Noise performance of various Modulation schemes.

previous discussion we assumed that transmission of deterministic signals over a channel & we did not emphasize the central role played by concept of "randomness" in communication. The word "random" means unpredictable. If the receiver at end of channel knew in advance the message output from the originating source, there would be no need for communication. So there is a randomness in the message source. Moreover, transmitted signals always accompanied by noise introduced in the system (S/m). These noise waveforms are also unpredictable. process of communication becomes challenging in presence of unwanted electrical noise.

The receiver input, in general consists of (message) signal ~~put~~ plus noise, possibly with comparable power levels. The purpose of receiver is to produce the desired signal with a signal-to-noise ratio that is above a specified value.

→ Receiver model and Figure of merit: Linear modulation.

To undertake an analysis of noise in CW modulation S/m^s, foremost task is to model receiver noise as additive, white and Gaussian. This assumption enable us to obtain basic understanding of the way in which noise effects the performance of receiver.

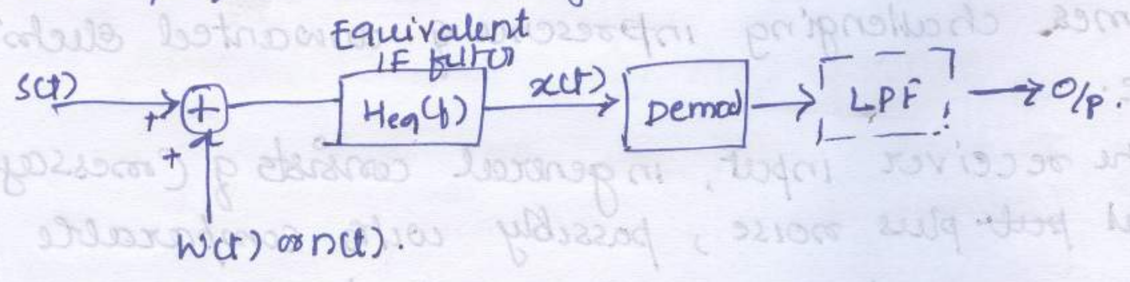
For modelling Receiver, following points must be kept in mind:

* The model provides an adequate description of the form of receiver noise that is of common c
* The model accounts for the inherent filtering and modulation characteristics of the system.
* The model is simple enough for stastistical analysis of the system to be possible.

Receiver model

consider ~~the~~ superheterodyne receiver

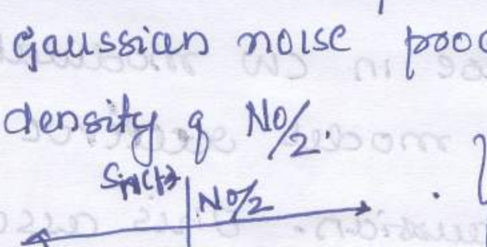
simplified model of receiver is shown in Fig. 1. for noise performance analysis.



Here, Heq(f) is equivalent IF filter which actually represents the cascade filtering characterstics of the RF, mixer, & IF sections of superheterodyne receiver.

s(t) is desired modulated carrier

w(t) or n(t) – represents a sample functions of white Gaussian noise process with the two sided spectral density of $N_0/2$.



AWGN → white - spectral density prou for all freq.
→ prob density function – Gaus
→ A → additive in nature.

not distribution
It is PSD of w(t).

$H_{eq}(f)$ to be an ideal narrowband, bandpass filter with passband between $f_c - W$ to $f_c + W$ for double side band modulation schemes (DSB-SC, AM).

For case of SSB, we take filter passband either between $f_c - W$ and f_c (LSB) or f_c & $f_c + W$ (USB).

→ The transmission bandwidth B_T is $2W$ for double side band modulation schemes, whereas it is W for the case of SSB. ~~Also for:~~

→ For present case, f_c represents the carrier frequency measured at the mixer output. that is $f_c = f_{IF}$.

The i/p to detector is $x(t) = s(t) + n(t)$, where $n(t)$ is the sample function of a bandlimited (NB) white noise process $N(t)$ with the PSD $S_N(f) = N_0/2$ over the passband of $H_{eq}(f)$.

Figure of merit

The performance of analog communication s/m are measured in terms of Signal-to-Noise Ratio.

we define two types of SNR.

(i) $(SNR)_o$ = The output signal to noise ratio is defined as

$$(SNR)_o = \frac{\text{Average power of message at receiver output}}{\text{Average noise power at the receiver o/p.}}$$

(ii) $(SNR)_{ref}$ = The input reference signal to noise ratio is defined as.

$$= \frac{[\text{Average power of modulated message signal at receiver input}]}{[\text{Average noise power in the message BW at receiver i/p}]}$$

$(SNR)_{r\,or\,I}$ can be viewed as the output signal-to-noise ratio which results from baseband or direct transmission of message without any modulation as shown in Fig. 2

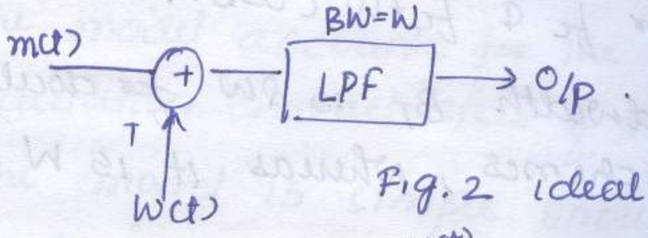


Fig. 2 ideal baseband receiver.

Here m(t) is baseband signal with the same power as modulated wave.

