y)} = \overline{x} \cdot \overline{y}$$

$$\overline{(x \cdot y)} = \overline{x} + \overline{y}$$

⑥ Identity law

$$x + 0 = x$$

$$x \cdot 0 = 0$$

$$x + 1 = 1$$

$$x \cdot 1 = x$$

⑦ Complementation law

$$\overline{T} = F \quad \overline{0} = 1$$

$$\overline{F} = T \quad \overline{1} = 0$$

$$\boxed{x \cdot \overline{x} = 0}$$

$$\boxed{x + \overline{x} = 1}$$

⑧ Involution law

$$\overline{\overline{x}} = x$$

Logic Gates:

→ 1 or more I/P

But

only 1 O/P.

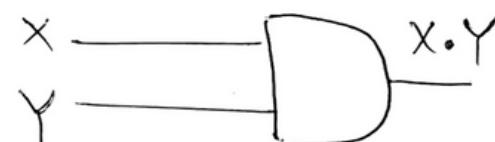
Basic Gates: AND, OR & NOT

Universal Gates: NAND, NOR

Special Gates: XOR, XNOR

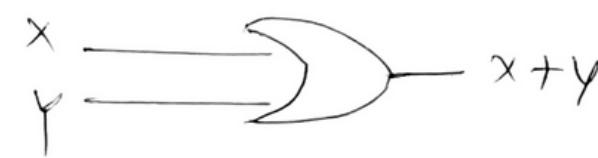
- AND Gate [O/P is T/High if all I/P are True]

X	Y	$X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1



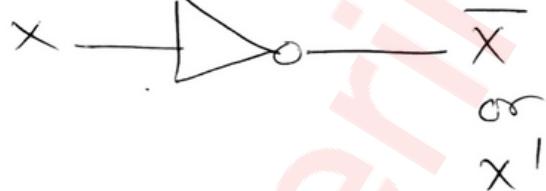
- OR Gate [O/P is 'T' if any one I/P is 'T']

X	Y	$X + Y$
0	0	0
0	1	1
1	0	1

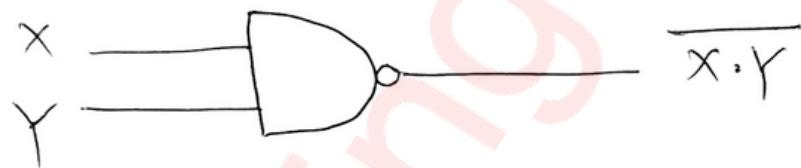


- NOT Gate [$0 \rightarrow 1$ & $1 \rightarrow 0$]

<u>X</u>	<u>\overline{X} or x'</u>
0	1
1	0

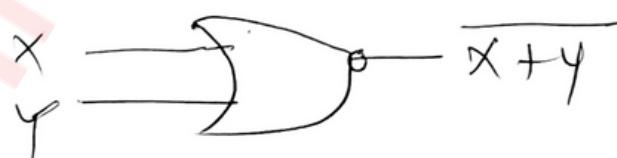


- NAND Gate [Complement of 'AND']



<u>X</u>	<u>Y</u>	<u>$\overline{X \cdot Y}$</u>
0	0	1
0	1	1
1	0	1
1	1	0

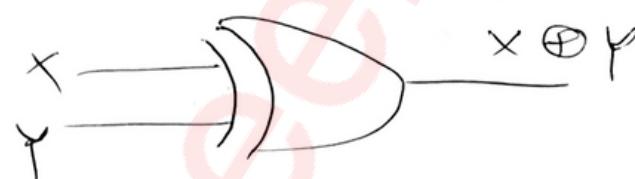
- NOR Gate [Complement of 'OR']



<u>X</u>	<u>Y</u>	<u>$\overline{X+Y}$</u>
0	0	1
0	1	0
1	0	0
1	1	0

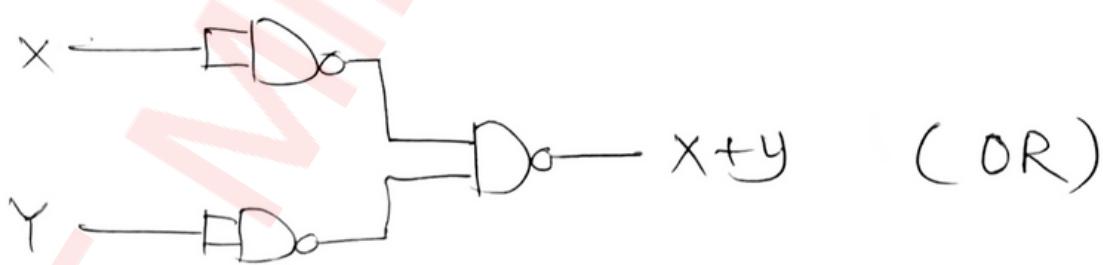
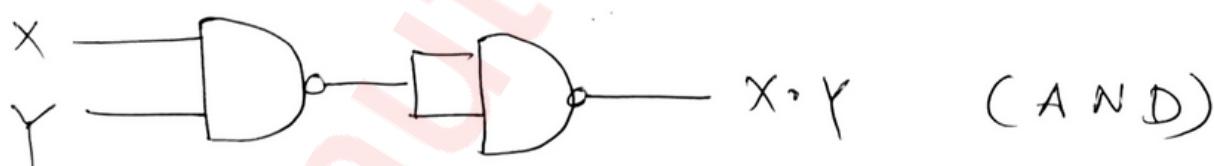
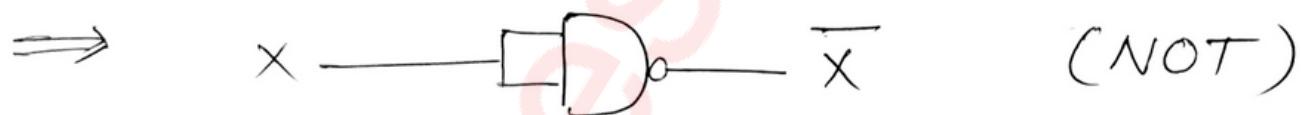
° XOR Gate [O/p is True if "Different"]
 "01"
 "10"

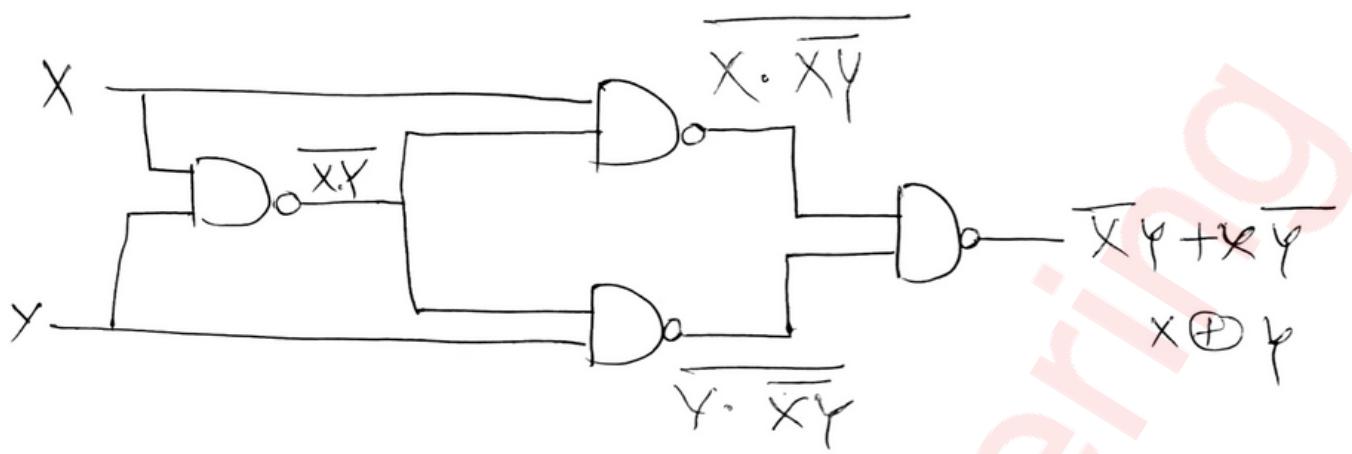
<u>X</u>	<u>Y</u>	<u>$X \oplus Y$</u>
0	0	0
0	1	1
1	0	1
1	1	0



$$X \oplus Y = X \cdot \bar{Y} + \bar{X} \cdot Y$$

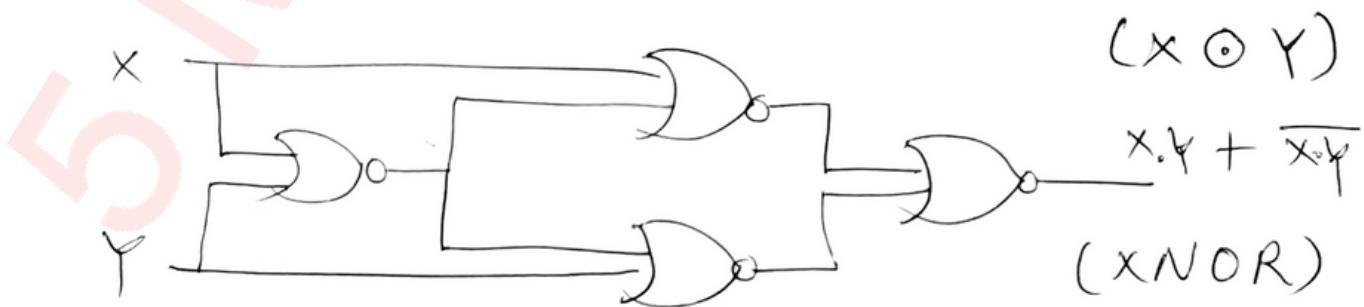
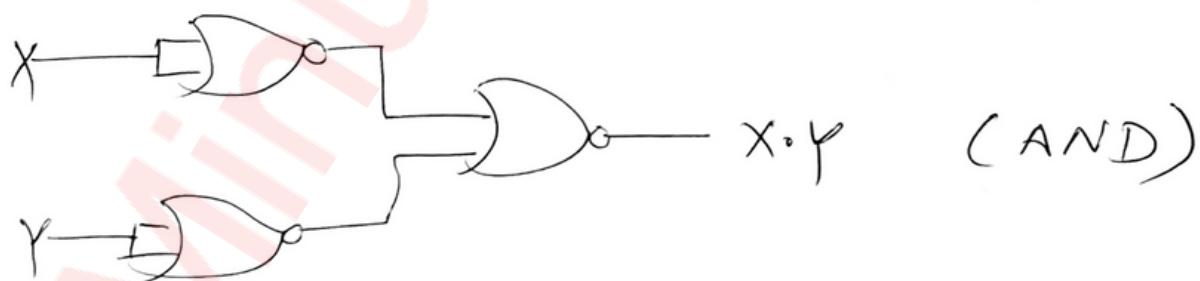
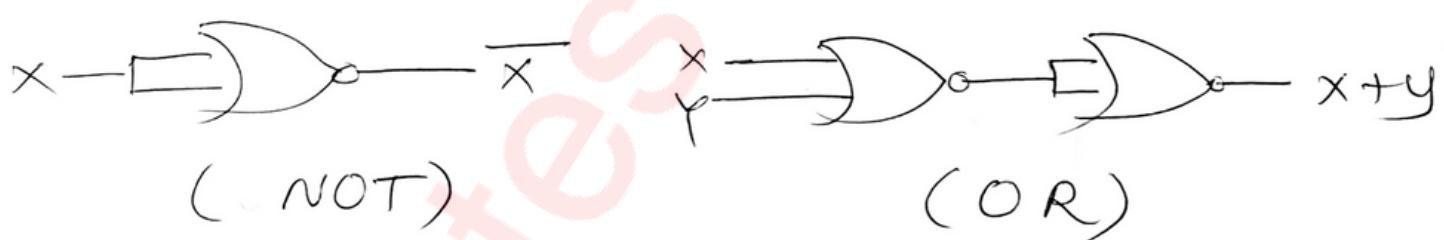
⇒ Realization of NOT, OR, AND & XOR using NAND gate.



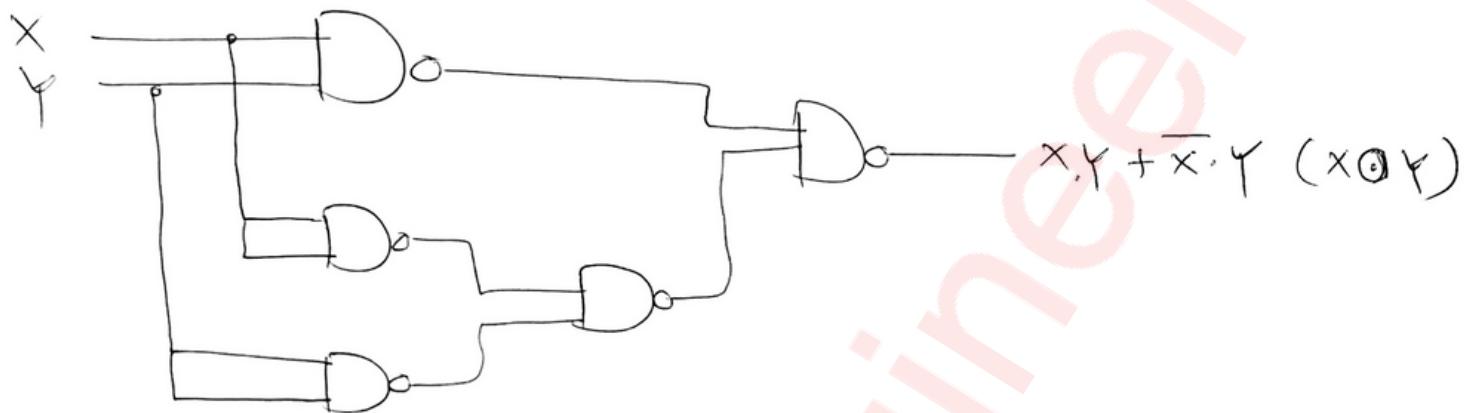


X	Y	$x \cdot y$	$\overline{x} \cdot \overline{y}$	$\overline{x} \cdot \overline{y}$	$\overline{y} \cdot \overline{x}$	$(x \oplus y)$
0	0	0	1	1	1	0
0	1	0	1	0	0	1
1	0	0	0	1	1	1
1	1	1	1	1	1	0

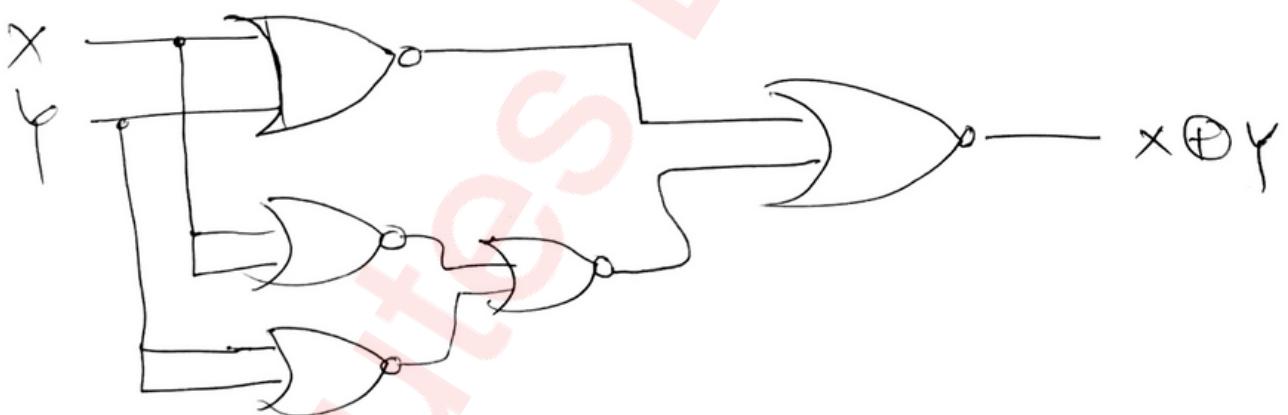
Similarly using NOR gate.



(XNOR using NAND)



(XOR using NOR)



Boolean Expression



→ Expression that can tell when $O/P \rightarrow 0$ or 1

→ SOP [Sum of Product] "1"

→ POS [Product of sum] "0"

X	Y	O/P	$O(X,Y) = \sum_m (0, 1)$
0	0	1	$O(X,Y) = \prod_M (2, 3)$
0	1	1	
1	0	0	
1	1	0	

↙ POS

Eg: $\begin{array}{c} X \\ \hline 0 \\ 0 \\ 1 \\ 1 \end{array} \quad \begin{array}{c} Y \\ \hline 0 \\ 1 \\ 0 \\ 1 \end{array} \quad \begin{array}{c} O/P \\ \hline 0 \\ 1 \\ 1 \\ 0 \end{array} = X + Y$

X	Y	O/P
0	0	0
0	1	1
1	0	1
1	1	1

$$SOP = XY + \bar{X}Y + X\bar{Y}$$

↑ ↑ ↑ ↓
Sum of Product

Sum of Product

→ Expression have:

Product terms sumed
(AND) (OR)

Eg: $x\bar{y} + \bar{y}z + \bar{x}\bar{y}\bar{z}$

↑
2 literals

↑
3 literals

[complement or non-complement] allowed ✓

⇒ Minterm = $[x\bar{y}, \bar{y}z, \bar{x}\bar{y}\bar{z}]$

$f(x, y, z)$

X

X

✓

Binary

0 0 0

0 0 1

0 1 0

0 1 1

1 0 0

1 0 1

1 1 0

1 1 1

Minterm

$\bar{a}\bar{b}\bar{c} \rightarrow m_0$

$\bar{a}\bar{b}c \rightarrow m_1$

$\bar{a}b\bar{c} \rightarrow m_2$

$\bar{a}bc \rightarrow m_3$

$a\bar{b}\bar{c} \rightarrow m_4$

$a\bar{b}c \rightarrow m_5$

$ab\bar{c} \rightarrow m_6$

$abc \rightarrow m_7$

Canonical logic forms

→ CSOP: $XYZ + X\bar{Y}Z + \bar{X}\bar{Y}Z (\checkmark)$

$XY + \bar{X}Y\bar{Z} (x)$
 $Y + X + Z (x)$

$XY(\bar{Z} + Z) + \bar{X}Y\bar{Z}$

↓ ↓

$[XYZ + XY\bar{Z} + \bar{X}\bar{Y}\bar{Z}] \checkmark \text{ CSOP}$

• POS [Product of sum]

⇒ OR product

terms → $(x+y) \cdot (y+z)$

$(x+y), (y+z)$

↑ / ↑ ↑
 (OR) (AND)

⇒ Maxterm = $[\bar{X}, \bar{Z}, \bar{X}\bar{Z}]$

$f(x,z) \quad (\times) \quad (\times) \quad (\checkmark)$

0 → X
 1 → \bar{X}
 (POS)

0 → \bar{X}
 1 → X
 (SOP)

$\circ \underline{CPOS} :- (\bar{x} + y + z) \cdot (x + \bar{y} + \bar{z}) \quad (\checkmark)$
 - $(\bar{x} + \bar{y}) \cdot (x + \bar{y} + \bar{z}) \quad (x)$
 $(\bar{x}) \cdot (\bar{y}) \cdot (y + \bar{z}) \quad (x)$

$(\bar{x} + \bar{y} + \bar{z} \cdot z) \cdot (x + \bar{y} + \bar{z})$
 \downarrow
 $(\bar{x} + \bar{y} + \bar{z}) (\bar{x} + \bar{y} + z) \cdot (x + \bar{y} + \bar{z})$
 $\quad \quad \quad (\checkmark)$

\rightarrow Given $n = 3$
 $\uparrow (x, y, z)$
 Boolean variables

① Total Combinations $= 2^n$

i.e $n=2 (x, y)$

00, 01, 10, 11

$n=3 (x, y, z)$

000 - - - 111

② Total Boolean fns $= 2^{2^n}$

$n=2 (x, y) \rightarrow 2^{2^2} = 16$

$n=3 (x, y, z) \rightarrow 2^{2^3} = 2^8 = 256$

\rightarrow Complement of a f^n .

$$\Rightarrow f(x, Y, z, 0, 1, \cdot, +)$$

$$f^1(\bar{x}, \bar{Y}, \bar{z}, 1, 0, +, \cdot)$$

e.g.: $\bar{x} + y + z \xrightarrow{\text{C}} x \cdot \bar{Y} \cdot \bar{z}$

e.g.: $xy + \bar{x}\bar{y} \leftarrow [\text{EX NOR}]$

$$(\bar{x} + \bar{y}) \cdot (x + y)$$

$$(\bar{x} \cdot x) + (\bar{x} \cdot Y) + (\bar{Y} \cdot x) + (\bar{Y} \cdot Y)$$

$$(\bar{x} \cdot Y) + (x \cdot \bar{Y}) \leftarrow [\text{EXOR}]$$

Duality Theorem

$$\bullet \longleftrightarrow + (\checkmark)$$

$$\oplus \longleftrightarrow \odot (\checkmark)$$

$$0 \longleftrightarrow 1 (\checkmark)$$

$$x \longleftrightarrow \bar{x} (x)$$

$$\sim \longleftrightarrow \sim (\checkmark)$$

$$\text{eg: } \textcircled{1} \times Y \bar{Z} + \bar{X} Y Z + X Y Z$$

↓ ↓ ↓ ↓ ↓

$$(X+Y+\bar{Z}) \cdot (\bar{X}+Y+Z) \cdot (X+Y+Z)$$

$$\textcircled{2} \quad X Y Z + \bar{X} Y \bar{Z} = 1$$

↓ ↓ ↓ ↓

$$(X+Y+Z) \cdot (\bar{X}+Y+\bar{Z}) = 0$$

• Self dual fn:

→ If dual is equal to I/p fn.

$$\text{eg: } (AB + BC + CA)$$

$$(A+B) \cdot (B+C) \cdot (C+A)$$

↓

$$AB + AC + B + BC$$

$$B(A+1+C) + AC$$

↓

$$(B(1) + AC) (C+A)$$

$$(BC + BA + AC + AC)$$

$$\Rightarrow AB + BC + AC$$

Q → No. of self dual fns -

$$\rightarrow n = 2$$

- ① Total combinations (0, 1) = 2^n
- ② Total fns possible = 2^{2^n}
- ③ Total self dual fns possible = $\left[2^{2^{n-1}}\right]$

Eg:

