Note: We will start at 12:53 pm ET

Course Summary:

Date	Details	
Mon Feb 1, 2021	18-441/741 Lecture 1	12:50pm to 2:50pm
Wed Feb 3, 2021	18-441/741 Lecture 2	12:50pm to 2:50pm
Mon Feb 8, 2021	18-441/741 Lecture 3	12:50pm to 2:50pm
Wed Feb 10, 2021	18-441/741 Lecture 4	12:50pm to 2:50pm
Fri Feb 12, 2021	18-441/741 Recitation 1 (Hybrid) -- Project-1 Intro -- Zoom / In-person (A-L)	12:50pm to 1:40pm
Sun Feb 14, 2021	Quiz 1	due by 11:59pm
Mon Feb 15, 2021	18-441/741 Lecture 5	12:50pm to 2:50pm
→ Wed Feb 17, 2021	18-441/741 Lecture 6	12:50pm to 2:50pm
Mon Feb 22, 2021	18-441/741 Lecture 7	12:50pm to 2:50pm
Wed Feb 24, 2021	18-441/741 Lecture 8	12:50pm to 2:50pm
Fri Feb 26, 2021	18-441/741 Recitation 2 (Hybrid) -- Project 2 Intro -- Zoom / In-person (M-Z)	12:50pm to 1:40pm
Sun Feb 28, 2021	Quiz 2	due by 11:59pm
	Project 1	due by 11:59pm

18-441/741: Computer Networks
Lecture 6: Physical Layer IV

Swarun Kumar

Physical Layer: Outline

- Digital networks
- Characterization of Communication Channels
- Fundamental Limits in Digital Transmission
- Line Coding
- Modems and Digital Modulation
- Error Detection and Correction (cotd.)
- Wired PHY 101
- Wireless PHY 101

Recap: CRC = Polynomial Codes

- Do "Long Division" on (mod 2) polynomials
- Let i(x) denote information bits in polynomial form
- Then:

$$\begin{array}{r} q(x) \\ g(x)\ \overline{\big)\ x^{n-k}i(x)} \\ \underline{\phantom{x^{n-k}i(x)}} \\ r(x) \end{array}$$

$x^{n-k}i(x) \longrightarrow$ Codeword

Add

$r(x) \longrightarrow$ $x^{n-k}i(x)$ + r(x)

The *Pattern* in Polynomial Coding

- All codewords satisfy the following **pattern**:

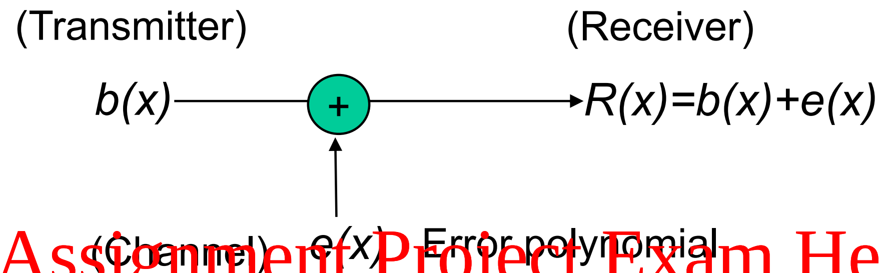
in modular

$$b(x) = x^{n-k}i(x) + r(x) = q(x)g(x) + r(x) + r(x) = q(x)g(x)$$

- All codewords are a multiple of *g(x)*!
- Receiver should divide received n-tuple by g(x) and check if remainder is zero

- If remainder is non-zero, then received n-tuple is not a codeword

Undetectable error patterns



- *e(x)* has 1's in error locations & 0's elsewhere

- Receiver divides the received polynomial *R(x)* by *g(x)*
- Undetectable error: If *e(x)* is a multiple of *g(x)*, that is, *e(x)* is a non-zero codeword, then

$$R(x) = b(x) + e(x) = q(x)g(x) + q'(x)g(x)$$

- *The set of undetectable error polynomials is the set of nonzero code polynomials*
- *Choose the generator polynomial so that selected error patterns can be detected.*

Designing good polynomial codes

- Select generator polynomial so that likely error patterns are not multiples of *g(x)*
- *Detecting Single Errors*

  - $e(x) = x^i$ for error in location $i+1$
  - If *g(x)* has more than 1 term, it cannot divide x^i

- *Detecting Double Errors*
  - $e(x) = x^i + x^j = x^i(x^{j-i}+1)$ where $j>i$

  - If *g(x)* has more than 1 term, it cannot divide x^i
  - If g(x) is a *primitive* polynomial, it cannot divide x^m+1 for all $m<2^{n-k}-1$ (Need to keep codeword length less than $2^{n-k}-1$)
  - Primitive polynomials can be found by consulting coding theory books