For purpose of comparing different modulation systems, we use the Figure of merit (FOM) defined as.

$$FOM = \frac{(SNR)_o}{(SNR)_{r\,or\,i}}$$

FOM as defined above provides a normalised $(SNR)_o$ performance of various modulation-demodulation schemes; larger value of FOM, better is noise performance of the given communication system.

→ ~~Let us~~ Before analyzing SNR, the expected output of ~~a~~ idealised detector for given input as ~~narrowband~~ narrowband signal quantized as below.

If $x(t)$ is real narrowband bandpass signal than it is expressed as

$$x(t) = \begin{cases} x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) \\ A(t)\cos[\omega_c t + \varphi(t)] \end{cases}$$

$x_I(t)$ → inphase
$x_Q(t)$ – quadrature } components.

$A(t)$ – envelope
$\varphi(t)$ – phase.

→ We analyze the performance of a coherent detector, envelope detector, phase detector and frequency detector. When signals such as $x(t)$ are given as input, outputs of the (idealized) detectors can be expressed

	$x(t)$ is i/p to an ideal	Detector o/p proportional to
1)	Coherent detector	$x_I(t)$.
2)	ED	$A(t)$.
3)	Phase det	$\varphi(t)$.
4)	Freq det	$\frac{1}{2\pi}\frac{d\varphi(t)}{dt}$

Here $x(t)$ — could be used to represent any of the four types of linear modulated signals (DSB-SC, SSB, VSB, AM).

or any one of type angle mod sig (PM or FM).

Components of linear and angle modulated signals

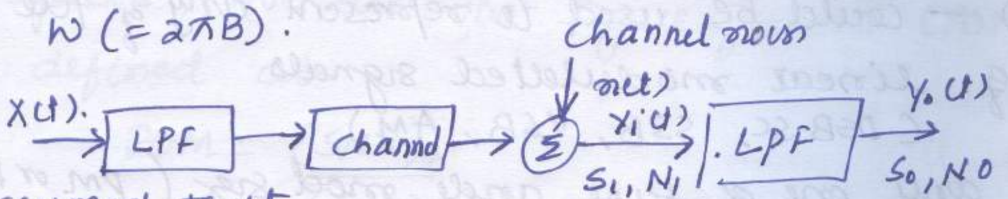
	Signal	$x_I(t)$	$x_Q(t)$	$A(t)$	$\varphi(t)$
1	DSB-SC $A_c m(t)\cos\omega_c t$	$A_c m(t)$	zero	$A_c\lvert m(t)\rvert$	0 $m(t)>0$ π $m(t)<0$
2	AM $A_c[1+\mu m(t)]\cos\omega_c t$ $A_c[1+\mu m(t)]\geq 0$	$A_c[1+\mu m(t)]$	zero	$A_c[1+\mu m(t)]$	zero
3	SSB $\frac{A_c}{2}m(t)\cos\omega_c t \pm \frac{A_c}{2}\hat{m}(t)\sin\omega_c t$	$\frac{A_c m(t)}{2}$	$\pm\frac{A_c \hat{m}(t)}{2}$	$\frac{A_c}{2}\sqrt{m^2(t)+\hat{m}^2(t)}$	$\tan^{-1}\left[\frac{-\hat{m}(t)}{m(t)}\right]$
5	Freq mod $A_c\cos[\omega_c t+\varphi(t)]$ $\varphi(t)=2\pi k_f\int_{-\infty}^{t} m(\tau)d\tau$	$A_c\cos\varphi(t)$	$A_c\sin\varphi(t)$	A_c	$2\pi k_f\int_{-\infty}^{t} m(\tau)d\tau$

SNR in Baseband comm s/ms.

↳ in baseband comm s/ms, signal is transmitted directly without modulation. The results obtained for baseband systems serve as basis for comparing with other systems.

Fig. 3 shows a simple analog baseband system. For a baseband system, the receiver is a low-pass filter that passes the message while reducing noise at the output. Obviously, the filter should reject all noise frequency components that fall outside the message band.

We assume that the LPF is ideal with bandwidth $W (= 2\pi B)$.



Assumed that

∧ message signal $X(t)$ is a zero-mean ergodic random process band limited to W with power spectral density $S_{XX}(W)$.

The channel is assumed to be distortionless over the message band so that

$\boxed{X_o(t) = X(t-t_d)}$ where t_d is the time delay of s/m

The average output signal power S_o is

$$S_o = E[X_o^2(t)] = E[X_o^2(t-t_d)] \quad E \to \text{Expectation.}$$

$$= \frac{1}{2\pi}\int_{-W}^{W} S_{XX}(W)\,dW = S_X = S_i$$

$\mu_x = E(x)$ → gives Average value

$\underbrace{\sigma_x^2 = E[x^2]}$ → called second moment → provides mean square value when $\mu_x = 0$

S_X → is average signal power

S_i → is the signal at I/P receiver ↳ these are statistical averages

$$E(x) = \int_{-\infty}^{\infty} x f(x)$$

↳ x is random variable

$f(x)$ → prob density function

$$\boxed{E[x^2] = \int_{-\infty}^{\infty} x^2 f(x).}$$

Average noise power N_0 is

$$N_0 = E[n_0^2(t)] = \frac{1}{2\pi}\int_{-W}^{W} S_{nn}(w)\, dw \qquad S_{nn}(w) = \frac{\eta_0}{2}$$

$$= \frac{1}{2\pi}\int_{-W}^{W} \eta/2 \, dw - \eta \frac{W}{2\pi} = \eta B \quad \text{BW=}$$

The white noise assumption simplifies the calculations & provides essential aspects of analysis.

