CSE 211: Discrete Mathematics **(Due: 27/12/19)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework4 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Nonhomogeneous Linear Recurrence Relations (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.

(Solution)

Step 1: $a_n = a_n^{(h)} + a_n^{(p)}$

Step 2: Char eq: $r - 3 = 0$, so $r = 3$

Step 3: $a_n^{(h)} = \alpha 3^n$ 1

Step 4: $a_n^{(p)} = C.2^n$, substitute this to the recurrence relation.

Step 5: $C.2^n = 3C.2^{n-1} + 2^n$, divide both sides to 2^{n-1}

Step 6: $2C = 3C + 2$, so $C = -2$

Step 7: Thus, $a_n^{(p)} = -2.2^n = -2^{n+1}$ 2

Step 8: If we combine 1 and 2, $a_n = \alpha 3^n - 2^{n+1}$

Step 9: So, just -2^{n+1} is not the solution.

(b) Find the solution with $a_0 = 1$.

(Solution)

Step 1: $a_n = \alpha 3^n - 2^{n+1}$, and $a_0 = 1$.

Step 2: $a_0 = \alpha - 2 = 1$, thus $\alpha = 3$.

Step 3: $a_n = 3.3^n - 2^{n+1} = 3^{n+1} - 2^{n+1}$

Problem 2: Linear Recurrence Relations (35 points)

Find all solutions of the recurrence relation $a_n = 7a_{n-1}$ - $16a_{n-2}$ + $12a_{n-3}$ + n4^n with a_0 = -2, a_1 = 0, and $a_2 = 5$.

(Solution)

Step 1: $a_n = a_n^{(h)} + a_n^{(p)}$

Step 2: Char eq: $r^3 - 7r^2 + 16r - 12 = 0$
We can observe that 2 is one of the roots.
This means that the equation above can be divided by (r-2).
$r^3 - 7r^2 + 16r - 12 = (r-2)(r^2 - 5r + 6) = (r-2)^2(r-3)$
$r = 2, 2, 3$

Step 3: $a_n^{(h)} = \alpha_1.2^n + \alpha_2.n.2^n + \alpha_3.3^n$ ①

Step 4: $a_n^{(p)} = C.n4^n$, substitute this to the recurrence relation.
$C.n4^n = 7C.n4^{n-1} - 16Cn4^{n-2} + 12Cn4^{n-3} + n4^n$, divide this equation to n.
$C4^n = 7C4^{n-1} - 16C4^{n-2} + 12C4^{n-3} + 4^n$, divide this equation to 4^{n-3}
$64C = 112C - 64C + 12C + 64$, so $C = 16$

Step 5: $a_n^{(p)} = 16n4^n = n4^{n+2}$ ②

Step 6: If we combine ① and ②, $a_n = \alpha_1.2^n + \alpha_2.n.2^n + \alpha_3.3^n + n4^{n+2}$

Step 7: $a_0 = \alpha_1 + \alpha_3 = -2$
$a_1 = 2\alpha_1 + 2\alpha_2 + 3\alpha_3 = -64$
$a_2 = 4\alpha_1 + 8\alpha_2 + 9\alpha_3 = -507$

Step 8: Multiply a_1 with -4 and add to a_2
$4\alpha_1 + 3\alpha_3 = 251$
$\alpha_1 + \alpha_3 = -2$
$\alpha_1 = 257, \alpha_3 = -259, \alpha_2 = \frac{199}{2}$

Step 9: $a_n = 257.2^n + \frac{199}{2}.n.2^n - 259.3^n + n4^{n+2}$

Problem 3: Linear Homogeneous Recurrence Relations (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.

(Solution)

Step 1: $r^n = 2r^{n-1} - 2r^{n-2}$

Step 2: $r^2 - 2r + 2 = 0$

Step 3: $\Delta = b^2 - 4ac = -4$

Step 4: $X_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2.a} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$

Step 5: $X_{1,2} = 1 + i, 1 - i$

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

(Solution)

Step 1: $a_n = \alpha(1+i)^n + \beta(1-i)^n$

Step 2: $a_0 = \alpha + \beta = 1$

Step 3: $a_1 = \alpha(1+i) + \beta(1-i) = 2$

Step 4: Multiply a_0 with i-1 to get rid of β

Step 5: $a_0 = \alpha(i-1) + \beta(i-1) = i-1$

Step 6: $a_0 + a_1 = \alpha(2i) = i+1$, so $\alpha = \frac{i+1}{2i}$

Step 7: Put α value to the equation 2, so $\beta = \frac{i-1}{2i}$

Step 8: So, $a_n = (\frac{i+1}{2i})(1+i)^n + (\frac{i-1}{2i})(1-i)^n$