Standard Generator Polynomials

CRC = cyclic redundancy check

- CRC-8: $= x^8 + x^2 + x + 1$ ATM

- CRC-16: $= x^{16} + x^{15} + x^2 + 1$

  $= (x+1)(x^{15} + x + 1)$ Bisync

- CCITT-16: $= x^{16} + x^{12} + x^5 + 1$ HDLC, XMODEM, V.41

- CCITT-32: IEEE 802, DoD, V.42

$$= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$$

Hamming Codes

- Class of *error-correcting* codes
- Capable of correcting all *single-error* patterns
- Provably optimal for 1-bit errors
- Very less redundancy, e.g. 1-bit error proof – adds O(log n) bits of redundancy for n bit sequences

m=3 Hamming Code

- Information bits are b_1, b_2, b_3, b_4
- Equations for parity checks b_5, b_6, b_7

$$b_5 = b_1 + b_3 + b_4$$

$$b_6 = b_1 + b_2 + b_4$$

$$b_7 = + b_2 + b_3 + b_4$$

- There are 2^4=16 codewords
- (0,0,0,0,0,0,0) is a codeword

My "simple" proof of optimality

Assume you got the following 7 bit sequences and make the following checks:

$$b_5 = b_1 \qquad\quad + b_3 + b_4$$

$$b_6 = b_1 + b_2 \qquad\quad + b_4$$

$$b_7 = \qquad + b_2 + b_3 + b_4$$

Case	b_5 match	b_6 match	b_7 match
No error			
b_1 flipped			
b_2 flipped			
b_3 flipped			
b_4 flipped			
b_5 flipped			
b_6 flipped			
b_7 flipped			

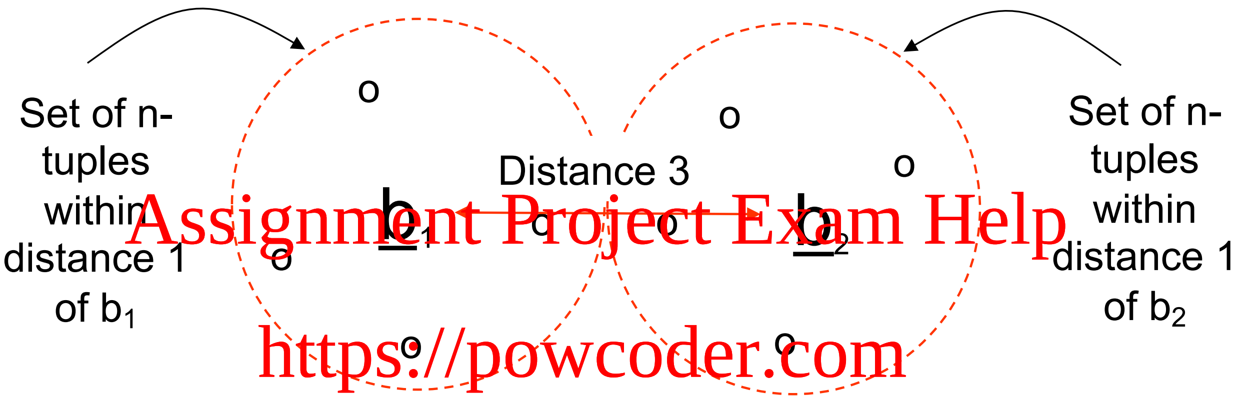
My ”simple” proof of optimality

Assume you got the following 7 bit sequences and make the following checks:

$$b_5 = b_1 \quad + b_3 + b_4$$
$$b_6 = b_1 + b_2 \quad + b_4$$
$$b_7 = \quad + b_2 + b_3 + b_4$$

Case	b_5 match	b_6 match	b_7 match
No error	✓	✓	✓
b_1 flipped	X	X	✓
b_2 flipped	✓	X	X
b_3 flipped	X	✓	X
b_4 flipped	X	X	X
b_5 flipped	X	✓	✓
b_6 flipped	✓	X	✓
b_7 flipped	✓	✓	X

Why is Hamming a "good code"?



- Two valid bit sequences have a minimum distance of 3 bit flips
- Spheres of distance 1 around each codeword do not overlap
- If a single error occurs, the resulting n-tuple will be in a unique sphere around the original codeword
- Thus, receiver can correct erroneous reception back to original codeword