The output signal to noise ratio is

$$\left[\frac{S}{N}\right]_0 = \frac{S_0}{N_0} = \frac{S_i}{\eta B}.$$

<u>Ex:-</u> consider an analog baseband communication s/m with additive noise. The transmission channel is assumed to be distortionless. and psd of white noise $\eta/2$ is 10^{-9} watt per hertz (W/Hz). The signal to be transmitted is an audio signal with 4-KHz bandwidth. At the receiver end, an RC low-pass filter with a 3-dB Bandwidth of 8 kHz is used to ~~transmit~~ limit the noise power at the output. calculate the output noise power.

<u>Sol:-</u>

WKT → frequency response $H(\omega)$ of RC Low-pass filter with 3-dB BW of 8KHz is given by

$$H(\omega) = \frac{1}{1+j\omega/\omega_0} \qquad \boxed{\omega_0 = \frac{1}{RC}} \text{ — cut-off freq}$$

$$\omega_0 = 2\pi \times 8 \times 1000$$

$$N_0 = E[n_0^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \eta/2\, |H(\omega)|^2\, d\omega$$

~~property~~
~~If $R_x(\tau)$ is Auto Correlation function~~

important property

If $R_X(\tau)$ is autocorrelation of wide-sense stationary random process $X(t)$.

Then $R_X(\tau) = E[x(t)\, x(t+\tau)]$

τ ↳ which is ~~diffn~~ function of delay parameter or scanning parameter

then Fourier transform of $R_X(\tau)$ provides power spectral density PSD ⇒

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \, \mathrm{Exp}(-j 2\pi f \tau) d\tau.$$

Transmission of Random process $X(t)$ through linear filter $h(t)$ is related as

$$X(t) \longrightarrow \boxed{\begin{matrix}\text{impulse resp} \\ h(t)\end{matrix}} \longrightarrow Y(t)$$

$$\boxed{Y(t) = X(t) * h(t).}$$

$R_X(\tau)$ ↓ PSD → $S_X(\tau)$.

$H(f)$ ↓ $|H(f)|^2$

$R_Y(\tau)$ ↓ $S_Y(f)$.

$$\boxed{S_Y(f) = \int_{-\infty}^{\infty} |H(f)|^2 S_X(f) \, df.}$$

$$= \frac{\eta}{2} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{1 + (\omega/\omega_0)^2} d\omega.$$

$$\boxed{\frac{1}{4} \eta \omega_0 = \frac{1}{4} \times 2 \times 10^{-9} (2\pi) \times (8) \times 10^{3} = 25.24 W.}$$

Exp: 2

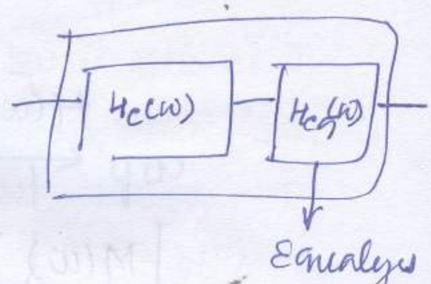
consider an analog baseband communication system with additive white noise having power spectral density $\eta/2$ and distorting channel having frequency response

$$H_c(\omega) = \frac{1}{1 + j\omega/W}$$

Sol:-

The distortion is eqnalized by a receiver filter having frequency response.

$$H_{eq}(\omega) = \begin{cases} \frac{1}{H_c(\omega)} & 0 \le |\omega| \le W \\ 0 & \text{otherwise.} \end{cases}$$



obtain expression for o/p SNR

$$S_o = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H_c(\omega)|^2 |H_{eq}(\omega)|^2 S_{xx}(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-W}^{W} S_{xx}(\omega)\, d\omega = S_x$$

$$N_o = \frac{1}{2\pi} \int_{-\infty}^{\infty} \eta/2\, |H_{eq}(\omega)|^2 d\omega$$

$$= \frac{\eta}{2\pi} \int_0^W \left[1 + \left(\frac{\omega}{W}\right)^2\right] d\omega = \frac{\eta}{2\pi}\left(\frac{4}{3}W\right) = \frac{4}{3}\eta\beta$$

$$\left[\frac{S}{N}\right]_o = \frac{S_o}{N_o} = \frac{S_x}{\frac{4}{3}\eta B} = \frac{3}{4}\frac{S_x}{\eta B}$$

Ex:- 3

A signal $m(t) = A_c \cos w_c t$ is corrupted by additive white gaussian noise $n(t)$ with zero mean & psd $= \frac{N_0}{2}$ W/rad/sec. Find an expression for output SNR after the signal $m(t) + n(t)$ is applied to an LTI filter with impulse response $h(t) = e^{-t} u(t)$

output signal power $= \int_{-\infty}^{\infty} |M(\omega)|^2 |H(\omega)|^2 d\omega.$

$H(\omega) = \frac{1}{1+j\omega} \Rightarrow |H(\omega)|^2 = \frac{1}{1+\omega^2}$

$|M(\omega)| = A_c \pi [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$

O/P signal power is given as

$S_x(\omega) |H(\omega)|^2 = A_c^2 \pi^2 \left[\frac{1}{1+w_c^2} + \frac{1}{1+w_c^2} \right]$

$= \frac{2 A_c^2 \pi^2}{1+w_c^2}$

output noise power $= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{N_0}{2} \frac{1}{1+w^2} d\omega$

