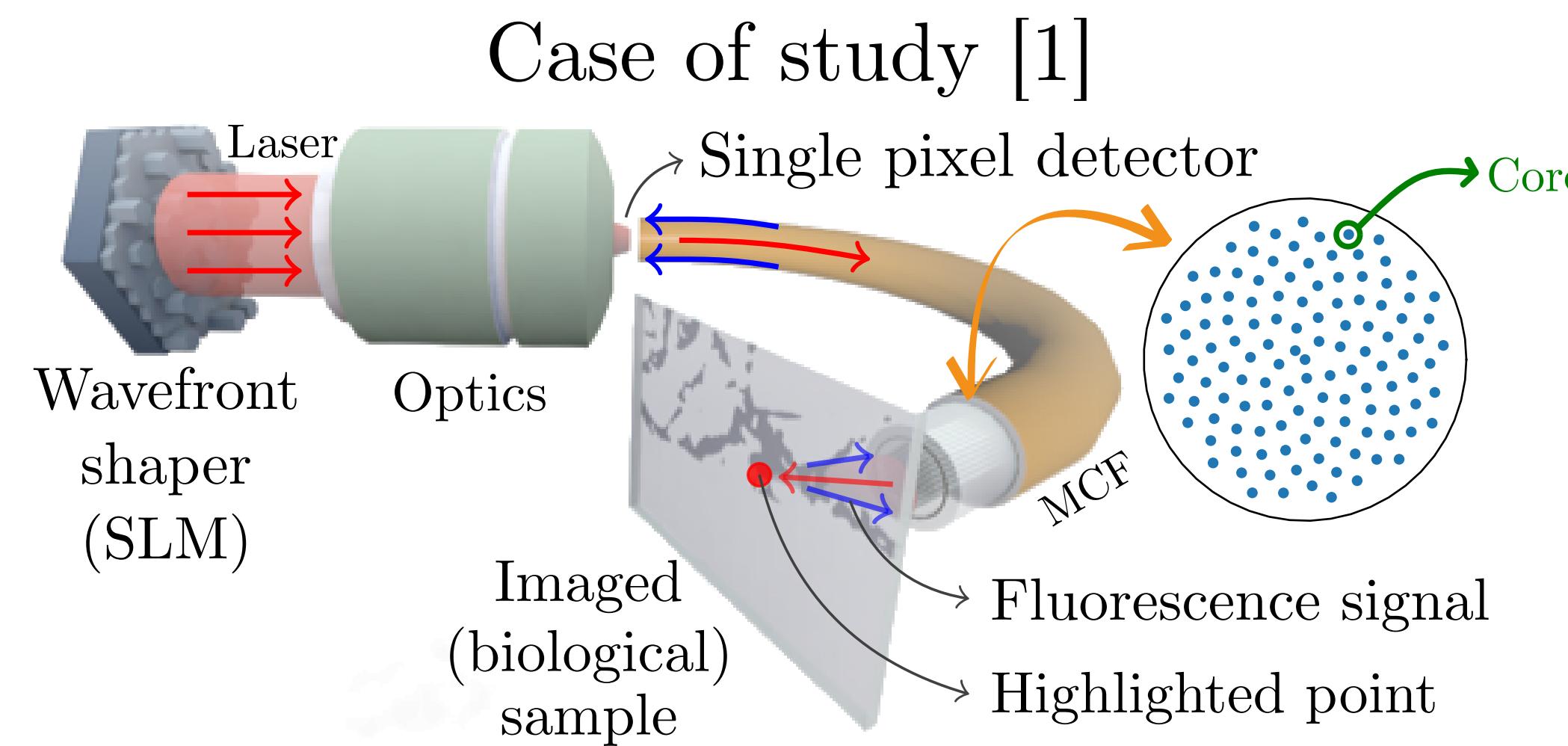