X	Y
0	0
0	1
1	0
1	1

$$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 2^n = 2^2 = 4$$

② Total fns = $2^2 = 2^4 = 16$

-
- 1
- ✓ A
- ✓ B
- ✓ \bar{A}
- ✓ \bar{B}
- A B
- $\bar{A} B$

$$A + B$$

$$\bar{A} + B$$

$$A + \bar{B}$$

$$\bar{A} + \bar{B}$$

$$AB + \bar{A}\bar{B}$$

$$\bar{A}B + A\bar{B}$$

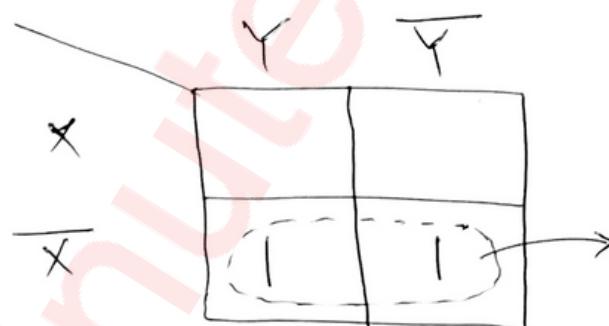
③ Total self dual fns = $2^{2^{n-1}}$
= 2^2
 $\Rightarrow = 4$

◦ Minimization [Simplification]

- Boolean Laws
- Karnaugh map (kmap)

◦ Kmap :- $f(x, y) = \sum m(0, 1)$

$$\begin{array}{c|cc|c}
 & \overline{x} & x & f \\
 \hline
 0 & 0 & 0 & 1 \\
 0 & 0 & 1 & 1 \\
 1 & 1 & 0 & 0 \\
 1 & 1 & 1 & 0
 \end{array}
 \rightarrow \overline{x}\overline{y} + \overline{x}y \\
 \Rightarrow \overline{x}(y + \overline{y}) \\
 \Rightarrow \overline{x}$$



y is changing
(y, \bar{y})

But x is constant
as ($\underline{\bar{x}}$)

Eg: $n=3 f(x, y, z)$

$\rightarrow \sum m(1, 3, 6, 7)$

A	BC		$\bar{B}\bar{C}$		BC		$\bar{B}\bar{C}$	
	00	01	11	10	00	01	11	10
0	0	1	1	2	0	1	1	2
1	4	5	7	6	4	5	7	6

Gray code pattern

Eg: $n=4 f(A, B, C, D)$

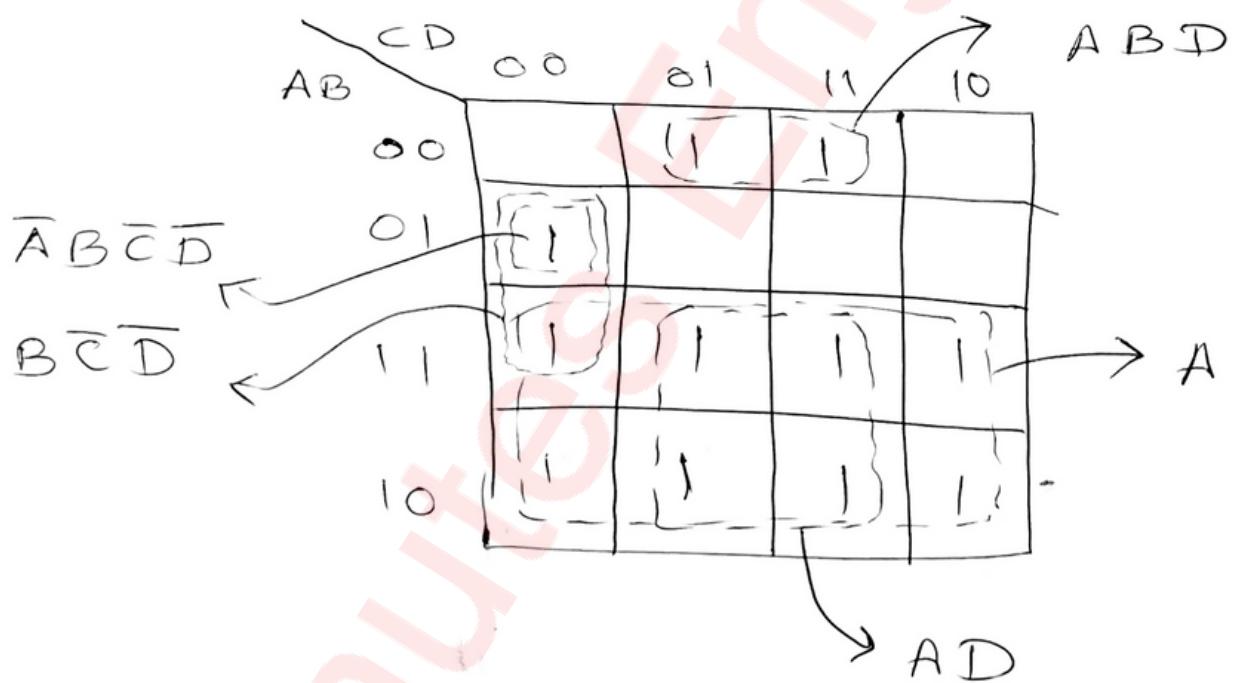
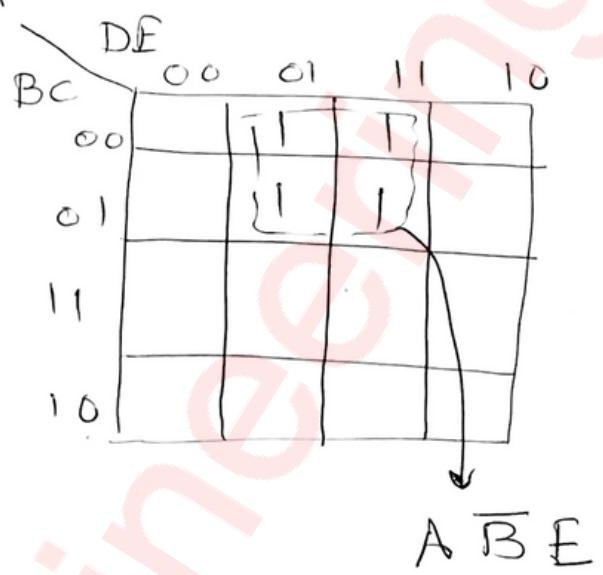
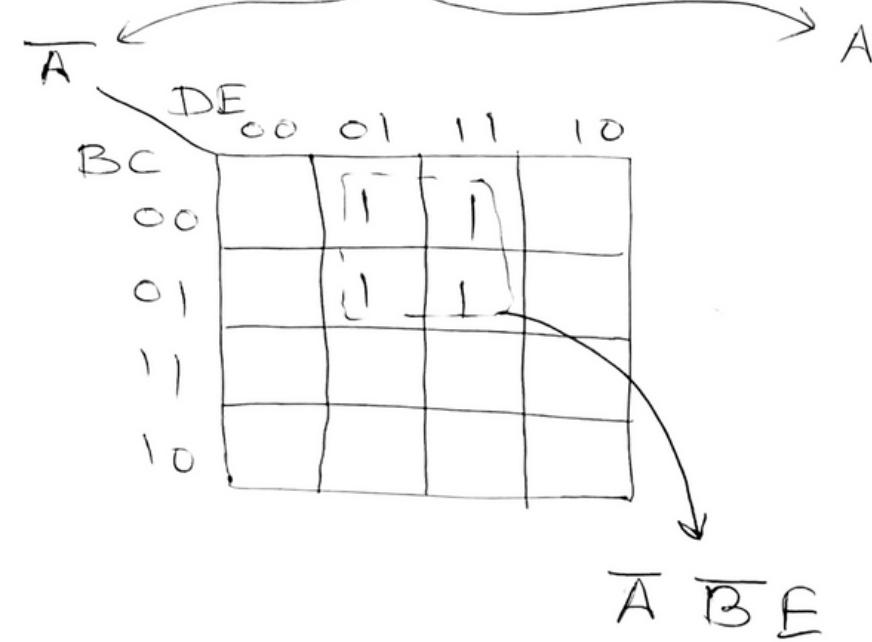
$\rightarrow \sum m(0, 2, 5, 7, 8, 10, 13, 15)$

AB	CD		$\bar{C}\bar{D}$		CD		$C\bar{D}$	
	00	01	11	10	00	01	11	10
$\bar{A}\bar{B}$	00	1	1	0	1	1	3	2
$\bar{A}B$	01	4	1	1	5	1	7	6
AB	11	12	1	1	13	1	15	14
$A\bar{B}$	10	1	8	9	11	1	10	

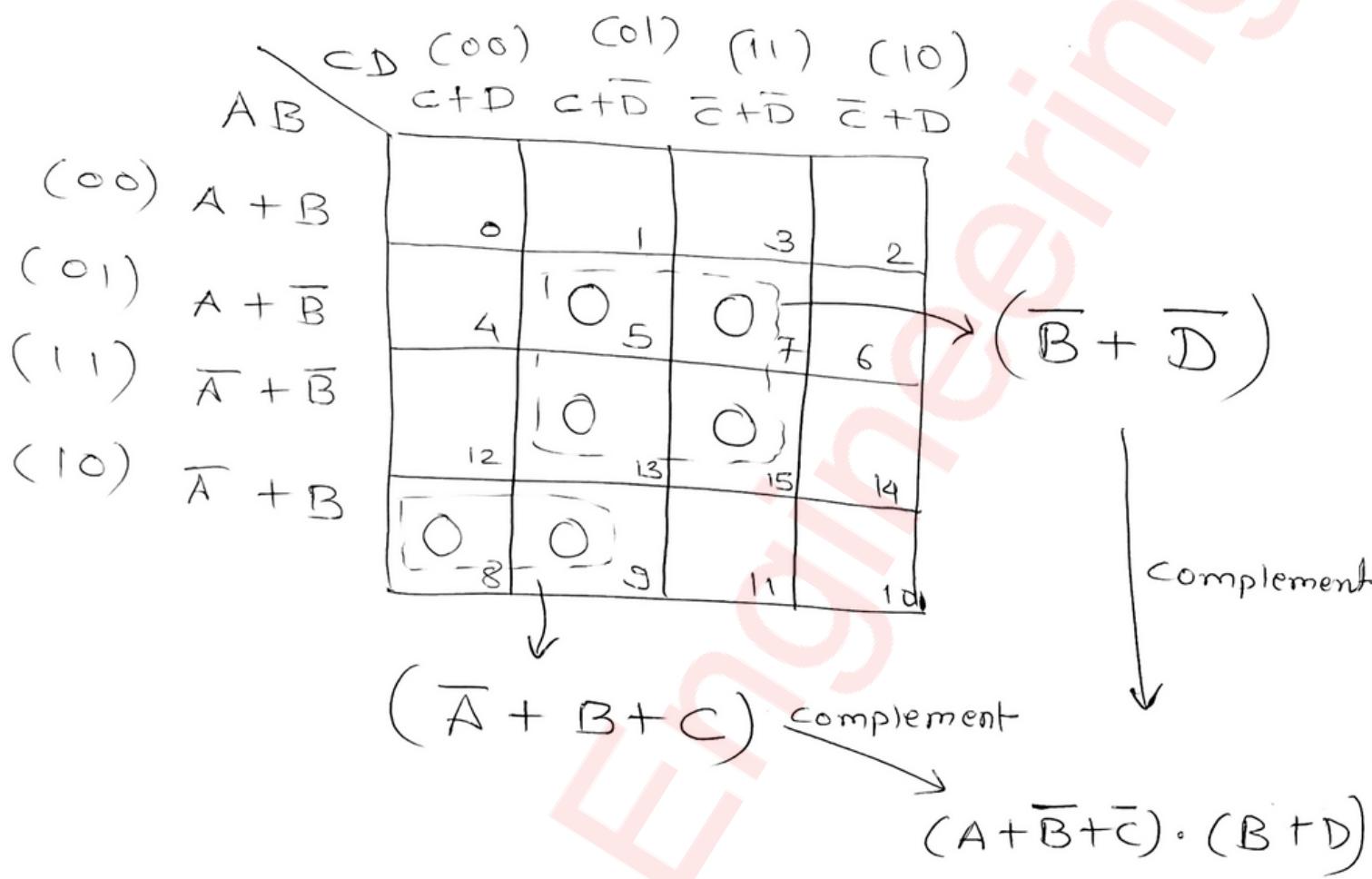
Don't care (x) \rightarrow can be used to simplify

But we focus mostly on 1 | no separate group of (x) is formed.

$n=5$ (A, B, C, D, E)



Kmap for POS $(A \ B \ C \ D)$



④ Prime Implicants

$$\rightarrow B\bar{C}D, AB\bar{D}, A\bar{C}\bar{D}, ABC$$

③ Essential Prime Implicants

$$\rightarrow B\bar{C}D, A\bar{C}\bar{D}, ABC$$

5 Minutes

		CD	00	01	11	10
		A \ B	00	1	3	2
00	00	0	1	3	2	
01	01	4	{1, 5}	7	6	
11	11	1	{1, 12}	{1, 13}, {1, 15}, {1, 14}		
10	10	8	9	{1, 11}	10	

$$\sum m(5, 13, 14, 15)$$

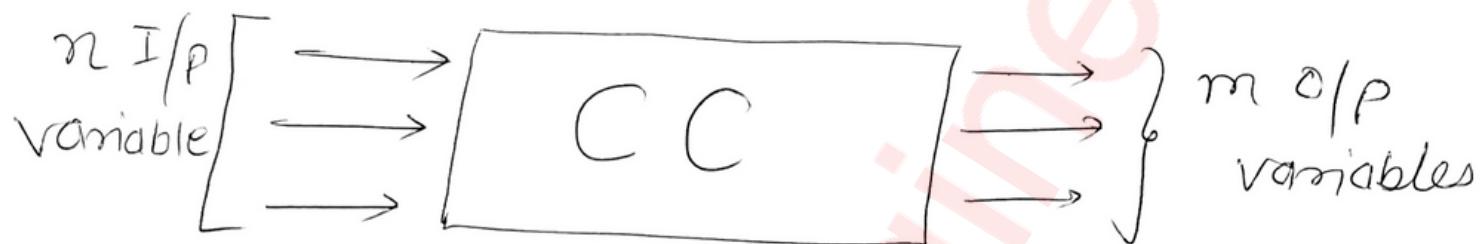
Combinational Circuit

→ (AND, OR, NOT, NAND, NOR)

→ (I/P & O/P variables)

→ Current combination of I/P

→ NO memory unit



$$O/P = F(I/P)$$

Eg:- I/P: 5ME & Guide & GD

O/P: Result

① Truth table

GP	5 ME	Guide	Result
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

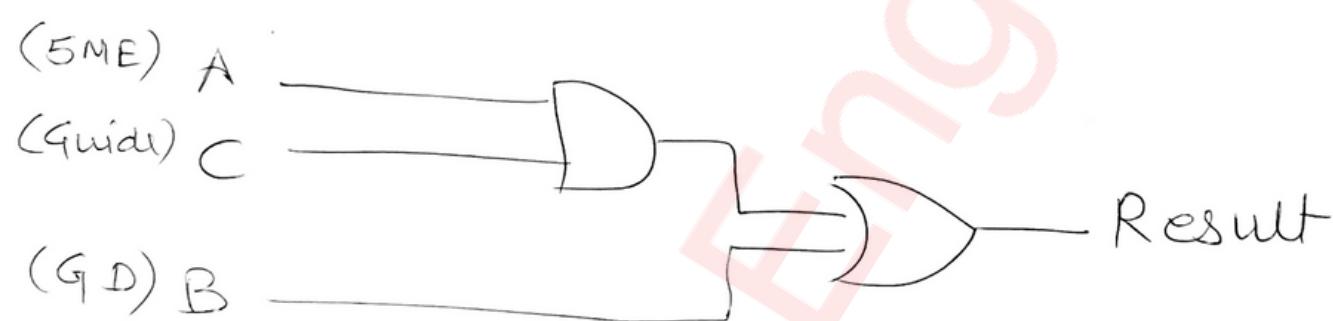
② Use K-map

		B	C	GD, Guide			
		A	5ME	00	01	11	10
0				0	1	1	1
1				4	5	7	6

5ME Guide
(A C)

GD (B)

$$R \Rightarrow A C + B$$

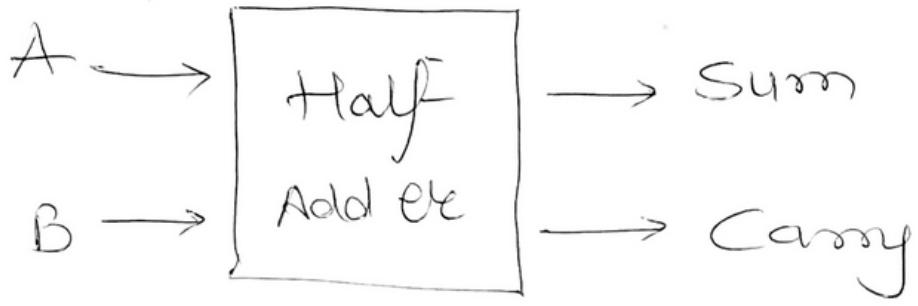


- Adder : CC to perform addition of nos.

→ Half adder (works on 2 bits)
addition

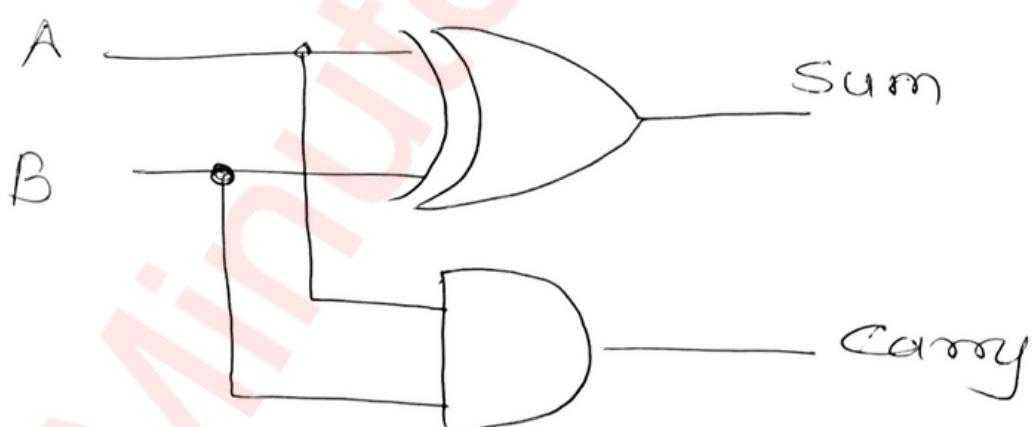
→ full adder (works on 3 bits)
additions

$\begin{array}{r} 0 \\ + 0 \\ \hline 0 \end{array}$	$\begin{array}{r} 0 \\ + 1 \\ \hline 1 \end{array}$	$\begin{array}{r} 1 \\ + 0 \\ \hline 1 \end{array}$	$\begin{array}{r} 1 \\ + 1 \\ \hline 0 \end{array}$
0 T C	0 T C R	1 T C R	1 T C R



<u>A</u>	<u>B</u>	<u>sum</u>	<u>carry</u>
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

sum $\xrightarrow{\text{XOR}}$ XOR
 carry $\xrightarrow{\text{AND}}$ AND



• full addle



A	B	CI
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

sum	CO
0	0
1	0
1	0
0	1
1	0
0	1
1	1
1	1

→ Complement

Sum

A	BC			
	00	01	11	10
0	0	1	3	1'2
1	1'4	5	1'7	6

carry

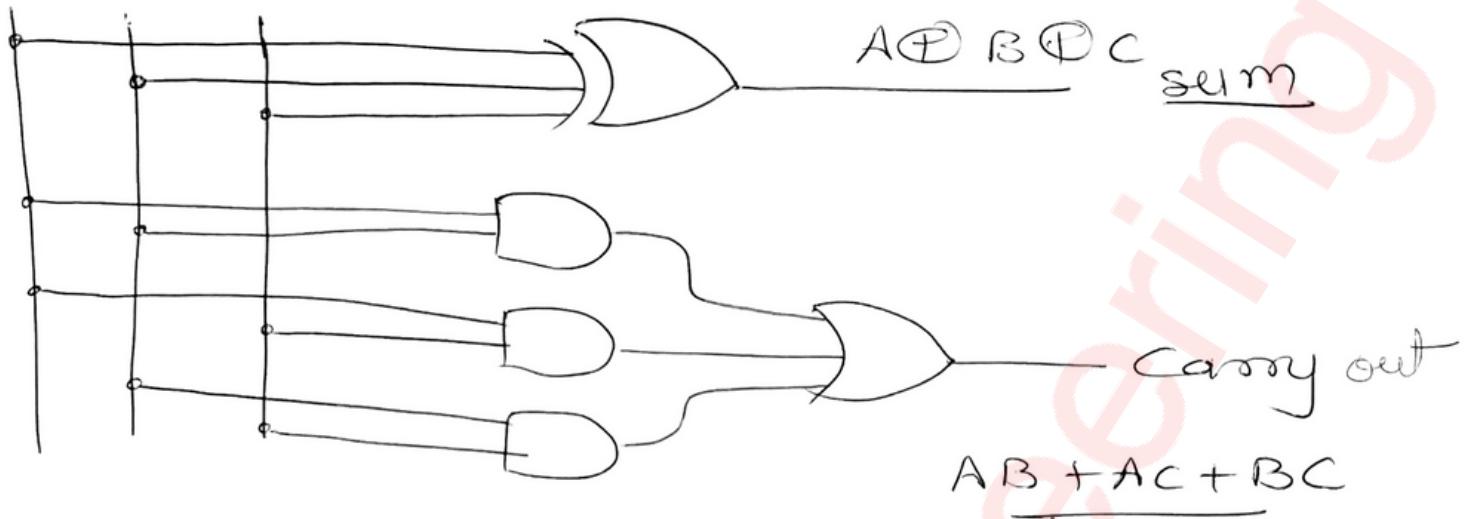
A	BC			
	00	01	11	10
0	0	1	1'3	2
1	4	5	7	6