$= \frac{N_0}{4}$

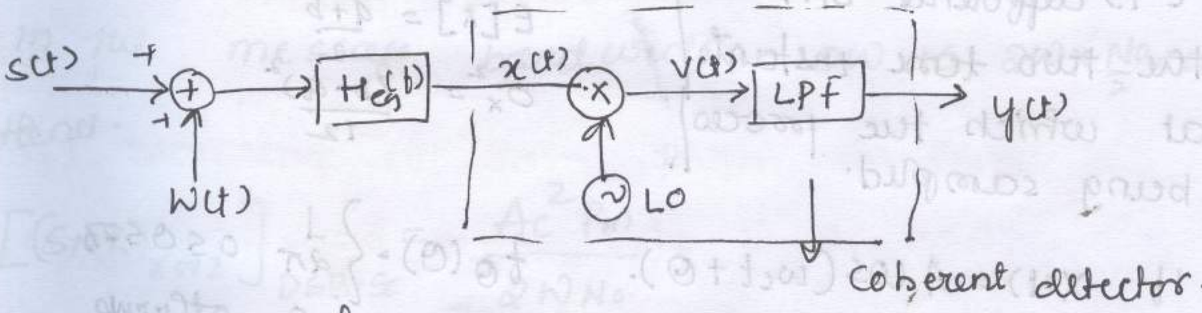
$(SNR)_0 = \frac{8 A_c^2 \pi^2}{(1+w_c^2) N_0}.$

Noise performance in coherent demodulation

DSB-SC

The receiver model for coherent detection of DSB-SC signals is shown in Fig. 4. The DSB-SC signal is
$s(t) = A_c m(t) \cos \omega_c t$

Let us assume $m(t)$ to be sample function of a WSS process $M(t)$ with power spectral density $S_M(f)$, limited to $\pm W$ Hz.



Coherent Detection of DSB-SC.

The carrier, $A_c \cos(\omega_c t)$, which is independent of the message $m(t)$ is actually a sample function of the process $A_c \cos(\omega_c t + \Theta)$ where Θ is a random variable, uniformly distributed in the interval 0 to 2π.

With a random phase added to carrier term, $R_s(\tau)$
The autocorrelation function of process $s(t)$ is given by.

$R_s(\tau) = \frac{A_c^2}{2} R_M(\tau) \cos(\omega_c \tau)$.

$R_M(\tau)$. $\rightarrow$ is autocorrelation function of message process.

$\rightarrow$ carrier case $\rightarrow$ $\cos(\omega_c t + \Theta)$. $\rightarrow$ $\Theta \rightarrow$ is random variable having uniform dist

ACF of a random process $X(t)$ is a function of 2 variables $R_X(t_k, t_i)$ t_k & t_i is given by

$$R_X(t_k, t_i) = E[\underbrace{X(t_k)}_{x}\underbrace{X(t_i)}_{y}]$$

$$R_X(t_k, t_i) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{X_k X_i}(x,y)\,dx\,dy$$

For case of wide sense stationary

$m_X(t) = \mu_X$ = mean

&

$R_X(t_k, t_i) = R_X(t_k - t_i)$.

$R_X(\tau) = E[X(t+\tau)X(t)]$

τ is difference btn the two time instants at which the process being sampled.

If $X(t) = A\cos(\omega_c t + \Theta)$.

$R_X(t_1, t_2)$

$$= E[A\cos(\omega_c t_1 + \Theta)\, A\cos(\omega_c t_2 + \Theta)]$$

$$= \frac{A^2}{2}\{E[\cos(\omega_c(t_1+t_2)+2\Theta)] + \cos(\omega_c(\underbrace{t_2-t_1}_{\tau})\}$$

$E[\cos(\omega_c(t_1+t_2)+2\Theta)] \rightarrow 0$

$E[2] \rightarrow 2$

expectation of constant is constant itself

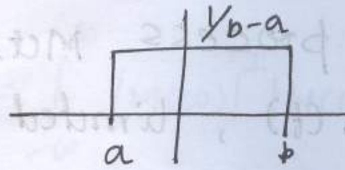
$$= \boxed{\frac{A^2}{2}\cos\omega_c\tau.}$$

1) uniform distribution

A random variable X is said to be uniformly distributed in the interval $a \le x \le b$ if

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{Elsewhere} \end{cases}$$

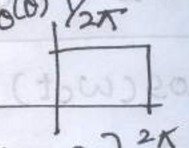
A plot of $f_X(x)$ is shown



$E[X] = \frac{a+b}{2}$

$\sigma_X^2 = \frac{(b-a)^2}{12}$

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi} & 0 \le \theta \le 2\pi \\ 0, & \text{otherwise} \end{cases}$$



this is only funda about ACF & Uniform distribution

Fourier transform of $R_S(\tau)$ yields $S_S(f)$ given by. $R_S(\tau) = \frac{A_c^2}{2}\underbrace{R_m(\tau)\cos\omega_c\tau}$

$$S_S(f) = \frac{A_c^2}{4}[S_M(f-f_c) + S_M(f+f_c)]$$

Let P_M denotes the message power, where.

$$P_M = \int_{-\infty}^{\infty} S_M(f)\,df = \int_{-W}^{W} S_M(f)\,df.$$

then $\int_{-\infty}^{\infty} S_S(f)\,df = 2\,\frac{A_c^2}{4}\int_{-W}^{f_c+W} S_M(f-f_c)\,df = \frac{A_c^2 P_M}{2}$

That is, average power modulated signal $s(t)$. is $\frac{A_c^2 P_M}{2}$. With the (two sided) noise power spectral density of $\frac{N_0}{2}$, the average noise power in the message bandwidth $2W$ is $2W \times \frac{N_0}{2} = WN_0$

Hence.

$$\left[(SNR)_{r\,or\,I}\right]_{DSB\text{-}SC} = \frac{A_c^2 P_M}{2WN_0}.$$

So to find FOM - we require $(SNR)_o$. The input to the detector is $x(t) = s(t) + n(t)$.