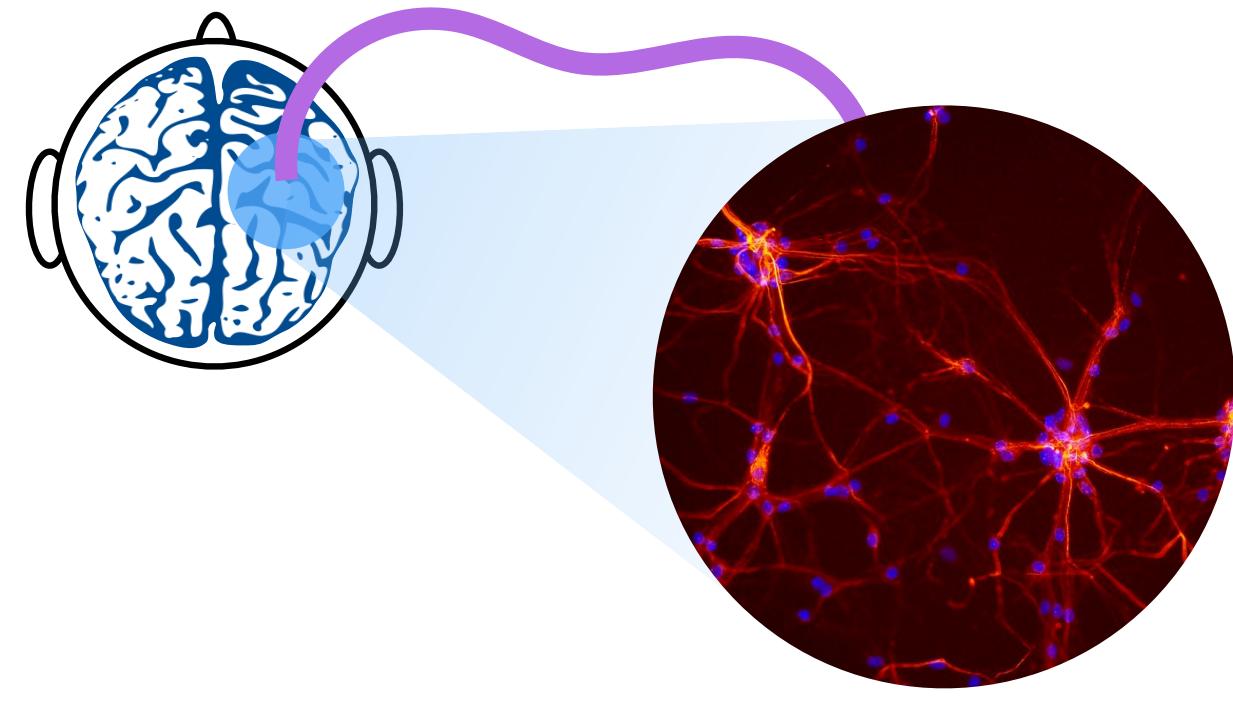


