$$MAE = \begin{cases} -(\hat{y}-y), & \text{if } \hat{y} < y \\ \hat{y}-y, & \text{if } \hat{y} > y \\ 0, & \text{if } \hat{y} = y \end{cases}$$

When ŷ < y:

$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon)-(\hat{y}-\epsilon)} = \frac{-[(\hat{y}+\epsilon)-y] - [-[(\hat{y}-\epsilon)-y]]}{(\hat{y}+\epsilon)+(-\hat{y}+\epsilon)}$$

$$= \frac{[-(\hat{y}+\epsilon)+y]+[(\hat{y}-\epsilon)-y]}{(\hat{y}-\hat{y})+(\epsilon+\epsilon)} = \frac{(-\hat{y}-\epsilon+y)+(\hat{y}-\epsilon-y)}{2\epsilon}$$

$$= \frac{(-\hat{y}+\hat{y})+(-\epsilon-\epsilon)+(y-y)}{2\epsilon} = \frac{-2\epsilon}{2\epsilon} = -1$$

When ŷ > y:

$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon)-(\hat{y}-\epsilon)} = \frac{[(\hat{y}+\epsilon)-y] - [(\hat{y}-\epsilon)-y]}{(\hat{y}+\epsilon)+(-\hat{y}+\epsilon)}$$

$$= \frac{[(\hat{y}+\epsilon)-y]+[-(\hat{y}-\epsilon)+y]}{(\hat{y}-\hat{y})+(\epsilon+\epsilon)} = \frac{(\hat{y}+\epsilon-y)+(-\hat{y}+\epsilon+y)}{2\epsilon}$$

$$= \frac{(\hat{y}-\hat{y})+(\epsilon+\epsilon)+(-y+y)}{2\epsilon} = \frac{2\epsilon}{2\epsilon} = 1$$

When ŷ = y:

$$\frac{J(\hat{y}+\epsilon, y) - J(\hat{y}-\epsilon, y)}{(\hat{y}+\epsilon)-(\hat{y}-\epsilon)} = \frac{0-0}{(\hat{y}+\epsilon)+(-\hat{y}+\epsilon)}$$

$$= \frac{0}{(\hat{y}-\hat{y})+(\epsilon+\epsilon)} = \frac{0}{2\epsilon} = \frac{0}{2(0)} = \frac{0}{0} = \text{undefined!}$$