$$\frac{J(\hat{y}+\epsilon,y)-J(\hat{y}-\epsilon,y)}{(\hat{y}+\epsilon)-(\hat{y}-\epsilon)} = \frac{[(\hat{y}+\epsilon)-y]^2-[(\hat{y}-\epsilon)-y]^2}{(\hat{y}+\epsilon)+(-\hat{y}+\epsilon)}$$

$$= \frac{[(\hat{y}+\epsilon)^2-2y(\hat{y}+\epsilon)+y^2]-[(\hat{y}-\epsilon)^2-2y(\hat{y}-\epsilon)+y^2]}{(\hat{y}-\hat{y})+(\epsilon+\epsilon)}$$

$$= \frac{[\hat{y}^2+2\hat{y}\epsilon+\epsilon^2-2y(\hat{y}+\epsilon)+y^2]-[\hat{y}^2-2\hat{y}\epsilon+\epsilon^2-2y(\hat{y}-\epsilon)+y^2]}{2\epsilon}$$

$$= \frac{[\hat{y}^2+2\hat{y}\epsilon+\epsilon^2-2y(\hat{y}+\epsilon)+y^2]+[-\hat{y}^2+2\hat{y}\epsilon-\epsilon^2+2y(\hat{y}-\epsilon)-y^2]}{2\epsilon}$$

$$= \frac{(\hat{y}^2-\hat{y}^2)+[2\hat{y}\epsilon+2\hat{y}\epsilon]+[\epsilon^2-\epsilon^2]+[-2y(\hat{y}+\epsilon)+2y(\hat{y}+\epsilon)]+[y^2-y^2]}{2\epsilon}$$

$$= \frac{4\hat{y}\epsilon+[-2y(\hat{y}+\epsilon)+2y(\hat{y}-\epsilon)]}{2\epsilon} = \frac{4\hat{y}\epsilon+(-2y\hat{y}-2y\epsilon)+(2y\hat{y}-2y\epsilon)}{2\epsilon}$$

$$= \frac{4\hat{y}\epsilon+(-2y\hat{y}+2y\hat{y})+(-2y\epsilon-2y\epsilon)}{2\epsilon} = \frac{4\hat{y}\epsilon-4y\epsilon}{2\epsilon} = \frac{4(\hat{y}\epsilon-y\epsilon)}{2\epsilon}$$

$$= \frac{2(\hat{y}\epsilon-y\epsilon)}{\epsilon} = \frac{2\epsilon(\hat{y}-y)}{\epsilon} = 2(\hat{y}-y)$$