$$MAE = \begin{cases} -(\hat{y} - y), & \text{if } \hat{y} < y \\ \hat{y} - y, & \text{if } \hat{y} > y \\ 0, & \text{if } \hat{y} = y \end{cases}$$

When ŷ < y:

$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon) - (\hat{y}-\epsilon)} = \frac{-[(\hat{y}+\epsilon) - y] - [-[(\hat{y}-\epsilon) - y]]}{(\hat{y}+\epsilon) + (-\hat{y}+\epsilon)}$$

$$= \frac{[-(\hat{y}+\epsilon) + y] + [(\hat{y}-\epsilon) - y]}{(\hat{y}-\hat{y}) + (\epsilon+\epsilon)} = \frac{(-\hat{y} - \epsilon + y) + (\hat{y} - \epsilon - y)}{2\epsilon}$$

$$= \frac{(-\hat{y}+\hat{y}) + (-\epsilon-\epsilon) + (y-y)}{2\epsilon} = \frac{-2\epsilon}{2\epsilon} = -1$$

When ŷ > y:

$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon) - (\hat{y}-\epsilon)} = \frac{[(\hat{y}+\epsilon) - y] - [(\hat{y}-\epsilon) - y]}{(\hat{y}+\epsilon) + (-\hat{y}+\epsilon)}$$

$$= \frac{[(\hat{y}+\epsilon) - y] + [-(\hat{y}-\epsilon) + y]}{(\hat{y}-\hat{y}) + (\epsilon+\epsilon)} = \frac{(\hat{y} + \epsilon - y) + (-\hat{y} + \epsilon + y)}{2\epsilon}$$

$$= \frac{(\hat{y}-\hat{y}) + (\epsilon+\epsilon) + (-y+y)}{2\epsilon} = \frac{2\epsilon}{2\epsilon} = 1$$

When ŷ = y:

$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon) - (\hat{y}-\epsilon)} = \frac{0 - 0}{(\hat{y}+\epsilon) + (-\hat{y}+\epsilon)}$$

$$= \frac{0}{(\hat{y}-\hat{y}) + (\epsilon+\epsilon)} = \frac{0}{2\epsilon} = \frac{0}{2(0)} = \frac{0}{0} = \text{undefined!}$$

$$\frac{\partial MAE}{\partial \hat{y}} = \begin{cases} -1, & \text{if } \hat{y} < y \\ 1, & \text{if } \hat{y} > y \\ 0, & \text{if } \hat{y} = y \end{cases}$$