$X \sim U(0,1)$
$Y \sim U(0,1)$ $\Rightarrow Z = \max(X,Y)$

$\cancel{F_X(x)}$
$\cancel{F_Y(y)}$

$$F_Z(z) = P(Z \leqslant z) = P(\max(X,Y) \leqslant z) = P(X \leqslant z)\,P(Y \leqslant z) = F_X(z)\,F_Y(z)$$

$$\Downarrow$$

$$f_Z(z) = \frac{dF_Z(z)}{dz} = f_X(z)\,F_Y(z) + f_Y(z)\,F_X(z) =$$

حل داریم:

$$f_X(x)=\begin{cases}1 & 0\leq x\leq 1\\ 0 & ow\end{cases}, \quad F_X(x)=\begin{cases}0 & x<0\\ x & 0\leq x\leq 1\\ 1 & x\geq 1\end{cases}$$

$$f_Y(y)=\begin{cases}1 & 0\leq x\leq 1\\ 0 & ow\end{cases} \qquad F_Y(y)=\begin{cases}0 & y<0\\ x & 0\leq y\leq 1\\ 1 & y\geq 1\end{cases}$$

$$\Rightarrow f_Z(z)=I(0\leq z\leq 1)\, 1\times z + z\, I(0\leq z\leq 1) = 2z\, I(0\leq z\leq 1)$$

$$\boxed{f_Z(z)=2z\, I(0\leq z\leq 1)}$$

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$W = \min(X, Y)$

$0 < w < 1$

$F_W(w) = P(W \leq w) = P(\min(X,Y) \leq w) = 1 - P(\min(X,Y) \geq w) = 1 - (1-w)^2$

می دانیم $P(\min(X,Y) > w) = P(X > w, Y > w) = P(X \geq w)\, P(Y \geq w) = (1 - F_X(w))(1 - F_Y(w))$

$$= \begin{cases} 0 & w < 0 \\ (1-w)^2 & 0 < w \leq 1 \\ 0 & w \geq 1 \end{cases} = I(0 < w < 1)\,(1-w)^2$$

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$$Y = \frac{2}{X} + 3 \Rightarrow f_Y(y) = f_X(x)\left|\frac{d}{dy} g^{-1}(y)\right|$$

$$Y - 3 = \frac{2}{X} \Rightarrow x = \frac{2}{Y-3}, \quad \frac{dx}{dy} = \frac{-2}{(Y-3)^2}$$

$$f_Y(y) = f_X\left(\frac{2}{Y-3}\right)\frac{-2}{(Y-3)^2} = I(0 \leq y \leq 1)\left(\frac{2}{Y-3}\right)^2\left(\frac{4}{Y-3} + \frac{3}{2}\right)\frac{-2}{Y-3} =$$

$$\frac{4x-2}{(Y-3)^3}\left(\frac{8+3Y-9}{2Y-6}\right) = \frac{-8}{2(Y-3)^4}(3Y-1) = \frac{-4}{(y-3)^4}(3y-1)$$

$$E[y] = \int y f_y(y)\,dy = \int_0^1 \frac{-4y}{(y-3)^4}(3y-1)\,dy = \left.\frac{12y^2 - 38y + 38}{y^3 - 9y^2 + 27y - 27}\right]_0^1$$

$$= \frac{-5}{54}$$

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$$Var(y) = E\left[\left(y + \frac{5}{54}\right)^2\right] = E\left[y^2 + \frac{10}{54}y + \frac{25}{(54)^2}\right]$$

or

$$Var(y) = E[(y-\mu)^2]^2 = E[y^2] - (E[y])^2 = \checkmark$$

$$E[y^2] = \int y^2 f_y(y)\,dy = \int_0^1 \frac{-4y^2}{(y-3)^4}(3y-1)\,dy = \frac{216\ln(3) - 216\ln(2) - 89}{78}$$

(3) $X \sim U(\frac{-\pi}{2}, \pi) \Rightarrow f_X(x) = \begin{cases} \frac{1}{\pi + \frac{\pi}{2}} & \frac{-\pi}{2} < x < \pi \\ 0 & \text{ow} \end{cases} = \begin{cases} \frac{2}{3\pi} & \frac{-\pi}{2} < x < \pi \\ 0 & \text{ow} \end{cases}$

$Y = \sin x \Rightarrow x = \sin^{-1}(y) \Rightarrow \frac{dx}{dy} = \frac{1}{\sqrt{1-y^2}}$

$f_Y(y) = \frac{2}{3\pi} \frac{1}{\sqrt{1-y^2}} \cancel{I(0 < y < 1)} \; I(-1 < y \leq 1)$

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