Where $n(t)$ is narrow band noise quantity.

So $n(t)$ is expressed as inphase and quadrature component.

$x(t) = A_c m(t)\cos\omega_c t + n_I(t)\cos\omega_c t - n_s \sin(\omega_c t)$

Assuming that the LO output is $\cos(\omega_c t)$. the output $v(t)$ of the multiplier in detector is given by.

$$v(t) = \frac{1}{2}A_c m(t) + \frac{1}{2}n_I(t) + \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos(2\omega_c t) - \frac{1}{2}n_s(t) A_c \sin(2\omega_c t)$$

As LPF rejects the spectral components centered around $2f_c$, we have

So we have

$$\boxed{y(t) = \frac{1}{2} A_c m(t) + \frac{1}{2} n_I(t)}$$

We observe that,

1> signal & noise which are additive at i/p to detector are additive even at the output of the detector.

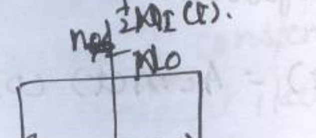
ii> coherent detector completely rejects the quadrature component $n_Q(t)$.

iii> If noise spectral density is flat at the detector input over the passband (f_c+W, f_c-W) then it is flat over passband $(-W, W)$ at the detector output

[$n_I(t)$ has flat spectrum in the range $-W$ to W]

→ As message component at the o/p is $\frac{1}{2} A_c m(t)$. the average message power at the output is

$\frac{A_c^2}{4} P_m$



→ As spectral density of the in-phase noise component is N_0 for $|f| \le W$, the average noise power at the receiver output is → $\left(\frac{1}{2}\right)^2 \times 2 W N_0 = \frac{W N_0}{2}$.

$$\left[(SNR)_0\right]_{DSB\text{-}SC} = \frac{(A_c^2/4) P_m}{W N_0/2}$$

$$SNR_O = \frac{A_c^2 P_M}{2WN_0}$$ we obtain

$$[FOM]_{DSB\text{-}SC} = \frac{(SNR)_o}{(SNR)_{ref}} = 1$$

<u>For case</u> – SSB modulation.

Assuming that LSB has been transmitted, we can write $s(t)$ as follows.

$$s(t) = \frac{A_c}{2} m(t)\cos(\omega_c t) + \frac{A_c}{2}\hat{m}(t)\sin(\omega_c t)$$

where $\hat{m}(t)$ is HT of $m(t)$.

$$s(t) = \frac{A_c}{2} M(t)\cos(\omega_c t) + \frac{A_c}{2}\hat{M}(t)\sin(\omega_c t)$$

We can show that the generalized case autocorrelation function of $s(t)$, $R_s(\tau)$ is given by

$$R_s(\tau) = \frac{A_c^2}{4}\left[R_M(\tau)\cos(\omega_c\tau) + \hat{R}_M(\tau)\sin(\omega_c\tau)\right]$$

$\rightarrow$ <u>from ACF property</u>

$\hat{R}_m(t)$ is the Hilbert transform of $R_M(t)$.

Hence the average signal power

$$R_s(0) = \frac{A_c^2}{4} P_m \rightarrow \text{mean square value}$$

$$\& \ (SNR)_r = \frac{A_c^2 P_M}{4WN_0}.$$

property of ACF

1) The mean square value of the random process may be obtained from $R_X(\tau)$ - ACF simply putting $\tau = 0$.

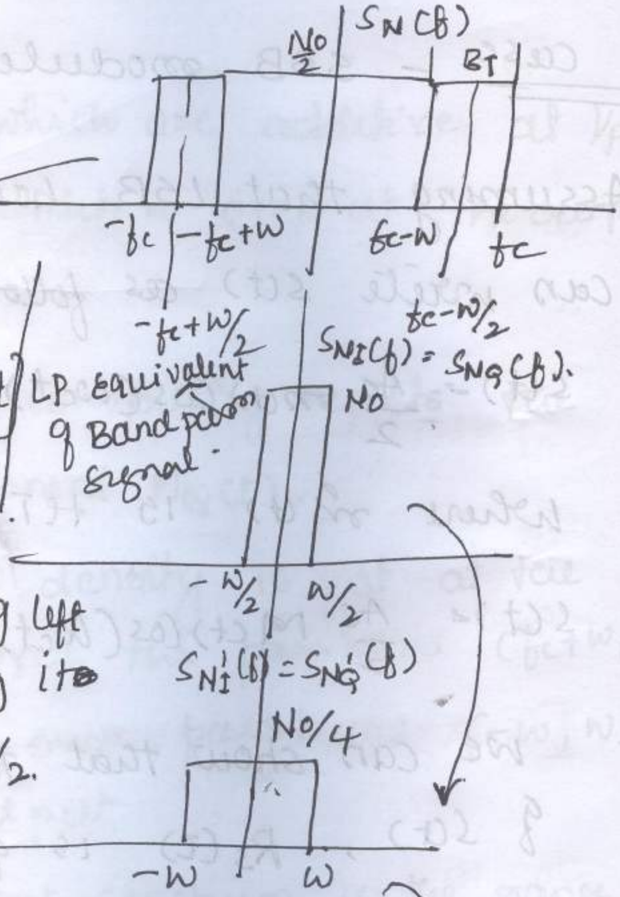
$$R_X(0) = E[X^2(t)]$$

$$R_X(\tau) = E[x(t)x(t)]$$

Let $n(t) = n_I(t)\cos w_c t - n_Q(t)\sin w_c t$.

