$$E[x^2(t)] = E[x(t)\,x(t+0)] = R_{xx}(0) = 12 \; *$$

$$\max \underbrace{\text{avg Power output}} = \max E[\text{Power output}] = \max E\left[|Y(t)|^2\right] =$$

$$\max E[Y(t)\,Y^*(t+0)] = \max R_{YY}(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{xx}(\omega)\,|H(\omega)|^2\,d\omega =$$

$$\leq \frac{1}{2\pi} \max |H(\omega)|^2 \int_{-\infty}^{+\infty} S_{xx}(\omega)\,d\omega = \frac{1}{2\pi}\max\left(\frac{1}{6}\right)^2 \underbrace{R_{xx}(0)}^{*} = \frac{1}{6\pi}$$

$$\frac{1}{36} = \frac{1}{6^2}$$

البته توجه داریم که ~~...~~ ماکزیمم $|H(\omega)|$ وقتی بدست می‌آید که $\omega = 0$ باشد [illegible]

حال ~~...~~ می‌خواهیم $S_x(\omega)$ را بدست آوریم پس داریم

$$S_{xx}(\omega) = \;?$$

فرمول
اسلاید صفحه ۳۱

$$\Rightarrow R_{xx}(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{xx}(\omega)\,d\omega = 12$$

$$\int_{-\infty}^{+\infty} S_x(\omega)\,d\omega = 12 \times 2\pi = 24\pi \Rightarrow S_x(\omega) = 24\pi\,\delta(\omega)$$

زیرا تابع دلتا معکوس ~~...~~ فوریه عدد ۱ است.

~~$R_{YY}(t,t)$~~ ②

$Y(t) = x(t+a) - x(t-a) \Rightarrow h(t) = \delta(t+a) - \delta(t-a) = h^*(t)$ real signal ②

الف

حل دادیم

$R_{YY}(\tau) = R_{YX}(\tau) * h(\tau)$

$R_{YY}(t_1,t_2) = R_{YX}(t_1,t_2) * h^*(t_2)$

$R_{YX}(t_1,t_2) = R_{XX}(t_1,t_2) * h(t_1) = R_{XX}(t_1,t_2) * [\delta(t_1+a) - \delta(t_1-a)] =$

$= R_{XX}(t_1,t_2) * \delta(t_1+a) - R_{XX}(\tau) * \delta(t_1-a)$

$= \int R_{XX}(t_1-\tau, t_2)\, \delta(\tau+a)\, d\tau - \int R_{XX}(t_1-\tau, t_2)\, \delta(\tau-a)\, d\tau$

$R_{XX}(t_1+a, t_2) - R_{XX}(t_1-a, t_2)$

$R_{YY}(t_1,t_2) = [R_{XX}(t_1+a,t_2) - R_{XX}(t_1-a,t_2)] * \underbrace{h(t_2)}$ $[\delta(t_2+a) - \delta(t_2-a)]$

$= R_{XX}(t_1+a, t_2+a) - R(t_1+a, t_2-a)$

$- R_{XX}(t_1-a, t_2+a) + R_{XX}(t_1-a, t_2-a)$

$= R_X(\tau) - R_X(\tau+2a) - R_X(\tau-2a) + R_X(\tau)$ ✓

ب

2

$$h(t) = \delta(t+a) - \delta(t-a)$$

$$H(\omega) = e^{-j\omega a} - e^{j\omega a} = \cancel{\cos(a\omega) - j\sin(a\omega)} \; -2j\sin(a\omega)$$

$$\Rightarrow S_Y(\omega) = S_X(\omega)\,|H(\omega)|^2 = 4S_X(\omega)\sin^2(a\omega)$$

③ پراسس $x(t)$ نرمال است یعنی

$$E\{x, x; \tau\} \to F^2(x) \text{ as } \tau \to \infty$$

حال اگر بخواهیم Mean Ergodic باشد باید داشته باشیم $\mu_x(t) = E[x(t,e)]$ ~~$\mu_x(t)$~~ μ_x را به صورت تخمین

$$\hat{x}_T(e_0) = \frac{1}{2T}\int_{-T}^{T} x(t, e_0)\,dt \Rightarrow E[\hat{x}_T(e_0)] = \frac{1}{2T}\int_{-T}^{T} E[x(t,e)]\,dt = \frac{1}{2T}\int_{-T}^{T} \mu_x\,dt = \mu_x$$

