MODULE *Contracts*

This spec captures the behaviour of commitment transactions on the two sides of a Lightning channel.

We model the various kinds of outputs a commitment transactions will have over its lifetime.

The state of the commitment transaction changes in reponse to the various actions like supersede, publish, etc are taken by parties.

We also do not deal with the communication protocol between nodes for creating and updating commitment transactions. This spec will only focuss on the various commitment transaction and their lifecycle in response to interaction between parties and the blockchain.

We ignore the details of how transactions are signed and just mark transactions as signed. This lets us focus on the specifying the behaviour of the commitment transactions without dealing with lower level complexities.

The model defines the intial balance from *alice* to bob. TLA+ will handle situations where channels are balanced and when all the balance is on the other side.

TODO: Add *HTLCs*! Now that will be fun!

EXTENDS *Integers*, *TLC*, *Sequences*, *FiniteSets*

CONSTANTS

CSV,	The csv value to use in contracts
Height,	The height up to which we run the spec
NumTxs,	The number of commitment txs we want
InitialBalance	Initial balances for *alice* and bob

Current channel contracts only ever have two parties

$Party \triangleq \{\text{“alice”}, \text{“bob”}\}$

For the first revocation we only need two keys per party

$NumKey \triangleq 2$

Set of all keys

$Key \triangleq \{\langle p, k\rangle : p \in Party, k \in 0 \,..\, NumKey\}$

Value to capture missing *CSV* in output

$NoCSV \triangleq \text{CHOOSE } c : c \notin 0 \,..\, CSV$

Multisig outputs without *CSV* encumberance

$MultiSig \triangleq Party \times Party \times \{NoCSV\}$

Multisig outputs with *CSV* encumberance

$MultiSigWithCSV \triangleq Party \times Party \times \{CSV\}$

$P2WKH$ outputs, without encumbrance

$P2WKH \triangleq Key$

Set of all signatures for all commit txs. The signature in real world is related to the commit transaction. However, we leave out this complication of how the signature is generated. If there is a signature by a key on a tx, it is assumed it is correctly signed as per bitcoin's requirements

$Sig \triangleq \{\langle p, k\rangle : p \in Party, k \in 0 \,..\, NumKey - 1\}$

Value to capture unsigned transactions

$NoSig \triangleq \text{CHOOSE } s : s \notin Sig$

$$\begin{aligned} CT \triangleq\ & [index \mapsto 0 \,..\, NumTxs, \\ & multisig \mapsto MultiSigWithCSV,\ pk \mapsto P2WKH, \\ & local\_sig \mapsto Sig \cup \{NoSig\}, \\ & remote\_sig \mapsto Sig \cup \{NoSig\}, \\ & balance \mapsto -InitialBalance \,..\, InitialBalance] \end{aligned}$$

$PublishId \triangleq \{\langle p, i, h\rangle : p \in Party, i \in 0 \,..\, NumTxs, h \in 0 \,..\, Height\}$

$NoSpend \triangleq \langle\rangle$

VARIABLES

$alice\_cts$,	Commitment tx for *alice*
bob_cts,	Commitment tx for bob
$alice\_brs$,	Breach remedy transactions for *alice*
bob_brs,	Breach remedy transactions for bob
$mempool\_ct$,	The CT txs that have been broadcasted.
$published\_ct$	The CT that has been included in a block and confirmed.

$vars \triangleq \langle alice\_cts, bob\_cts, alice\_brs, bob\_brs, mempool\_ct, published\_ct\rangle$

Helper function to get other party

$OtherParty(party) \triangleq \text{CHOOSE } p \in Party : p \neq party$

Create a commitment transaction given the party, index and key to use.

$$\begin{aligned} CreateCT(party, index, key\_num, balance) \triangleq\ & [index \mapsto index, \\ & multisig \mapsto \langle party, OtherParty(party), CSV\rangle, \\ & pk \mapsto \langle party, key\_num\rangle, \\ & local\_sig \mapsto NoSig, \\ & remote\_sig \mapsto \langle OtherParty(party), key\_num\rangle, \\ & balance \mapsto balance] \end{aligned}$$

$Init \triangleq$

Once sided channel to start with

$\land\ alice\_cts = \{CreateCT(\text{“alice”},\ 0,\ 0,\ InitialBalance)\}$
$\land\ bob\_cts = \{CreateCT(\text{“bob”},\ 0,\ 0,\ 0)\}$
$\land\ alice\_brs = \{\}$
$\land\ bob\_brs = \{\}$
$\land\ mempool\_ct = \{\}$
$\land\ published\_ct = NoSpend$

$TypeInvariant\ \triangleq$
$\land \forall\, ct \in alice\_cts \cup bob\_cts :$
$\land\ ct.index \in 0\,..\,NumTxs$
$\land\ ct.local\_sig \in Sig \cup \{NoSig\}$
$\land\ ct.remote\_sig \in Sig \cup \{NoSig\}$
$\land\ ct.pk \in P2WKH$
$\land\ ct.multisig \in MultiSigWithCSV$
$\land \forall\, br \in alice\_brs \cup bob\_brs :$
$\land\ br.index \in 0\,..\,NumTxs$
$\land\ br.pk \in P2WKH$
$\land\ mempool\_ct \in \text{SUBSET}\ PublishId$
$\land\ published\_ct \in PublishId \cup \{NoSpend\}$