(w.r.t f_c, $n(t)$ does not have locally symmetric spectrum).

For SSB modulation



$n(t) =$

$$\underbrace{n_I(t)\cos[2\pi(f_c - W/2)t]}_{\downarrow\; n_I'(t)} - n_Q(t)\,\underbrace{\sin[2\pi(f_c - W/2)t]}_{\downarrow\; n_Q'(t)}$$

↳ obtained by shifting left by $W/2$ & shifting it right by $W/2$ and adding the spectra

$$y(t) = \frac{1}{4}A_c m(t) + \frac{1}{2}n_I'(t).$$

↓ O/p signal power $\rightarrow \frac{A_c^2 P_M}{16}$

↳ noise power $= \left(\frac{1}{4}\right)^2 \times \frac{N_0}{\cancel{2}} \cancel{2}W$

$= \frac{W N_0}{4}$

$$(SNR)_{0,SSB} = \frac{A_c^2 P_M}{16} \times \frac{4}{W N_0} = \frac{A_c^2 P_M}{4 W N_0}$$

$$(FOM)_{SSB} = \frac{(SNR)_0}{(SNR)_r} = 1$$

<u>Under synchronous detection</u>, SNR performance of DSB-SC & SSB and baseband are identical when both s/ms operate with the same ~~digital~~ signal to noise ratio at i/p of their detectors

noise performance in AM – Envelope detectors

AM signals are normally envelope detected, though coherent detection can also be used for message recovery. This is mainly because envelope detector is simpler to implement as compared to coherent detector.

The transmitted signal $s(t)$ is given by

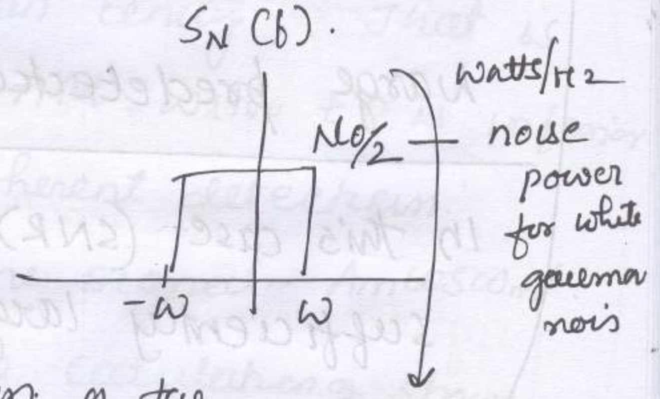
$$s(t) = A_c[1 + \mu m(t)] \cos \omega_c t$$

The average signal power in $s(t)$

$$= \frac{A_c^2[1 + \mu^2 P_M]}{2}, \text{ hence}$$

$(SNR)_r$, DSB-LC or AM given as follows

$$= \frac{A_c^2[1 + \mu^2 P_M]}{2 W N_0}$$



$\frac{N_0}{2} \times 2W = WN_0$

~~Using the inphase.~~

Using the inphase and quadrature component descriptions of the narrowband noise, the quantity at envelope detector input, $x(t)$, can be written as

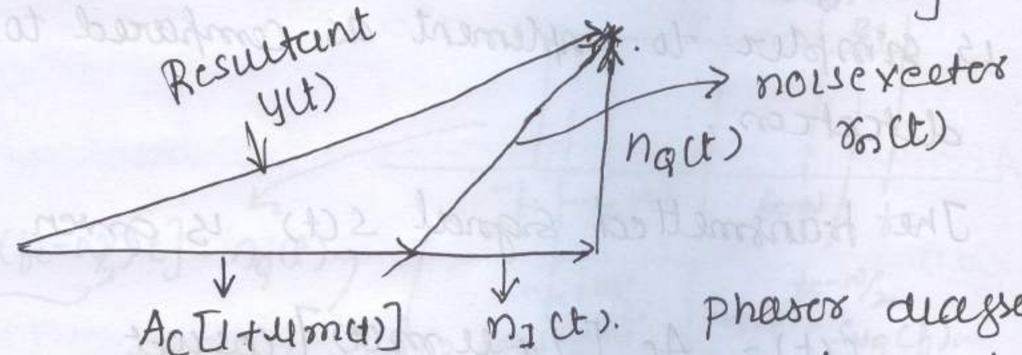
$$x(t) = s(t) + n_I(t) \cos \omega_c t - n_Q(t) \sin \omega_c t$$

$$= [A_c + A_c \mu m(t) + n_I(t)] \cos \omega_c t - n_Q(t) \sin \omega_c t$$

↳ this can be shown in phasors

The receiver output $y(t)$ is Envelope of the i/p quantity $x(t)$. That is

$$y(t) = \{[A_c + A_c \mu m(t) + n_I(t)]^2 + n_Q^2(t)\}^{1/2}$$



Phasor diagram analyze the ED

→ So noise performance of ED is analyzed for two different cases namely.

