Time: 20 mins

Name: Std. Number:

Quiz 8 (UMVUE and Hypothesis Testing)

1. Let $X_1, \ldots, X_5$ be a random sample from Bernoulli distribution with pdf f, find the uniform minimal variance unbiased estimator(UMVUE) of θ.

$$f(x|\theta) = \theta^x(1-\theta)^{1-x}, x = 0, 1, \theta \in (0, 1) \tag{1}$$

2. Suppose the mean GPA of all graduate students of Computer Engineering from Sharif University was 17.34 in 2017. The admission committee plans to look at the records in the year 2021 to see if the mean GPA has changed. The sample from 64 students has a mean of 17.46 and standard deviation of 1.0.
   (a) State the null and the alternative hypothesis for this investigation.
   (b) With $\alpha = 0.05$ what can we conclude?[1]
   (c) What does the p-value mean?

Solution:

1.

We can rewrite the pdf as: $f(x;\theta) = \exp\{\log(\frac{\theta}{1-\theta}) \cdot x + \log(1-\theta)$

This belongs to exponential family with $~~x=0~~$ $x = 0$ or 1

$p(\theta) = \log\frac{\theta}{1-\theta}$, $k(x) = x$.

$\therefore T = \sum_{i=1}^{n} k(x_i) = \sum_{i=1}^{n} X_i$ is a complete and sufficient stat for θ.

Note $E(\bar{X}) = E(X_i) = \theta$, where $\bar{X}$ is a function of T, $\bar{X} = \frac{T}{n}$.

$\therefore$ By Lehmann-Scheffé theorem, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the UMVUE for θ.

2.

(a) $H_0 : \mu = 17.34, H_1 : \mu \neq 17.34$

(b) The sample size is large enough so the sampling distribution is approx. normal.
$t = 8 \times (17.46 - 17.34)/1.0 = 0.96$ (two-tail)
$p - value = 2 \times P(Z > 0.96) = 2 \times 0.16 = 0.32$
$p - value > 0.05$ therefore we fail to reject the hypothesis.

(c) It means that we are 32% confident that the differences in results have just occured by chance.

[1] use http://www.z-table.com/ for calculations.