When (ŷ - y) < 0:

$$\frac{J(\hat{y}+\epsilon,y)-J(\hat{y}-\epsilon,y)}{(\hat{y}+\epsilon)-(\hat{y}-\epsilon)}=\frac{-[(\hat{y}+\epsilon)-y]-[-[(\hat{y}-\epsilon)-y]]}{(\hat{y}+\epsilon)+(-\hat{y}+\epsilon)}$$

$$=\frac{[-(\hat{y}+\epsilon)+y]+[(\hat{y}-\epsilon)-y]}{(\hat{y}-\hat{y})+(\epsilon+\epsilon)}=\frac{(-\hat{y}-\epsilon+y)+(\hat{y}-\epsilon-y)}{2\epsilon}$$

$$=\frac{(-\hat{y}+\hat{y})+(-\epsilon-\epsilon)+(y-y)}{2\epsilon}=\frac{-2\epsilon}{2\epsilon}=-1$$

When (ŷ - y) > 0:

$$\frac{J(\hat{y}+\epsilon,y)-J(\hat{y}-\epsilon,y)}{(\hat{y}+\epsilon)-(\hat{y}-\epsilon)}=\frac{[(\hat{y}+\epsilon)-y]-[(\hat{y}-\epsilon)-y]}{(\hat{y}+\epsilon)+(-\hat{y}+\epsilon)}$$

$$=\frac{[(\hat{y}+\epsilon)-y]+[-(\hat{y}-\epsilon)+y]}{(\hat{y}-\hat{y})+(\epsilon+\epsilon)}=\frac{(\hat{y}+\epsilon-y)+(-\hat{y}+\epsilon+y)}{2\epsilon}$$

$$=\frac{(\hat{y}-\hat{y})+(\epsilon+\epsilon)+(-y+y)}{2\epsilon}=\frac{2\epsilon}{2\epsilon}=1$$