1. Context : Towards Lensless Endoscopy

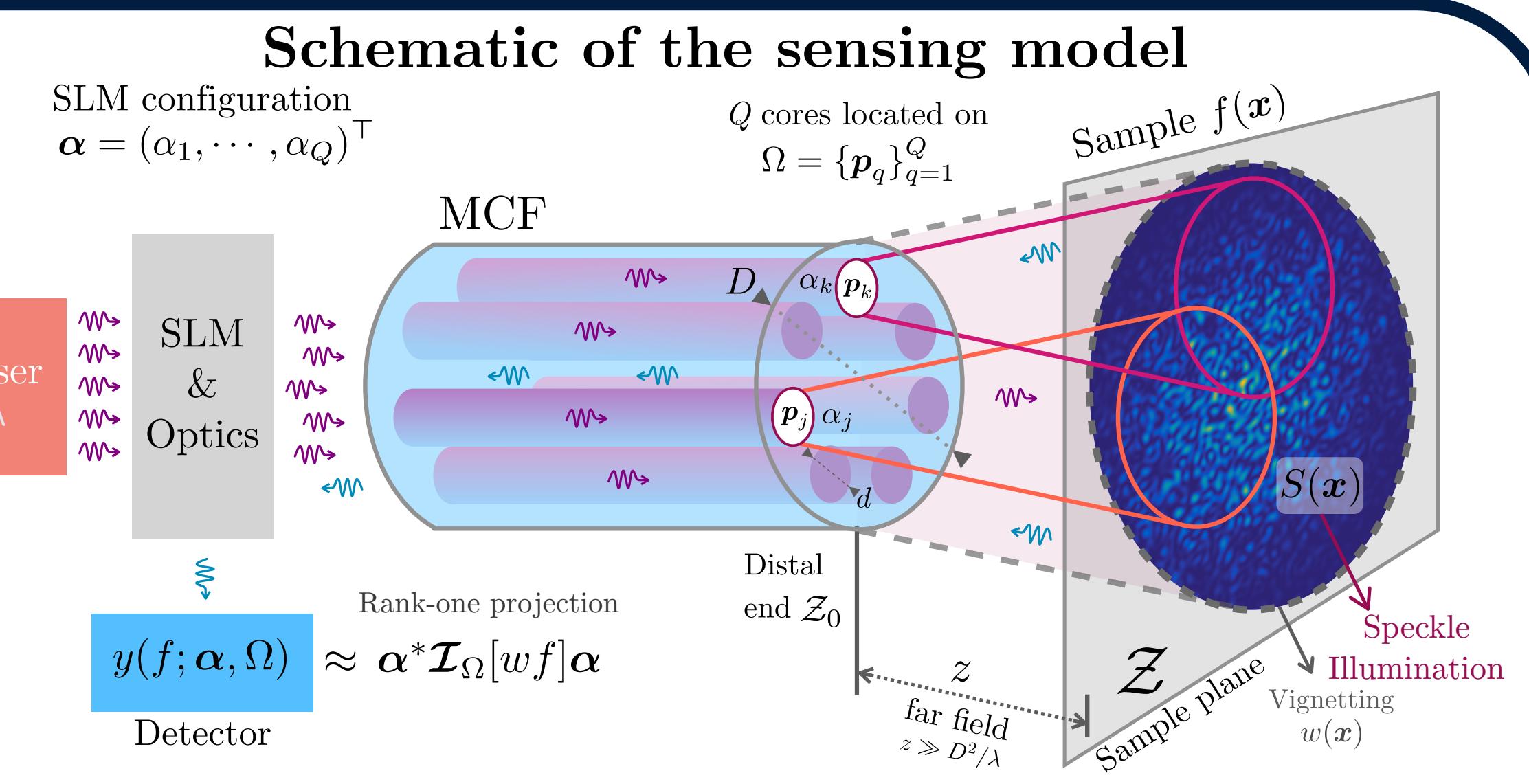


How to see neurons firing?

- ▷ SLM controls the light injected in each core of the MCF.
- ▷ The cores are arranged in a Golden Fermat's spiral [1].
- ▷ The light reflected/reemitted by the sample is integrated in a single-pixel detector.

Solution:

→ Lensless endoscopy!



- ▷ The cores define a set Ω critical for the sensing \Leftrightarrow speckle illumination.
- ▷ Far-field assumption: MCF diameter \ll Sample-Distal End distance.
- ▷ Field-of-view \Leftrightarrow speckle illumination diameter \Leftrightarrow core mode field.

2. Interferometric sensing model

Complex light wavefield emitted by core q

$$E_z(\mathbf{x}, \mathbf{p}_q) = \alpha_q e^{ikz} e^{-\frac{i k \| \mathbf{x} \|^2}{2z}} \sqrt{w(\mathbf{x})} e^{-\frac{2\pi i}{\lambda z} \mathbf{p}_q^\top \mathbf{x}}$$

M different illuminating speckles

$$S(\mathbf{x}; \boldsymbol{\alpha}) \approx w(\mathbf{x}) \left| \sum_{q=1}^Q \alpha_q e^{-\frac{2\pi i}{\lambda z} \mathbf{p}_q^\top \mathbf{x}} \right|^2$$

Single speckle illumination

$$\begin{aligned} \text{intensity of } Q \text{ coded core wavefields} \\ y(f; \boldsymbol{\alpha}, \Omega) &= \int_{\mathbb{R}^2} S(\mathbf{x}; \boldsymbol{\alpha}) f(\mathbf{x}) d\mathbf{x} \\ &= \sum_{j,k=1}^Q \alpha_j^* \alpha_k \int_{\mathbb{R}^2} e^{\frac{2\pi i}{\lambda z} (\mathbf{p}_j - \mathbf{p}_k)^\top \mathbf{x}} w(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} \\ &= \boldsymbol{\alpha}^* \mathcal{I}_\Omega[wf] \boldsymbol{\alpha} \in \mathbb{R}_+ \Rightarrow \text{linear in } f \end{aligned}$$

