02239 Data Security Protocol Security I

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Overview of Problem Areas

Example: Alice wants to tell her bank to transfer 1000 Kr. to Bob.

- What are the involved goals?
 - ★ Authentication/Integrity
 - ★ Confidentiality/Privacy
- Involved Cryptographic Protocols: could be
 - ★ TLS
 - ★ The banking application
 - ★ Some login like MitID (also over TLS? Same session?)
- Implementation
 - ★ Crypto API
 - ★ All the non-crypto aspects, like parsing message formats.

Mathematical Abstraction





- A clearly defined game
 - ★ "winnable" is a clearly defined
- Like in chess, it is still very complex for automated analysis
 - ★ astronomical or infinite size of search trees
 - ★ computers are sometimes better than humans at it...
- Mind the gap
 - ★ Be clear about the abstractions and assumptions made
 - ★ Separation of concerns

Protocol Security

"Logical Hacking" and Security Proofs

- What is an "attack"? (and what is not?)
- How can we automatically find attacks?
- How can we prove the security of a system?
 - ... not just with respect to currently known attacks, but against any attacks!
 - ★ Is that even possible?
 - ★ Can we do that even automatically?
- How can we build systems that are secure?

This requires a precise definitions of

- the systems in questions
- its goals
- the assumptions (in particular, the intruder)

Main Takeaways from Protocol Security

- You may easily overlook security problems if you are not precise about: what the system does, what the security goals are, and what the intruder can do.
- Even if you are precise on this, manual analysis can overlook problems. Tools can often find something you overlooked.
 - ★ In particular one should by default assume that participants could be dishonest and all dishonest people work together.
- You can often easily avoid a lot of problems by being explicit in messages about who is communicating and what the message is supposed to say
- You can often easily make a system resilient against guessing attacks, even though users use weak passwords.

Today's Program

1 Alice and Bob

Alice and Bob

Alice and Bob notation

 aka Message Sequence Charts aka Protocol Narrations

- A -> B: {NA,A}pk(B) B -> A: {NA,NB}pk(A)
- $A \rightarrow B: \{NB\}pk(B)$
- popular informal notation for protocols

AnB

- A formal language based on Alice and Bob notation
 - ★ Defining the roles of the protocol and their initial knowledge
 - Indirectly defining the intruder's initial knowledge
 - Indirectly defining how agents execute the protocol
 - ★ Defining the security goals of the protocol
- OFMC: Open-Source Fixedpoint Model-Checker.
 - ★ Automatically finding attacks in protocols
 - ★ AnB is one input language for OFMC.

Capture-the-flag challenges in AnB/OFMC: given a flawed protocol, find a fix that OFMC cannot attack anymore—without changing initial knowledge and goals of the protocol.

Example

Step-by-step development of a protocol for the following scenario:

- A (Alice) and B (Bob) want a secure connection with each other, but have no prior security relationship.
- There is a server s and both A and B each have with s a shared secret key sk(A, s) and sk(B, s), respectively.
- s should now help A and B establish a secure connection, i.e., a shared secret key KAB that they can use to communicate with each other.
- This only works if s is honest (why?)

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk:
Knowledge:
   A: A, B, s, sk(A, s);
   B: A, B, s, sk(B, s);
   s: A, B, s, sk(A, s), sk(B, s);
Actions:
   A \rightarrow s : A, B
   s \rightarrow A : KAB
   A \rightarrow B : KAB
Goals:
   KAB secret between A, B, s
   A authenticates s on KAB
```

B authenticates s on KAB

 A, B are variables of type Agent: they can be instantiated with any agent name during the run of the protocol

```
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Types:
   Agent A, B, s;
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Knowledge:
   A: A, B, s, sk(A, s);
   B: A, B, s, sk(B, s);
   s: A, B, s, sk(A, s), sk(B, s);
Actions:
   A \rightarrow s : A, B
   s \rightarrow A : KAB
   A \rightarrow B : KAB
Goals:
   KAB secret between A, B, s
```

A authenticates s on KAB
B authenticates s on KAB

- A, B are variables of type Agent: they can be instantiated with any agent name during the run of the protocol
 - ★ ... including the intruder i
 - ★ The intruder can thus play the role of A or B

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Protocol: KeyExchange
Types:
    Agent A, B, s;
    Symmetric_key KAB;
    Function sk;
Knowledge:
    A: A, B, s, sk(A, s);
    B: A, B, s, sk(B, s);
    s: A, B, s, sk(A, s), sk(B, s);
```

