In the name of GOD.
Sharif University of Technology
Stochastic Processes CE 695 Fall 2021 H.R. Rabiee

Homework 2 (Stationary Stochastic Processes, Ergodicity, Stochastic Analysis of Systems)

1. Consider an LTI system with system function :
$H(s) = \frac{1}{s^3+6s^2+11s+6}$
The input to this system is a WSS process $X(t)$ with $E\{X^2(t)\} = 12$. Find $S_X(\omega)$ such that the average power of output is maximum.

2. If $y(t) = x(t+a) - x(t-a)$ Show that
   - $R_y(\tau) = 2R_x(\tau) - R_x(\tau+2a) - R_x(\tau-2a)$
   - $S_y(\omega) = 4S_x(\omega)sin^2(a\omega)$

3. Show that if a process is normal and distribution-ergodic, then it's also mean-ergodic.

4. Suppose $y(t)$ is a WSS process such that

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

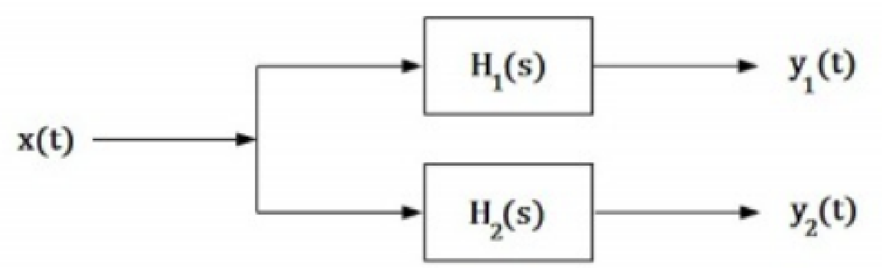
and $x(t)$ another WSS process with mean zero and

$$C_{x,x}(\tau) = \delta(\tau) + 4e^{-|\tau|}$$

. Calculate

   (a) $R_{x,y}(\tau)$
   (b) $S_{x,y}(\omega)$

5. Let $x(t)$ be a real valued, continous time, zero mean WSS random process with correlation function $R_{xx}(\tau)$ and power spectrum $S_{xx}(\omega)$. Suppose $x(t)$ is the input to two real valued LTI systems as depicted below, producing two new processes $y_1(t)$ and $y_2(t)$. Find $C_{y_1y_2}(\tau)$ and $S_{y_1y_2}(\omega)$



6. Let $X(t)$ be a WSS process with correlation function

$$R_x(\tau) = 1 - |\tau|, if - 1 \leq \tau \leq 1$$

It's known that when $X(t)$ is input to a system with transfer function $H(\omega)$, the system output $Y(t)$ has a correlation function

$$R_Y(\tau) = \frac{sin\pi\tau}{\pi\tau}$$

Find the transfer function $H(\omega)$.

7. Consider a WSS process $X(t)$ with autocorrelation function

$$R_X(\tau) = sinc(\pi\tau)$$

The process is sent to an LTI system, with input-output relationship

$$2\frac{d^2}{dt^2}Y(t) + 2\frac{d}{dt}Y(t) + 4Y(t) = 3\frac{d^2}{dt^2}X(t) - 3\frac{d}{dt}X(t) + 6X(t)$$

Find the autocorrelation function $R_Y(\tau)$