Collecting M measurements $\Leftrightarrow M$ SLM codes $\{\boldsymbol{\alpha}_m\}_{m=1}^M$

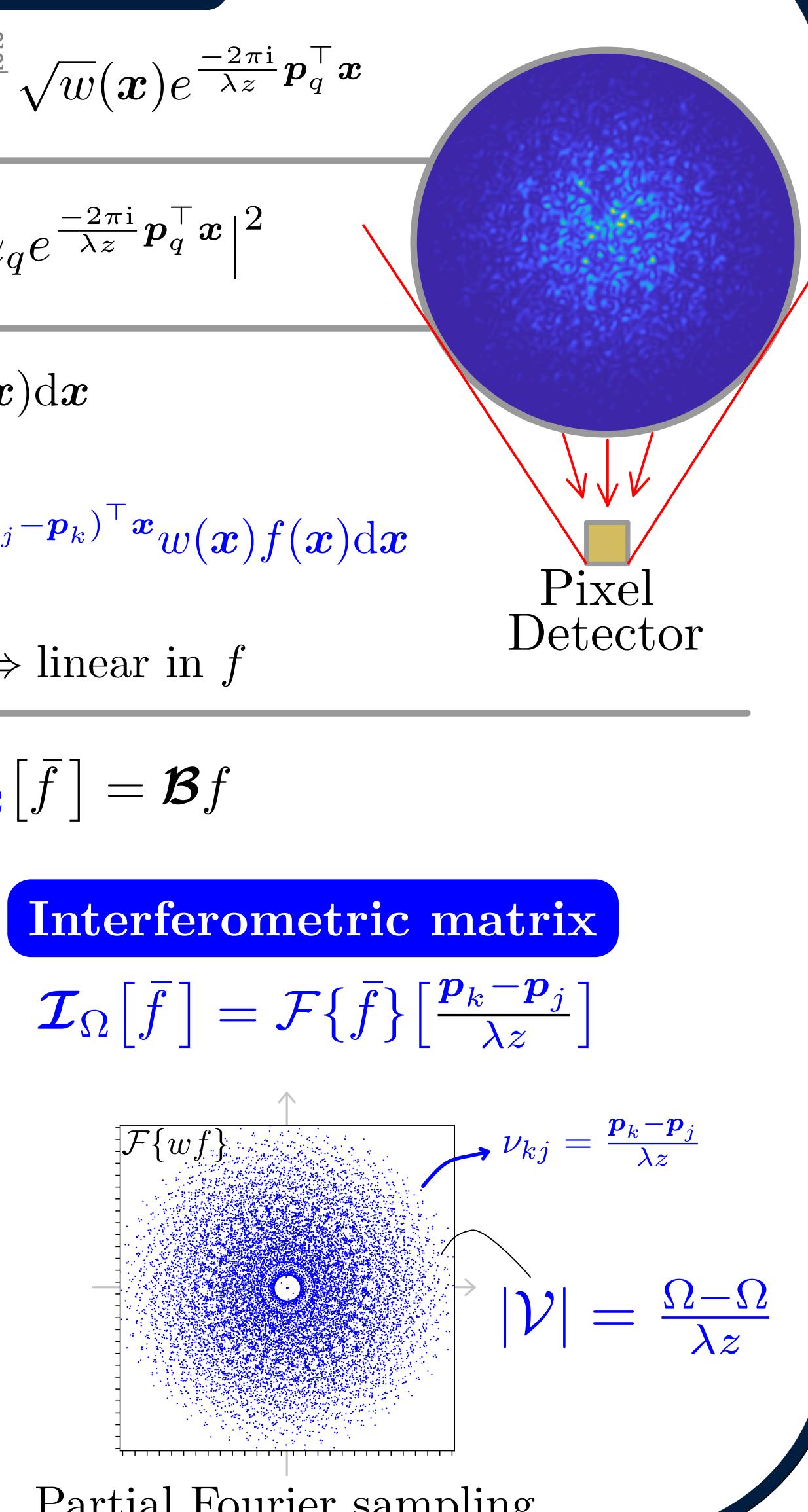
$$(\mathcal{A}[\mathbf{H}])_m := \boldsymbol{\alpha}_m^* \mathbf{H} \boldsymbol{\alpha}_m$$