(1) Large SNR at detector i/p (input)
(2) Weak SNR at " i/p "

Large predetection SNR

In this case (SNR) at input to detector is sufficiently large, we can approximate y(t) as

$$y(t) \approx \underbrace{A_c + A_c \mu m(t) + n_I(t)}_{\text{three quantities.}} \quad \text{neglect } n_Q(t)$$

→ A_c → DC term due to transmitted carrier

→ $A_c \mu m(t)$ – proporational to the message

→ $n_I(t)$ – inphase noise component

in the final o/p DC is blocked. Hence Average signal power at the o/p is given by $A_c^2 \mu^2 P_m$.

o/p noise power is equal to $- 2WN_0$.

$$\left[(SNR)_0\right]_{AM} = \frac{A_c^2 \mu^2 P_m}{2WN_0}$$

It is to be noted that signal & noise are additive at detector output and power spectral density of the output noise is flat over the message bandwidth. from previous eqns. $(SNR)_i$ & $(SNR)_0$.

$$\boxed{FOM = \frac{\mu^2 P_m}{1+\mu^2 P_m}}$$

P_m → Avg message power.

The FOM of ED is less than unity. That is noise performance of ~~DSB~~ AM with ED is inferior to that of DSB-SC with coherent detection. Assuming $m(t)$ to be tone signal - $A_m \cos \omega_m t$. & $\boxed{\mu' = \mu A_m}$ - simple calculation shows that $(FOM)_{Am}$ is $\rightarrow \frac{\mu'^2}{2+\mu'^2}$

With maximum value $\mu = 1$, we find that the $(FOM)_{Am}$ is $\frac{1}{3}$. That is other factors being equal, AM has to transmit three times as much power as DSB-SC, to achieve the same quality of noise performance of course, ~~this is the price one has to pay~~

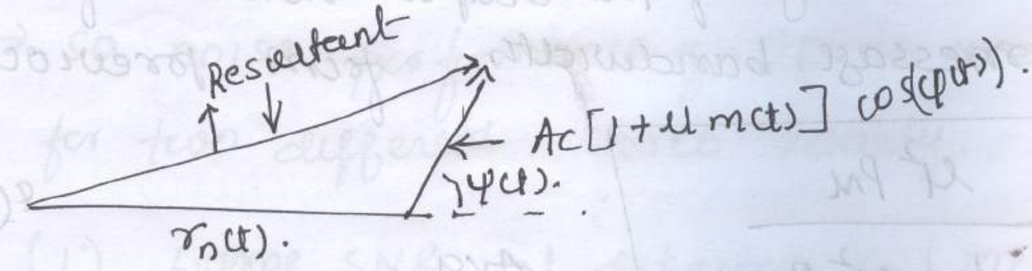
This is price one has to pay for simplicity in receiver.

ii) case – weak predetection SNR

noise term dominates.

Let from phasor diagram → $n(t) = r_n(t)\cos[w_c t + \psi$

→ construing the phasor diagram using $r_n(t)$ as the reference phasor fig

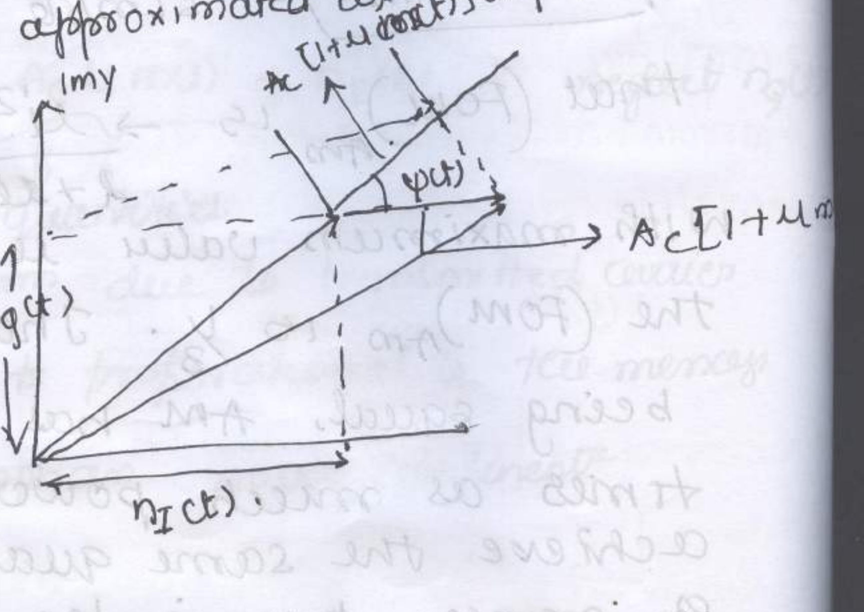


Envelope detector output can be ~~expressed as~~ approximated as

$$y(t) \simeq r_n(t) + A_c\cos[\varphi(t)] + A_c\mu m(t)\cos[\varphi(t)]$$

because assumed $(S/N)_c << 1$. so ED resultant signal is primary dominated by Envelope of noise signal. so it is approximated as

how it is arrived



so in approximated y(t).

there is no term strickly proporational to m(t).

the last term in the y(t) contains the message signal m(t) – multiplied by the noise quantity cos(ψ(t)), → which is random; that is the message signal is helplessly ~~~~ mutilated or deteriorated beyond any hope of signal recovery.

Also it is to be noted that signal and noise are no longer additive at detector output. As such, $(SNR)_0$ is not meaningful.

→ The loss of message at Low SNR is called.

