
MODULE *Contracts*

This spec captures the behaviour of commitment transactions on the two sides of a Lightning channel.

We model the various kinds of outputs a commitment transactions will have over its lifetime.

The state of the commitment transaction changes in reponse to the various actions like supercede, spend, revoke etc are taken.

We also do not deal with the communication protocol between nodes for creating and updating commitment transactions. This spec only focusses on the various commitment transaction created, revoked, spent to open, close, force close or penalise.

We ignore the details of how transactions are signed and just mark transactions as signed. This lets us focus on the specifying the behaviour of the commitment transactions without dealing with lower level complexities.

EXTENDS *Integers*,
 TLC,
 Sequences,
 FiniteSets

CONSTANTS

<i>CSV</i> ,	The csv value to use in contracts
<i>Height</i> ,	The height up to which we run the spec
<i>NumTxs</i>	The number of commitment txs we want

Sequences utility

$SeqOf(set, n) \triangleq$

All sequences up to length n with all elements in set. Includes empty sequence.

UNION $\{[1 \dots m \rightarrow set] : m \in 0 \dots n\}$

$ToSet(s) \triangleq$

The image of the given sequence s . $Cardinality(ToSet(s)) \leq Len(s)$ see [https://en.wikipedia.org/wiki/Image_\(mathematics\)](https://en.wikipedia.org/wiki/Image_(mathematics))
 $\{s[i] : i \in \text{DOMAIN } s\}$

$Contains(s, e) \triangleq$

TRUE iff the element $e \in ToSet(s)$.

$\exists i \in 1 \dots Len(s) : s[i] = e$

Current channel contracts only ever have two parties

$Party \triangleq \{ \text{"alice"}, \text{"bob"} \}$

For the first revocation we only need two keys per party

$NumKey \triangleq 2$

Set of all keys

$$Key \triangleq \{\langle p, k \rangle : p \in Party, k \in 0 \dots NumKey\}$$

Value to capture missing CSV in output

$$NoCSV \triangleq \text{CHOOSE } c : c \notin 0 \dots CSV$$

Multisig outputs without CSV encumbrance

$$MultiSig \triangleq Party \times Party \times \{NoCSV\}$$

Multisig outputs with CSV encumbrance

$$MultiSigWithCSV \triangleq Party \times Party \times \{CSV\}$$

P2PKH outputs, without encumbrance

$$P2PKH \triangleq Key$$

Set of all signatures for all commit txs. The signature in real world is related to the commit transaction, however, leave out this complication of how the signature is generated. If there is a signature by a key on a tx, it is assumed it is correctly signed as per bitcoin's requirements

$$Sig \triangleq \{\langle p, k \rangle : p \in Party, k \in 0 \dots NumKey - 1\}$$

Value to capture unsigned transactions

$$NoSig \triangleq \text{CHOOSE } s : s \notin Sig$$

$$CT \triangleq [index \mapsto 0 \dots NumTxs, \\ multisig \mapsto MultiSigWithCSV, pk \mapsto P2PKH, \\ local_sig \mapsto Sig \cup \{NoSig\}, \\ remote_sig \mapsto Sig \cup \{NoSig\}]$$

VARIABLES

alice_cts,
bob_cts

$$vars \triangleq \langle alice_cts, bob_cts \rangle$$

Helper function to get other party

$$OtherParty(party) \triangleq \text{CHOOSE } p \in Party : p \neq party$$

$$CreateCT(party, index, key_num) \triangleq \\ [index \mapsto index, \\ multisig \mapsto \langle party, OtherParty(party), CSV \rangle, \\ pk \mapsto \langle OtherParty(party), key_num \rangle, \\ local_sig \mapsto NoSig, \\ remote_sig \mapsto \langle OtherParty(party), key_num \rangle]$$

$$Init \triangleq$$

$$\wedge \text{alice_cts} = \{ \text{CreateCT}(\text{"alice"}, 0, 0) \}$$

$$\wedge \text{bob_cts} = \{ \text{CreateCT}(\text{"bob"}, 0, 0) \}$$

We don't define transactions using a function because using variables as functions become hard to work with in TLA+.

$$\text{TypeInvariant} \triangleq$$

$$\wedge \forall ct \in \text{alice_cts} :$$

$$\wedge ct.\text{index} \in 0 \dots \text{NumTxs}$$

$$\wedge ct.\text{local_sig} \in \text{Sig} \cup \{ \text{NoSig} \}$$

$$\wedge ct.\text{remote_sig} \in \text{Sig} \cup \{ \text{NoSig} \}$$

$$\wedge ct.\text{pk} \in \text{P2PKH}$$

$$\wedge ct.\text{multisig} \in \text{MultiSigWithCSV}$$

Create first commitment transactions for given parties

$$\text{SupercedeCommitmentTx}(\text{index}) \triangleq$$

$$\wedge \text{index} = \text{Cardinality}(\text{alice_cts})$$

$$\wedge \text{Cardinality}(\text{alice_cts}) > 0 \wedge \text{Cardinality}(\text{bob_cts}) > 0$$

$$\wedge \text{Cardinality}(\text{alice_cts}) < \text{NumTxs} \wedge \text{Cardinality}(\text{bob_cts}) < \text{NumTxs}$$

$$\wedge \text{alice_cts}' = \text{alice_cts} \cup \{ \text{CreateCT}(\text{"alice"}, \text{index}, 1) \}$$

$$\wedge \text{bob_cts}' = \text{bob_cts} \cup \{ \text{CreateCT}(\text{"bob"}, \text{index}, 1) \}$$

Party p spends their commitment transaction.

If the tx is the latest commitment transaction it is successfully spend.

If not, it gives the other party a chance to spend the breach remedy tx.

$$\text{SpendCommitmentTx}(p, tx)$$

$$\text{Next} \triangleq$$

$$\exists \text{index} \in 0 \dots \text{NumTxs} : \text{SupercedeCommitmentTx}(\text{index})$$

$$\text{Spec} \triangleq \text{Init} \wedge \Box [\text{Next}]_{\langle \text{vars} \rangle}$$