Asymptotic problem $\mathbb{E}[\mathcal{A}^* \mathcal{A} \mathcal{I}_\Omega] \neq \mathcal{I}_\Omega$ if $\text{tr } \mathcal{I}_\Omega \neq 0$

Solution

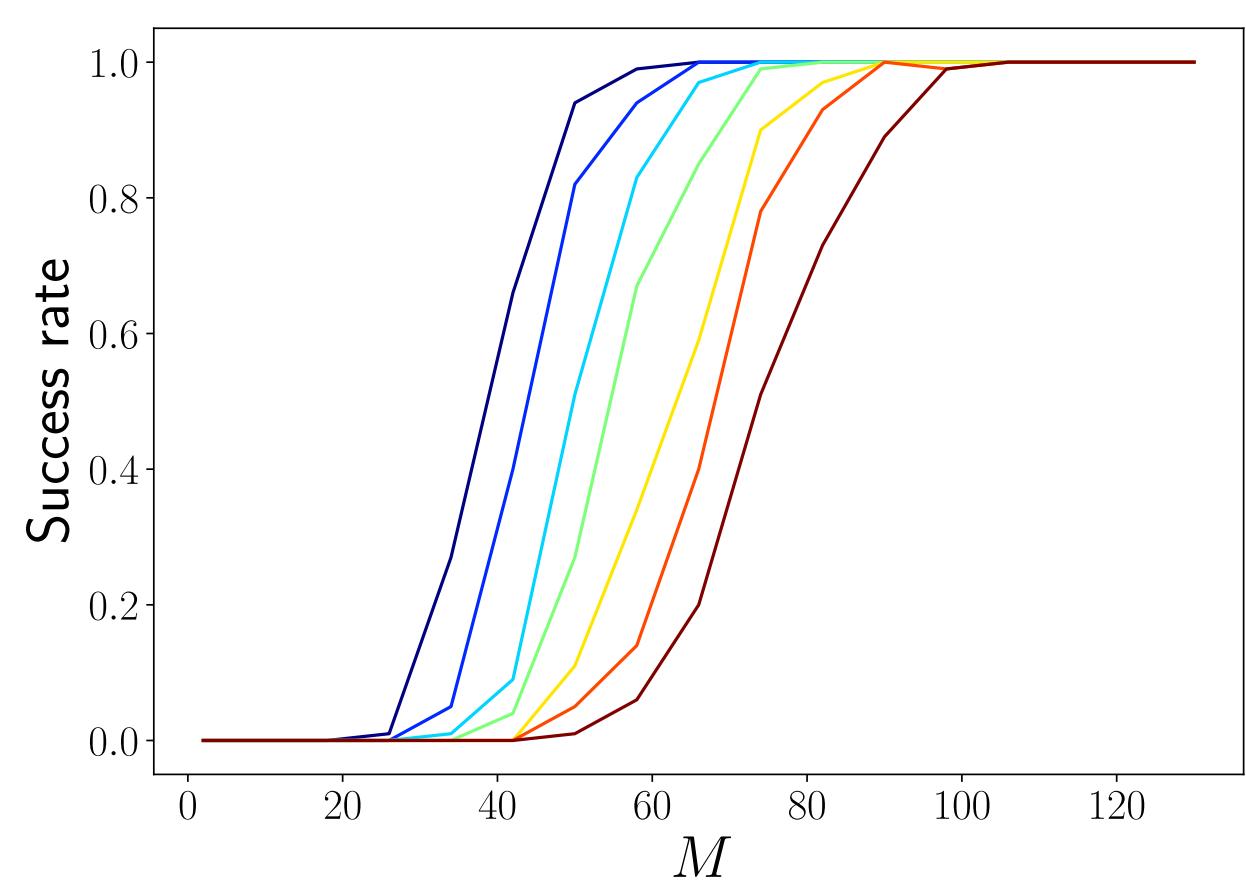
→ Diagless ROP

$$(\mathcal{A}_{\text{dl}}[\mathbf{H}])_m := \boldsymbol{\alpha}_m^* (\mathbf{H} - \text{diag } \mathbf{H}) \boldsymbol{\alpha}_m \Leftrightarrow \text{debiasing the measurements } \mathbf{y}$$