$$A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + \bar{A}BC$$

$\underbrace{\quad\quad\quad}_{\text{Sum}} = A \oplus B \oplus C$

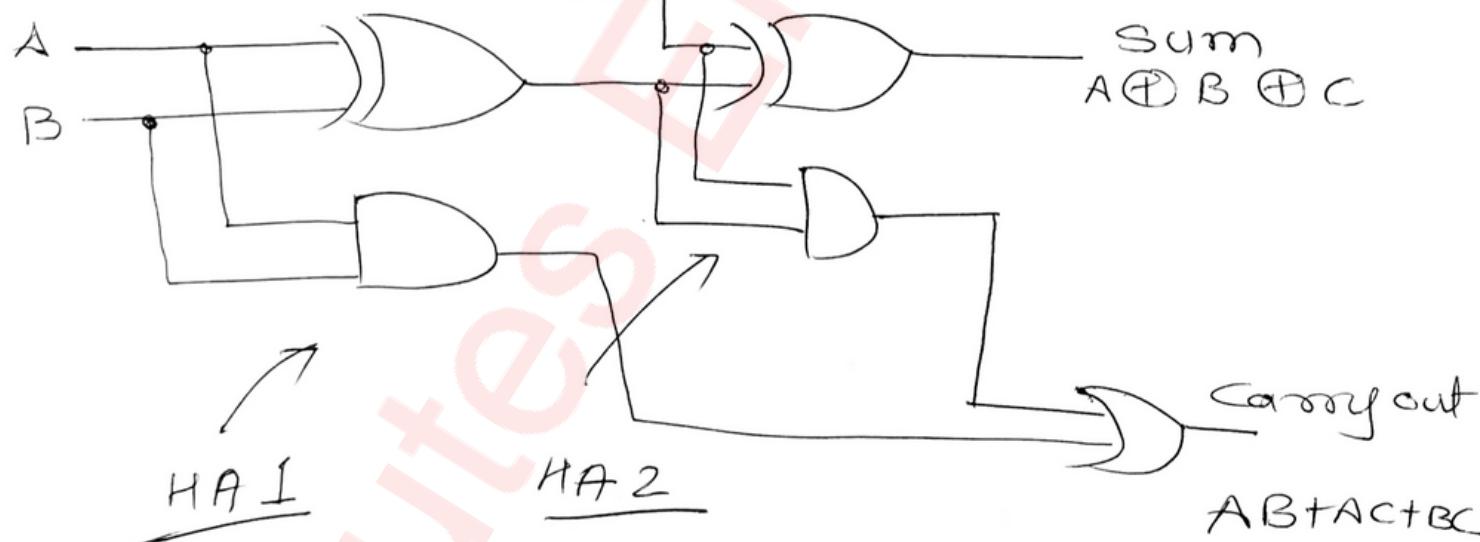
$$AC + AB + BC \\ \text{carry} =$$

A B C

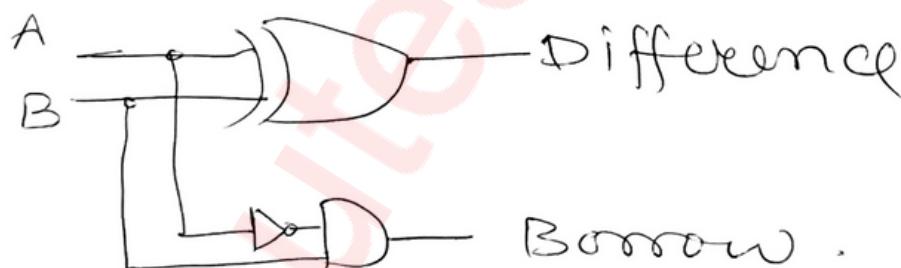
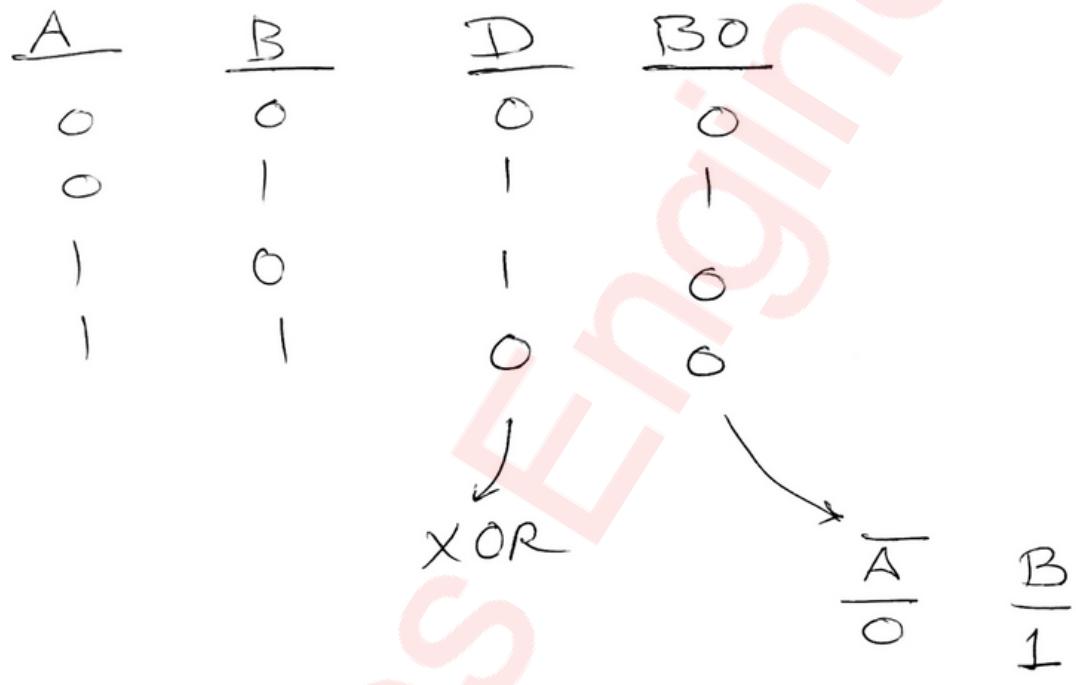
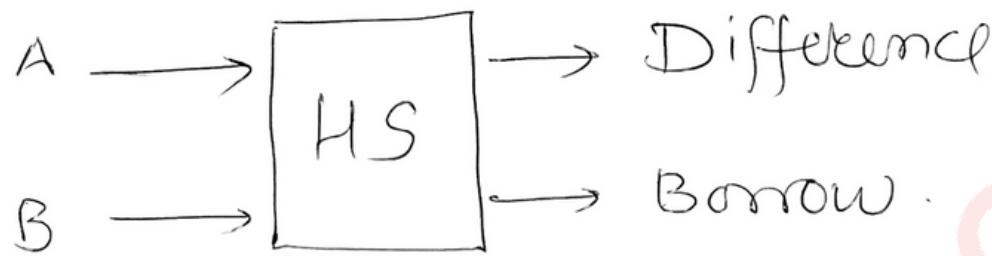


Now implementing the behaviour of Full adder with 2 half adder

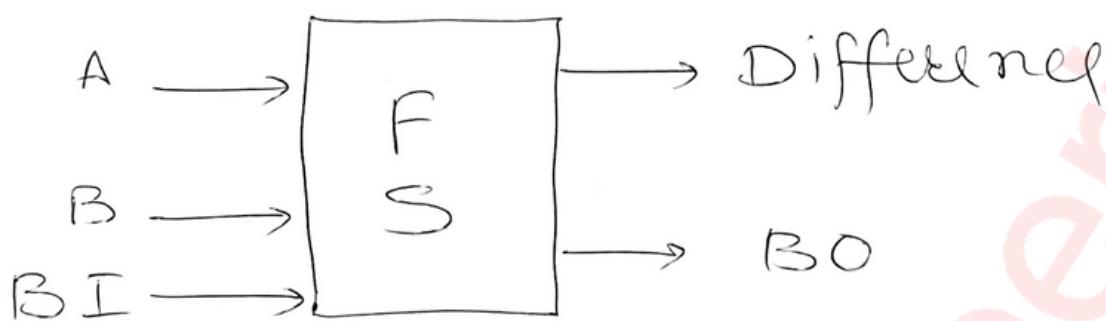
C_I



Half Subtractor

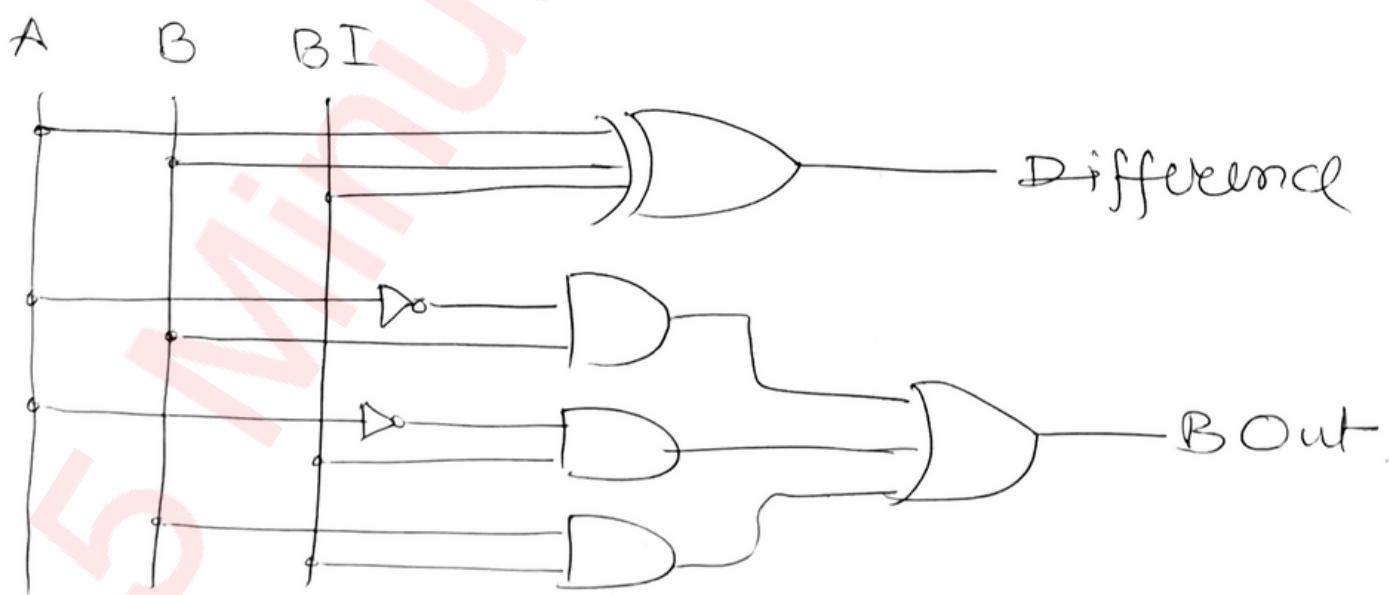


• full subtractor

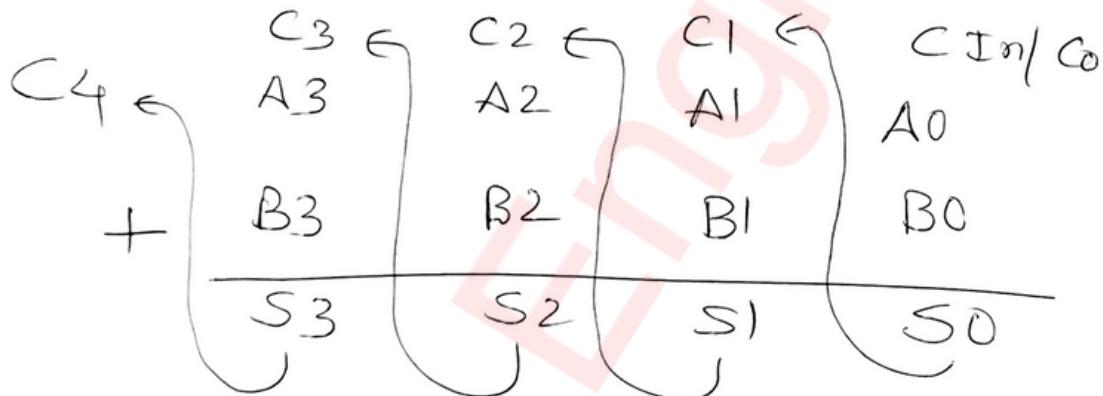
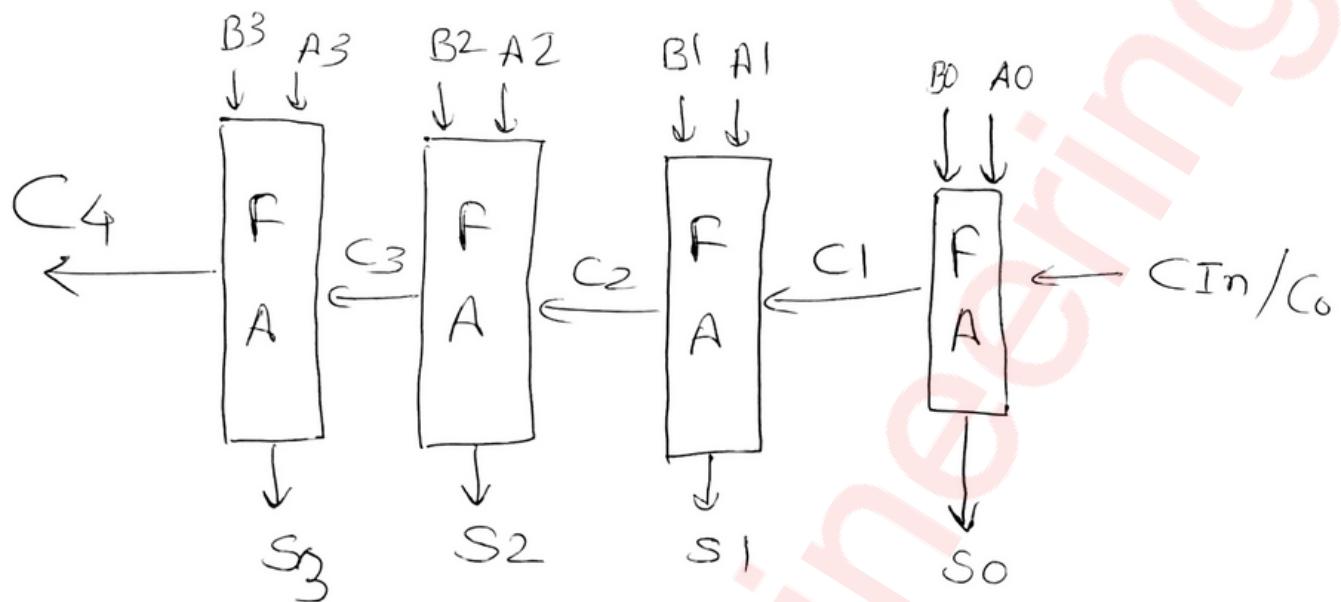


A	B	BI	D	BO
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	-
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

compliment

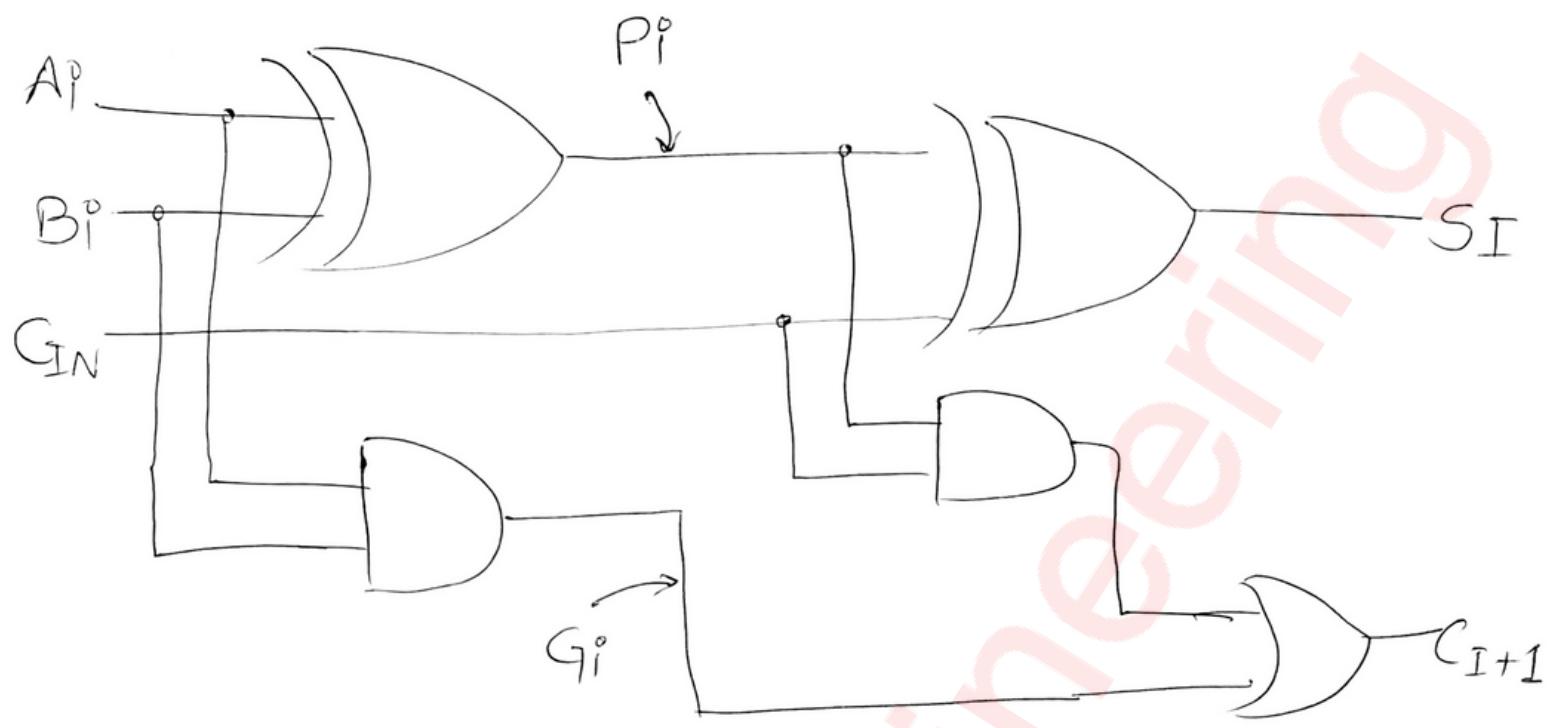


• Ripple Adder (4 bit adder)



• Carry propagation
Lag / Delay Problem

Slow we use Look ahead carry adder
Idea is to get C_1, C_2, C_3 from C_0 itself.



$$P_i^o = A_i^o \oplus B_i^o$$

$$G_i^o = A_i^o B_i^o$$

$$S_i^o = P_i^o \oplus C_i^o \quad | \quad A_i^o \oplus B_i^o \oplus C_i^o$$

$$C_{i+1}^o = G_i^o + P_i^o C_i^o$$

$$C_0 = 0$$

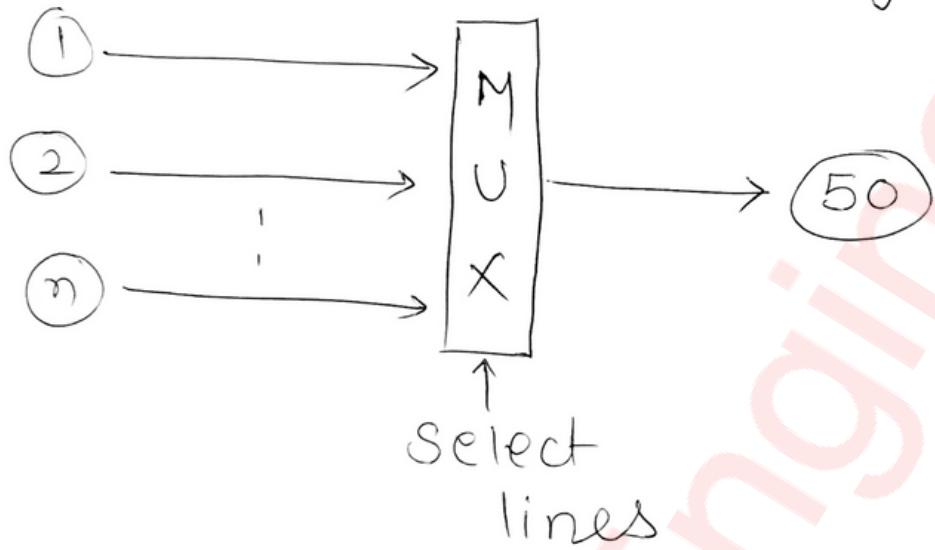
$$C_1 = G_0 + P_0 C_0$$

$$C_2 = G_1 + P_1 G_0 + P_1 P_0 C_0$$

$$C_3 = G_2 + P_2 G_1 + P_2 P_1 P_0 C_0$$

$$C_4 = G_3 + P_3 G_2 + P_3 P_2 P_1 P_0 C_0$$

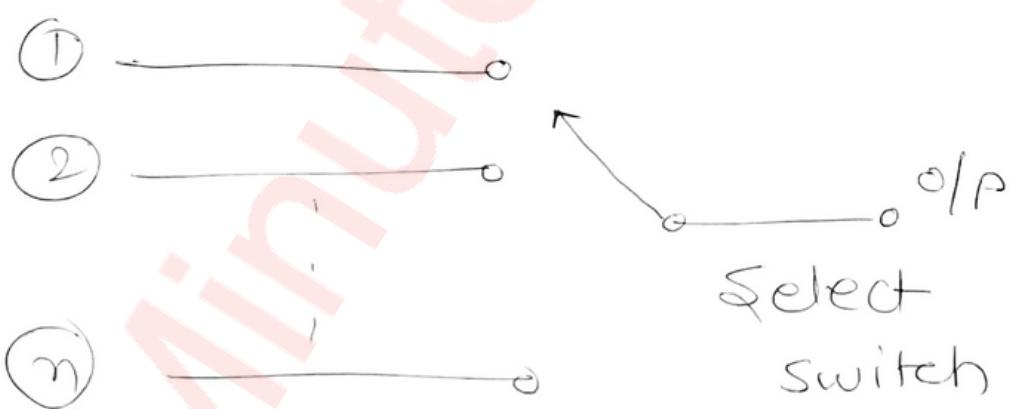
◦ Multiplexer [Trending CC]

