1-

False: The process is not ergodic in the mean because the ensemble mean does not equal the time-average of a realization of the process $x(t)$. The ensemble mean of the process $x(t)$ is 0. The time-average of a realization of the process $x(t)$ is the particular value of A obtained in that realization

True: If a WSS process $x(t)$ with mean μ_x and autocorrelation function $R_{xx}(\tau)$ is the input to a stable LTI system with frequency response $H(j\omega)$, then the output process has mean $\mu_y = H(j0)\mu_x$ and autocorrelation function $R_{yy}(\tau) = h * \overleftarrow{h} * R_{xx}(\tau)$. Since the output process has a constant mean and autocorrelation function that depends only on the lag τ, the process $y(t)$ is also WSS.

2-

Solution: The transfer function $H(s)$ representing this linear, constant coefficient differential equation is given by

$$H(s) = \frac{b}{s+a}$$

The PSD (function of $j\omega$) and complex PSD (function of s) of the output process $x(t)$ are

$$\begin{aligned} S_{xx}(j\omega) &= \frac{4}{1+\omega^2} \\ S_{xx}(s) &= \frac{4}{(1+s)(1-s)} \end{aligned}$$

We know that the complex PSDs of the input process $w(t)$ and the output process $x(t)$ are related as follows:

$$S_{xx}(s) = H(s)H(-s)S_{ww}(s)$$

Since the complex PSD of the input process $w(t)$ is $S_{ww}(s) = 1$, we can write

$$\begin{aligned} S_{xx}(s) &= H(s)H(-s) \\ \frac{4}{(1+s)(1-s)} &= \frac{b^2}{(a+s)(a-s)} \end{aligned}$$

From the above we recognize that $b = \pm 2$ and $a = 1$.