Exercise 1

For the Brock and Mirman model, find the value of A in the policy function. Show that your value is correct.

For this case find an algebraic solution for the policy function $k_{t+1} = \Phi(k_t, z_t)$. Sources for hints are Stokey et al. (1989, exercise 2.2, p. 12) and Sargent (1987, exercise 1.1, p. 47).

0.1 Brock and Mirman's Model

Households solve the following dynamic program:

$$V(K_t, z_t) = max_{K_{t+1}} ln(e^{z_t} K_t^{\alpha} - K_{t+1}) + \beta E_t\{V(K_{t+1}, z_{t+1})\}$$

The associated Euler equaation is:

$$\frac{1}{e^{z_t} K_t^{\alpha} - K_{t+1}} = \beta E_t\{\frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}} K_{t+1}^{\alpha} - K_{t+2}}\}$$

The law of motion for z is:

$$z_{t+1} = \rho z_t + \epsilon_t; \, \epsilon_t \approx i.i.d(0, \sigma^2)$$

Verify that the policy function is of the following form:

$$K_{t+1} = A e^{z_t} K_t^{\alpha}$$

In order to determine the policy function, we need to find the value of A as a function of the model's parameters. Lets expand the euler equations:

$$\frac{1}{e^{z_t} K_t^{\alpha} - K_{t+1}} = \beta E_t\{\frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}} K_{t+1}^{\alpha} * (1-A)}\}$$

$$\iff \frac{1}{e^{z_t} K_t^{\alpha} - K_{t+1}} = \beta E_t\{\frac{\alpha K_{t+1}^{\alpha-1}}{K_{t+1}^{\alpha} * (1-A)}\}$$

$$\iff \frac{1}{e^{z_t} K_t^{\alpha} - K_{t+1}} = \beta E_t\{\frac{\alpha}{K_{t+1}(1-A)}\}$$

$$\iff \frac{1}{e^{z_t} K_t^{\alpha} - K_{t+1}} = \beta \frac{\alpha}{K_{t+1}(1-A)}$$

$$\iff 1 = \beta * \alpha \frac{e^{z_t} K_t^{\alpha} - K_{t+1}}{K_{t+1}(1-A)}$$

$$\iff A = \beta * \alpha$$

Exercise 2

$$u = a \log(1 - l_t) + \log(c_t)$$

$$F = K_t^{\alpha} L_t^{1-\alpha} e^{z}$$

$$du/dc = \frac{1}{c_t}$$

$$du/dl = -\frac{a}{1 - l_t}$$

$$E1\ 0 = T_t - c_t + k_t - k_{t+1} + (1 - \tau)\left(k_t\left(-\delta + r_t\right) + l_t w_t\right)$$

$$E2\ 0 = \frac{\beta\left((1 - \tau)\left(-\delta + r_{t+1}\right) + 1\right)}{c_{t+1}} - \frac{1}{c_t}$$

$$E3\ 0 = -\frac{a}{1 - l_t} + \frac{w_t(1 - \tau)}{c_t}$$

$$E4\ 0 = -\frac{\alpha k_t^{\alpha} l_t^{1-\alpha} e^{z}}{k_t} + r_t$$

$$E5\ 0 = -\frac{k_t^{\alpha} l_t^{1-\alpha}(1 - \alpha) e^{z}}{l_t} + w_t$$

$$E6\ 0 = T_t - \tau\left(k_t\left(-\delta + r_t\right) + l_t w_t\right)$$

Exercise 3

$$u = a \log(1 - l_t) + \frac{c_t^{1-\gamma} - 1}{1 - \gamma}$$

$$F = k_t^{\alpha} l_t^{1-\alpha} e^{z}$$

$$du/dc = \frac{c_t^{1-\gamma}}{c_t}$$

$$du/dl = -\frac{a}{1 - l_t}$$

$$E1\ 0 = T_t - c_t + k_t - k_{t+1} + (1 - \tau)\left(k_t\left(-\delta + r_t\right) + l_t w_t\right)$$

$$E2\ 0 = \frac{\beta c_{t+1}^{1-\gamma}\left((1 - \tau)\left(-\delta + r_{t+1}\right) + 1\right)}{c_{t+1}} - \frac{c_t^{1-\gamma}}{c_t}$$

$$E3\ 0 = -\frac{a}{1 - l_t} + \frac{c_t^{1-\gamma} w_t(1 - \tau)}{c_t}$$

$$E4\ 0 = -\frac{\alpha k_t^{\alpha} l_t^{1-\alpha} e^{z}}{k_t} + r_t$$

$$E5\ 0 = -\frac{k_t^{\alpha} l_t^{1-\alpha}(1 - \alpha) e^{z}}{l_t} + w_t$$

$$E6\ 0 = T_t - \tau \left(k_t \left(-\delta + r_t\right) + l_t w_t\right)$$

Exercise 4

$$u = \frac{a \left(\left(1 - l_t\right)^{1-\xi} - 1\right)}{1 - \xi} + \frac{c_t^{1-\gamma} - 1}{1 - \gamma}$$

$$F = \left(\alpha k_t^{\eta} + l_t^{\eta} \left(1 - \alpha\right)\right)^{\frac{1}{\eta}} e^{z}$$

$$du/dc = \frac{c_t^{1-\gamma}}{c_t}$$

$$du/dl = \frac{a \left(1 - l_t\right)^{1-\xi} \left(\xi - 1\right)}{\left(1 - \xi\right) \left(1 - l_t\right)}$$

