

Algorithms: Design and Analysis, Part II

# Greedy Algorithms

A Scheduling Application: Handling Ties

#### Correctness Claim

Claim: Algorithm #2 (order jobs in nonincreasing order of ratio  $w_j/l_j$ ) is always correct. [Even with ties]

New Proof Plan: Fix arbitrary input of n jobs. Let  $\sigma =$  greedy schedule, let  $\sigma^* =$  any other schedule.

Will show  $\sigma$  at least as good as  $\sigma^* \Rightarrow$  Implies that greedy schedule is optimal.

### Correctness Proof

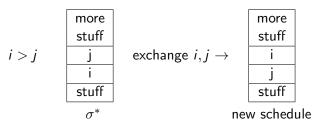
Assume: [Just by renaming jobs] Greedy schedule  $\sigma$  is just 1, 2, 3, ..., n (and so  $w_1/l_1 > w_2/l_2 > ... > w_n/l_n$ ).

Consider arbitrary schedule  $\sigma^*$ . If  $\sigma^* = \sigma$ , done.

Else recall  $\exists$  consecutive jobs i, j in  $\sigma^*$  with i > j. (From last time)

Note:  $i > j \Rightarrow w_i/l_i \le w_j/l_j \Rightarrow w_il_j \le w_jl_i$ .

Recall: Exchanging i&j in  $\sigma^*$  has net benefit of  $w_jl_i - w_il_j \ge 0$ .



## Correctness Proof

Upshot: Exchanging an "adjacent inversion" like i, j only makes  $\sigma^*$  better, and it decreases the number of inverted pairs .

Jobs i, j with i > j and i scheduled earlier

- $\Rightarrow$  After at most  $\binom{n}{2}$  such exchanges, can transform  $\sigma^*$  into  $\sigma$ .
- $\Rightarrow \sigma$  at least as good as  $\sigma^*$ .
- $\Rightarrow$  Greedy is optimal.

#### QED!