$x|\theta \sim \text{Geometric}(\theta)$

$$P_{x|\theta}(x|\theta) = \theta(1-\theta)^{x-1} \Rightarrow P_{x|\theta}(3|\theta) = \theta(1-\theta)^2$$

$$P_{x|\theta}(x|\theta)\, f_\theta(\theta) = \theta(1-\theta)^2\, 2\theta = 2\theta^2(1-\theta)^2$$

$$\Rightarrow \frac{d\left(\theta^2(1-\theta)^2\right)}{d\theta} = 2\theta(1-\theta)^2 - 2(1-\theta)\theta^2 = 0$$

$$\Rightarrow \hat{\theta}_{map} = \frac{1}{2}$$

(۲)

$$L(\theta) = \sum_{i=1}^{n} \log f(x_i|\theta) = \sum \left(\log\theta + \theta \log x_0 - (\theta+1)\log x_i\right)$$

$$n\log\theta + n\theta\log x_0 - (\theta+1)\sum \log x_i$$

مشتق $= \frac{n}{\theta} + n\log x_0 - \sum_{i=1}^{n}\log x_i = 0$

$$\cancel{\hat{\theta} = \frac{1}{\sum_{i=1}^{n}\log x_i - \log x_0}}$$

$$\hat{\theta} = \frac{1}{\frac{\sum_{i=1}^{n}\log x_i}{n} - \log x_0} = \frac{1}{\log x - \log x_0}$$