$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon) - (\hat{y}-\epsilon)} = \frac{[(\hat{y}+\epsilon) - y]^2 - [(\hat{y}-\epsilon) - y]^2}{(\hat{y}+\epsilon) + (-\hat{y}+\epsilon)}$$

$$= \frac{[(\hat{y}+\epsilon)^2 - 2y(\hat{y}+\epsilon) + y^2] - [(\hat{y}-\epsilon)^2 - 2y(\hat{y}-\epsilon) + y^2]}{(\hat{y}-\hat{y}) + (\epsilon+\epsilon)}$$

$$= \frac{[\hat{y}^2 + 2\hat{y}(\epsilon) + (\epsilon)^2 - 2y(\hat{y}+\epsilon) + y^2] - [\hat{y}^2 - 2\hat{y}(\epsilon) + (\epsilon)^2 - 2y(\hat{y}-\epsilon) + y^2]}{2 * (\epsilon)}$$

$$= \frac{[\hat{y}^2 + 2\hat{y}(\epsilon) + (\epsilon)^2 - 2y(\hat{y}+\epsilon) + y^2] + [-\hat{y}^2 + 2\hat{y}(\epsilon) - (\epsilon)^2 + 2y(\hat{y}-\epsilon) - y^2]}{2 * (\epsilon)}$$

$$= \frac{(\hat{y}^2 - \hat{y}^2) + [2\hat{y}(\epsilon) + 2\hat{y}(\epsilon)] + [(\epsilon)^2 - (\epsilon)^2] + [-2y(\hat{y}+\epsilon) + 2y(\hat{y}+\epsilon)] + [y^2 - y^2]}{2 * (\epsilon)}$$

$$= \frac{4\hat{y}(\epsilon) + [-2y(\hat{y}+\epsilon) + 2y(\hat{y}-\epsilon)]}{2 * (\epsilon)} = \frac{4\hat{y}(\epsilon) + [-2y(\hat{y}) - 2y(\epsilon)] + [2y(\hat{y}) - 2y(\epsilon)]}{2 * (\epsilon)}$$

$$= \frac{4\hat{y}(\epsilon) + [-2y(\hat{y}) + 2y(\hat{y})] + [-2y(\epsilon) - 2y(\epsilon)]}{2 * (\epsilon)} = \frac{4\hat{y}(\epsilon) - 4y(\epsilon)}{2 * (\epsilon)} = \frac{4[\hat{y}(\epsilon) - (y(\epsilon)]}{2 * (\epsilon)}$$

$$= \frac{2[\hat{y}(\epsilon) - y(\epsilon)]}{\epsilon} = \frac{2(\epsilon)[\hat{y} - y]}{\epsilon} = 2(\hat{y} - y)$$