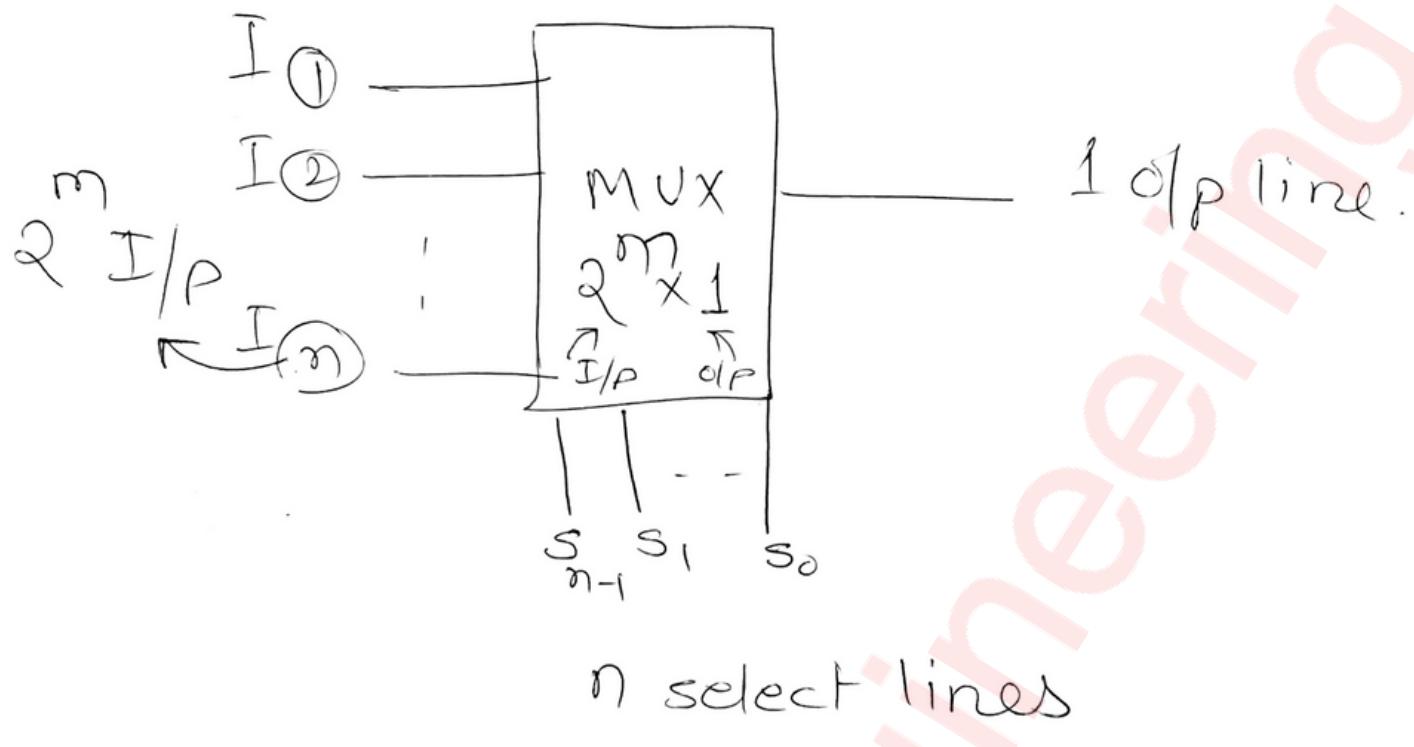


Multiple I/P lines

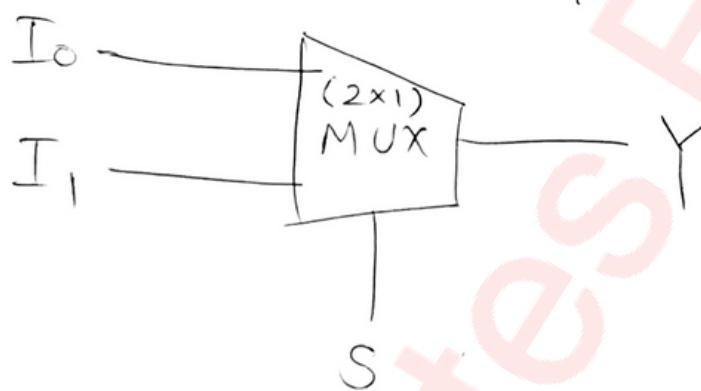
single o/p Line

[multiple option select one]



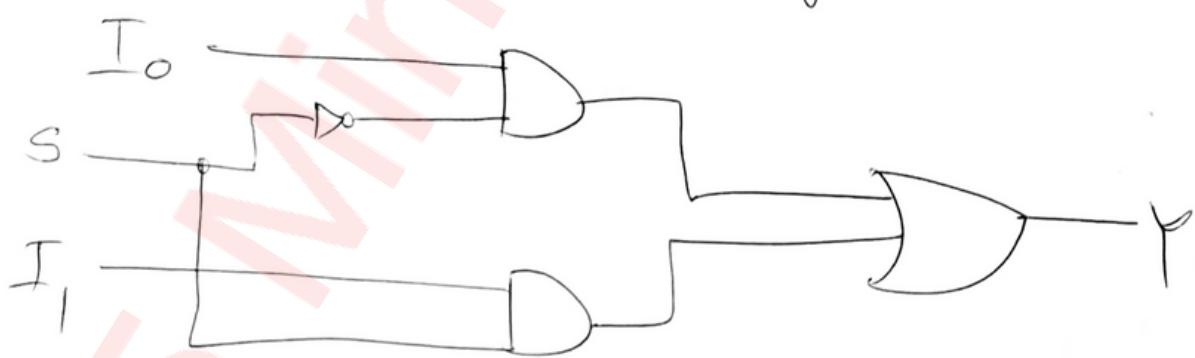


Two-to-One Multiplexer
 $\Rightarrow (2 \times 1) \text{ O/P}$

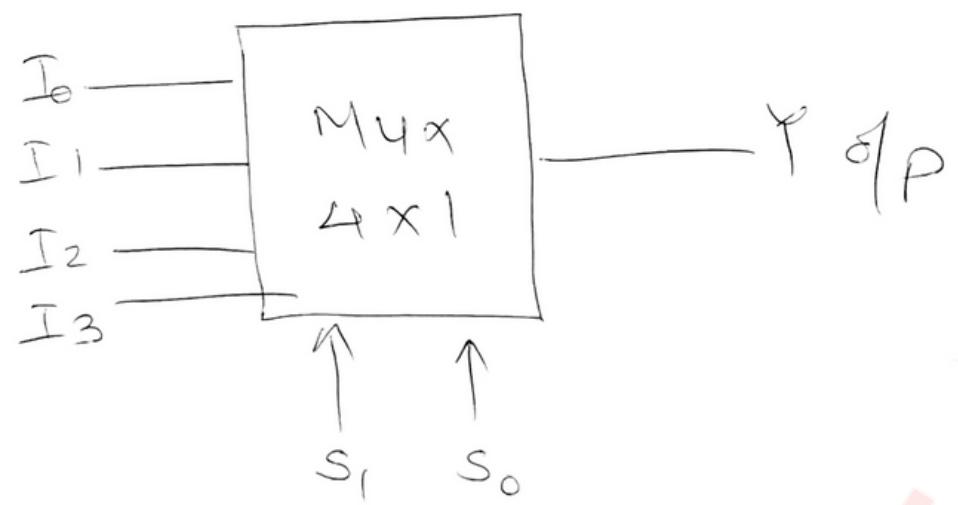


I/P	O/P
$S = 0$	$Y = I_0$
$S = 1$	$Y = I_1$

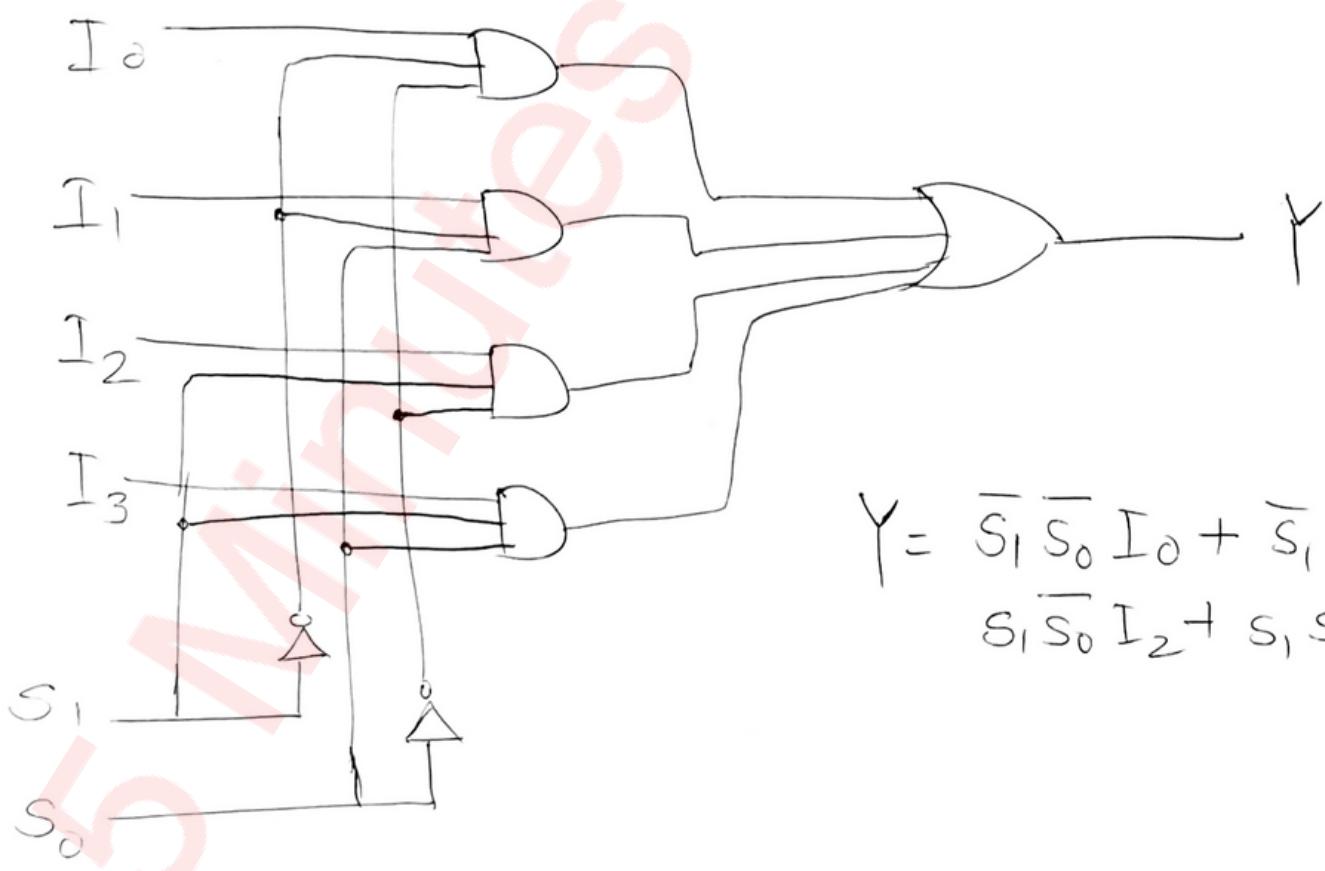
$$\text{eq}^n = Y = \bar{S}_0 I_0 + S_0 I_1$$



$\Rightarrow 4 \times 1 \text{ Mux}$

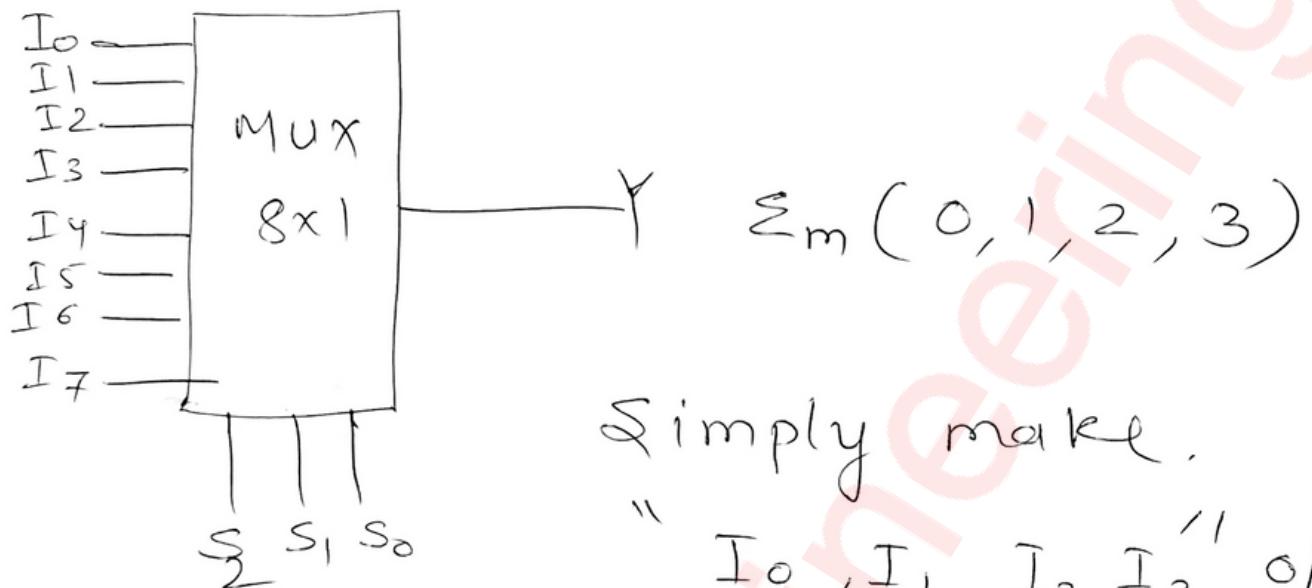


S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3



$$Y = \overline{S_1} \overline{S_0} I_0 + \overline{S_1} S_0 I_1 + S_1 \overline{S_0} I_2 + S_1 S_0 I_3$$

Eg: 8x1 Mux



Simply make.

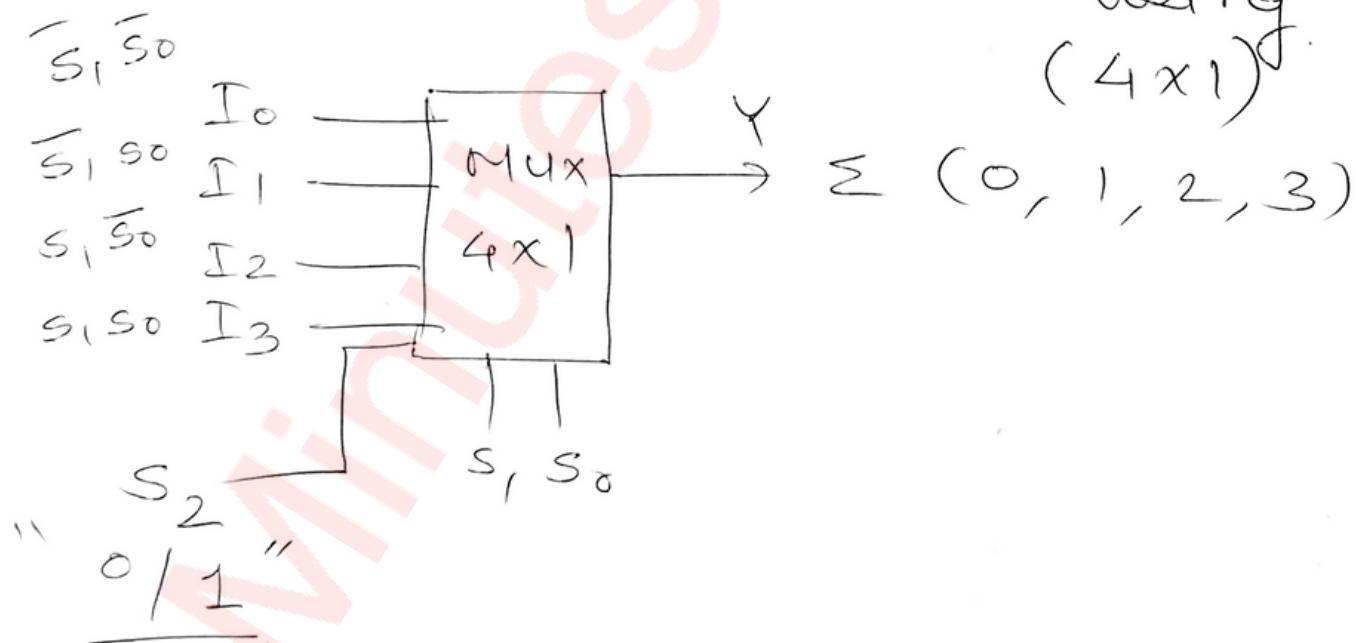
" I_0, I_1, I_2, I_3 " o/p

S_0	S_2	S_1	S_0
✓	0	0	0
✓	0	0	1
✓	0	1	0
✓	0	1	1

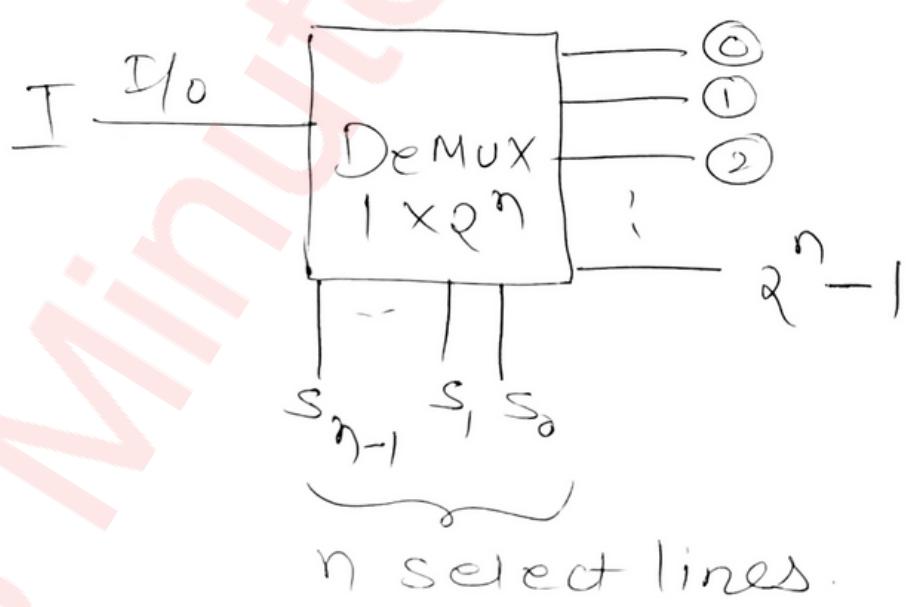
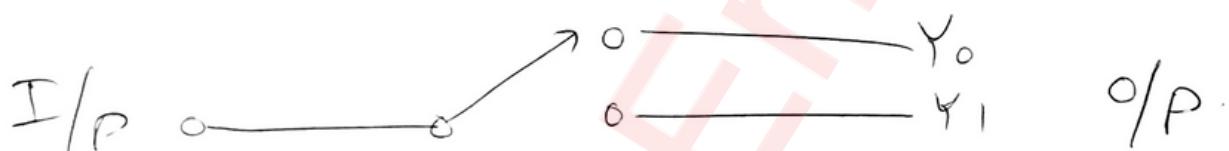
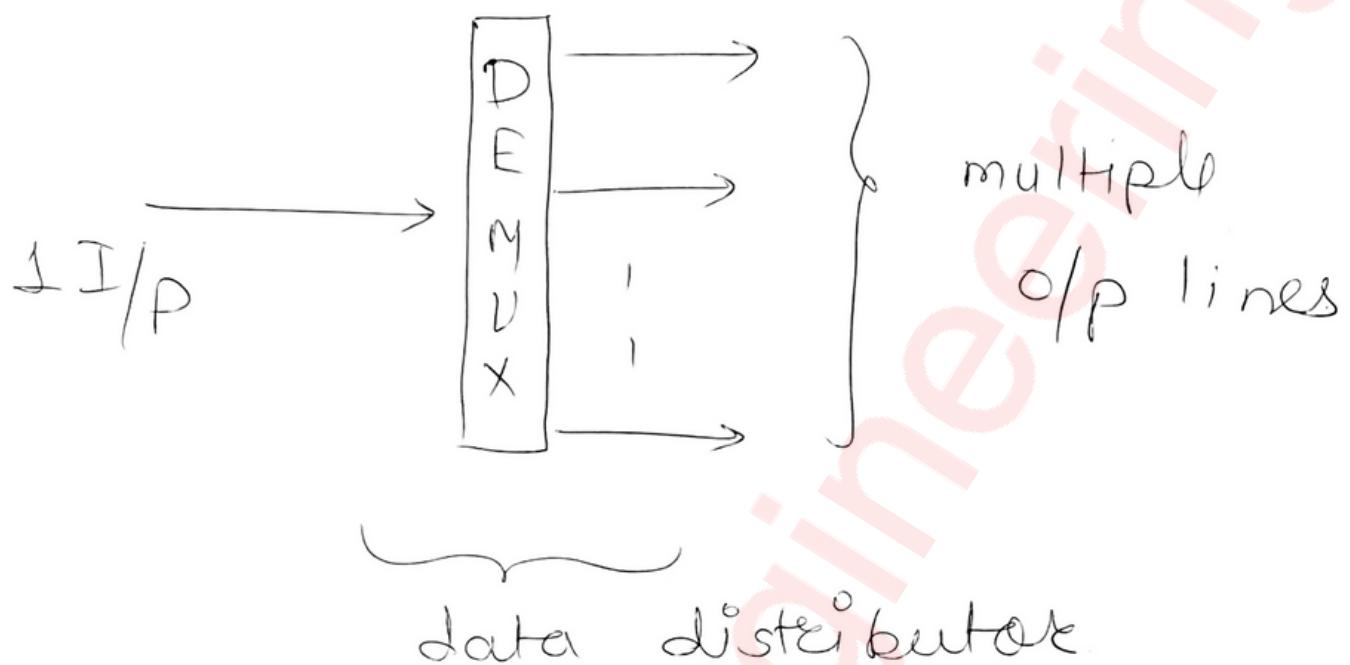
'OR' we can

Implement the same thing
using

(4x1)

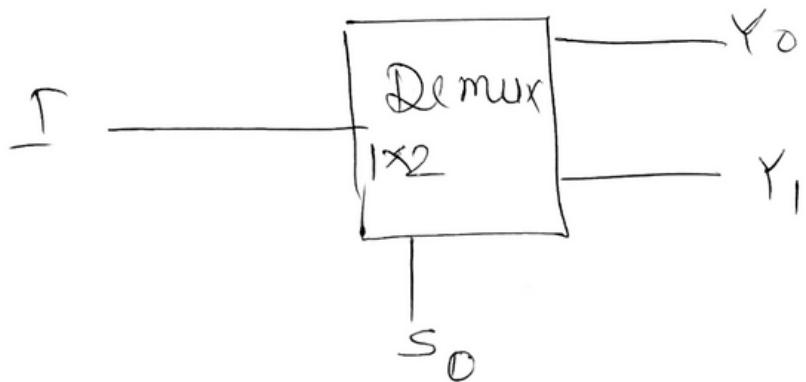


◦ De multiplexer [opposite of mux]