Actions:

 $A \rightarrow s : A, B$ $s \rightarrow A : KAB$ $A \rightarrow B : KAB$

Goals:

- A, B are variables of type Agent: they can be instantiated with any agent name during the run of the protocol
 - ★ ... including the intruder i
 - ★ The intruder can thus play the role of A or B
- s is a constant of type Agent: there is only one agent called s who will play in all sessions

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Protocol: KeyExchange
Types:
    Agent A, B, s;
    Symmetric_key KAB;
    Function sk;
Knowledge:
    A: A, B, s, sk(A, s);
    B: A, B, s, sk(B, s);
```

Actions:

 $A \rightarrow s : A, B$ $s \rightarrow A : KAB$ $A \rightarrow B : KAB$

Goals:

KAB secret between A, B, s A authenticates s on KAB B authenticates s on KAB

s: A, B, s, sk(A, s), sk(B, s);

- A, B are variables of type Agent: they can be instantiated with any agent name during the run of the protocol
 - ★ ... including the intruder i
 - ★ The intruder can thus play the role of A or B
- s is a constant of type Agent: there is only one agent called s who will play in all sessions
 - ★ the intruder cannot play the role of *s*
 - ★ s is thus a trusted third party

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk:
Knowledge:
   A: A, B, s, sk(A, s);
   B: A, B, s, sk(B, s);
   s: A, B, s, sk(A, s), sk(B, s);
Actions:
   A \rightarrow s : A, B
   s \rightarrow A : KAB
   A \rightarrow B : KAB
Goals:
   KAB secret between A, B, s
```

A authenticates s on KAB
B authenticates s on KAB

- *KAB* is a variable of type symmetric key.
 - ★ The value will be freshly created during the protocol run.

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk:
```

Knowledge:

A: A, B, s, sk(A, s);B: A, B, s, sk(B, s);s: A, B, s, sk(A, s), sk(B, s);

Actions:

 $A \rightarrow s : A, B$ $s \rightarrow A : KAB$ $A \rightarrow B : KAB$

Goals:

- KAB is a variable of type symmetric key.
 - ★ The value will be freshly created during the protocol run.
- sk is a user-defined function. We use it to model shared secret. keys of two agents that are fixed before the protocol run.

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk:
Knowledge:
   A: A, B, s, sk(A, s);
   B: A, B, s, sk(B, s);
   s: A, B, s, sk(A, s), sk(B, s);
Actions:
   A \rightarrow s : A, B
   s \rightarrow A : KAB
   A \rightarrow B : KAB
Goals:
   KAB secret between A, B, s
   A authenticates s on KAB
```

B authenticates s on KAB

- It is necessary to specify an initial knowledge for every role of the protocol.
 - ★ It determines how agents send and receive messages

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk:
Knowledge:
   A: A, B, s, sk(A, s);
   B: A, B, s, sk(B, s);
   s: A, B, s, sk(A, s), sk(B, s);
Actions:
   A \rightarrow s : A, B
   s \rightarrow A : KAB
   A \rightarrow B : KAB
Goals:
```

- It is necessary to specify an initial knowledge for every role of the protocol.
 - ★ It determines how agents send and receive messages
- Typically everybody knows all agent names.

```
Protocol : KeyExchange
Types :
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk;
```

Knowledge:

A: A, B, s, sk(A, s); B: A, B, s, sk(B, s);s: A, B, s, sk(A, s), sk(B, s);

Actions:

 $A \rightarrow s : A, B$ $s \rightarrow A : KAB$ $A \rightarrow B : KAB$

Goals:

- It is necessary to specify an initial knowledge for every role of the protocol.
 - ★ It determines how agents send and receive messages
- Typically everybody knows all agent names.
- A knows a secret key with the server: sk(A, s)
- B knows a secret key with the server: sk(B, s)
- s knows both sk(A, s) and sk(B, s)

```
Protocol : KeyExchange
Types :
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk;
```

Knowledge:

$$A: A, B, s, sk(A, s);$$

 $B: A, B, s, sk(B, s);$
 $s: A, B, s, sk(A, s), sk(B, s);$

Actions:

 $A \rightarrow s : A, B$ $s \rightarrow A : KAB$ $A \rightarrow B : KAB$

Goals:

- The idea of the protocol is to establish a fresh secret key KAB between A and B
 - ★ A and B initially do not have any key material with each other
 - ★ but both have a shared key with trusted third party s that can be used for establishing KAB.
- Question: why would this be impossible if we had an untrusted S instead of s?