و اگر پراسس distribution ergodic باشد داریم $F_x = P\{x(t) \leq x\}$

به طور مشابه

$$y_T = \frac{1}{2T}\int_{-T}^{T} y(t)\,dt = \frac{\sum_{i=1}^{n} \tau_i}{2T}$$

$$\lim_{T\to\infty} y_T = F(x) \Rightarrow x(t) = \text{distribution - ergodic}$$

4 الف

$$R_{XY}(-\tau) = R_{xx}(\tau) * h^*(-\tau)$$

پس باید $R_{xx}(\tau)$ و $h^*(\tau)$ را حساب کنیم.

$$C_{xx}(\tau) = R_{xx}(\tau) - \eta_x \eta_y \Rightarrow$$ صورت $E[x(t)] = 0$

$$\Rightarrow C_{xx}(\tau) = R_{xx}(\tau)$$

سیگنال Real است. $\underline{h^* = h}$

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$\Downarrow \mathcal{F}$$

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega) \Rightarrow (2 + j\omega) Y(\omega) = X(\omega) \Rightarrow Y(\omega) = \frac{1}{1+j\omega} X(\omega)$$

$$H(\omega) = \frac{1}{2+j\omega} \Rightarrow h(t) = e^{-2t} u(t)$$

$$R_{XY}(\tau) = (\delta(\tau) + 4e^{-|\tau|}) * e^{+2\tau} u(-\tau) = \int (\delta(k-\tau) + 4e^{-|k-\tau|}) e^{+2k} u(-k)\,dk$$

$$= \int_{-\infty}^{+\infty} \delta(k-\tau) e^{2k} u(-k)\,dk + \int_{\tau}^{0} 4e^{-k+\tau+2k} u(-k)\,dk + \int_{-\infty}^{\min(\tau,0)} 4e^{-\tau+k+2k} u(-k)\,dk =$$

$k \geq \tau,\ k \leq 0$; $k \leq \tau,\ k \leq 0$

$$e^{2\tau} u(-\tau) + 4e^{\tau} \left[e^{k}\right]_{\tau}^{0} + \frac{4}{3} e^{-\tau} \left[e^{3k}\right]_{-\infty}^{\min(\tau,0)}$$

$$= e^{2\tau} u(-\tau) + 4e^{\tau} - 4e^{2\tau} + \frac{4}{3} e^{-\tau} e^{3\min(\tau,0)}$$

ب 4

$$S_{xx}(w) = 1 + \frac{8}{1+w^2}$$

$$H(w) = \frac{1}{2+jw} = \frac{(2-jw)}{(2+jw)(2-jw)} = \frac{2-jw}{4+w^2} \Rightarrow H^*(w) = \frac{2+jw}{4+w^2}$$

$$S_{xY}(w) = S_{xx}(w)\, H^*(w) = \left(1 + \frac{8}{1+w^2}\right)\left(\frac{2+jw}{4+w^2}\right) = \frac{(1+w^2+8)(2+jw)}{(4+w^2)(1+w^2)}$$

x : real , $E[x(t)]=0$, $R_{xx}(t_1,t_2)=R_x(\tau)$

$E[y_1(t)]=0$, $E[y_2(t)]=0$

real $\Rightarrow Y_1 = Y_1^*$

real $\Rightarrow Y_2 = Y_2^*$

5

الف

$$C_{y_1y_2}(\tau)=R_{y_1y_2}(\tau)=E\left[Y_1(t)\,Y_2^*(t+\tau)\right]=$$

$$Y_2(t)=x(t)*h_2(t) = \int x(t-\tau)h_2(\tau)d\tau$$

$$E\left[\left(\int x(t-\tau)h_1(\tau)d\tau\right)Y_2(t+\tau)\right]$$

$$=E\left[\int x(t-k)Y_2(t+\tau)h_1(k)dk\right]=\int E\left[x(t-k)Y(t+\tau)\right]h_1(k)dk=$$

$$\int R_{xY_2}(\overset{t-k-t-\tau}{\cancel{\quad}})h_1(k)dk=R_{xY_2}(-\tau)*h_1(\tau)=\boxed{R_{xx}(-\tau)*h_1(\tau)*h_2(-\tau)}$$

$$R_{xY_2}(\tau)=E\left[x(t)Y_2(t+\tau)\right]=R_{xx}(\tau)*h_2^*(-\tau)=R_{xx}(\tau)*h_2(-\tau)$$