Physical Layer: Outline

- Digital networks
- Characterization of Communication Channels
- Fundamental Limits in Digital Transmission
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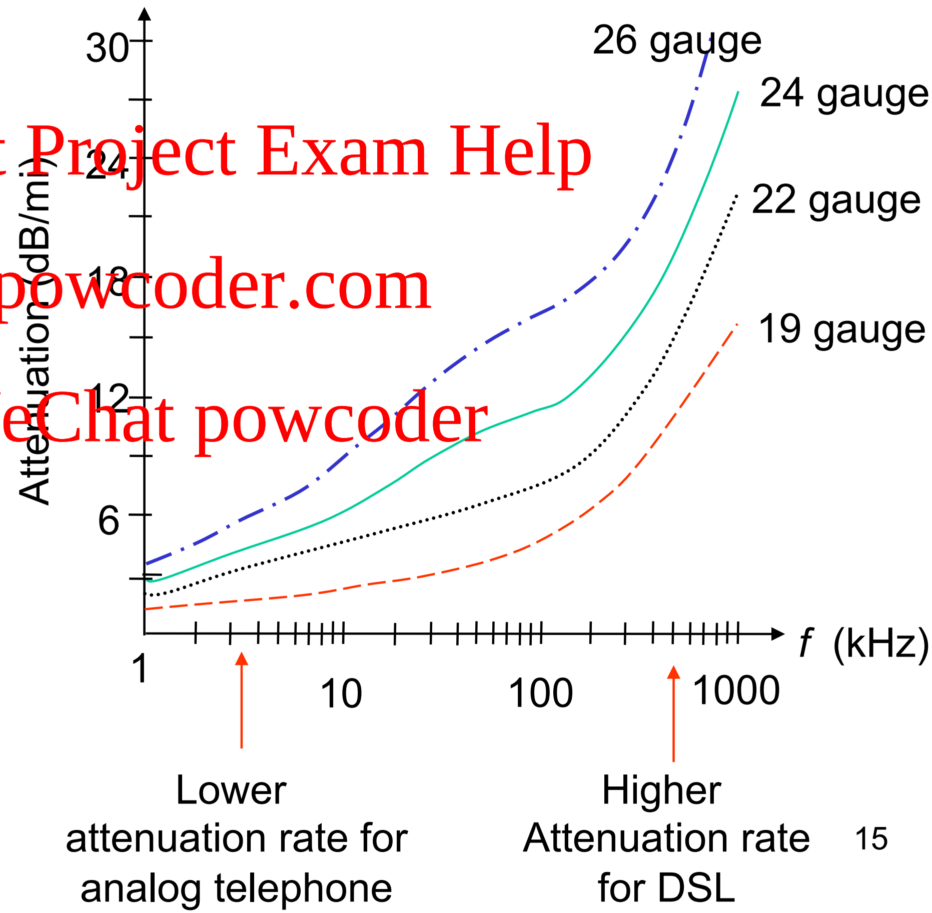
- Error Detection and Correction
- Wired PHY 101
- Wireless PHY 101

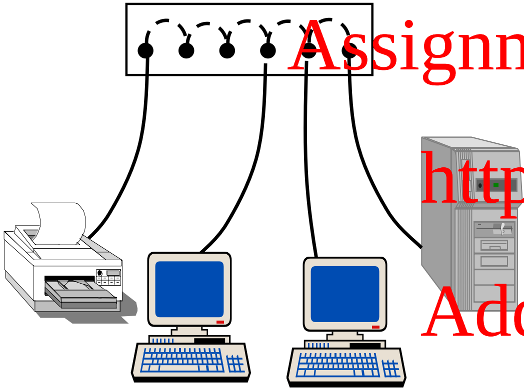
Twisted Pair

- Two insulated copper wires arranged in a regular spiral pattern to minimize interference
- Various thicknesses, e.g. 0.016 inch (24 gauge)
- Low cost
- Telephone subscriber loop from customer to CO
- Old trunk plant connecting telephone COs
- Intra-building telephone from wiring closet to desktop



Ethernet LANs

- Evolved from 10 -> 100 → 1000 Mbps to now 10Gbps
- All use twisted pair in some form!

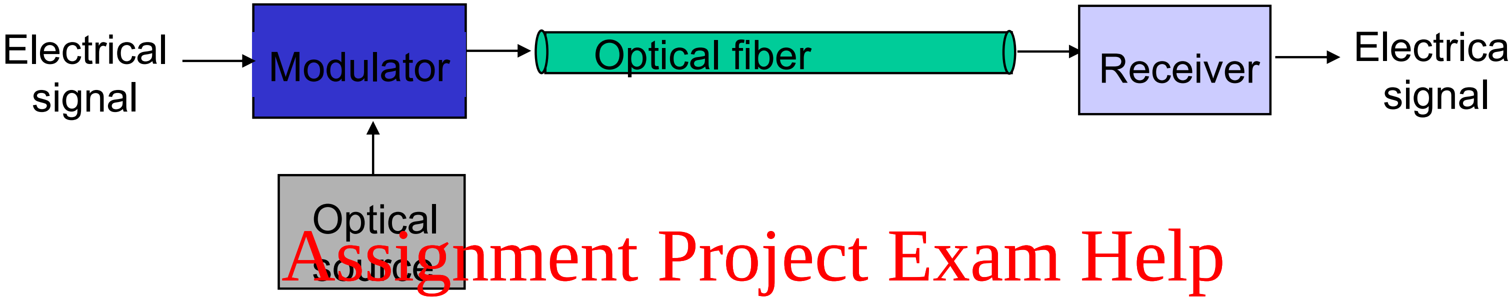


- 10BASE-T Ethernet

  – 10 Mbps, Baseband, Twisted pair
  – Two Cat3 pairs
  – Manchester coding, 100 meters

- 100BASE-T4 *Fast* Ethernet
  – 100 Mbps, Baseband, Twisted pair
  – Four Cat3 pairs
  – Three pairs for one direction at-a-time
  – 100/3 Mbps per pair;
  – 3B6T line code, 100 meters
- 1000BASE-T
  – 8b10b encoding, Four pairs