```
Protocol: KeyExchange

Types:

Agent A, B, s;

Symmetric_key KAB;

Function sk;

Knowledge:

A: A, B, s, sk(A, s);

B: A, B, s, sk(B, s);

s: A, B, s, sk(A, s), sk(B, s);

Actions:

A \rightarrow s: A, B
```

Goals:

- The knowledge section also determines the initial knowledge of the intruder:
 - ★ Say A = i and B = b for agent i in role A and honest b in role B.
 - ★ Then the intruder gets the knowledge of A under this instantiation: i, b, s, sk(i, s)
 - ★ The intruder thus also has a shared secret key with s!
 - ★ That's only fair: the intruder should know enough to play a protocol role as a normal user.

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk;
```

Knowledge:

A: A, B, s, sk(A, s);B: A, B, s, sk(B, s);s: A, B, s, sk(A, s), sk(B, s);

Actions:

 $A \rightarrow s : A, B$ $s \rightarrow A : KAB$ $A \rightarrow B : KAB$

Goals:

- The protocol starts by A contacting s stating the names of A and B
- Without crypto, there is no reliable information about senders and receivers.
- The intruder may intercept messages sent by honest agents, and insert arbitrary messages as if coming from any agent.
- A and B are not IP addresses. but unique identifiers (think domain name or user name/CPR).
- All agent names as public for now. Privacy: later lecture.

Protocol: KeyExchange
Types:
 Agent A, B, s;
 Symmetric_key KAB;
 Function sk;
Knowledge:
 A: A, B, s, sk(A, s);
 B: A, B, s, sk(B, s);
 s: A, B, s, sk(A, s), sk(B, s);

Actions:

 $egin{array}{lll} A &
ightarrow & s & : & A,B \ s &
ightarrow & A & : & \textit{KAB} \ A &
ightarrow & B & : & \textit{KAB} \end{array}$

Goals:

- The server generates a fresh shared key KAB for A and B.
 - ★ The entity first using a non-agent variable is the creator.
- Here, KAB is sent in clear text to A. This obviously is not secure in an intruder-controlled network.
- In the last step A forwards the key to B (also in clear...)
- The server cannot directly send the key to both A and B, because a message can only have one recipient who has to be the sender of the next message

```
Protocol: KeyExchange
Types:
   Agent A, B, s;
   Symmetric_key KAB;
   Function sk:
Knowledge:
   A: A, B, s, sk(A, s);
                                 • The secrecy goal: only A, B,
   B: A, B, s, sk(B, s);
                                    and s may know the key.
   s: A, B, s, sk(A, s), sk(B, s);

    The authentication goals: later

Actions:
   A \rightarrow s : A, B
   s \rightarrow A : KAB
   A \rightarrow B : KAB
Goals:
   KAB secret between A, B, s
   A authenticates s on B, KAB
```

B authenticates s on A, KAB

```
Running OFMC we get an attack:
Protocol: KeyExchange
Types:
                                     SUMMARY:
   Agent A, B, s;
                                       ATTACK_FOUND
   Symmetric_key KAB;
                                     GOAL:
   Function sk:
                                       secrets
Knowledge:
   A: A, B, s, sk(A, s);
   B: A, B, s, sk(B, s);
                                     ATTACK TRACE:
   s: A, B, s, sk(A, s), sk(B, s);
                                     i \rightarrow (s,1): x32,x31
Actions:
                                     (s.1) \rightarrow i: KAB(1)
   A \rightarrow s : A, B
                                     i can produce secret KAB(1)
   s \rightarrow A : KAB
   A \rightarrow B : KAB
                                     secret leaked: KAB(1)
Goals:
   KAB secret between A, B, s
                                     First: try to associate attack steps
   A authenticates s on B, KAB
```