$MaxIndex(party\_cts)\ \triangleq$
$(\text{CHOOSE}\ x \in party\_cts : \forall\, y \in party\_cts : x.index \geq y.index).index$

$LastCT(party\_cts)\ \triangleq$
$\text{CHOOSE}\ ct \in party\_cts : \forall\, y \in party\_cts : ct.index \geq y.index$

Create first commitment transactions for given parties

Breach remedy transactions are pre-signed transactions instead of they private key being sent over to the other party.

delta is the balance going from *alice* to bob. We allow negative balances to enable payments in other other direction.

Parties are free to keep creating CT even if FT is spent. They will not be usable, but the protocol does not disallow this.

$SupersedeCommitmentTx(index,\ delta)\ \triangleq$
$\land$
LET
$key\_index\ \triangleq\ 1$
$last\_alice\_ct\ \triangleq\ LastCT(alice\_cts)$
$last\_bob\_ct\ \triangleq\ LastCT(bob\_cts)$
IN
$\land\ index > MaxIndex(alice\_cts)$
$\land\ index > MaxIndex(bob\_cts)$
$\land\ last\_alice\_ct.balance - delta > 0$

$\land\ last\_bob\_ct.balance + delta > 0$
$\land\ alice\_cts' = alice\_cts \cup$
$\{CreateCT(\text{“alice”}, index, key\_index,$
$last\_alice\_ct.balance - delta)\}$
$\land\ bob\_cts' = bob\_cts \cup$
$\{CreateCT(\text{“bob”}, index, key\_index,$
$last\_alice\_ct.balance + delta)\}$
$\land\ alice\_brs' = alice\_brs \cup$
$\{[index \mapsto index, pk \mapsto \langle \text{“bob”}, key\_index\rangle]\}$
$\land\ bob\_brs' = bob\_brs \cup$
$\{[index \mapsto index, pk \mapsto \langle \text{“alice”}, key\_index\rangle]\}$
$\land$ UNCHANGED $\langle mempool\_ct, published\_ct\rangle$

Publish a commitment transaction to the blockchain. The commitment is first signed. The protocol allows all commitments to be published, what happens next depends on the status of the commitment transaction.

If the tx is the latest commitment transaction it is succesfuly spend.

If not, it gives the other party a chance to spend the breach remedy tx.

TODO: We only spec *CSV* (self) commitment transaction. We need to handle the non-*CSV* output being published and co-op closes.

$PublishCommitment(party, index, height) \triangleq$
$\land\ mempool\_ct = \{\}$
$\land\ mempool\_ct' = mempool\_ct \cup \{\langle party, index, height\rangle\}$
$\land$ UNCHANGED $\langle alice\_cts, bob\_cts, alice\_brs, bob\_brs, published\_ct\rangle$

Publish a breach remedy transaction in response to a commitment transaction.

party is publishing the breach remedy tx when it is on index CT, and the chain is on height.

This tx is immediately published on chain.

TODO: We skip the *BR* going through the mempool and confirm it immeidiately. This can be improved too.

$PublishBR(party, index, height) \triangleq$
LET $cts \triangleq$ IF $party = \text{“alice”}$ THEN $alice\_cts$ ELSE bob_cts
IN
$\land\ published\_ct = NoSpend$ — No CT is confirmed on chain yet
$\land\ mempool\_ct \neq \{\}$ — Only if some CT has been published
$\land\ \exists\, m \in mempool\_ct :$
$\land\ m[1] = OtherParty(party)$ — CT was broadcastt by the other party
$\land\ m[2] < MaxIndex(cts)$ — Revoked CT was broadcast
$\land\ m[2] = index$ — We need to use the *BR* from the same index
$\land\ height - m[2] < CSV$ — Can only publish *BR* if *CSV* hasn't expired
Record which index was published at what height
$\land\ published\_ct' = \langle party, index, height\rangle$
$\land$ UNCHANGED $\langle alice\_cts, bob\_cts, alice\_brs, bob\_brs, mempool\_ct\rangle$

$$
\begin{array}{l}
\mathit{Next} \triangleq \\
\quad \lor \exists\, i \in 0 \,..\, \mathit{NumTxs},\ d \in -\mathit{InitialBalance} \,..\, \mathit{InitialBalance} : \mathit{SupersedeCommitmentTx}(i,\ d) \\
\quad \lor \exists\, i \in 0 \,..\, \mathit{NumTxs},\ p \in \mathit{Party},\ h \in 0 \,..\, \mathit{Height} : \mathit{PublishCommitment}(p,\ i,\ h) \\
\quad \lor \exists\, i \in 0 \,..\, \mathit{NumTxs},\ p \in \mathit{Party},\ h \in 0 \,..\, \mathit{Height} : \mathit{PublishBR}(p,\ i,\ h)
\end{array}
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\mathit{Spec} \triangleq \mathit{Init} \land \Box[\mathit{Next}]_{\langle vars \rangle}
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