$$\frac{J(\hat{y}+eps,y)-J(\hat{y}-eps,y)}{(\hat{y}+eps)-(\hat{y}-eps)} = \frac{[(\hat{y}+eps)-y]^2-[(\hat{y}-eps)-y]^2}{(\hat{y}+eps)+(-\hat{y}+eps)}$$

$$= \frac{[(\hat{y}+eps)^2-2y(\hat{y}+eps)+y^2]-[(\hat{y}-eps)^2-2y(\hat{y}-eps)+y^2]}{(\hat{y}-\hat{y})+(eps+eps)}$$

$$= \frac{[\hat{y}^2+2\hat{y}(eps)+(eps)^2-2y(\hat{y}+eps)+y^2]-[\hat{y}^2-2\hat{y}(eps)+(eps)^2-2y(\hat{y}-eps)+y^2]}{2*(eps)}$$

$$= \frac{[\hat{y}^2+2\hat{y}(eps)+(eps)^2-2y(\hat{y}+eps)+y^2]+[-\hat{y}^2+2\hat{y}(eps)+-(eps)^2+2y(\hat{y}-eps)+-y^2]}{2*(eps)}$$

$$= \frac{(\hat{y}^2-\hat{y}^2)+[2\hat{y}(eps)+2\hat{y}(eps)]+[(eps)^2-(eps)^2]+[-2y(\hat{y}+eps)+2y(\hat{y}+eps)]+[y^2-y^2]}{2*(eps)}$$

$$= \frac{4\hat{y}(eps)+[-2y(\hat{y}+eps)+2y(\hat{y}-eps)]}{2*(eps)} = \frac{4\hat{y}(eps)+[-2y(\hat{y})-2y(eps)]+[2y(\hat{y})-2y(eps)]}{2*(eps)}$$

$$= \frac{4\hat{y}(eps)+[-2y(\hat{y})+2y(\hat{y})]+[-2y(eps)-2y(eps)]}{2*(eps)} = \frac{4\hat{y}(eps)-4y(eps)}{2*(eps)} = \frac{4[\hat{y}(eps)-(y(eps)]}{2*(eps)}$$

$$= \frac{2[\hat{y}(eps)-y(eps)]}{eps} = \frac{2(eps)[\hat{y}-y]}{eps} = 2(\hat{y}-y)$$

1