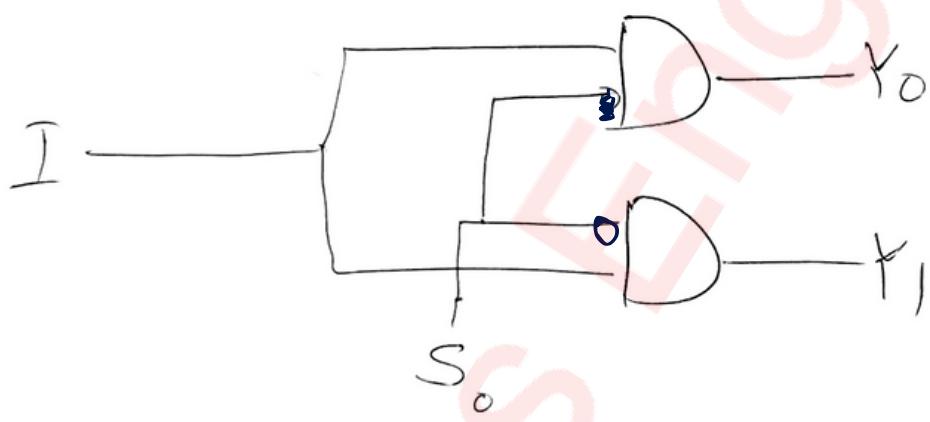


\Rightarrow

1x2 Demultiplexer

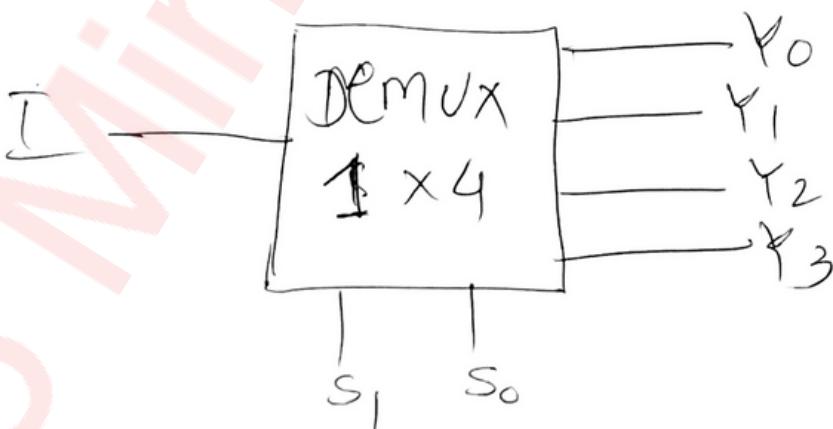


S_0	Y_0	Y_1
0	0	1
1	1	0



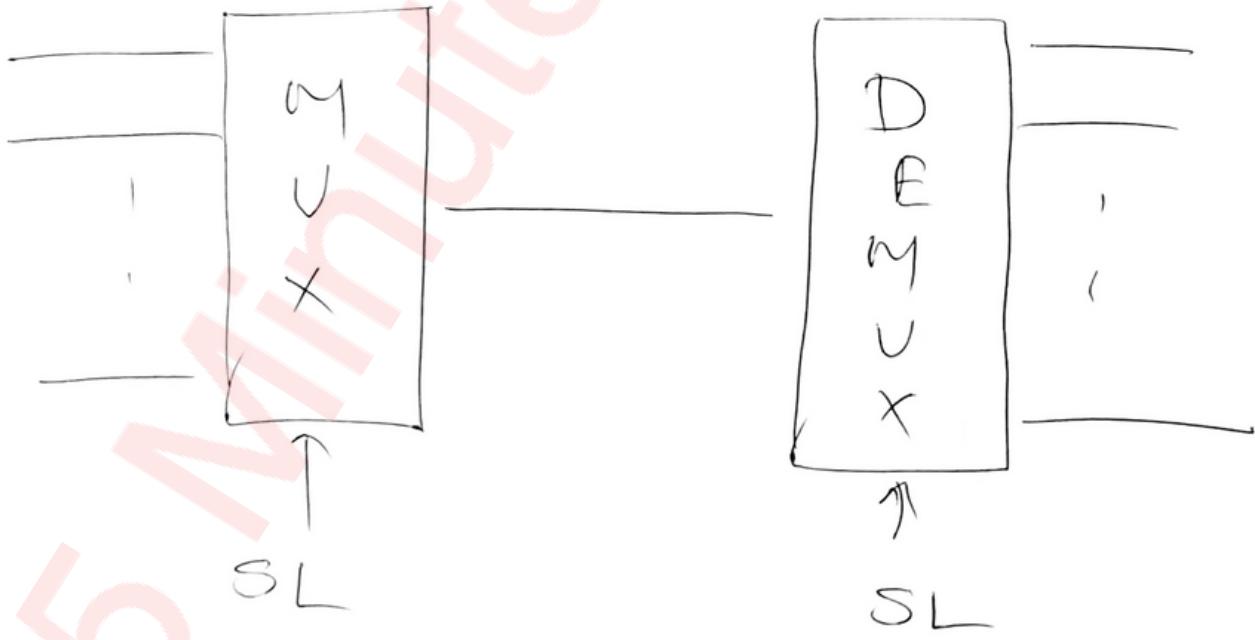
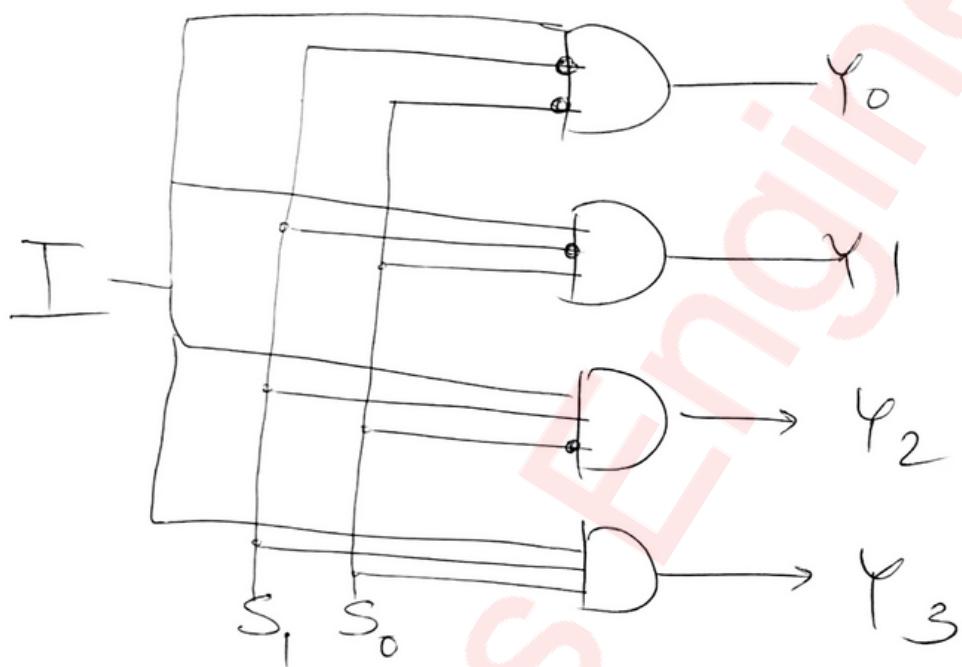
$$Y_0 = S_0 \quad Y_1 = \overline{S_0}$$

1x4 Demultiplexer

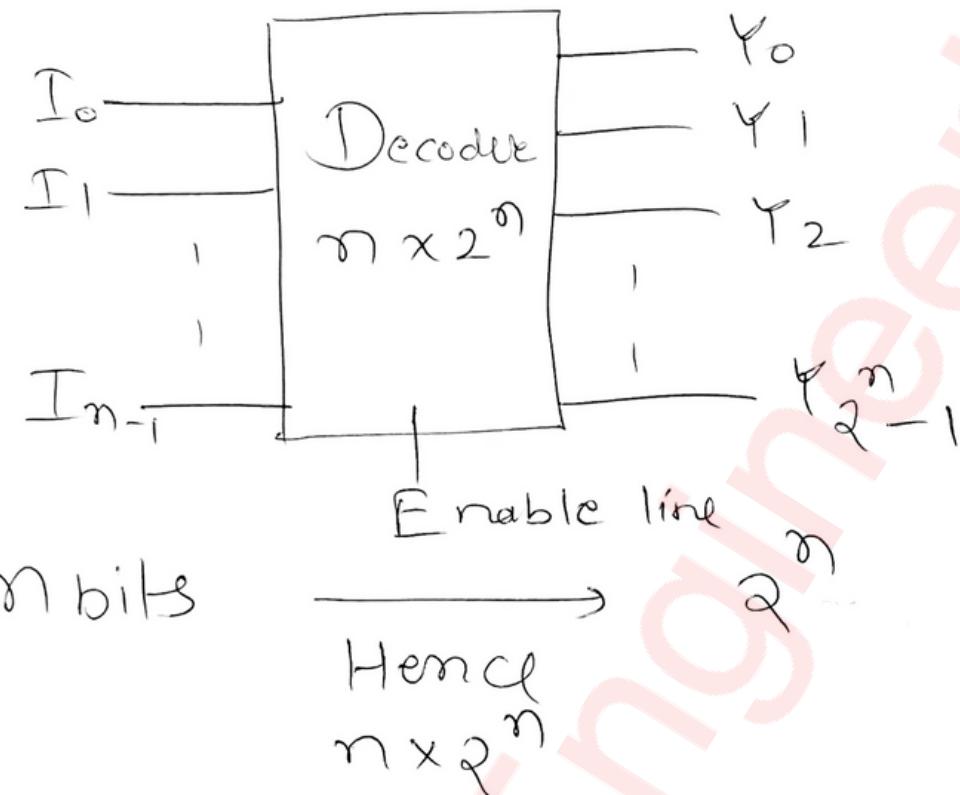


S_0	S_1
0	0
0	1
1	0
1	1

φ_3	φ_2	φ_1	φ_0
0	0	0	1
0	0	1	0
0	1	0	0
1	0	0	0



Decoder



Eg:	$n = 2$	$2^m = 2^2 = 4$
	$I_0 \ I_1$	Y_0
	0 1	Y_1
	1 0	Y_2
	1 1	Y_3

(Decoder) "SL \leftrightarrow I" (Demux)

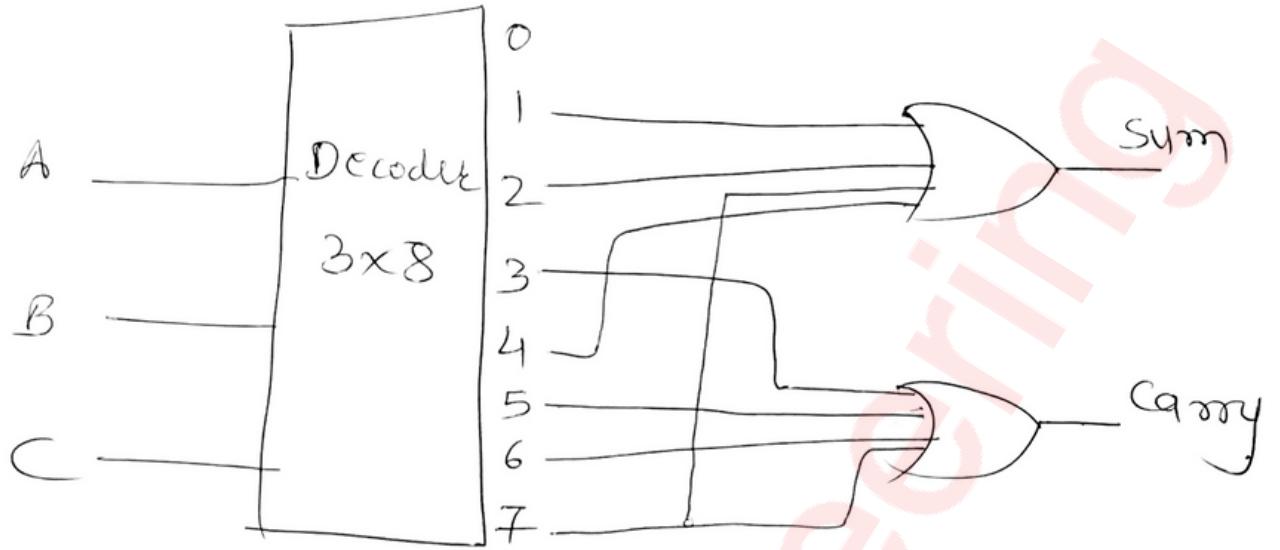
$$I \rightarrow EL$$

$$SL \rightarrow I$$

$$EL \rightarrow I$$

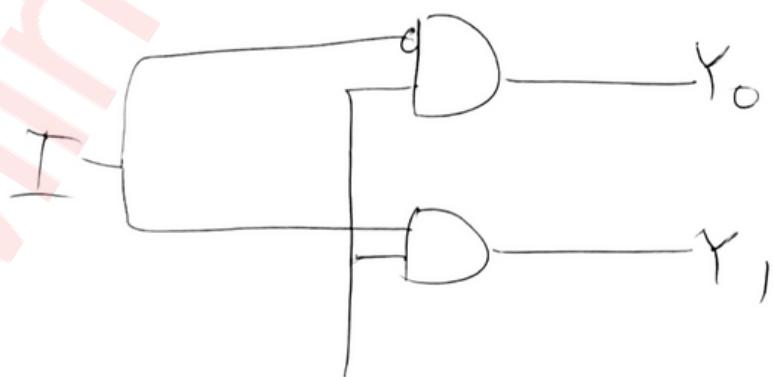
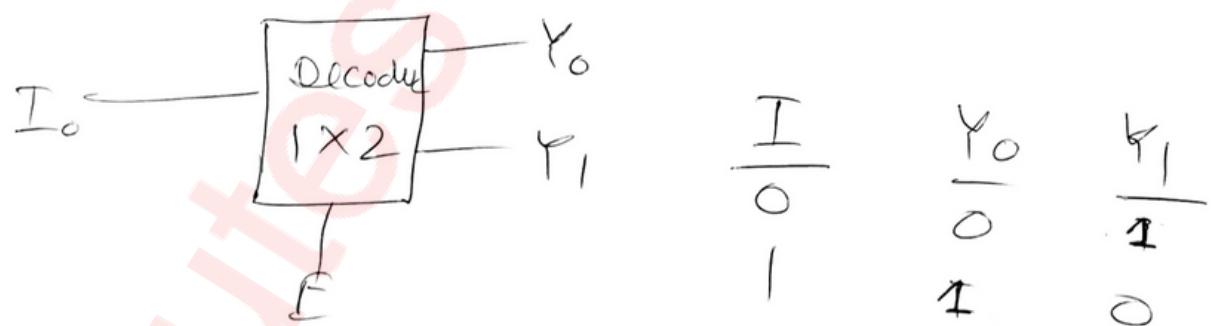
$$I \rightarrow SL$$

Eg

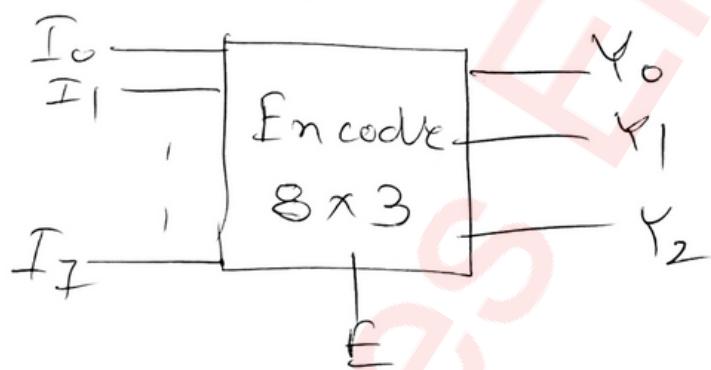
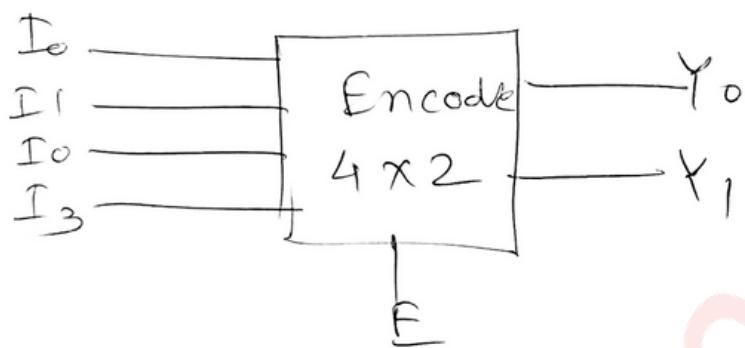
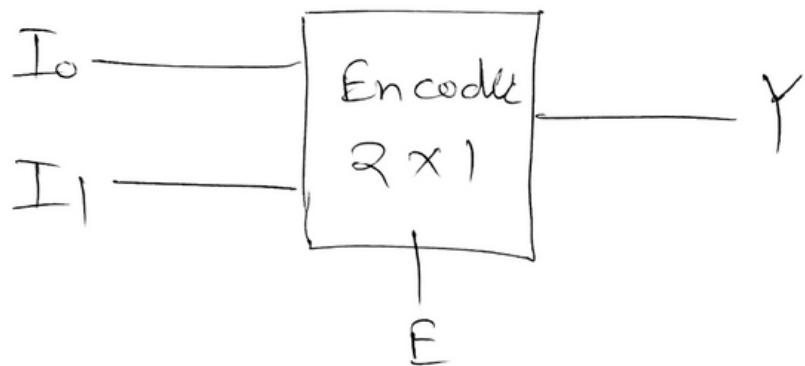


0	1	2	3	4	5	6	7	
$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$A\bar{B}\bar{C}$	ABC
000	001	010	011	100	101	110	111	111

Eg :- 1x2 Decoder



◦ Encoder



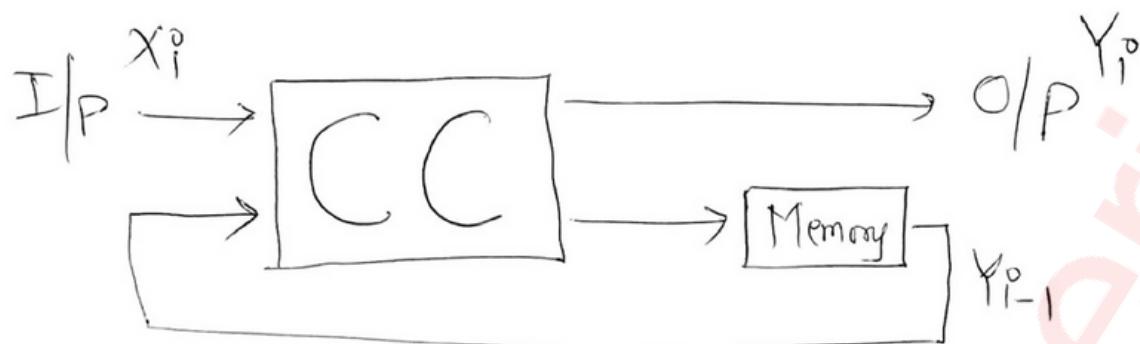
2^n I/P

n O/P

◦ Priority Encoder [Assign the priority to multiple I]

I_3	I_2	I_1	I_0	Y_1	Y_0
0	0	0	1	0	0
0	0	1	DC	0	1
0	1	DC	DC	1	0
DC	DC	DC	DC	1	1

Sequential Circuits = CC + Memory Element

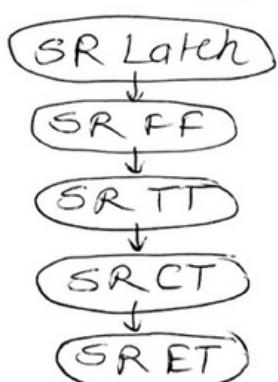
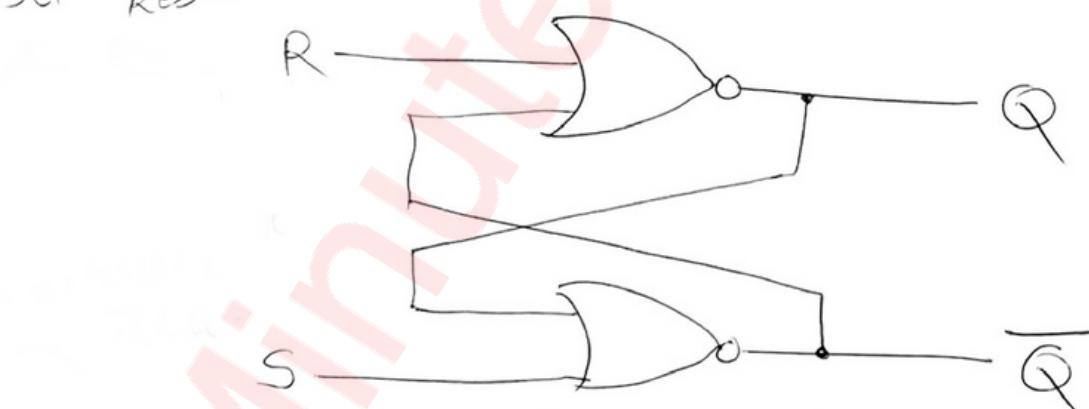


$$F(x_i, y_{i-1}) = y_i$$

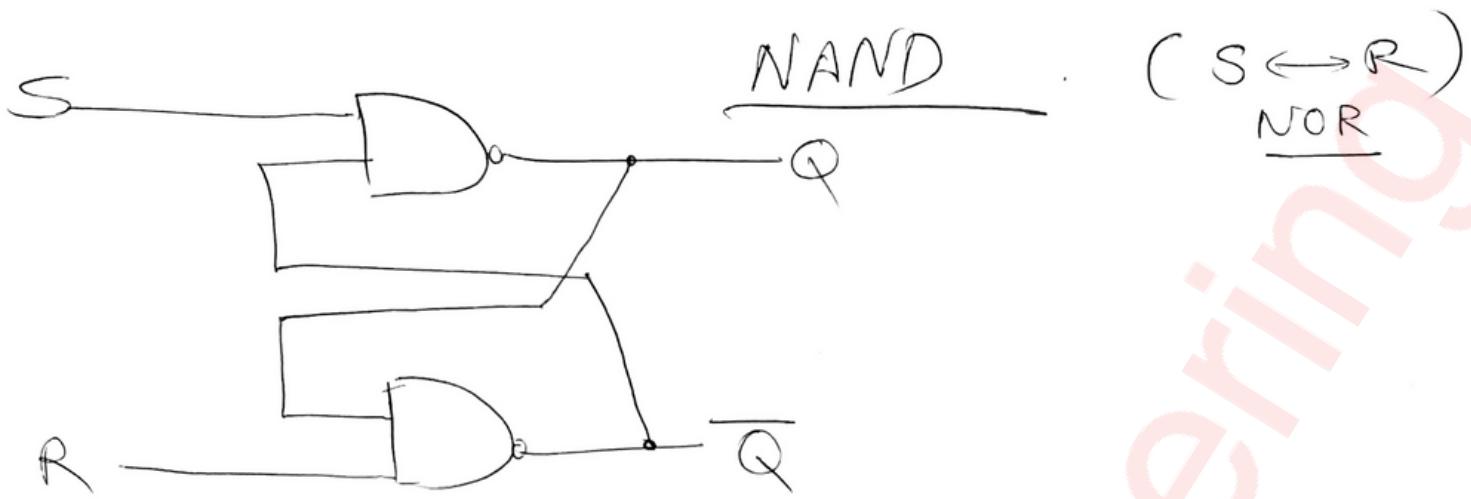
→ Latch (used to hold a bit)

↳ used in making of FFs.