Optical Fiber

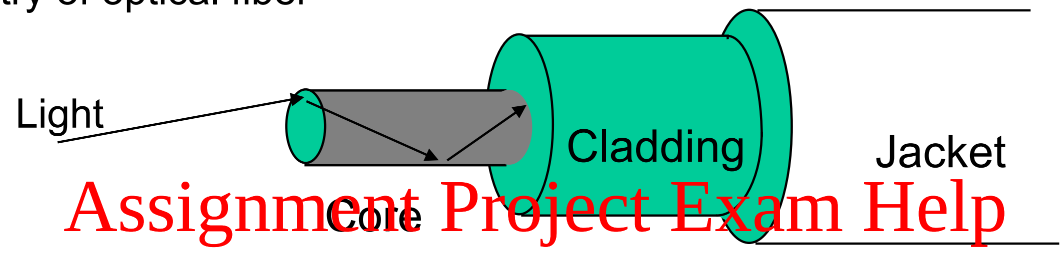


- Light sources (lasers, LEDs) generate pulses of light that are transmitted on optical fiber
  - Very long distances (>1000 km)
  - Very high speeds (>40 Gbps/wavelength)
  - Nearly error-free (BER of 10^{-15})
- Profound influence on network architecture
  - Dominates long distance transmission
  - Distance less of a cost factor in communications
  - Plentiful bandwidth for new services

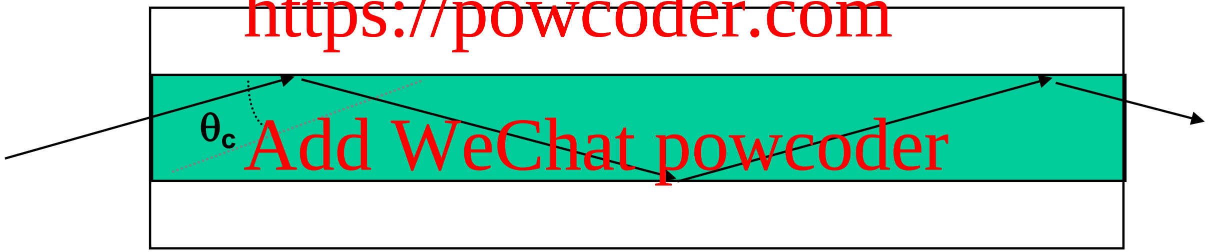
Transmission in Optical Fiber

don't fold

Geometry of optical fiber



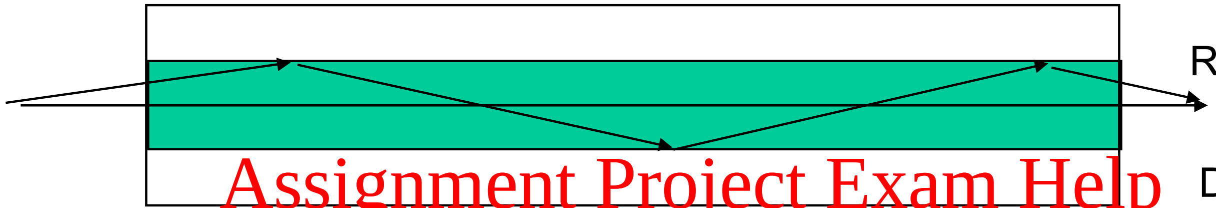
Total Internal Reflection in optical fiber



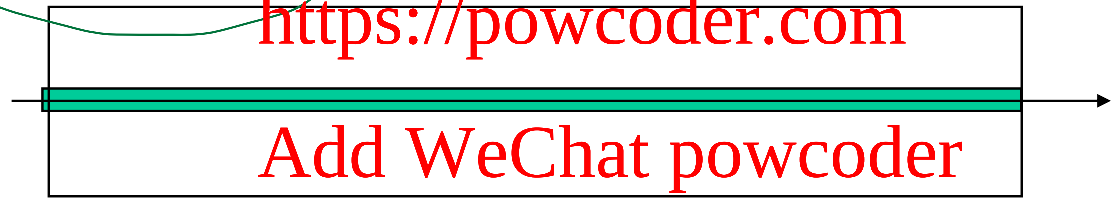
- Very fine glass cylindrical core surrounded by concentric layer of glass (cladding)
- Core has higher index of refraction than cladding
- Light rays incident at less than critical angle θ_c is completely reflected back into the core

