

# Sample of Funcon Specifications

The P<sub>L</sub>anCompS Project

Binding.cbs

## OUTLINE

- Binding
  - Environments
  - Current bindings
  - Scope
  - Recurse

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## Binding

- [ *Type* environments
- Alias* envs
- Datatype* identifiers
- Alias* ids
- Funcon* identifier-tagged
- Alias* id-tagged
- Funcon* fresh-identifier
- Entity* environment
- Alias* env
- Funcon* initialise-binding
- Funcon* bind-value
- Alias* bind
- Funcon* unbind
- Funcon* bound-directly
- Funcon* bound-value
- Alias* bound
- Funcon* closed
- Funcon* scope
- Funcon* accumulate
- Funcon* collateral
- Funcon* bind-recursively
- Funcon* recursive ]

*Meta-variables*    *T* <: values

## Environments

- Type* environments  $\rightsquigarrow$  maps(identifiers, values?)
- Alias* envs = environments

An environment represents bindings of identifiers to values. Mapping an identifier to ( ) represents that its binding is hidden.

Circularity in environments (due to recursive bindings) is represented using bindings to cut-points called **links**. Funcons are provided for making declarations recursive and for referring to bound values without explicit mention of links, so their existence can generally be ignored.

*Datatype* **identifiers** ::= { $\_ : \text{strings}$ } | **identifier-tagged**( $\_ : \text{identifiers}$ ,  $\_ : \text{values}$ )

*Alias* **ids** = **identifiers**

*Alias* **id-tagged** = **identifier-tagged**

An identifier is either a string of characters, or an identifier tagged with some value (e.g., with the identifier of a namespace).

*Funcon* **fresh-identifier** :  $\Rightarrow \text{identifiers}$

**fresh-identifier** computes an identifier distinct from all previously computed identifiers.

*Rule* **fresh-identifier**  $\rightsquigarrow$  **identifier-tagged**("generated", **fresh-atom**)

### Current bindings

*Entity* **environment**( $\_ : \text{environments}$ )  $\vdash \_ \longrightarrow \_$

*Alias* **env** = **environment**

The environment entity allows a computation to refer to the current bindings of identifiers to values.

*Funcon* **initialise-binding**( $X : \Rightarrow T$ ) :  $\Rightarrow T$   
 $\rightsquigarrow$  **initialise-linking**(**initialise-generating**(**closed**( $X$ )))

**initialise-binding**( $X$ ) ensures that  $X$  does not depend on non-local bindings. It also ensures that the linking entity (used to represent potentially cyclic bindings) and the generating entity (for creating fresh identifiers) are initialised.

*Funcon* **bind-value**( $I : \text{identifiers}$ ,  $V : \text{values}$ ) :  $\Rightarrow \text{environments}$   
 $\rightsquigarrow \{I \mapsto V\}$

*Alias* **bind** = **bind-value**

**bind-value**( $I, X$ ) computes the environment that binds only  $I$  to the value computed by  $X$ .

*Funcon* **unbind**( $I : \text{identifiers}$ ) :  $\Rightarrow \text{environments}$   
 $\rightsquigarrow \{I \mapsto ( )\}$

**unbind**( $I$ ) computes the environment that hides the binding of  $I$ .

*Funcon* **bound-directly**( $\_ : \text{identifiers}$ ) :  $\Rightarrow \text{values}$

**bound-directly**( $I$ ) returns the value to which  $I$  is currently bound, if any, and otherwise fails.

**bound-directly**( $I$ ) does *not* follow links. It is used only in connection with recursively-bound values when references are not encapsulated in abstractions.

$$\begin{array}{l}
\text{Rule} \quad \frac{\text{lookup}(\rho, l) \rightsquigarrow (V : \text{values})}{\text{environment}(\rho) \vdash \text{bound-directly}(l : \text{identifiers}) \longrightarrow V} \\
\text{Rule} \quad \frac{\text{lookup}(\rho, l) \rightsquigarrow ( )}{\text{environment}(\rho) \vdash \text{bound-directly}(l : \text{identifiers}) \longrightarrow \text{fail}}
\end{array}$$

$$\begin{array}{l}
\text{Funcon} \quad \text{bound-value}(l : \text{identifiers}) : \Rightarrow \text{values} \\
\qquad \rightsquigarrow \text{follow-if-link}(\text{bound-directly}(l))
\end{array}$$

$$\text{Alias} \quad \text{bound} = \text{bound-value}$$

$\text{bound-value}(l)$  inspects the value to which  $l$  is currently bound, if any, and otherwise fails. If the value is a link,  $\text{bound-value}(l)$  returns the value obtained by following the link, if any, and otherwise fails. If the inspected value is not a link,  $\text{bound-value}(l)$  returns it.

$\text{bound-value}(l)$  is used for references to non-recursive bindings and to recursively-bound values when references are encapsulated in abstractions.

## Scope

$$\text{Funcon} \quad \text{closed}(X : \Rightarrow T) : \Rightarrow T$$

$\text{closed}(X)$  ensures that  $X$  does not depend on non-local bindings.

$$\begin{array}{l}
\text{Rule} \quad \frac{\text{environment}(\text{map}( )) \vdash X \longrightarrow X'}{\text{environment}( ) \vdash \text{closed}(X) \longrightarrow \text{closed}(X')} \\
\text{Rule} \quad \text{closed}(V : T) \rightsquigarrow V
\end{array}$$

$$\text{Funcon} \quad \text{scope}( _ : \text{environments}, _ : \Rightarrow T) : \Rightarrow T$$

$\text{scope}(D, X)$  executes  $D$  with the current bindings, to compute an environment  $\rho$  representing local bindings. It then executes  $X$  to compute the result, with the current bindings extended by  $\rho$ , which may shadow or hide previous bindings.

