

When $(\hat{y} - y) < 0$:

$$\begin{aligned}
\frac{J(\hat{y} + \epsilon, y) - J(\hat{y} - \epsilon, y)}{(\hat{y} + \epsilon) - (\hat{y} - \epsilon)} &= \frac{-[(\hat{y} + \epsilon) - y] - [-[(\hat{y} - \epsilon) - y]]}{(\hat{y} + \epsilon) + (-\hat{y} + \epsilon)} \\
&= \frac{[-(\hat{y} + \epsilon) + y] + [(\hat{y} - \epsilon) - y]}{(\hat{y} - \hat{y}) + (\epsilon + \epsilon)} = \frac{(-\hat{y} - \epsilon + y) + (\hat{y} - \epsilon - y)}{2\epsilon} \\
&= \frac{(-\hat{y} + \hat{y}) + (-\epsilon - \epsilon) + (y - y)}{2\epsilon} = \frac{-2\epsilon}{2\epsilon} = -1
\end{aligned}$$

When $(\hat{y} - y) > 0$:

$$\begin{aligned}
\frac{J(\hat{y} + \epsilon, y) - J(\hat{y} - \epsilon, y)}{(\hat{y} + \epsilon) - (\hat{y} - \epsilon)} &= \frac{[(\hat{y} + \epsilon) - y] - [(\hat{y} - \epsilon) - y]}{(\hat{y} + \epsilon) + (-\hat{y} + \epsilon)} \\
&= \frac{[(\hat{y} + \epsilon) - y] + [-(\hat{y} - \epsilon) + y]}{(\hat{y} - \hat{y}) + (\epsilon + \epsilon)} = \frac{(\hat{y} + \epsilon - y) + (-\hat{y} + \epsilon + y)}{2\epsilon} \\
&= \frac{(\hat{y} - \hat{y}) + (\epsilon + \epsilon) + (-y + y)}{2\epsilon} = \frac{2\epsilon}{2\epsilon} = 1
\end{aligned}$$