$\boxed{1-2}$

$L(\lambda) = \prod_{i=1}^{n} \lambda f(x_i | \lambda) = \lambda^n \exp\left(-\sum_{i=1}^{n} \lambda x_i\right)$

$\log L(\lambda) = n \log \lambda - \sum_{i=1}^{n} \lambda x_i \Rightarrow \underset{\lambda}{\arg\max} \, \log L(\lambda) = \frac{\partial \log L(\lambda)}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0$

$\Rightarrow \lambda = \dfrac{n}{\sum_{i=1}^{n} x_i}$

$$P(\lambda) \sim G(\alpha, B) = \frac{B^\alpha}{(\alpha-1)!} \lambda^{\alpha-1} e^{-B\lambda}$$

$$P(x|\lambda) = \lambda^n \exp\left(-\lambda \sum_{i=1}^{n} x_i\right) \Rightarrow P(\lambda|x) \propto P(x|\lambda) P(\lambda)$$

$$\propto \lambda^n \exp\left(-\lambda \sum_{i=1}^{n} x_i\right) \frac{B^\alpha}{(\alpha-1)!} \lambda^{\alpha-1} e^{-B\lambda}$$

$$\propto \lambda^{n+\alpha-1} \exp\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right)$$

$$= Gamma\left(n+\alpha, \; B + \sum_{i=1}^{n} x_i\right)$$

$$MAP \Rightarrow \underset{\lambda}{argmax} \; P(\lambda|x) \propto \underset{\lambda}{argmax} \; \lambda^{n+\alpha-1} \exp\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right)$$

$$\Rightarrow (n+\alpha-1)\lambda^{n+\alpha-2} \exp\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right) + \lambda^{n+\alpha-1}\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right) \exp\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right) = 0$$

$$(n+\alpha-1)\exp\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right) = \left(B + \sum_{i=1}^{n} x_i\right) \exp\left(-\lambda\left(B + \sum_{i=1}^{n} x_i\right)\right) \lambda$$

$$\boxed{\lambda = \frac{n+\alpha-1}{B + \sum_{i=1}^{n} x_i}}$$