$$E1\ 0 = T_t - c_t + k_t - k_{t+1} + \left(1 - \tau\right) \left(k_t \left(-\delta + r_t\right) + l_t w_t\right)$$

$$E2\ 0 = \frac{\beta c_{t+1}^{1-\gamma} \left(\left(1 - \tau\right) \left(-\delta + r_{t+1}\right) + 1\right)}{c_{t+1}} - \frac{c_t^{1-\gamma}}{c_t}$$

$$E3\ 0 = \frac{a \left(1 - l_t\right)^{1-\xi} \left(\xi - 1\right)}{\left(1 - \xi\right) \left(1 - l_t\right)} + \frac{c_t^{1-\gamma} w_t \left(1 - \tau\right)}{c_t}$$

$$E4\ 0 = -\frac{\alpha k_t^{\eta} \left(\alpha k_t^{\eta} + l_t^{\eta} \left(1 - \alpha\right)\right)^{\frac{1}{\eta}} e^{z}}{k_t \left(\alpha k_t^{\eta} + l_t^{\eta} \left(1 - \alpha\right)\right)} + r_t$$

$$E5\ 0 = w_t - \frac{l_t^{\eta} \left(1 - \alpha\right) \left(\alpha k_t^{\eta} + l_t^{\eta} \left(1 - \alpha\right)\right)^{\frac{1}{\eta}} e^{z}}{l_t \left(\alpha k_t^{\eta} + l_t^{\eta} \left(1 - \alpha\right)\right)}$$

$$E6\ 0 = T_t - \tau \left(k_t \left(-\delta + r_t\right) + l_t w_t\right)$$

Exercise 5

$$u = \frac{c_t^{1-\gamma} - 1}{1 - \gamma}$$

$$F = k_t^{\alpha} \left(l_t e^{z}\right)^{1-\alpha}$$

$$du/dc = \frac{c_t^{1-\gamma}}{c_t}$$

$$du/dl = 0$$

Equilibrium equations:

$$E1\ 0 = T_t - c_t + k_t - k_{t+1} + \left(1 - \tau\right) \left(k_t \left(-\delta + r_t\right) + l_t w_t\right)$$

$$E2\ 0 = \frac{\beta c_{t+1}^{1-\gamma} \left(\left(1 - \tau\right) \left(-\delta + r_{t+1}\right) + 1\right)}{c_{t+1}} - \frac{c_t^{1-\gamma}}{c_t}$$

$$E3\ 0 = \frac{c_t^{1-\gamma} w_t (1-\tau)}{c_t}$$

$$E4\ 0 = -\frac{\alpha k_t^{\alpha} (l_t e^z)^{1-\alpha}}{k_t} + r_t$$

$$E5\ 0 = -\frac{k_t^{\alpha} (l_t e^z)^{1-\alpha} (1-\alpha)}{l_t} + w_t$$

$$E6\ 0 = T_t - \tau (k_t (-\delta + r_t) + l_t w_t)$$

Steady state equations:

$$E1\ 0 = T - c + (1-\tau)(k(-\delta + r) + lw)$$

$$E2\ 0 = \frac{c^{1-\gamma} (-\beta ((1-\tau)(-\delta + r) + 1) + 1)}{c}$$

$$E3\ 0 = \frac{c^{1-\gamma} w (1-\tau)}{c}$$

$$E4\ 0 = r - \frac{K^{\alpha} \alpha (Le^z)^{1-\alpha}}{K}$$

$$E5\ 0 = -\frac{K^{\alpha} (Le^z)^{1-\alpha} (1-\alpha)}{L} + w$$

$$E6\ 0 = T - \tau (k(-\delta + r) + lw)$$

Exercise 6

Steady state equations:

$$u\ 0 = \frac{a\left((1-l)^{1-\xi} - 1\right)}{1-\xi} + \frac{c^{1-\gamma} - 1}{1-\gamma}$$

$$F\ 0 = K^{\alpha} (Le^z)^{1-\alpha}$$

$$du/dc\ 0 = \frac{c^{1-\gamma}}{c}$$

$$du/dl\ 0 = \frac{a (1-l)^{1-\xi} (\xi - 1)}{(1-\xi)(1-l)}$$

$$E1\ 0 = T - c + (1-\tau)(k(-\delta + r) + lw)$$

$$E2\ 0 = \frac{c^{1-\gamma} (-\beta ((1-\tau)(-\delta + r) + 1) + 1)}{c}$$

$$E3\ 0 = \frac{a (1-l)^{1-\xi} (\xi - 1)}{(1-\xi)(1-l)} + \frac{c^{1-\gamma} w (1-\tau)}{c}$$

$$E4\ 0 = r - \frac{K^{\alpha}\alpha\left(Le^{z}\right)^{1-\alpha}}{K}$$

$$E5\ 0 = -\frac{K^{\alpha}\left(Le^{z}\right)^{1-\alpha}(1-\alpha)}{L} + w$$

$$E6\ 0 = T - \tau\left(k\left(-\delta + r\right) + lw\right)$$

Exercise 7

For the steady state of the baseline tax model in section 3.7.3 use numerican differentiation to solve for the full set of comparative statics and sign them where possible. y   Find x for y k,l,y ,w ,r ,T,i,c and x ,,,,,,a,z . Using the same parameter values as in excercise 6, solve for the numerical values of the comparative statics.