Multimode & Single-mode Fiber

Multimode fiber: multiple rays follow different paths



Single-mode fiber: only direct path propagates in fiber



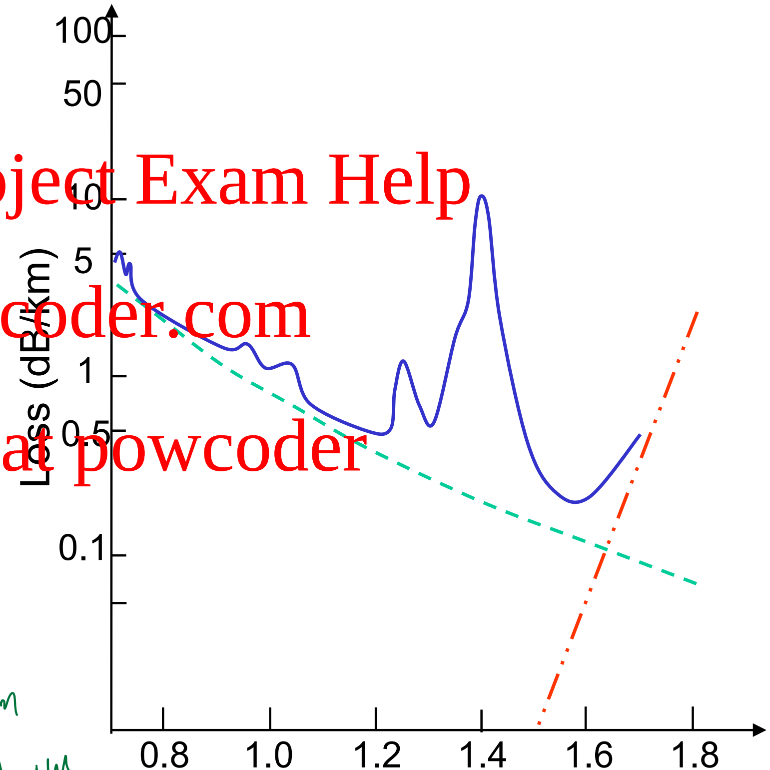
- Multi Mode: Thicker core, shorter reach
  - Rays on different paths interfere causing dispersion & limiting bit rate
- Single Mode: Very thin core supports only one mode (path)
  - More expensive lasers, but achieves very high speeds

Huge Available Bandwidth

- Optical range from λ_1 to $\lambda_1+\Delta\lambda$ contains bandwidth

$$B = f_1 - f_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_1 + \Delta\lambda}$$

$$= \frac{v}{\lambda_1}\left\{\frac{\Delta\lambda / \lambda_1}{1 + \Delta\lambda / \lambda_1}\right\} \cong \frac{v\Delta\lambda}{\lambda_1^2}$$



why v, not c ?

lights has diff speed in diff medium.

Quiz Question

How much optical fiber bandwidth is available between:

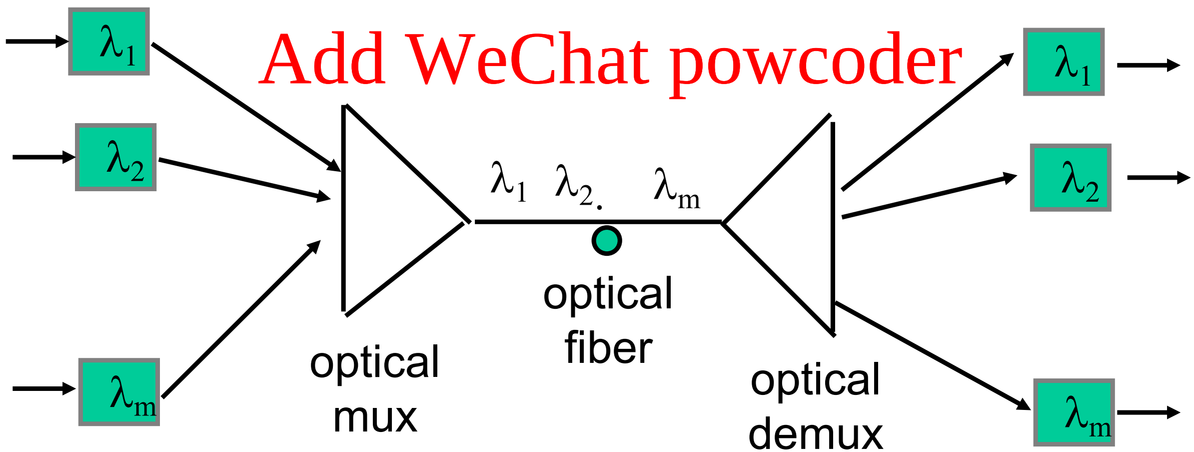
λ_1 = 1450 nm and $\lambda_1 + \Delta\lambda$ =1650 nm:

$\Delta\lambda = 200$ nm

Answer: $$B = \frac{2(10^8)\,\mathrm{m/s}\;200\,\mathrm{nm}}{(1450\,\mathrm{nm})^2} \approx 19\ \mathrm{THz}$$