B authenticates s on A, KAB

with protocol steps

```
A \rightarrow s : A, B

s \rightarrow A : KAB

A \rightarrow B : KAB

i \rightarrow (s,1): x32,x31

(s,1) \rightarrow i: KAB(1)

i can produce secret KAB(1)
```

- OFMC uses internal variables like x32 and x31 for things the intruder can arbitrarily choose.
 - \star Here, the intruder can choose any agent names for A and B
- KAB(1) means a fresh key that was generated by an honest agent the number (1) is to make it unique.
- (s,1) means server in session 1 (sometimes an attack may involve several sessions/runs of the protocol)
- i is the intruder
- 1 Here the intruder contacts the server s posing as some agent x32 (role A) who wants to talk to x31 (role B).
- 2 The server generates a new key KAB(1) for x32 and x31 and sends it.
- 3 The intruder sees this key, violating secrecy.

How to Encrypt this?

```
ofmc: Protocol not executable:
At the following state of the knowledge:
A->s: A,B
...one cannot compose the
s->A: {| KAB |}sk(A,s) following message:
A->B: {| KAB |}sk(B,s) {|KAB|}sk(B,s)
sk(B,s)
```

• $\{|KAB|\}_{sk(A,s)}$ means symmetric encryption of KAB with key sk(A,s).

lsk

- The server can do that, knowing sk(A, s).
- However A cannot produce $\{|KAB|\}_{sk(B,s)}$ for B.
- OFMC rejects this specification since A cannot generate a message that the protocol tells her to send.
 - ★ In the error message you can see what OFMC tried: the message $\{|KAB|\}_{sk(B,s)}$ is not known to A, and neither is sk(B,s) nor the entire function sk.

- In the second version, s generates both encrypted messages.
 - \star A cannot decrypt the second one, but she can forward it to B.
- This is now a meaningful specification, but OFMC finds an attack:

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- This is now a meaningful specification, but OFMC finds an attack:
 - ★ The intruder again chooses two agent names, and the server generates encrypted keys for them.
 - ★ The intruder forwards the part for x401 as required in the protocol.

 $\{|KAB(1)|\}_{(sk(x401,s))}$

 $i \rightarrow (x401,1): \{|KAB(1)|\}_{(sk(x401,s))}$

- In the second version, s generates both encrypted messages.
 - \star A cannot decrypt the second one, but she can forward it to B.
- This is now a meaningful specification, but OFMC finds an attack:
 - ★ The intruder again chooses two agent names, and the server generates encrypted keys for them.
 - ★ The intruder forwards the part for x401 as required in the protocol.
 - ★ So how does this represent an attack?

- Adding the agent names in clear text to the messages allows to see what's going wrong:
 - ★ To s, the intruder claims to be x32
 - ★ To B (x401), the intruder claims to be x30
- Thus there is confusion between B and s as to who A is This violates the goal

B authenticates s on A, KAB;

- Adding the agent names in clear text to the messages allows to see what's going wrong:
 - ★ To s, the intruder claims to be x32
 - ★ To B (x401), the intruder claims to be x30
- Thus there is confusion between *B* and *s* as to who *A* is This violates the goal
 - B authenticates s on A, KAB;
- Suppose x32=i, then the intruder can see KAB(1) while B thinks he shares KAB(1) with x30.

GOAL: weak auth

```
A->s: A,B ATTACK TRACE:
s->A: {| B,KAB |}sk(A,s), i -> (s,1): x401,x30
```

```
{| A,KAB |}sk(B,s) (s,1) -> i: {|x30,KAB(1)|}_(sk(x401,s)),
A->B: {| A,KAB |}sk(B,s) {|x401,KAB(1)|}_(sk(x30,s))
```

```
i -> (x401,1): {|x30,KAB(1)|}_(sk(x401,s)
```

- Third version adds the name of the other party to the encrypted message.
- There is an attack, but it is a bit hard to see what is wrong.
- Let us replace the variables in the attack trace with concrete agent names a and b.

```
GOAL: weak_auth

A->s: A,B

ATTACK TRACE:

s->A: {| B,KAB |}sk(A,s), i -> (s,1): a,b

{| A,KAB |}sk(B,s) (s,1) -> i: {|b,KAB(1)|}_(sk(a,s)),

A->B: {| A,KAB |}sk(B,s) {|a,KAB(1)|}_(sk(b,s))

i -> (a,1): {|b,KAB(1)|}_(sk(a,s))
```