◦ SR Latch [2 cross coupled NOR gates, NAND]



S	R	Q _n	Q _{n+1}
0	0	0	1] same/no change.
0	1	1	0] Reset
1	0	0	1] set
1	1	1	X] Random

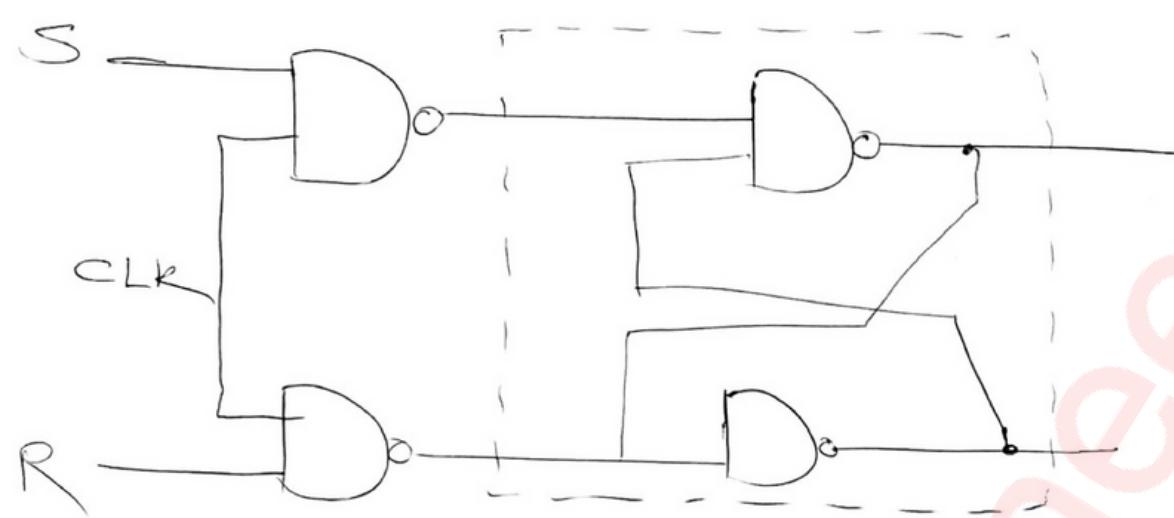


S	R	Q_n	Q_{n+1}	
0	0	0	X	Invalid / Random
0	0	1	0	Set
0	1	1	1	Reset
1	0	0	0	
1	0	1	1	
1	1	0	0	Same / No change
1	1	1	1	

→ S R Q_{n+1}

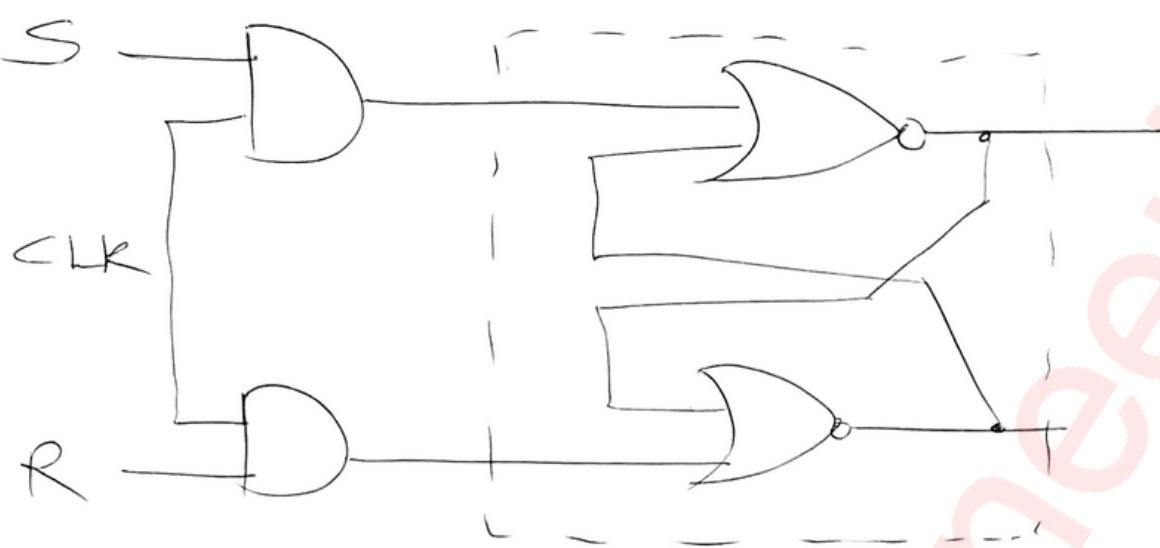
← 0	0	X	Invalid (X)
← 0	1	1	Set
← 1	0	0	Reset
← 1	1	1	No change

SR Flip-Flop (NAND)



<u>S</u>	<u>R</u>	<u>CLK</u>	<u>Q_{n+1}</u>
0	0	T/1	$\frac{Q_n+1}{Q_n}$ [Same / Hold]
0	1	T/1	0 [Reset]
1	0	T/1	1 [Set]
-	-	T/1	X [Invalid]
X	X	NT (0)	Q _n

(NOR Gate) SR FF



S	R	CLK	QnH	Qn
0	0	T/I	[Same/Hold]	Qn
0	1	T/I	0 [Reset]	
1	0	T/I	1 [Set]	
-	-	T/I	X [Invalid]	
X	X	NT		Qn

Same as NOR latch
situation.

SR FF Excitation Table

<u>S</u>	<u>R</u>	<u>Q_n</u>	<u>Q_{n+1}</u>
0	0	0	0] Same/hold
0	0	1	1]
0	1	0	0] Reset
0	1	1	0]
1	0	0	1] Set
1	0	1	1]
1	1	0	X] Invalid.
1	1	1	X]

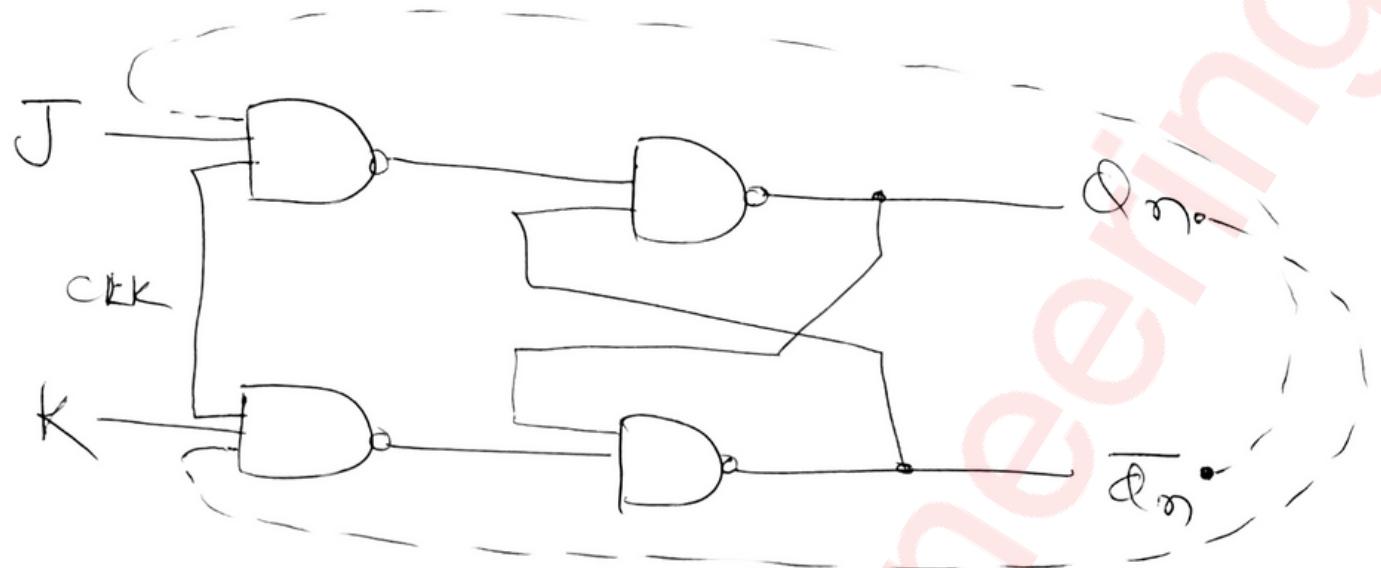
$S \xrightarrow{RQ_n}$

	00	01	11	10
0	0	1	3	2
1	4	5	X	6

$$Q_{n+1} = S + \overline{R} Q_n \rightarrow \text{characteristic eq}^n.$$

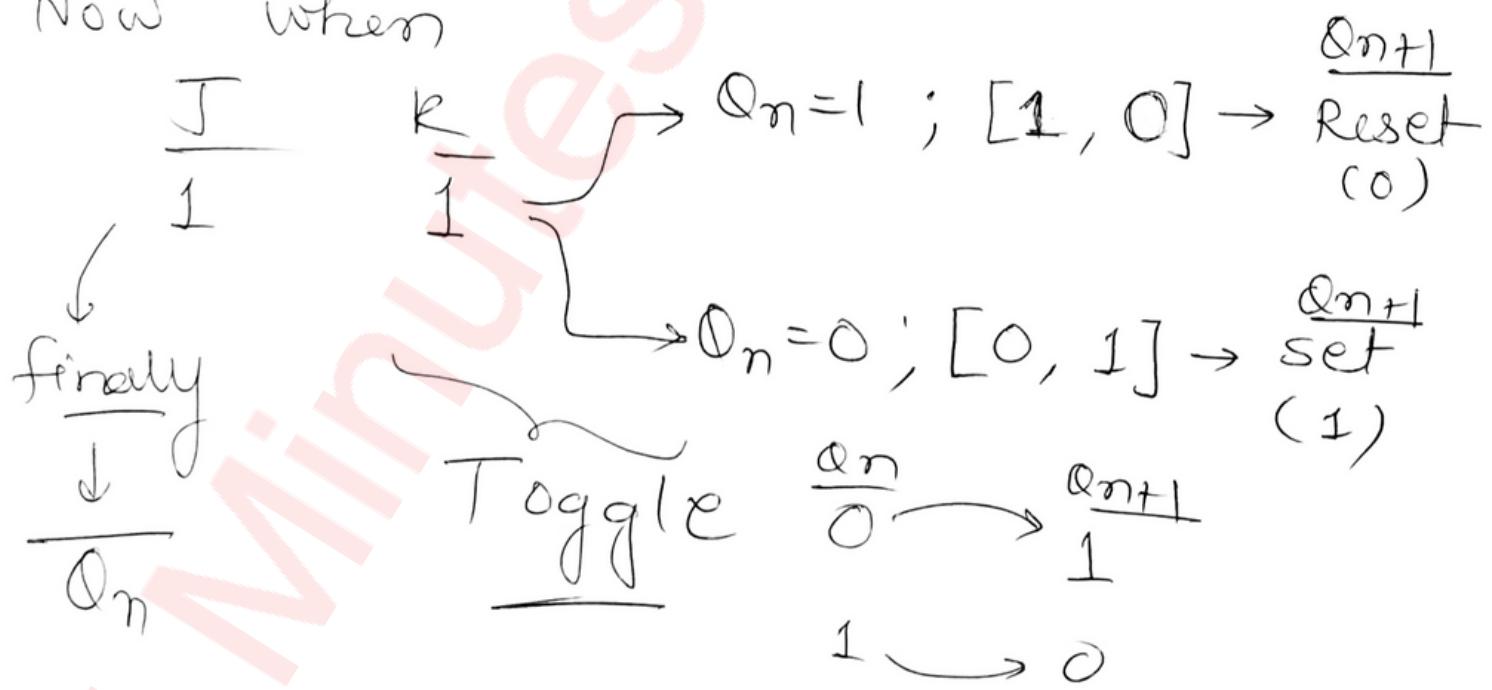
<u>Q_n</u>	<u>Q_{n+1}</u>	<u>S</u>	<u>R</u>
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

J_K flip flop



S	R	<u>Q_{n+1}</u>
0	0	Hold
0	1	Reset
1	0	Set
1	1	Invalid ← focuses / fix this.

Now when



J K flip flop Characteristic Table

<u>J</u>	<u>K</u>	<u>Q_n</u>	<u>Q_{n+1}</u>
0	0	0]	0 Hold
0	0	1]	1
0	1	0]	0 Reset
0	1	1]	0
1	0	0]	1 set
1	0	1]	1
1	1	0]	1
1	1	1]	0 \bar{Q}

$J \quad K Q_n$

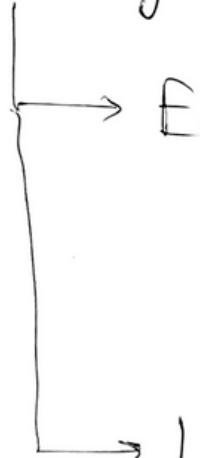
<u>J</u>	<u>K</u>	00	01	11	10
0	0	0	1	3	2
1	1	4	5	7	6

$$\Rightarrow Q_{n+1} = J\bar{Q}_n + \bar{K}Q_n \rightarrow \text{char. eqn}$$

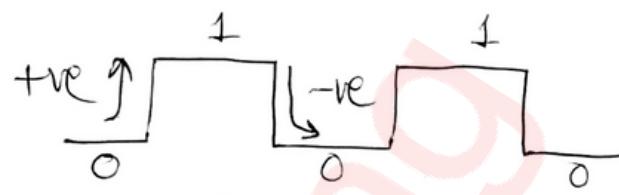
• Excitement table

<u>Q_n</u>	<u>Q_{n+1}</u>	<u>J</u>	<u>K</u>
0	0	0	X
0	1	1	X
1	0	X	X
1	1	X	0

→ Triggering



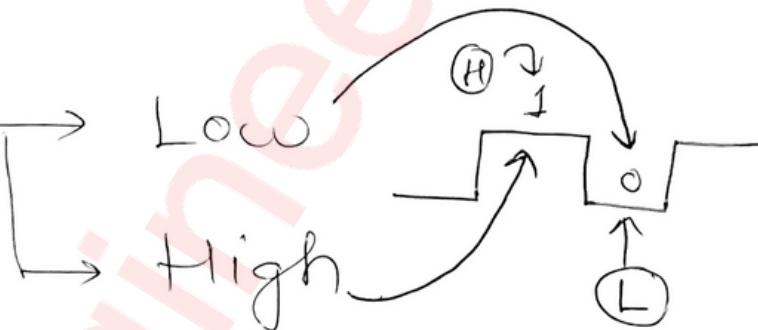
Edge → Negative
Positive



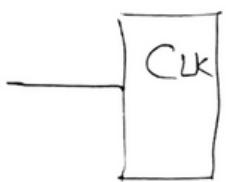
Level →

Low

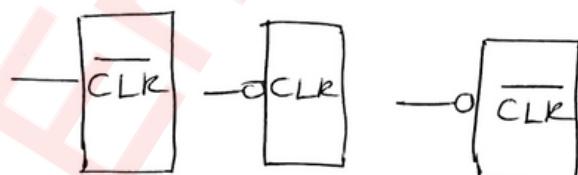
High



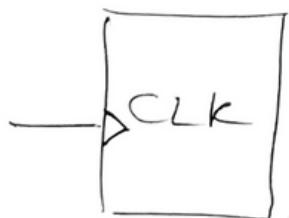
+ve level



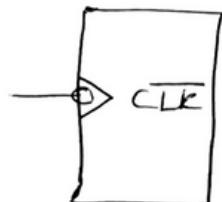
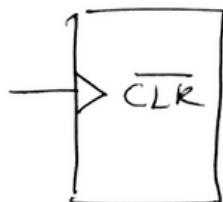
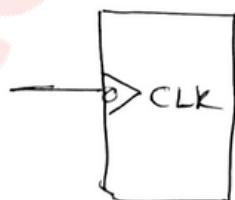
-ve level



+ve edge



-ve Edge



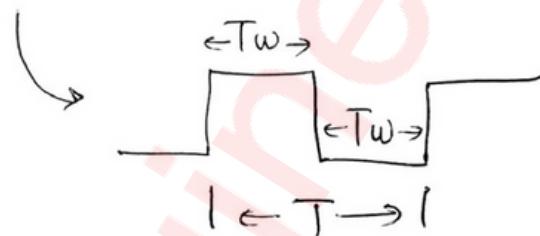
Race condition in JK ff

→ ① $K = J = 1$ [Toggle]

