

1-2 $L(\lambda) = \prod_{i=1}^n \lambda f(x_i | \lambda) = \lambda^n \exp(-\sum_{i=1}^n \lambda x_i)$

$$\log L(\lambda) = n \log \lambda - \sum_{i=1}^n \lambda x_i \Rightarrow \arg \max_{\lambda} \log L(\lambda) = \frac{\partial \log L(\lambda)}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^n x_i = 0$$

$$\Rightarrow \lambda = \frac{n}{\sum_{i=1}^n x_i}$$

$$P(\lambda) \sim G(\alpha, B) = \frac{B^\alpha}{(\alpha-1)!} \lambda^{\alpha-1} e^{-B\lambda}$$

برای راقی حسابات منطقی نیست $\leftarrow \begin{cases} B = \frac{1}{\theta} \\ \alpha = k \end{cases}$

1-3

$$P(\lambda|x) = \lambda^n \exp(-\lambda \sum_{i=1}^n x_i) \Rightarrow P(\lambda|x) \propto P(x|\lambda) P(\lambda)$$

$$\propto \lambda^n \exp(-\lambda \sum_{i=1}^n x_i) \frac{B^\alpha}{(\alpha-1)!} \lambda^{\alpha-1} e^{-B\lambda}$$

$$\propto \lambda^{n+\alpha-1} \exp(-\lambda (B + \sum_{i=1}^n x_i))$$

$$= \text{Gamma}(n+\alpha, B + \sum_{i=1}^n x_i)$$

$$\text{MAP} \Rightarrow \underset{\lambda}{\text{argmax}} P(\lambda|x) \propto \underset{\lambda}{\text{argmax}} \lambda^{n+\alpha-1} \exp(-\lambda (B + \sum_{i=1}^n x_i))$$

$$\Rightarrow (n+\alpha-1) \lambda^{n+\alpha-2} \exp(-\lambda (B + \sum_{i=1}^n x_i)) + \lambda^{n+\alpha-1} (-\lambda (B + \sum_{i=1}^n x_i)) \exp(-\lambda (B + \sum_{i=1}^n x_i)) = 0$$

$$\cancel{(n+\alpha-1) \exp(-\lambda (B + \sum_{i=1}^n x_i))} = \cancel{(B + \sum_{i=1}^n x_i) \exp(-\lambda (B + \sum_{i=1}^n x_i))} \lambda$$

$$\lambda = \frac{n+\alpha-1}{B + \sum_{i=1}^n x_i}$$