- Third version adds the name of the other party to the encrypted message.
- From s's point of view: role A is played by a, role B by b.

```
GOAL: weak_auth

A->s: A,B

ATTACK TRACE:

s->A: {| B,KAB |}sk(A,s), i -> (s,1): a,b

{| A,KAB |}sk(B,s) (s,1) -> i: {|b,KAB(1)|}_(sk(a,s)),

A->B: {| A,KAB |}sk(B,s) {|a,KAB(1)|}_(sk(b,s))

i -> (a,1): {|b,KAB(1)|}_(sk(a,s))
```

- Third version adds the name of the other party to the encrypted message.
- From s's point of view: role A is played by a, role B by b.
- From a's point of view: role A is played by b, role B is played by a. This violates again the authentication goal between B and s.

```
GOAL: weak_auth

A->s: A,B

ATTACK TRACE:

s->A: {| B,KAB |}sk(A,s), i -> (s,1): a,b

{| A,KAB |}sk(B,s) (s,1) -> i: {|b,KAB(1)|}_(sk(a,s)),

A->B: {| A,KAB |}sk(B,s) {|a,KAB(1)|}_(sk(b,s))

i -> (a,1): {|b,KAB(1)|}_(sk(a,s))
```

- Third version adds the name of the other party to the encrypted message.
- From s's point of view: role A is played by a, role B by b.
- From a's point of view: role A is played by b, role B is played by a. This violates again the authentication goal between B and s.
- In many scenarios, it is a serious problem if the intruder can confuse agents about the role they play.

Fourth Version

GOAL: strong_auth

- i -> (b,2): a,b,{|a,b,KAB(1)|}_(sk(b,s))
 Fourth version: in all encrypted messages we write both A and B-the ordering avoids the confusion.
 - ★ Alternative: have to tags init and resp to make clear which one is the initiator A and who is the responder B.

Fourth Version

GOAL: strong_auth

- Fourth version: in all encrypted messages we write both A and B-the ordering avoids the confusion.
 - ★ Alternative: have to tags init and resp to make clear which one is the initiator A and who is the responder B.
- In the attack, the intruder sends the last message a second time to b.
 - \star For b, this is a completely new protocol run—note (b,1) vs. (b,2)
 - ★ This is a replay attack: *b* is made to accept something a second time that was actually only said once by *s*.

```
GOAL: strong_auth
ATTACK TRACE:

i -> (s,1): a,b

{|A,B,KAB|}sk(B,s)

A->B: {|A,B,KAB|}sk(B,s)

i -> (b,1): a,b,{|a,b,KAB(1)|}_(sk(a,s)),

i -> (b,1): a,b,{|a,b,KAB(1)|}_(sk(b,s))

i -> (b,2): a,b,{|a,b,KAB(1)|}_(sk(b,s))
```

- Fourth version: in all encrypted messages we write both A and B-the ordering avoids the confusion.
 - ★ Alternative: have to tags init and resp to make clear which one is the initiator A and who is the responder B.
- In the attack, the intruder sends the last message a second time to b.
 - \star For b, this is a completely new protocol run—note (b,1) vs. (b,2)
 - ★ This is a replay attack: *b* is made to accept something a second time that was actually only said once by *s*.
- Replay can often be exploited, for instance:
 - ★ a bank transfer that was ordered once is executed many times
 - ★ an agent is made to accept an old broken key

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```
GOAL: strong_auth
ATTACK TRACE:

i -> (s,1): a,b

{|A,B,KAB|}sk(B,s)

A->B: {|A,B,KAB|}sk(B,s)

i -> (b,1): a,b,{|a,b,KAB(1)|}_(sk(a,s)),

i -> (b,1): a,b,{|a,b,KAB(1)|}_(sk(b,s))

i -> (b,2): a,b,{|a,b,KAB(1)|}_(sk(b,s))
```

- Note strong_auth at GOAL: this appears in OFMC whenever the agreement on the names and data is correct, but something has been accepted more often than it was said (a replay attack).
- One can turn off the replay detection and just ask for the pure agreement by changing the goal to weak authentication:

```
A weakly authenticates s on B,KAB;
B weakly authenticates s on A,KAB;
```

 One can turn off the replay detection and just ask for the pure agreement by changing the goal to weak authentication:

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B weakly authenticates s on A,KAB;
```

```
A->s: A,B
s->A: {|A,B,KAB|}sk(A,s),
{|A,B,KAB|}sk(B,s)
A->B: {|A,B,KAB|}sk(B,s)
```

 One can turn off the replay detection and just ask for the pure agreement by changing the goal to weak authentication:

```
A weakly authenticates s on B, KAB;
B weakly authenticates s on A, KAB;
```

Then OFMC will output:

```
Open-Source Fixedpoint Model-Checker version 2024
Verified for 1 sessions
Verified for 2 sessions
^C
```

- Here ^C means that I pressed Control-C to stop, because it will go on forever when no attack is found, checking more and more sessions.
- For the purposes of this course it is fine to step after two sessions, and you can do this in OFMC directly with the option --numSess 2

Fifth Version

- The best way to solve replay is to use challenge response:
 - ★ Participants create a fresh random number like NA and NB.
 - ★ They are included in encrypted messages to prove that the encryption is not older than the fresh numbers.

Fifth Version

```
Number NA,NB;
...
B->A: NB
A->s: A,B,NA,NB
SUMMARY:
NO_ATTACK_FOUND
{|A,B,KAB,NA,NB|}sk(A,s),
{|A,B,KAB,NA,NB|}sk(B,s)
A->B: {|A,B,KAB,NA,NB|}sk(B,s)
```