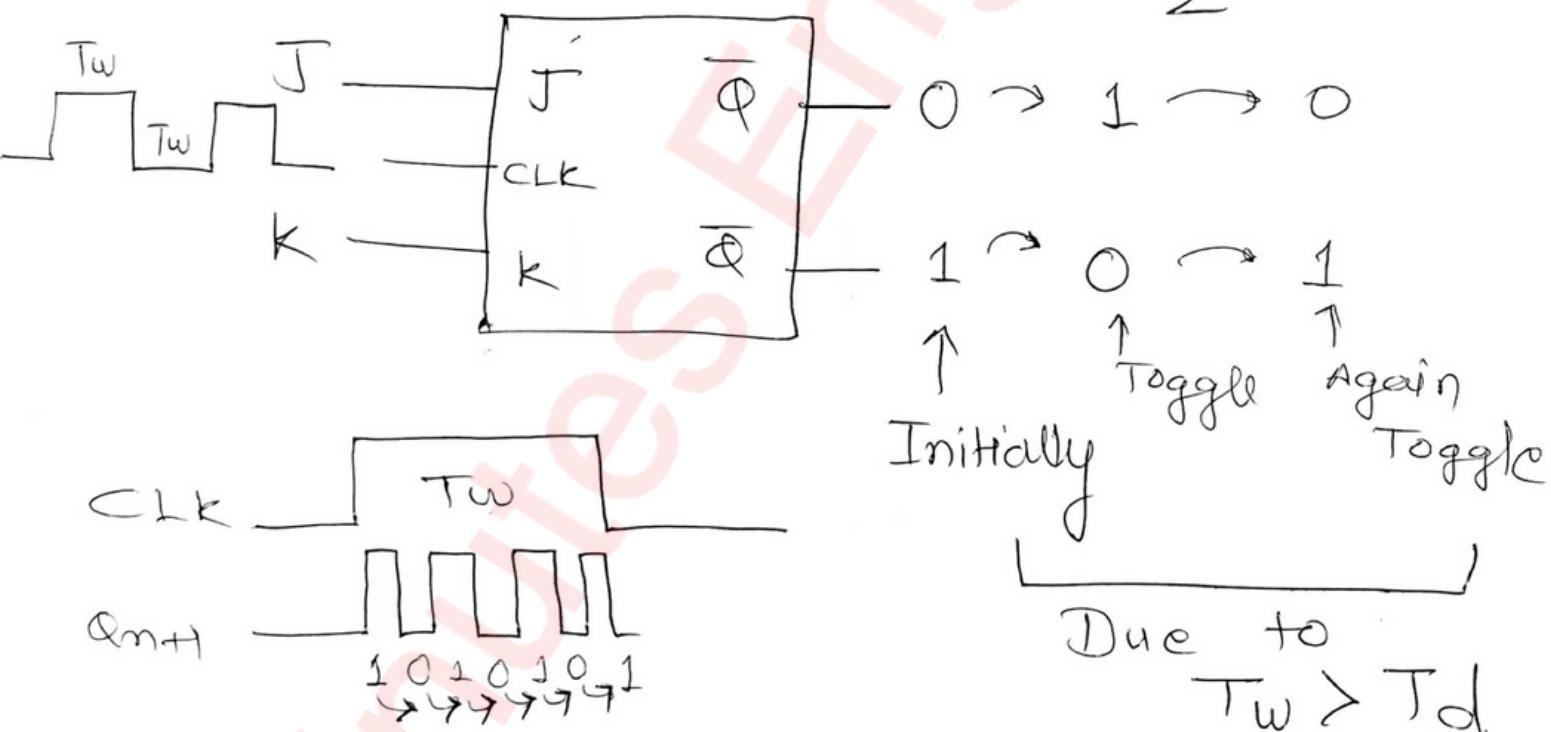
→ ② Level Triggered

→ ③ $T_d < T_w$

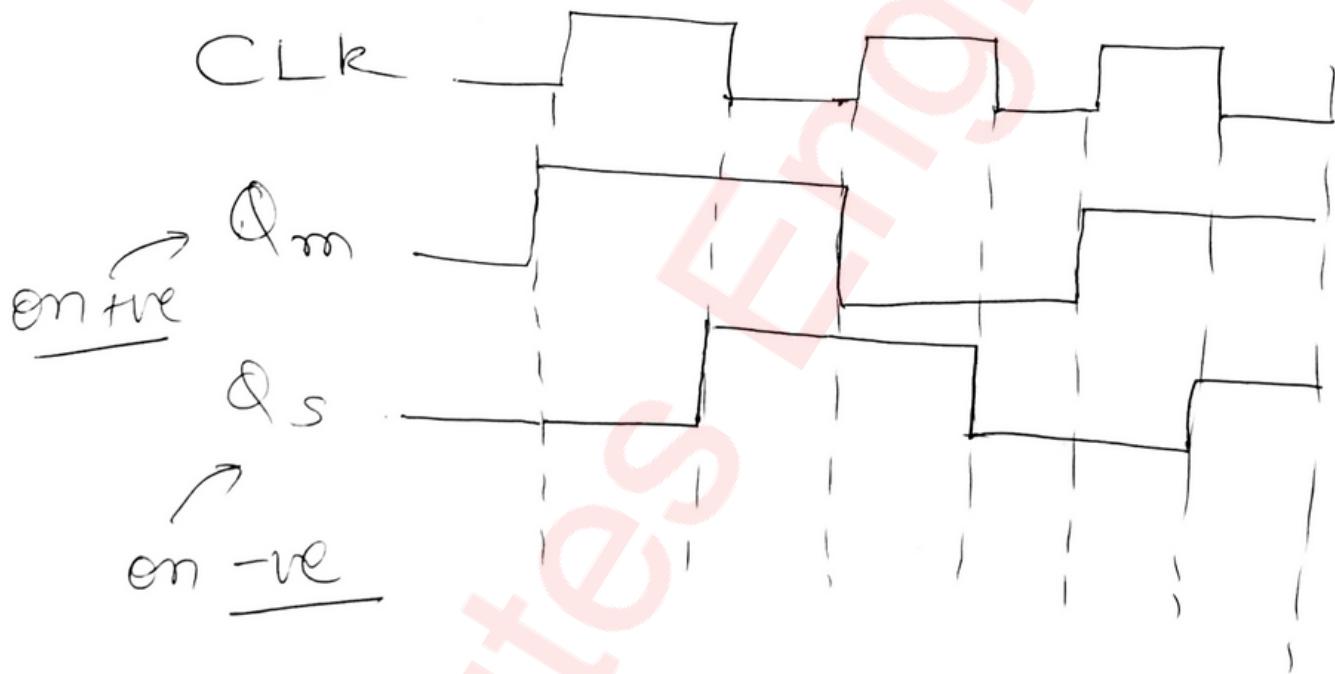
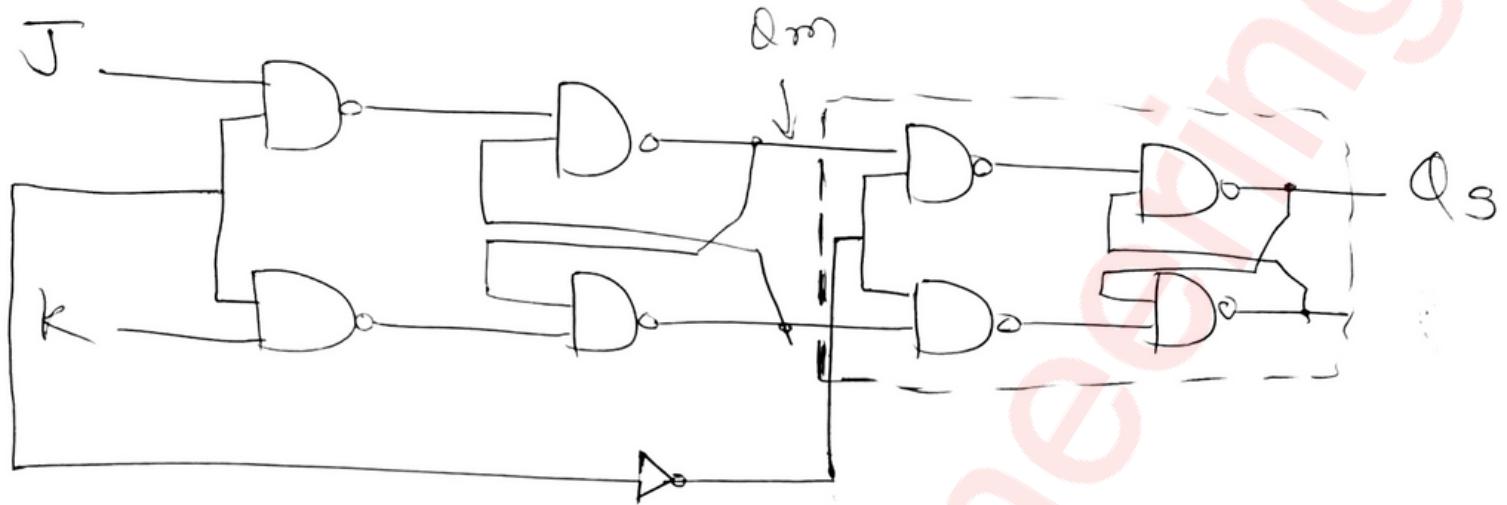
↓
delay



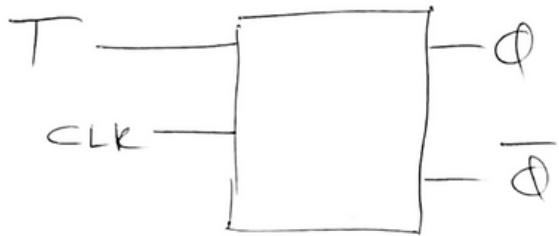
$$T_w = \frac{1}{2} (T)$$



Master slave JK FF



T flip flop



$T \quad Q_{n+1}$
 $0 \rightarrow Q_n \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Toggling behavior}$
 $1 \rightarrow \overline{Q_n}$

T	Q_n	Q_{n+1}
0	0	0
0	1	1
1	0	1
1	1	0

as $T = 0$ [No Toggle]

as $T = 1$

[Toggle]

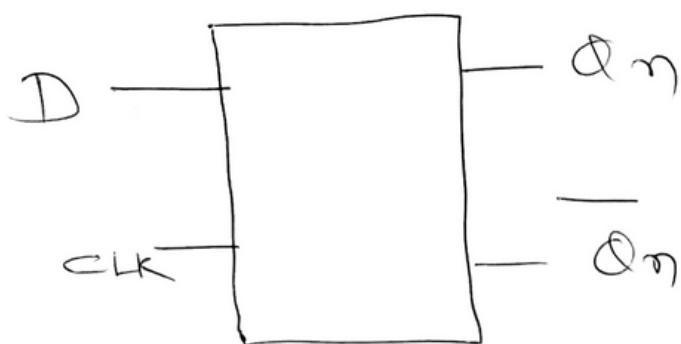
$$Q_{n+1} = T \oplus Q_n$$

Q_n	Q_{n+1}	T
0	0	0
0	1	-
1	0	1
1	1	0

- \downarrow mirror

5 Minutes

D Flip flop



D	Q_{n+1}
0	0
1	1

Same as D

D	Q_n	Q_{n+1}
0	0	0
0	1	0
↑	0	1
1	1	1

Same as D

$Q_{n+1} = D$

characteristic eqn.

Q	Q_{n+1}	D
0	0	0
0	1	-
1	0	0
1	1	1

SR → D ff conversion

O/P → Dff characteristic Table

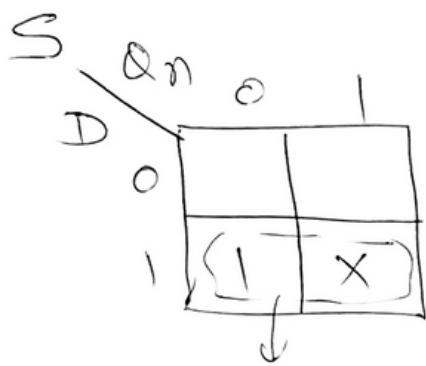
I/P → SR Excitation Table

<u>Q_n</u>	<u>Q_{n+1}</u>	<u>S</u>	<u>R</u>
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

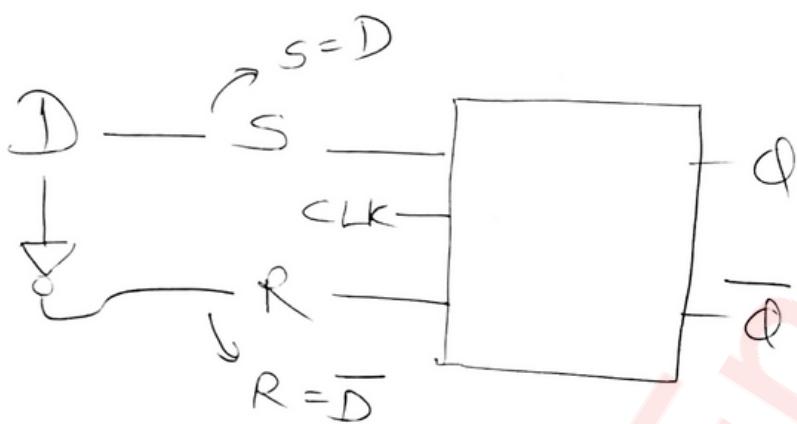
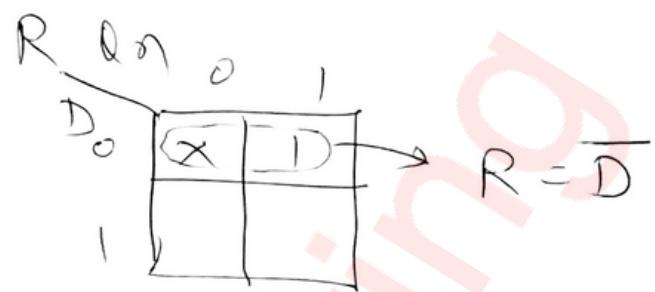
<u>D</u>	<u>Q_n</u>	<u>Q_{n+1}</u>
0	0	0
0	1	0
1	0	1
1	1	1

<u>D</u>	<u>Q_n</u>	<u>Q_{n+1}</u>	<u>S</u>	<u>R</u>
0	0	0	0	X
0	1	0	0	1
1	0	1	1	0
1	1	1	X	0

Refer to
ET (SR)



$$\boxed{S = D}$$



$\Rightarrow T \rightarrow JK \text{ ff conversion.}$

$\downarrow (ET)$ $\downarrow (CT)$

$\frac{Q_n}{0}$	$\frac{Q_{n+1}}{0}$	T
0	0	0
0	1	1
1	0	1
1	1	0

J	K	$\frac{Q_n}{0}$	$\frac{Q_{n+1}}{0}$
0	0	0	1
0	0	1	0
0	1	1	1
0	1	0	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

\downarrow] Samp.

\downarrow] Reset

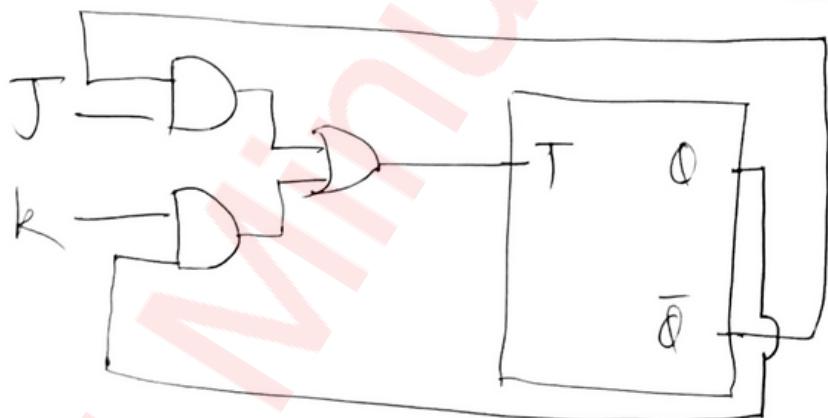
\downarrow] Set

\downarrow] Toggle

<u>J</u>	<u>k</u>	$\overline{Q_n}$	$\overline{Q_{n+1}}$	<u>T</u>
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1



$$T = J\bar{Q}_n + KQ_n$$

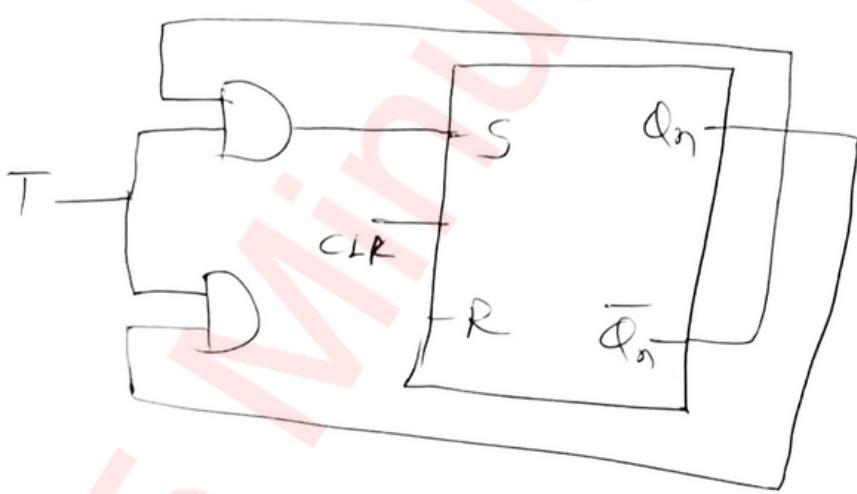
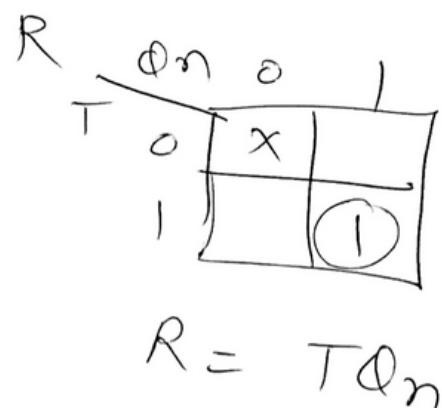
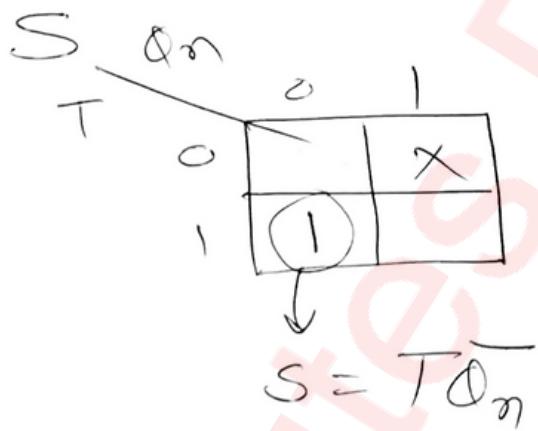


SR - T ff Conversion

\downarrow
 (ET)

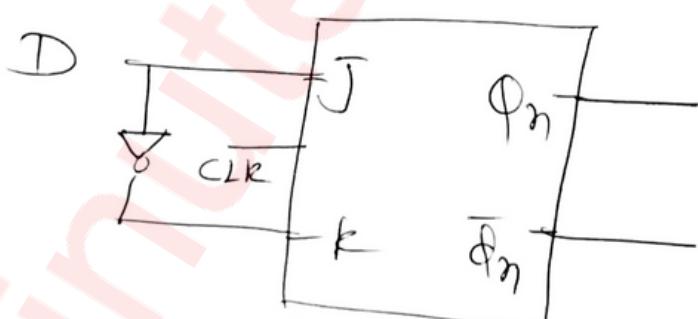
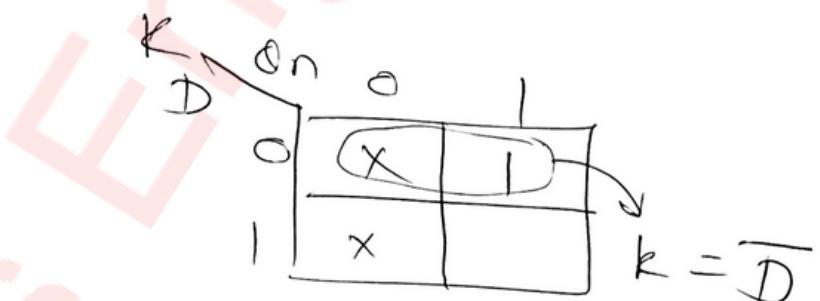
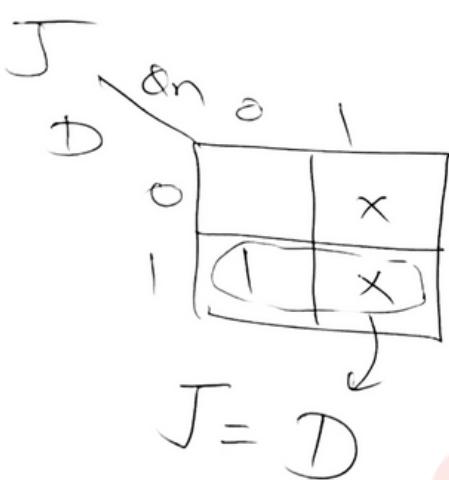
\downarrow
 (CT)

<u>T</u>	<u>Q_n</u>	<u>Q_{n+1}</u>	<u>S</u>	<u>R</u>
0	0	0	0	X
0	1	1	X	0
1	0	1	1	0
1	1	0	0	1



JK - D ff conversion

<u>D</u>	<u>Q_n</u>	<u>Q_{n+1}</u>	<u>J</u>	<u>K</u>
0	0	0	0	X
0	1	0	X	1
1	0	1	1	X
1	1	1	X	0



• $SR \rightarrow JK$ Conversion

↓ ↓

(ET) (CT)

$SR(ET)$

$\overline{Q_n} \quad \overline{Q_{n+1}}$

$0 \quad 0$

$0 \quad 1$

$1 \quad 0$

$1 \quad 1$

$S \quad R$

$0 \quad X$

$0 \quad 0$

$1 \quad 0$

$0 \quad 1$

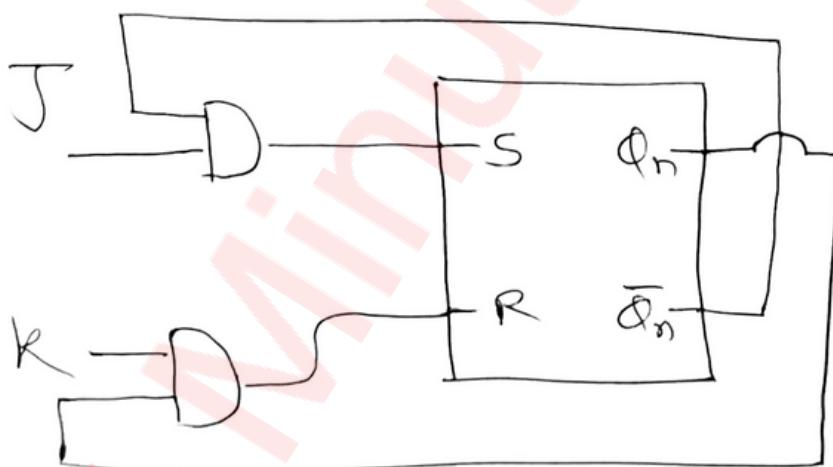
J	K	Q_n	Q_{n+1}	S	R
0	0	0	0	0	X
0	0	0	1	X	0
0	0	1	1	0	0
0	1	0	0	0	X
0	1	0	1	0	1
1	0	0	1	1	0
1	0	1	1	X	0
1	1	0	1	1	0
1	1	1	0	0	1

S	$J\bar{Q}_n$	00	01	10	11
0	0	X			
1	1	X			

$$S = J\bar{Q}_n$$

R	$K\bar{Q}_n$	00	01	11	10
0	0	X			
1	1		X		

$$R = K\bar{Q}_n$$



• $JK \rightarrow SR$ Conversion
 \downarrow \downarrow
 (ET) (CT)

S	R	Q_n	Q_{n+1}	J	K
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	X	1
1	0	0	1	1	X
1	0	1	1	X	0
1	1	0	X	X	0
1	1	1	X	X	X

$JK(ET)$

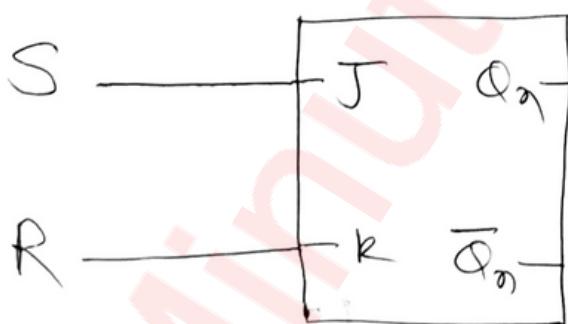
$Q_n Q_{n+1}$	00	01	10	11
00	0	1	1	X
01	1	0	X	1
10	X	1	0	0
11	0	X	1	X

J	$R Q_n$	00	01	11	10
0	S	X	X		
1	R	(1)	X	X	X

$$J = S$$

R	$R Q_n$	00	01	11	10
0	S	X		1	X
1	R	X		X	X

$$R = R$$



• Counters: count / frequency / occurred no. of times.
[Store & Display]

- Eg:
- Washing machine
 - Microwave oven
 - Traffic lights

→ n-bit counter

↓ have
n flip flops [0 to $2^n - 1$]

◦ Types

↳ Synchronous (parallel counter)
↳ Asynchronous (Ripple counter)

Eg: $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5$

↓
Mod 6 Counter

↑
unique states

Q: at 10 clk pulse what's the state.