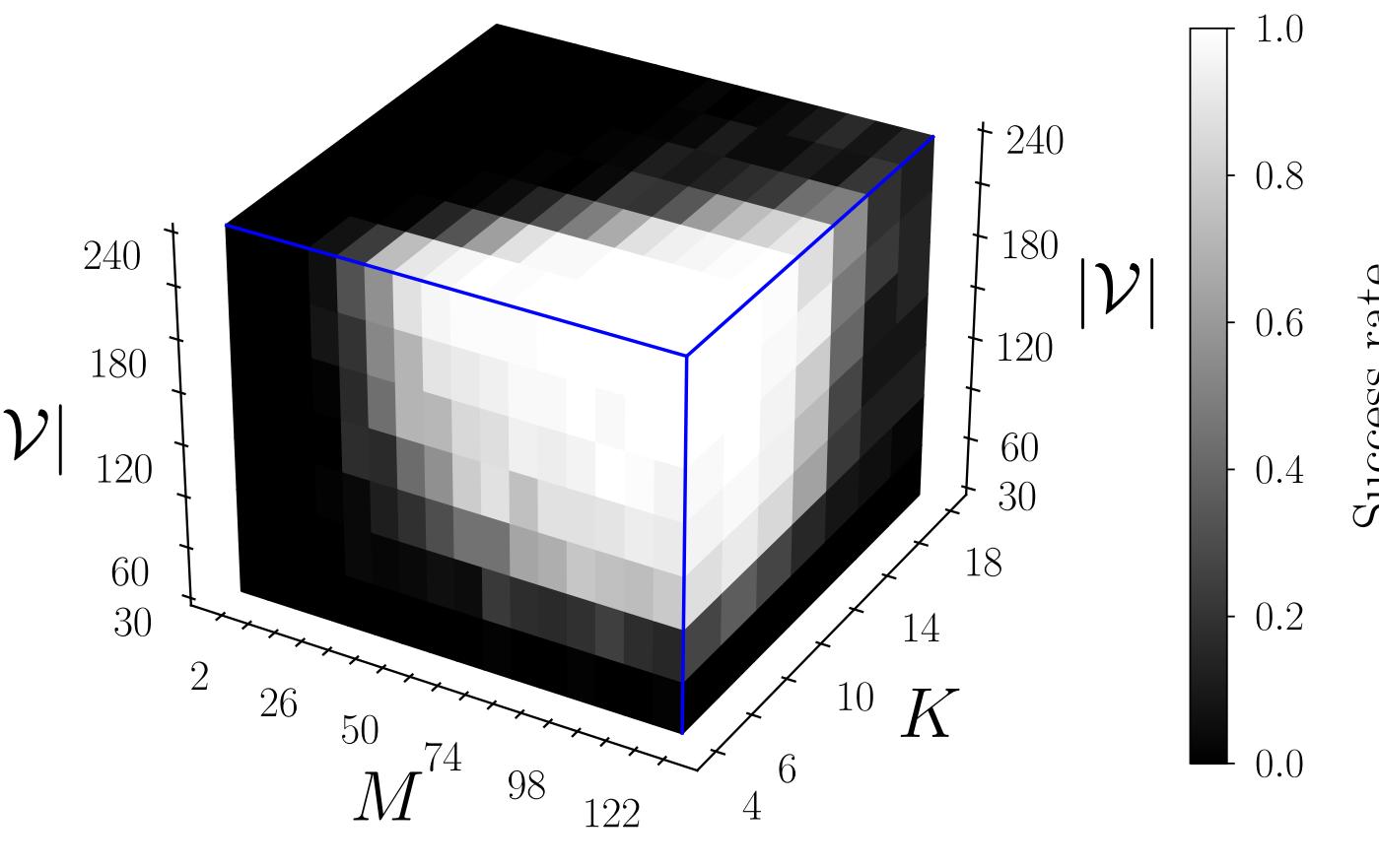
4. Numerical experiments

Recover 1-D signal solving $\hat{f} = \arg \min_f \|y - \mathcal{B}f\|_2$ s.t. $\|f\|_1 \leq K$, with $\begin{cases} p_q \in \mathbb{R} \sim \mathcal{U}[0, 127] \\ \boldsymbol{\alpha} \text{ i.i.d. } \mathcal{N}(\mathbf{0}, \mathbf{I}_Q) \end{cases}$



Transition curves obtained with $|V| = 240 \lesssim N = 256$.