Wavelength-Division Multiplexing

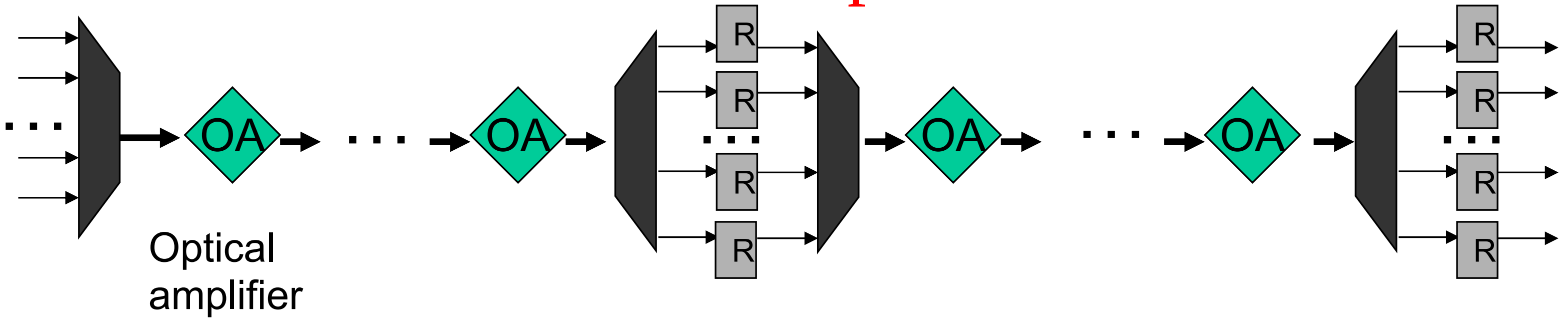
- Different wavelengths carry separate signals
- Multiplex into shared optical fiber
- Each wavelength like a separate circuit
- A single fiber can carry 160 wavelengths, 10 Gbps per wavelength: 1.6 Tbps!



How Do We Extend Range

- Use combinations of optical amplifiers and regenerators
- More amplifiers than regenerators (why?)

cheaper



Physical Layer: Outline

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- Error Detection and Correction
- Wired PHY 101
- Wireless PHY 101

Wireless vs. Wired

- Wireless is “flaky”
  - Environment, people, mobility affects signals

- Wireless is a *broadcast* medium
  - Collisions!
  - Interference
  - Noise

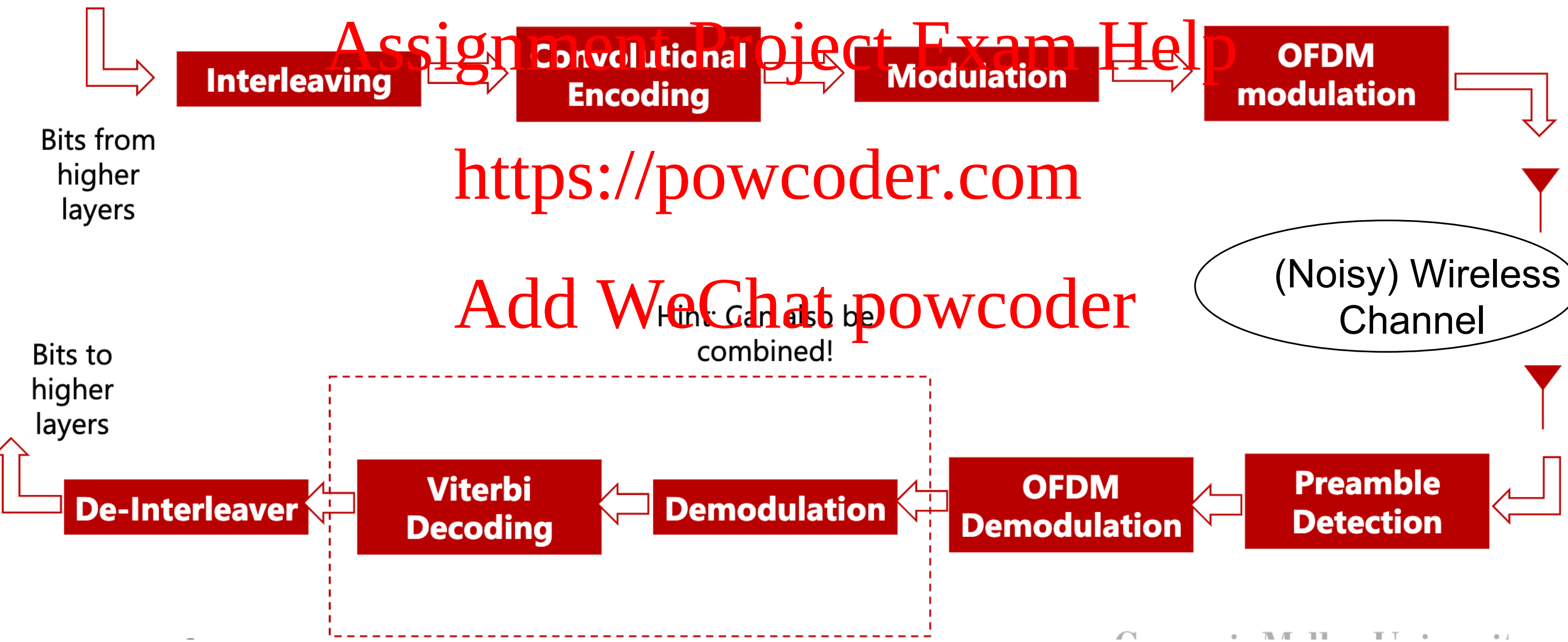
- Wireless is half-duplex
  - Only transmit or receive.. Not both