$\text{closed}(\text{scope}(\rho, X))$  ensures that  $X$  can reference only the bindings provided by  $\rho$ .

$$\begin{array}{l}
\text{Rule} \quad \frac{\text{environment}(\text{map-override}(\rho_1, \rho_0)) \vdash X \longrightarrow X'}{\text{environment}(\rho_0) \vdash \text{scope}(\rho_1 : \text{environments}, X) \longrightarrow \text{scope}(\rho_1, X')} \\
\text{Rule} \quad \text{scope}( _ : \text{environments}, V : T) \rightsquigarrow V
\end{array}$$

$$\text{Funcon} \quad \text{accumulate}( _ : (\Rightarrow \text{environments})^*) : \Rightarrow \text{environments}$$

$\text{accumulate}(D_1, D_2)$  executes  $D_1$  with the current bindings, to compute an environment  $\rho_1$  representing some local bindings. It then executes  $D_2$  to compute an environment  $\rho_2$  representing further local bindings, with the current bindings extended by  $\rho_1$ , which may shadow or hide previous current bindings. The result is  $\rho_1$  extended by  $\rho_2$ , which may shadow or hide the bindings of  $\rho_1$ .

$\text{accumulate}( _ , _ )$  is associative, with  $\text{map}( )$  as unit, and extends to any number of arguments.

$$\begin{array}{l}
\text{Rule} \quad \frac{D_1 \longrightarrow D'_1}{\text{accumulate}(D_1, D_2) \longrightarrow \text{accumulate}(D'_1, D_2)} \\
\text{Rule} \quad \text{accumulate}(\rho_1 : \text{environments}, D_2) \rightsquigarrow \text{scope}(\rho_1, \text{map-override}(D_2, \rho_1)) \\
\text{Rule} \quad \text{accumulate}( ) \rightsquigarrow \text{map}( ) \\
\text{Rule} \quad \text{accumulate}(D_1) \rightsquigarrow D_1 \\
\text{Rule} \quad \text{accumulate}(D_1, D_2, D^+) \rightsquigarrow \text{accumulate}(D_1, \text{accumulate}(D_2, D^+))
\end{array}$$

*Funcon*  $\text{collateral}(\rho^* : \text{environments}^*) : \Rightarrow \text{environments}$   
 $\rightsquigarrow \text{checked map-unite}(\rho^*)$

$\text{collateral}(D_1, \dots)$  pre-evaluates its arguments with the current bindings, and unites the resulting maps, which fails if the domains are not pairwise disjoint.

$\text{collateral}(D_1, D_2)$  is associative and commutative with  $\text{map}()$  as unit, and extends to any number of arguments.

## Recurse

*Funcon*  $\text{bind-recursively}(I : \text{identifiers}, E : \Rightarrow \text{values}) : \Rightarrow \text{environments}$   
 $\rightsquigarrow \text{recursive}(\{I\}, \text{bind-value}(I, E))$

$\text{bind-recursively}(I, E)$  binds  $I$  to a link that refers to the value of  $E$ , representing a recursive binding of  $I$  to the value of  $E$ . Since  $\text{bound-value}(I)$  follows links, it should not be executed during the evaluation of  $E$ .

*Funcon*  $\text{recursive}(SI : \text{sets}(\text{identifiers}), D : \Rightarrow \text{environments}) : \Rightarrow \text{environments}$   
 $\rightsquigarrow \text{re-close}(\text{bind-to-forward-links}(SI), D)$

$\text{recursive}(SI, D)$  executes  $D$  with potential recursion on the bindings of the identifiers in the set  $SI$  (which need not be the same as the set of identifiers bound by  $D$ ).

*Auxiliary Funcon*  $\text{re-close}(M : \text{maps}(\text{identifiers}, \text{links}), D : \Rightarrow \text{environments}) : \Rightarrow \text{environments}$   
 $\rightsquigarrow \text{accumulate}(\text{scope}(M, D), \text{sequential}(\text{set-forward-links}(M), \text{map}()))$

$\text{re-close}(M, D)$  first executes  $D$  in the scope  $M$ , which maps identifiers to freshly allocated links. This computes an environment  $\rho$  where the bound values may contain links, or implicit references to links in abstraction values. It then sets the link for each identifier in the domain of  $M$  to refer to its bound value in  $\rho$ , and returns  $\rho$  as the result.

*Auxiliary Funcon*  $\text{bind-to-forward-links}(SI : \text{sets}(\text{identifiers})) : \Rightarrow \text{maps}(\text{identifiers}, \text{links})$   
 $\rightsquigarrow \text{map-unite}(\text{interleave-map}(\text{bind-value}(\text{given}, \text{fresh-link}(\text{values})), \text{set-elements}(SI)))$

$\text{bind-to-forward-links}(SI)$  binds each identifier in the set  $SI$  to a freshly allocated link.

*Auxiliary Funcon*  $\text{set-forward-links}(M : \text{maps}(\text{identifiers}, \text{links})) : \Rightarrow \text{null-type}$   
 $\rightsquigarrow \text{effect}(\text{interleave-map}(\text{set-link}(\text{map-lookup}(M, \text{given}), \text{bound-value}(\text{given})), \text{set-elements}(\text{map-domain}(M))))$

For each identifier  $I$  in the domain of  $M$ ,  $\text{set-forward-links}(M)$  sets the link to which  $I$  is mapped by  $M$  to the current bound value of  $I$ .