- The best way to solve replay is to use challenge response:
 - ★ Participants create a fresh random number like NA and NB.
 - ★ They are included in encrypted messages to prove that the encryption is not older than the fresh numbers.
 - ★ We are done. However there is a better way to do this using Diffie-Hellman.

```
Protocol: KeyExchange
Types: Agent A,B,s;
       Number X,Y,g,Payload;
       Function sk:
Knowledge: A: A,B,s,sk(A,s),g;
           B: A,B,s,sk(B,s),g;
            s: A,B,s,sk(A,s),sk(B,s),g;
Actions:
A \rightarrow B: \exp(g,X)
B->s: {| A,B,\exp(g,X),\exp(g,Y) |}sk(B,s)
s\rightarrow A: \{|A,B,\exp(g,X),\exp(g,Y)|\}sk(A,s)
A->B: {| Payload |}exp(exp(g,X),Y)
Goals:
exp(exp(g,X),Y) secret between A,B;
Payload secret between A,B;
A authenticates B on exp(exp(g,X),Y);
B authenticates A on exp(exp(g,X),Y),Payload;
```

```
A->B: exp(g,X)
B->s: {| A,B,exp(g,X),exp(g,Y) |}sk(B,s)
s->A: {| A,B,exp(g,X),exp(g,Y) |}sk(A,s)
A->B: {| Payload |}exp(exp(g,X),Y)
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Diffie-Hellman:

- every agent generates a random X and Y
- they exchange $\exp(g, X) \mod p$ and $\exp(g, Y) \mod p$
 - ★ p is a large fixed prime number we omit in OFMC
 - \star g is a fixed generator of the group \mathbb{Z}_p^{\star}
 - \star Both p and g are public
 - \star we omit writing mod p in OFMC
- It is computationally hard to obtain X from $\exp(g,X) \mod p$
- However A and B have now a shared key $\exp(\exp(g, X), Y)$ mod $p = \exp(\exp(g, Y), X)$ mod p

	Classic	
Group	$\mathbb{Z}_p^{\star} = \{1,\ldots,p-1\}$	
Group Op.	$\times: \mathbb{Z}_p^{\star} \times \mathbb{Z}_p^{\star} \to \mathbb{Z}_p^{\star}$	
	(Mult. modulo p)	
Generator	$g \in \mathbb{Z}_p^{\star}$	
Secrets	$X, Y \in \{1, \ldots, p-1\}$	
Half keys	$g^X := \underbrace{g \times \ldots \times g}$	
	$g^Y := \dots$	
Full key	$(g^X)^Y = (g^Y)^X$	

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	Classic	Elliptic Curve (ECDH)
Group	$\mathbb{Z}_p^{\star} = \{1,\ldots,p-1\}$	Finite field \mathbb{F} of order n
Group Op.	$\times: \mathbb{Z}_p^{\star} \times \mathbb{Z}_p^{\star} \to \mathbb{Z}_p^{\star}$	$+: \mathbb{F} \times \mathbb{F} \to \mathbb{F}$
	(Mult. modulo <i>p</i>)	(not quite so intuitive)
Generator	$g \in \mathbb{Z}_p^{\star}$	g on curve
Secrets	$X, Y \in \{1, \ldots, p-1\}$	$X, Y \in \{1, \ldots, n-1\}$
Half keys	$g^X := \underbrace{g \times \ldots \times g}$	$X \cdot g := \underbrace{g + \ldots + g}$
	$g^Y := \dots$	$Y \cdot g := \dots$
Full key	$(g^X)^Y = (g^Y)^X$	$X \cdot Y \cdot g = Y \cdot X \cdot g$

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Secrets	$X, Y \in \{1, \ldots, p-1\}$	$X,Y\in\{1,\ldots,n-1\}$
Half keys	$g^X := \underbrace{g \times \ldots \times g}$	$g^X := \underbrace{g \times \ldots \times g}$
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Trick: write \times for the group operation also in ECDH.

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Typical size	thousand of bits	hundreds of bits

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 - \star The trusted party s does not even get to know the key
 - ▶ An honest but curious *s* cannot read messages between *A* and *B*.

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 - ★ Perfect Forward Secrecy: The intruder cannot read Payload even when learning sk(A,s) and sk(B,s) after the exchange.
- Do we even need the trusted party s then? Yes!
 - ★ exp(g,X) and exp(g,Y) are public
 - you may call them public keys (with X and Y the private keys)
 - ★ but they need to be authenticated (like public keys):
 - ▶ that exp(g,X) really comes from A
 - ▶ and exp(g,Y) really comes from B

Modeling Agents and Fixed Key-Infrastructures

- Normally variables (uppercase) like A,B,C,...
 - ★ can be played by any concrete (lowercase) agent like a,b,c,...,i
- Special agent: i the intruder
- Honest agent: constant like s for a trusted server
 - ★ Cannot be instantiated (especially the intruder), fixed in all protocol runs
- Given key infrastructures: use functions e.g.
 - \star sk(A,B) the shared key of A and B
 - \star pw(A,B) the password of A at server B
 - ★ pk(A) the public key of A
 - ▶ inv(K) is the private key that belongs to public key K.
 - Note inv and exp are a built-in function (do not declare as a function).
 - ★ Give every role the necessary initial knowledge

AnB: Things to Note

- Identifiers that start with uppercase: variables (E.g., A,B,KAB)
- Identifiers that start with lowercase: constants and functions (E.g., s,pre,sk)
- One should declare a type for all identifiers; OFMC can search for type-flaw attacks when using the option -untyped (in which case all types are ignored).
- The (initial) knowledge of agents MUST NOT contain variables of any type other than Agent.
 - ★ For long-term keys, passwords, etc. use functions like sk(A, B).
- Each variable that does not occur in the initial knowledge is freshly created during the protocol by the first agent who uses it.
 - ★ In the NSSK example, A creates NA, s creates KAB, B creates NB.

- Symmetric Cryptography:
 - ★ two (or more) agents share a secret key K
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 - ★ one can decrypt $\{M\}_K$ only when knowing K

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- Asymmetric Encryption (Public-key Encryption):
 - \star every agent A has a key-pair (K, inv(K)) consisting of
 - ▶ the public key *K* that everybody knows
 - ▶ the private key inv(K) that only A knows
 - ★ $\{M\}_K$ denotes asymmetric encryption of M with public key K.
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 - ★ $\{M\}_K$ denotes asymmetric encryption of M with public key K.
 - \star one can decrypt $\{M\}_K$ only when knowing inv(K)
- Digital Signatures
 - ★ Signing is "encryption" with a private key
 - ★ Signature checking is "decryption" with a public key
 - \star Thus, if (K, inv(K)) is the key pair of A then
 - \blacktriangleright $\{M\}_{inv(K)}$ can only be produced by A
 - ▶ but everybody can read *M* and check that it comes from *A*.

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- Timestamps
- Concatenation, i.e., a sequence of messages
 - \star written as M_1, M_2, M_3 for simplicity
 - ★ reality: quite complex encodings and source of mistakes see also: Mödersheim & Katsoris. A sound abstraction of the parsing problem, CSF 2014.