Outline - Wireless

- WiFi PHY
  - Wireless channel
  - OFDM
  - Multiple antennas (MIMO)

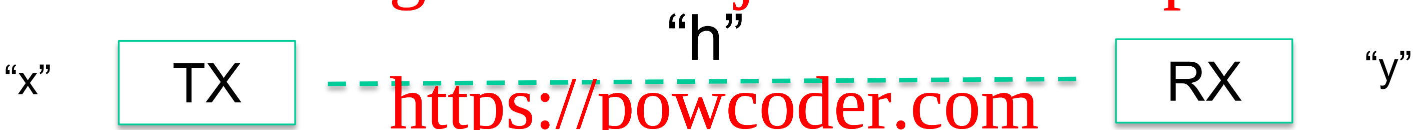
- Cellular Whirlwind (2G → 5G)

But hey, we already know Wi-Fi



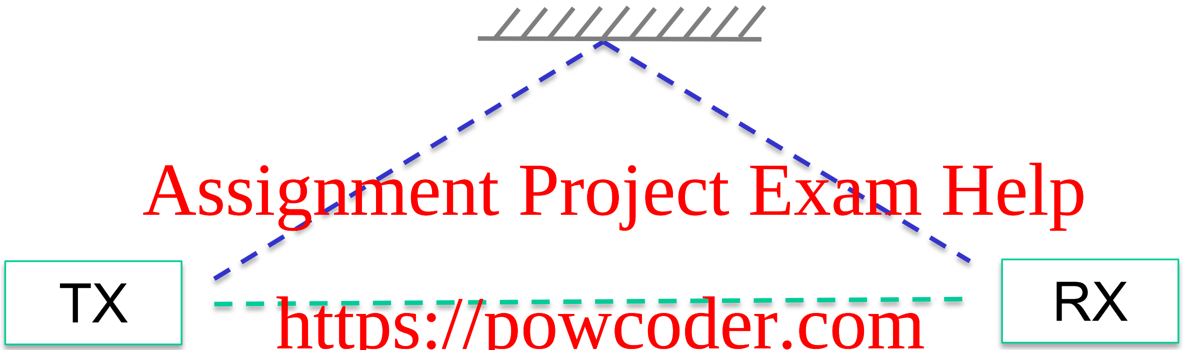
Wireless signals: Basic Equation

- In narrowband:



$$y(t) = hx(t) + n(t).$$

But in the real world…



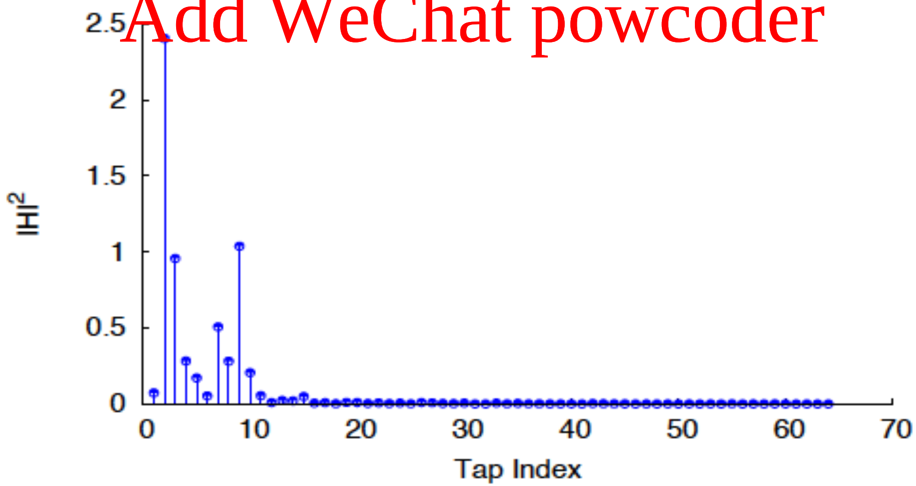
“Multipath”

Wireless signals

- More generally:

delay

$$y(t) = \sum_{i=0}^{i=k} h(i)\, s(t - i\tau)$$



Wireless signals

- But time is continuous!

$$y(t) = \int h(\tau)s(t-\tau) + n(t) = h(t) * s(t) + n(t)$$

$S \leftrightarrow X$

Challenges: How do I estimate h?

$$y(t) = hx(t) + n(t).$$

Send known x(t) as "preamble"

➔ h ≈ y(t)/x(t)

But… what *is* the channel?

- “Attenuation” & “Phase shift”

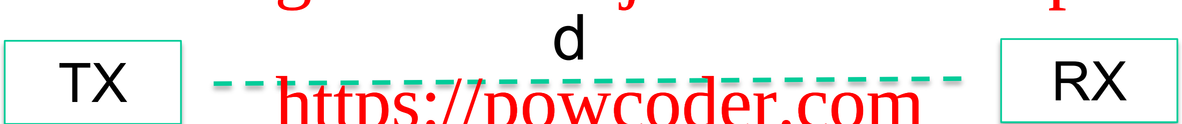
TX - - - - d - - - - RX

$$h = 1/d * e^{j2\pi d/\lambda}$$

- Consistent with $1/d^2$ power fading

But… what *is* the channel?