مسئله قبل

ب a

$$\cancel{S_{Y_1Y_2}(\omega) =} \; R_{Y_1Y_2}(\tau) = R_{xx}(-\tau) * h_2(-\tau) * h_1(\tau)$$

$$\Downarrow$$

$$S_{Y_1Y_2}(\tau) = S_{xx}(-\omega) \times H_2(-\omega) \times H_1(\omega)$$

6

$$\operatorname{sinc}(x) = \frac{\sin x\pi}{\pi x}$$

تابع sinc دو تعریف دارد که من با این تعریف حل کرده ام.

$$R_x(\tau) = 1 - |\tau|, \quad -1 \leqslant \tau \leqslant 1 = \Delta(t) = \operatorname{tri}(t)$$

$$S_x(\omega) = F\{\Delta(t)\} = F\{\operatorname{rect}(\tau) * \operatorname{rect}(\tau)\} = \operatorname{sinc}^2(\omega)$$

$$R_Y(\tau) = \overbrace{\frac{\sin \pi\tau}{\pi\tau}}^{=\operatorname{sinc}(\tau)} \Rightarrow S_Y(\omega) = \frac{1}{1}\operatorname{rect}\left(\frac{\omega}{1}\right) = \operatorname{rect}(\omega) = \Pi(\omega)$$

$$S_Y(\omega) = S_x(\omega)\,|H(\omega)|^2 \Rightarrow |H(\omega)|^2 = \frac{\pi(\omega)}{\operatorname{sinc}^2(\omega)} = \begin{cases} \dfrac{1}{\operatorname{sinc}^2(\omega)} & \omega \leqslant \frac{1}{2} \\ 0 & \omega > \frac{1}{2} \end{cases} = \begin{cases} \dfrac{\pi^2\omega^2}{\sin^2 \pi\omega} & \omega \leqslant \frac{1}{2} \\ 0 & \omega > \frac{1}{2} \end{cases}$$

$$H(\omega) = \begin{cases} \pm\dfrac{\pi\omega}{\sin \pi\omega} & \omega \leqslant \frac{1}{2} \\ 0 & \omega > \frac{1}{2} \end{cases}$$

V

$$2\frac{d^2y(t)}{dt^2} + 2\frac{dY(t)}{dt} + 4Y(t) = 3\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 6x(t)$$

$$2(jw)^2 Y(w) + 2jw\,Y(w) + 4Y(w) = 3(jw)^2 x(w) - 3jw\,x(w) + x(w)$$

$$Y(w)\{-2w^2 + 2jw + 4) = x(w)(-3w^2 - 3jw + 1)$$

$$\cancel{H(w)} = \quad Y(w) = x(w)\frac{(-3w^2 - 3jw + 1)}{(-2w^2 + 2jw + 4)} \Rightarrow H(w) = \frac{-3w^2 - 3jw + 1}{-2w^2 + 2jw + 4}$$

$$H(w) = \frac{(-3w^2 - 3jw + 1)}{(4 - 2w^2 + 2jw)} \times \frac{(4 - 2w^2 - 2jw)}{(4 - 2w^2 - 2jw)} = \frac{j(12w^3 - 14w) + 6w^4 - 20w^2 + 4}{4w^4 - 16w^2 + 16 - (2w)^2}$$

$$\Rightarrow H^*(w) = \frac{(6w^4 - 20w^2 + 4) + j(\cancel{12w^3}\,14w - 12w^3)}{\cancel{4w^4} - 16w^2 + 16}$$

$$|H(w)|^2 = \left(\frac{1}{\cancel{4w^4} - 16w^2 + 16}\right)^2 \left[(6w^4 - 20w^2 + 4)^2 - (14w - 12w^3)^2\right]$$

$R_x(\tau) = \text{sinc}(\pi\tau) \Rightarrow S_x(w) = \frac{1}{\pi}\,\text{rect}\left(\frac{w}{\pi}\right)$

$S_Y(w) = S_x(w) \times |H(w)|^2 = \checkmark$ هرچه معلوم

حست