<u>threshold Effect</u> : (same Effect found in FM)

That is there is some value ~~at~~ of input SNR, above which the Envelope detector operates satisfactorily whereas if the input SNR falls below this value, performance of the detector deteriorates quite rapidly. Actually threshold is not a unique point and we may have to use some reasonable criterion in arriving it.

Let R denote the random variable obtained by observing the process R(t) (of which r(t) is sample function) at some fixed point in time

It is quite reasonable to assume that the detector is operating well into the threshold region if $P[R \geq A_c] \geq 0.5$; where as, if the above probability is 0.01 or less, the detector performance is quite satisfactory.

Than one term new term can be defined

<u>Carrier-to-noise ratio</u>

$$\rho = \frac{\text{average carrier power}}{\text{average noise power in transmission Bandwidth}} = \frac{A_c^2/2}{2WN_0} = \frac{A_c^2}{4WN_0}$$

→ So we shall now compute the threshold SNR in terms of ρ defined above. As R is Rayleigh variable, we have (see background material)

$$f_R(r) = \frac{r}{\sigma_N^2} e^{-r^2/2\sigma_N^2} \longrightarrow$$ one of type of distribution as gaussian but it is always +ve side.

where $\sigma_N^2 = 2WN_0$

$$P[R \geq A_c] = \int_{A_c}^{\infty} f_R(r)\,dr$$

$$= e^{-A_c^2/4WN_0}$$

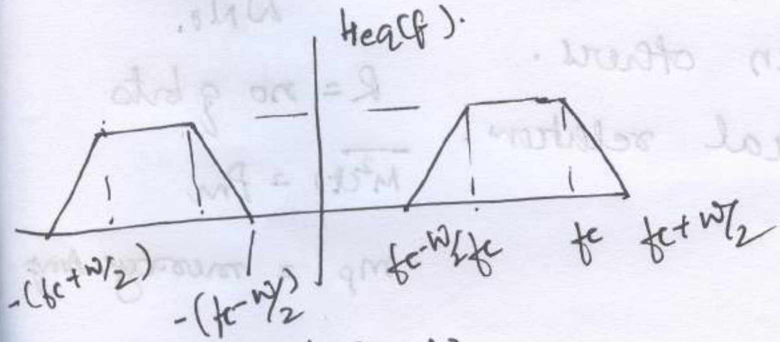
$$= e^{-\rho}$$

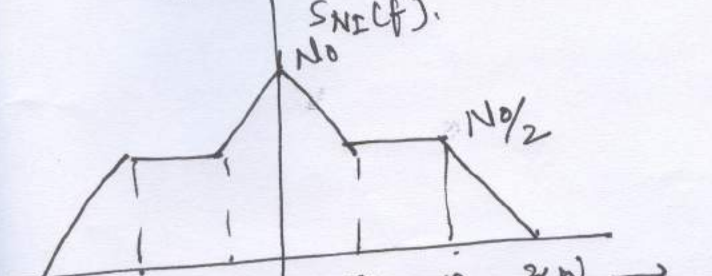
solving for ρ from $e^{-\rho} = 0.5$, we get $\rho = \ln 2 \doteq 0.6$

Similarly, from the condition $P[R \geq A_c] = 0.001$ $\doteq -1.6$

From above calculations we state that if $\rho \leq -1.6$ dB the receiver performance is controlled by noise & hence its output is not acceptable whereas for $\rho > 6.6$ dB the Effect noise and hence its output is not acceptable, whereas for $\rho \geq 6.6$ dB, the effect of noise is not deleterious, However, reasonable intelligibility and naturalness in voice reception requires a post detection SNR about 25 dB. That is satisfactory reception, we require a value of ρ much greater than what is indicated by threshold consideration, In other words, additive noise makes the signal quality unacceptable long before multiplicative noise mutilates it. Hence threshold effect is usually not a serious limit in AM transmission

EX: In receiver meant for demodulation of SSB signals Heq(f) has characteristics shown in Fig. Assuming that USB has been transmitted, Find FOM of the system





$S_{NI}(f)$

For SSB ~~mod~~ with coherent demod.

signal quantity at o/p $= \frac{A_c}{4} m(t)$

noise quantity at o/p $= \frac{n_I(t)}{2}$

o/p noise power $= \frac{1}{4}\int_{-W}^{W} S_{N_c}\, df$

$= \frac{5}{16} N_0 W$

$(SNR)_0 = \left(\frac{A_c^2 P_m}{16}\right) \Big/ \frac{5}{16} N_0 W$

$(SNR)_0 = \frac{A_c^2 P_m}{5 N_0 W}$ $\qquad (SNR)_r = \frac{A_c^2 P_m}{4 W N_0}$

$FOM = \frac{(SNR)_0}{(SNR)_r} = 4/5 = 0.8$

For case Frequency modulation

$$(FOM)_{FM} = \frac{3 k_f^2 P_M}{W^2}$$ → proporational frequency devia Δf.

For the case of single tone modulation it can be shown that

$$FOM = \frac{3}{2} \beta^2$$ β → mod index of fm

↳ by this FM performance will always be >> at Expense of Bandwidth. but in this threshold Effect has to be taken care. it much influence noise performance.

For the cases of PCM s/m.

$$(SNR)_0 = \frac{3L^2}{1+4(L^2-1)Q(\sqrt{\gamma/R})} \left(\frac{P_M}{m_p^2}\right).$$

$L = 2^R$.

$\gamma = \frac{S_r}{WN_0}$.

R = no of bits

$\overline{M^2(t)} = P_M$

m_p = message

↳ much superior than others.

because of Exponential relation.