If $K \nearrow \Rightarrow$ transition point (in M) \nearrow

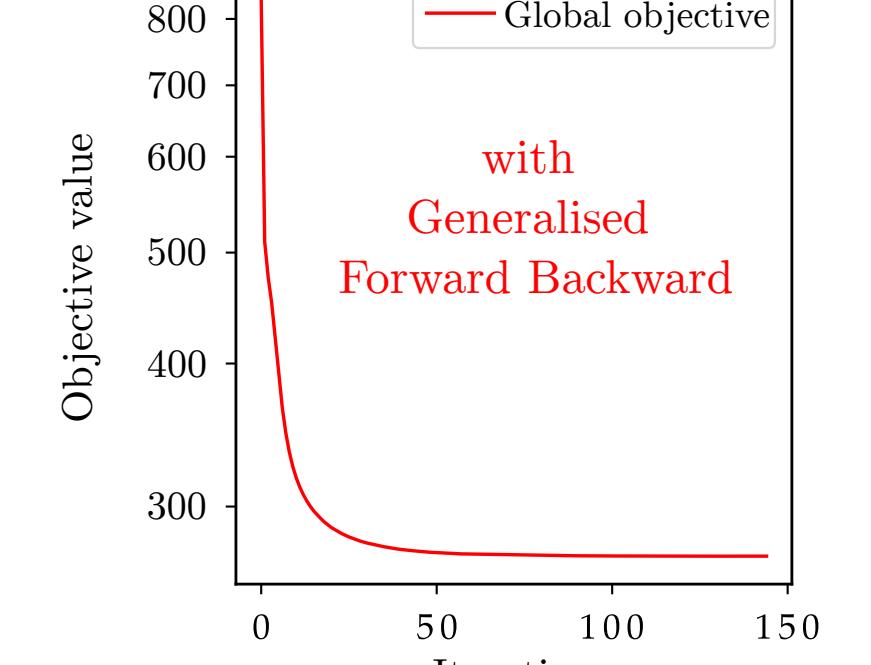
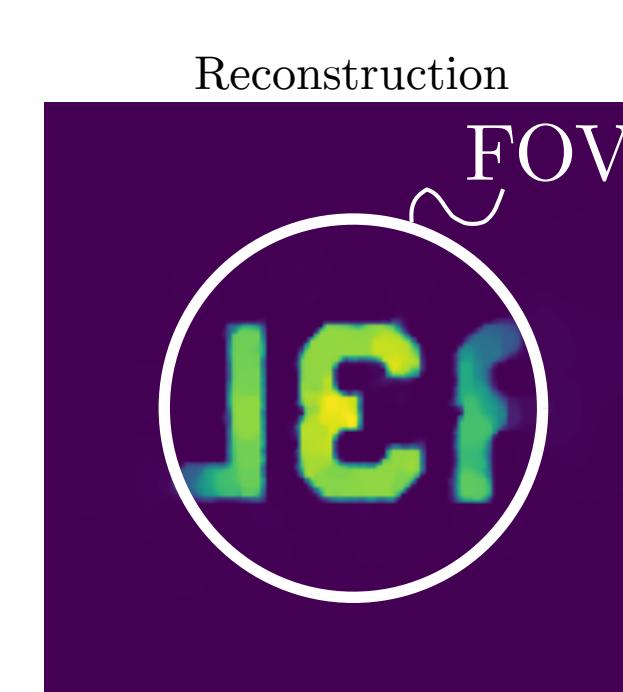
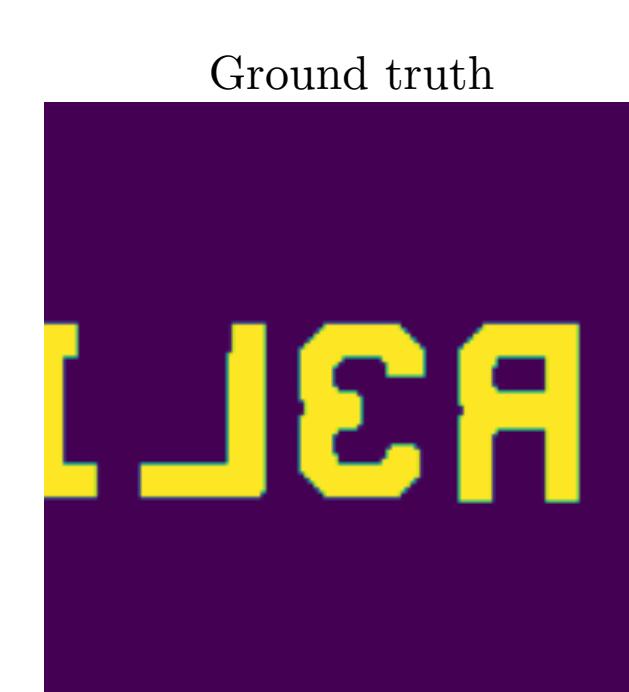


