

$$\begin{aligned}
& \frac{J(\hat{y} + eps, y) - J(\hat{y} - eps, y)}{(\hat{y} + eps) - (\hat{y} - eps)} = \frac{[(\hat{y} + eps) - y]^2 - [(\hat{y} - eps) - y]^2}{(\hat{y} + eps) + (-\hat{y} + eps)} \\
&= \frac{[(\hat{y} + eps)^2 - 2y(\hat{y} + eps) + y^2] - [(\hat{y} - eps)^2 - 2y(\hat{y} - eps) + y^2]}{(\hat{y} - \hat{y}) + (eps + eps)} \\
&= \frac{[\hat{y}^2 + 2\hat{y}(eps) + (eps)^2 - 2y(\hat{y} + eps) + y^2] - [\hat{y}^2 - 2\hat{y}(eps) + (eps)^2 - 2y(\hat{y} - eps) + y^2]}{2 * (eps)} \\
&= \frac{[\hat{y}^2 + 2\hat{y}(eps) + (eps)^2 - 2y(\hat{y} + eps) + y^2] + [-\hat{y}^2 + 2\hat{y}(eps) - (eps)^2 + 2y(\hat{y} - eps) - y^2]}{2 * (eps)} \\
&= \frac{(\hat{y}^2 - \hat{y}^2) + [2\hat{y}(eps) + 2\hat{y}(eps)] + [(eps)^2 - (eps)^2] + [-2y(\hat{y} + eps) + 2y(\hat{y} - eps)] + [y^2 - y^2]}{2 * (eps)} \\
&= \frac{4\hat{y}(eps) + [-2y(\hat{y} + eps) + 2y(\hat{y} - eps)]}{2 * (eps)} = \frac{4\hat{y}(eps) + [-2y(\hat{y}) - 2y(eps)] + [2y(\hat{y}) - 2y(eps)]}{2 * (eps)} \\
&= \frac{4\hat{y}(eps) + [-2y(\hat{y}) + 2y(\hat{y})] + [-2y(eps) - 2y(eps)]}{2 * (eps)} = \frac{4\hat{y}(eps) - 4y(eps)}{2 * (eps)} = \frac{4[\hat{y}(eps) - y(eps)]}{2 * (eps)} \\
&= \frac{2[\hat{y}(eps) - y(eps)]}{eps} = \frac{2(eps)[\hat{y} - y]}{eps} = 2(\hat{y} - y)
\end{aligned}$$