Stochastic Processes
Sharif University of Technology
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December 6, 2021
CE 40-695

Time: 20 mins

Name: Std. Number:

Quiz 5 (Gaussian processes)

Questions

1. Let $\{X(t), t \in \mathbb{R}\}$ ba a Gaussian process with covariance function $R_x(\tau)$, let's define the process $\{Y(t)\}$ as below :

$$Y(t) = X(t) - 0.4X(t-2)$$

   (a) (4 points) Compute covariance function for $Y(t)$ process. Is it stationary?

   (b) (2 points) Is $Y(t)$ a Gaussian process?

2. Assume that K_1 and K_2 kernels be represented as $K_i(x,y) = \Phi_i(x)^T\Phi_i(y)\ for\ i = 1,2$ where Φ_i is a mapping that maps input onto higher dimensional sapce. Then K_1 and K_2 are valid kernels.

   (a) (4 points) Find Φ_3 so that we can have $K_3(x,y) = K_1(x,y).K_2(x,y) = \Phi_3(x)^T\Phi_3(y)$(it means that multiplication of two valid kernels is a valid kernel).

   (b) (3 points) Find Φ_4 so that we can have and $K_4(x,y) = cK_1(x,y) = \Phi_4(x)^T\Phi_4(y), c > 0$

   (c) (7 points) Assume that we know $K_5(x,y) = K_1(x,y)^n$ is valid kernel. proof that $K(x,y) = e^{K_1(x,y)}$ is valid kernel.

   (Hint : use Taylor expansion)