Transition "cube".
 M ROPs of $\mathcal{I}_\Omega[wf] \in \mathbb{C}^{Q \times Q}$ for K -sparse f .
80 trials/pixel, success if SNR ≥ 40 dB.

5. Real experiments

Recover piecewise constant object solving

$$\hat{f} = \arg \min_f \frac{1}{2} \|y - \mathcal{B}f\|_2^2 + \lambda \|f\|_{\text{TV}} \text{ s.t. } f \geq 0$$



- ▷ Reconstruction limited by FOV
⇒ compute SNR in FWHM of envelope region.
- ▷ Both GT and estimate intensities are renormalized to $[0, 1]$.
- ▷ SNR: 11dB.

References

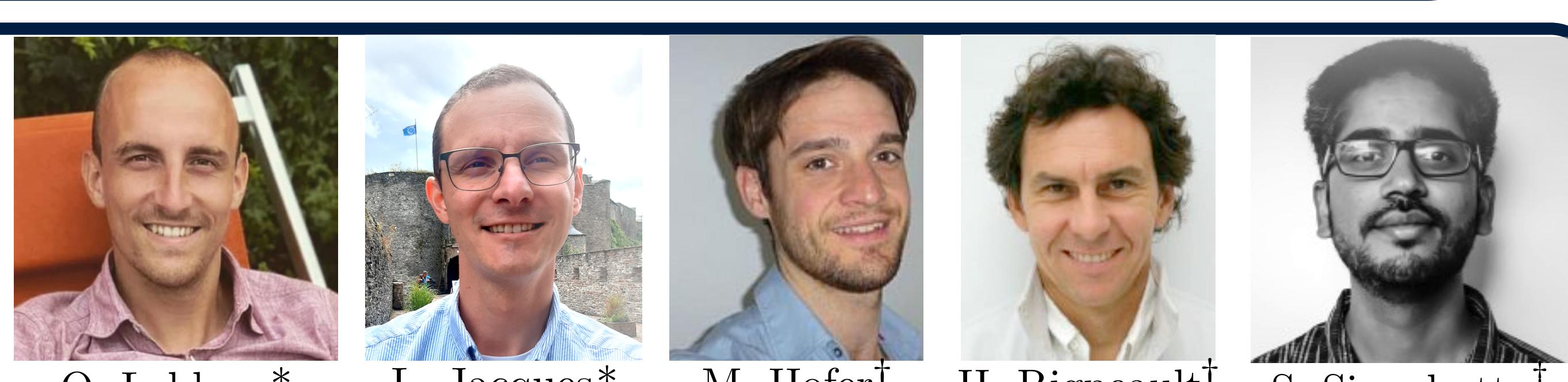
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Conclusion

- ▷ Forward problem close to phase retrieval, interferometric model, but still linear in f .
- ▷ LCM in f yields LCM in \mathcal{I}_Ω .
- ▷ Potential for computational cost reduction.
- ▷ Theoretical guarantees.
- ▷ First experimental results ⇒ Proof of concept.

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