$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3$

Simply multiple of '6' will take you to state '0' or value '0' ✓ 4

Like: 12, 18, 24, 30, 36. etc CLK pulses
 for '1' value 13, 19, 25, 31, 37 CLK pulses
 for '2' value 14, 20, 26, 32, 38 CLK pulses

Short cut

M CLK pulse for n mod counter

Simply: " 'mod n'" [If it starts from '0',]
 simply: " 'mod n'" [If it starts from '0',]

$$\text{eg: } 12 \bmod 6 = 0$$

$$13 \bmod 6 = 1$$

$$14 \bmod 6 = 2$$

Q: for mod n Counter [no. of ffs]

$$\text{use: } 2^k \geq n \quad \underline{\text{mod } 6 \leftarrow n}$$

$$2^k \geq 6$$

$$2^1 \geq 6 \times \quad 2^2 \geq 6 \times$$

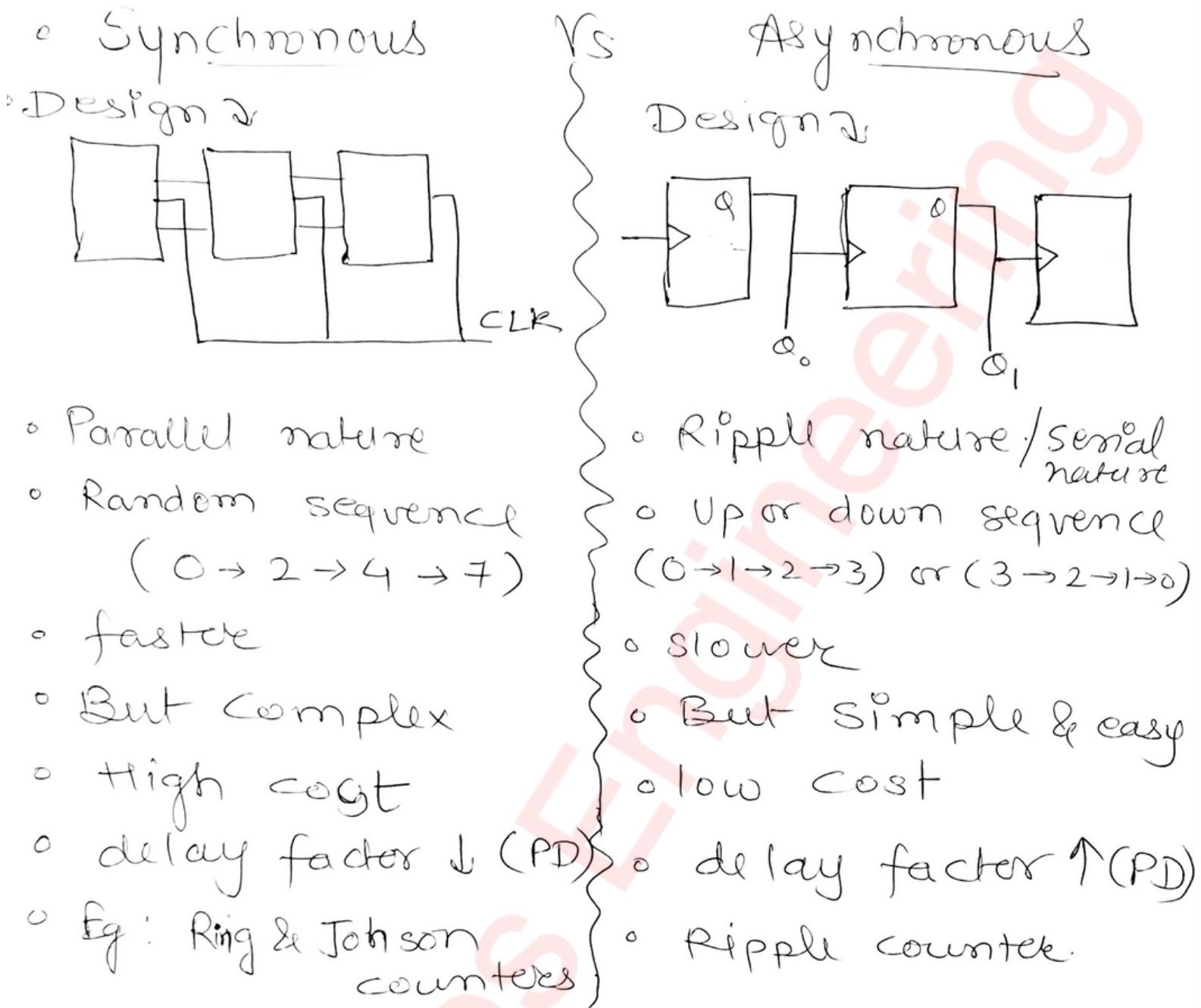
$$\boxed{k=3} \quad 2^3 \geq 6 \checkmark$$

$$\text{eg: } 0 \rightarrow 2 \rightarrow 4$$

so, consider the max bits for no. of ffs

0, 2, 4 : 3 bits but 9: 1001 (4 bits)

0, 10, 100



\Rightarrow



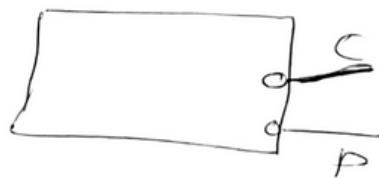
Q (O/P of ff)
 \downarrow
 Present (Set $\rightarrow 1$)
 \rightarrow clear (Reset $\rightarrow 0$)



or



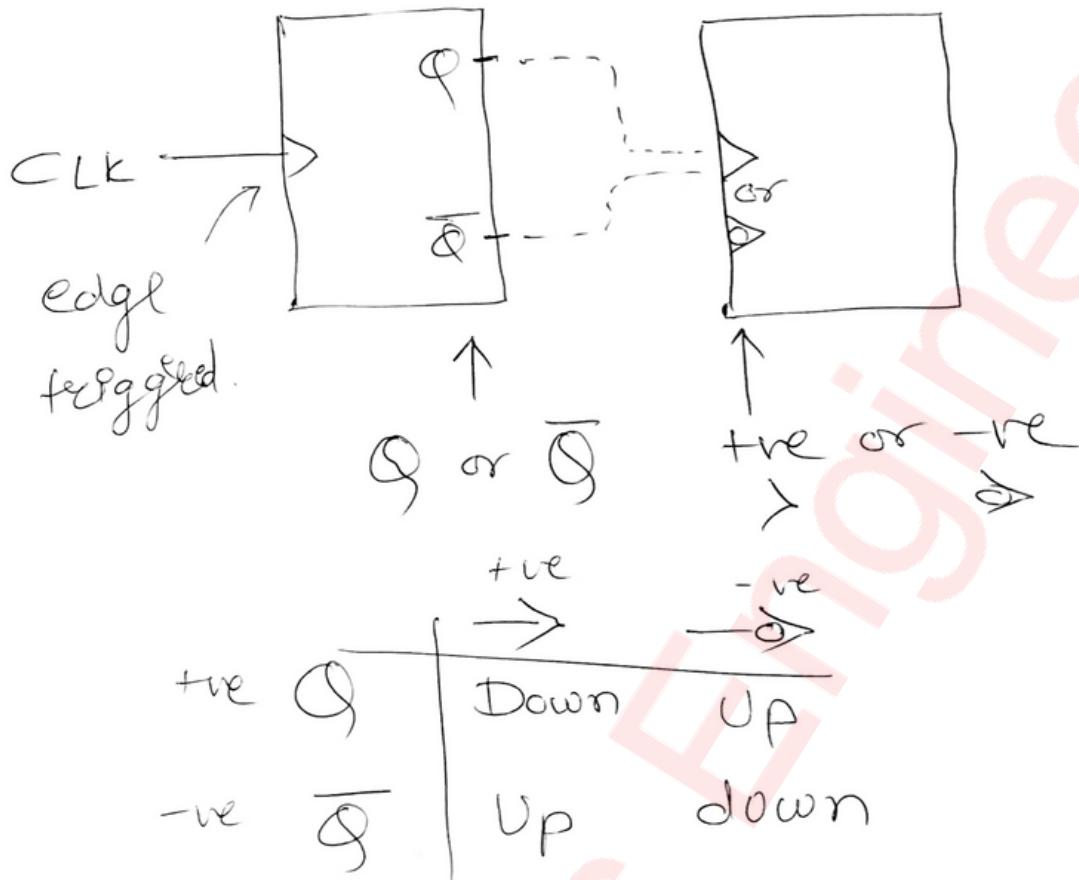
or



Low enabled.

Up & down counters

$(0 \rightarrow 1 \rightarrow 2 \rightarrow 3) \rightarrow (3 \rightarrow 2 \rightarrow 1 \rightarrow 0)$



Steps to design synchronous counter

- ① Let's say Given :- $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$ seq.
- ② no. of ff's : max no = 3 (bits required = 2)
hence 2 ff's [can use any ff]
- ③ Let's say we use 'D' ff.
- ④ will require excitation table of

D _{ff}	$\frac{Q_n}{0}$	$\frac{Q_{n+1}}{0} \rightarrow \frac{D}{0}$
0	0	0
1	1	0

⑤ Sequence (Present state & next state)

PS

NS

for 'D' values
refer (ET)

Q_1 Q_0

Q_1 Q_0

D_1 D_0

0) 0 0

3) 1 1

1 1

1) 0 1

0) 0 0

0 0

2) 1 0

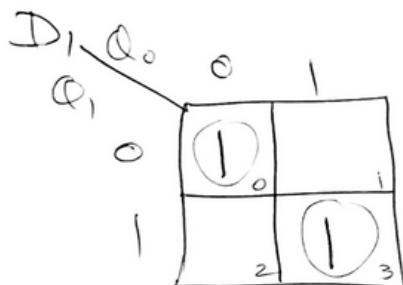
1) 0 1

0 1

3) 1 1

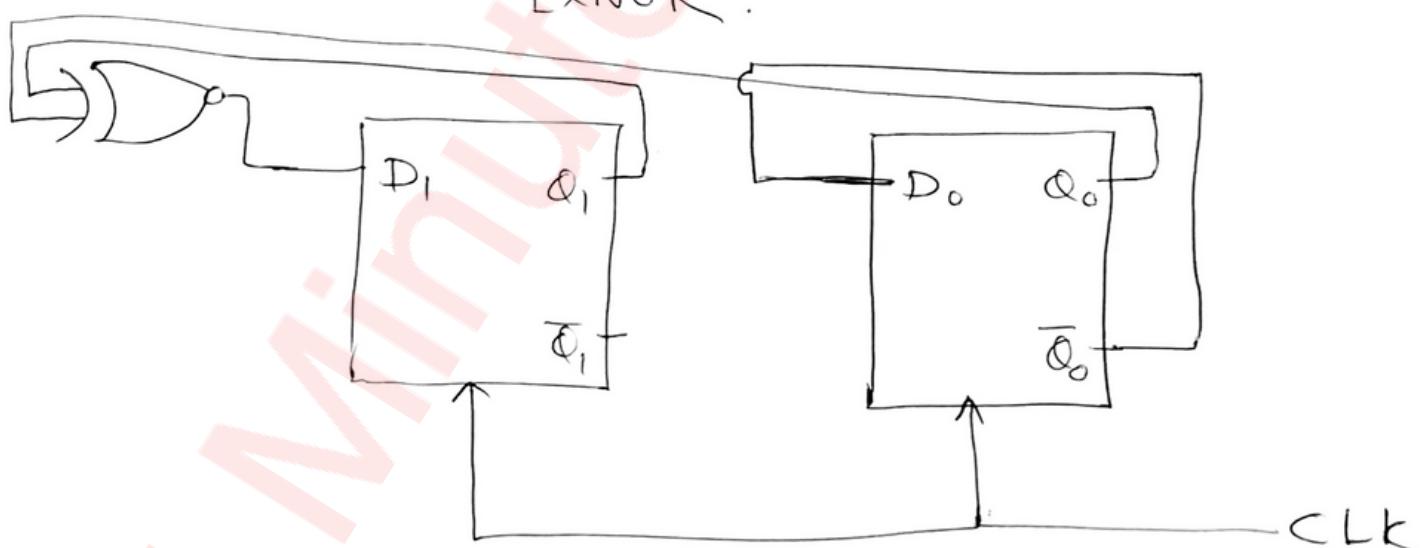
2) 1 0

1 0



$$\begin{aligned}
 D_1 &= \overline{Q_1} \overline{Q_0} + Q_1 Q_0 \\
 &= Q_1 \odot Q_0
 \end{aligned}$$

↑
EXNOR.

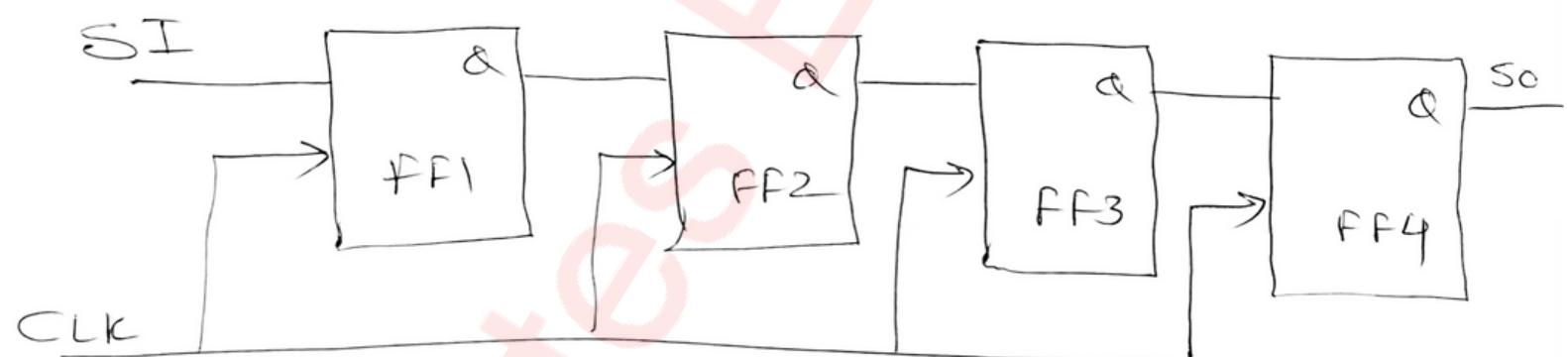


• Registers: Storage primary used using ff.
(D-ff) acts just as buffer memory.

Modes [Shift Registers]

- Serial In serial Out
- Serial In parallel Out
- Parallel In parallel Out
- Parallel In serial Out

① SISO



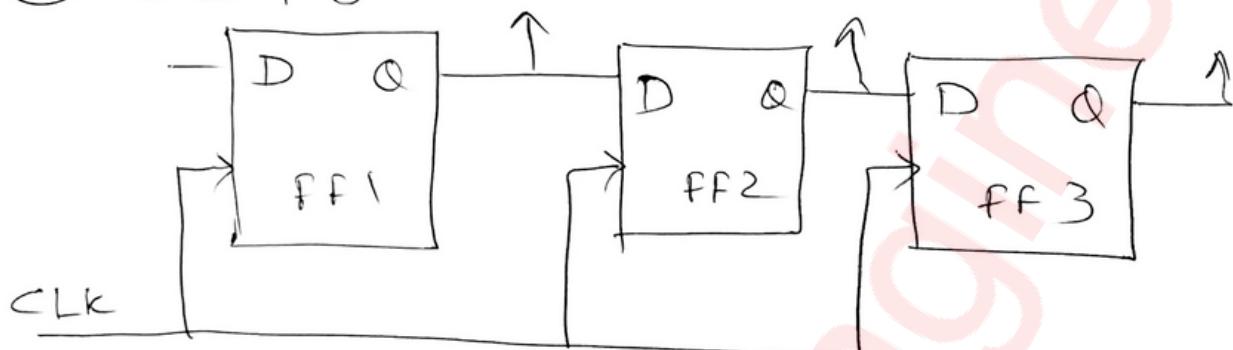
I/P	Q_3	Q_2	Q_1	Q_0	O/P
0	x	x	x	x	x
1	0	x	x	x	x
0	1	0	x	x	x
1	0	1	0	x	x
0	1	0	1	0	1
1	0	1	0	1	1

Y-axis: clock

- No. of clocks to write : 4 for 4 ff
i.e ' n ' for $\underline{n \text{ ff}}$
- No. of clock to Read : ' $\underline{n-1}$ '

$$\begin{aligned}\text{Total} &= n + n - 1 \\ &= 2n - 1\end{aligned}$$

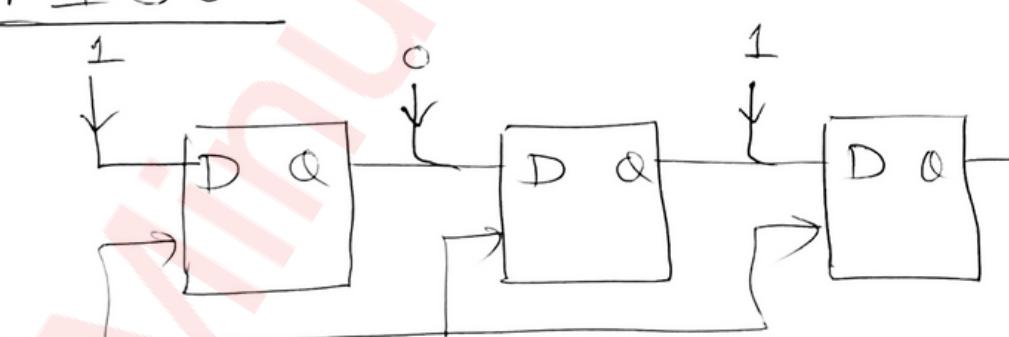
② SIPO



here I/p behaviour is same
so ' n ' clock cycle.

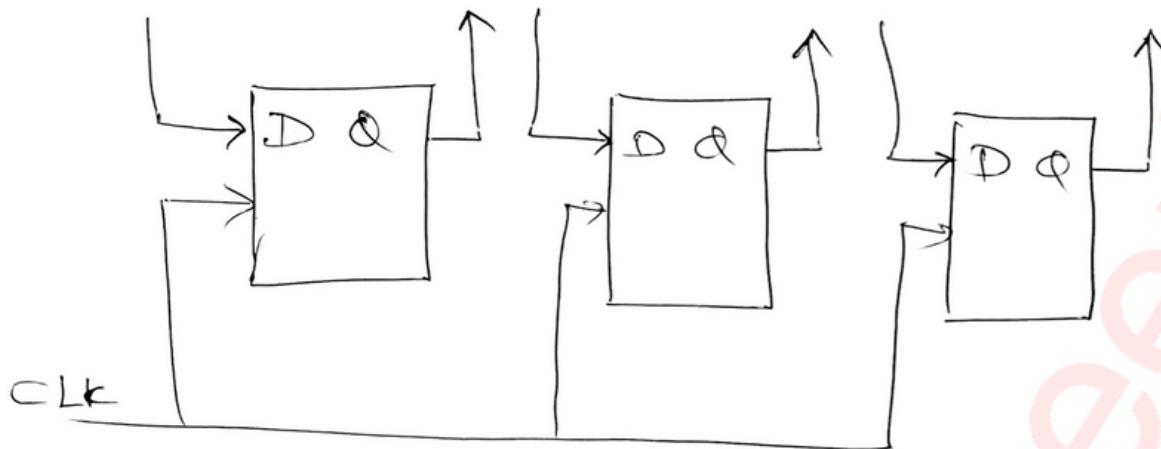
But o/p is parallel so ' n ' clk
 \therefore Total = ' \underline{n} '

③ PIPO



I/p behaviour parallel $\rightarrow 1$ clk
o/p behaviour serial $\rightarrow n-1$ clk
Total = ' \underline{n} '

④

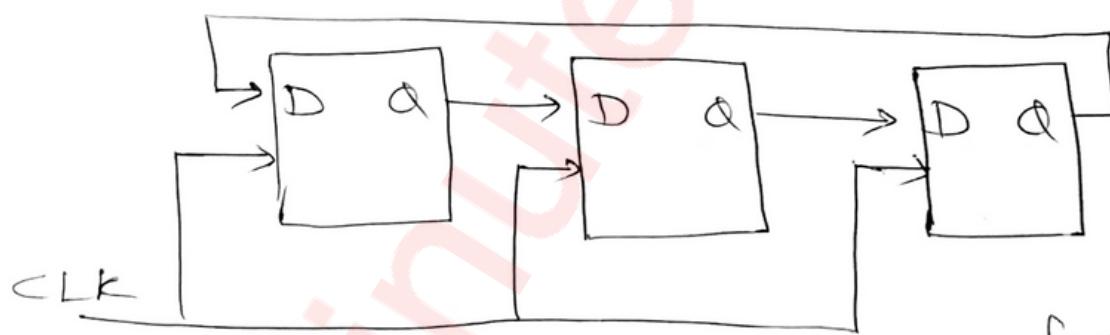
PIPO

I/P & O/P both parallel behaviour

So for write \rightarrow 1 CLK
 ——————
 Read \rightarrow 0 CLK

Total 1 ✓

- Ring counter [circular shift Reg]



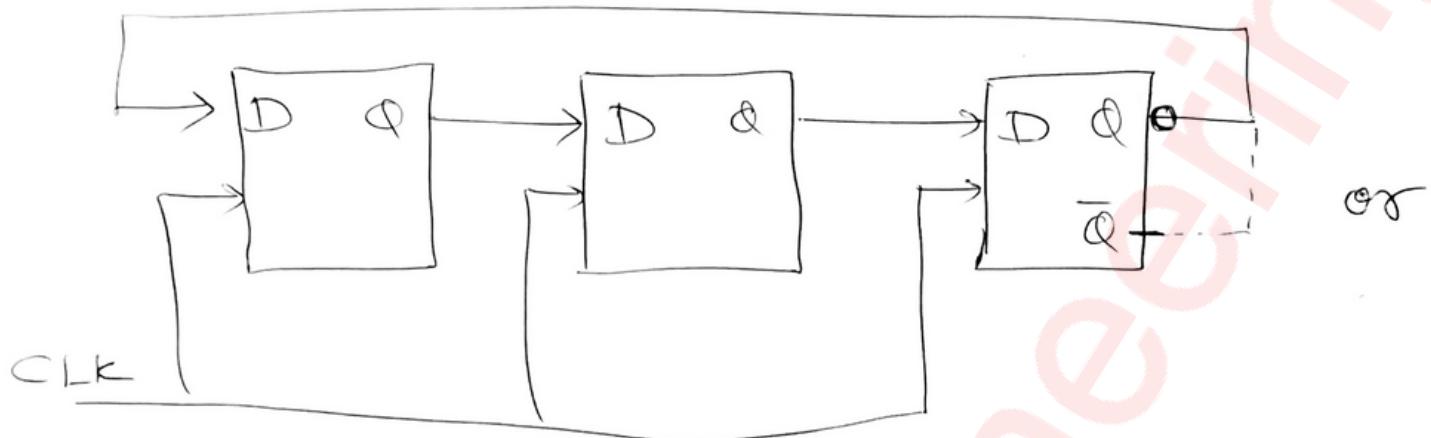
<u>CLK</u>	<u>Q₂</u>	<u>Q₁</u>	<u>Q₀</u>
0	1	0	0
1	0	1	0
2	0	0	1
0	1	0	0

for 'n' no. of ff
 $\rightarrow 2^n (0 \text{ to } 2^n - 1)$

\rightarrow But here we used only 'n' i.e. 3 state.

\rightarrow So $2^n - n$ unused state.

Johnson Counter



<u>CLK</u>	<u>Q₂</u>	<u>Q₁</u>	<u>Q₀</u>
0	0	0	0
1	1	0	0
2	1	1	0
3	1	1	1
4	0	1	1
5	0	0	1
0	0	0	0

so here we
get '6' states
Previously it
was just '3'
i.e $2(n)$

So for n bits → { Ideally (2^n count)
Ring (n count state)
Johnson ($2n$ state count)}