Stochastic Processes December 14, 2021
Sharif University of Technology CE 40-695
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Time: 20 mins

Name: Std. Number:

Quiz 6 (Sufficient Statistics, Estimation)

Questions

1. (50%) Assuming that X_1 to X_n are samples from the following PDF:

$$f(x;\theta) = \frac{1}{\theta^2} x e^{-\frac{x}{\theta}} I_{(0,\infty)}(x), \theta > 0$$

Where $I_{(0,\infty)}(x)$ is the indicator function for range $(0, \infty)$. Find a one dimansional sufficient statistics.

2. (50%) Assuming that X_1 to X_n are samples from the following PDF, and there are no restrictions on θ:

$$f(x \mid \theta) = \begin{cases} e^{\theta - x} & x \geq \theta \\ 0 & x < \theta \end{cases}$$

Find maximum likelihood estimation for θ.

Answers

1.

$$f(x;\theta) = \frac{1}{\theta^2} x e^{-\frac{x}{\theta}} I_{(0,\infty)} \theta > 0$$

$$f(x;\theta) = \prod_{i=1}^{n} \left(\frac{1}{\theta^2} x_i e^{-\frac{x_i}{\theta}} I_{(0,\infty)}(x_i) \right) = \theta^{-2n} \exp\left(-\sum_{i=1}^{n} x_i/\theta \right) \prod_{i=1}^{n} \left[I_{(0,\infty)}(x_i) x_i \right]$$

$$\mathit{Let}\ T(x) = \sum_{i=1}^{n} x_i$$

$$= \theta^{-2n} \exp(T(x)/\theta) \prod_{i=1}^{n} \left[I_{(0,\infty)}(x_i) x_i \right]$$

$$g(T(x) \mid \theta) = \theta^{-2n} \exp(T(x)/\theta), h(x) = \prod_{i=1}^{n} \left[I_{(0,\infty)}(x_i) x_i \right]$$

2.

$$x_{(1)} \triangleq \min \{x_1, \ldots, x_n\}$$

$$f(x \mid \theta) = \begin{cases} e^{\theta - x} & x \geq \theta \\ 0 & x < \theta \end{cases}$$

$$f(\underline{x} \mid \theta) = \prod_{i=1}^{n} f(x_i \mid \theta) = \begin{cases} e^{n\theta - \sum_{i=1}^{n} x_i} & x_{(1)} \geqslant \theta \\ 0 & o.w \end{cases}$$

$$\Longrightarrow \theta_{ML} = x_{(1)}$$