- “Attenuation” & “Phase shift”

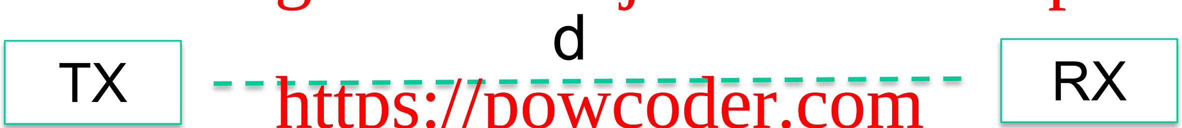


$$h = 1/d * e^{j2\pi d/\lambda}$$

- $d/\lambda = d*f/c = f*t$, where “t” is signal time

But… what *is* the channel?

- “Attenuation” & “Phase shift”

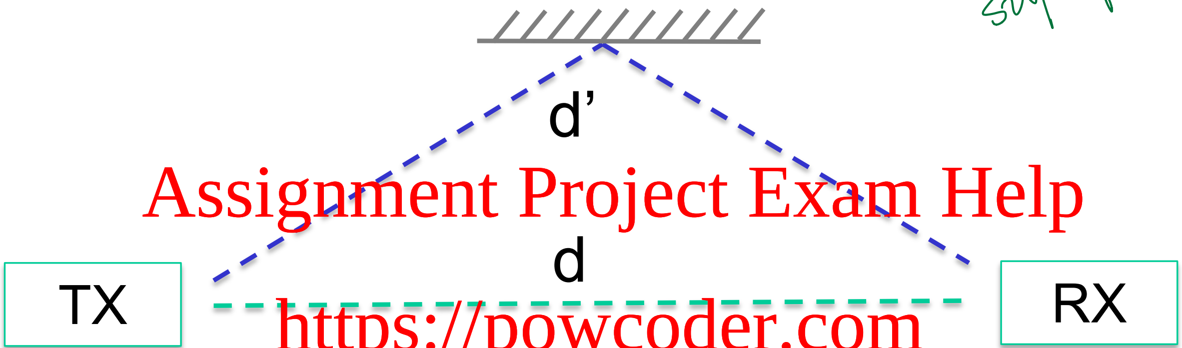


$$h = 1/d * e^{j2\pi d/\lambda} = 1/d * e^{j2\pi ft}$$

- $d/\lambda = d*f/c = f*t$, where “t” is signal time

How do channels capture multipath?

superposition



$$h = 1/d * e^{j2\pi d/\lambda} + 1/d' * e^{j2\pi d'/\lambda}$$

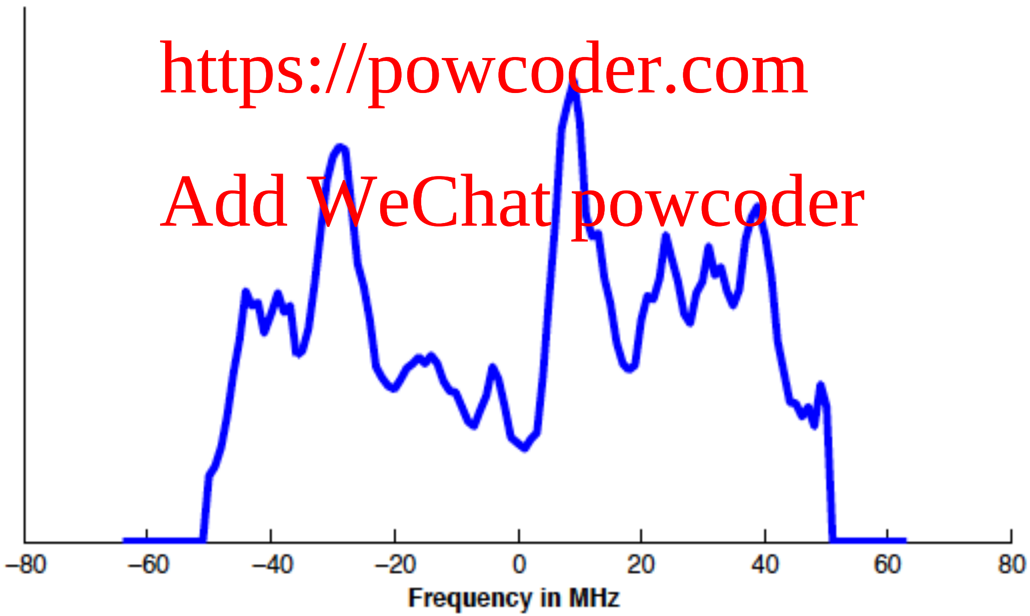
Channels can combine differently on different frequencies

→ Channels are frequency-selective

Challenge: Frequency Selective Fading

Fourier

$$y(t) = h(t) * s(t) + n(t) \Leftrightarrow Y(f) = H(f)S(f) + N$$



FDM

Frequency Division Multiplexing

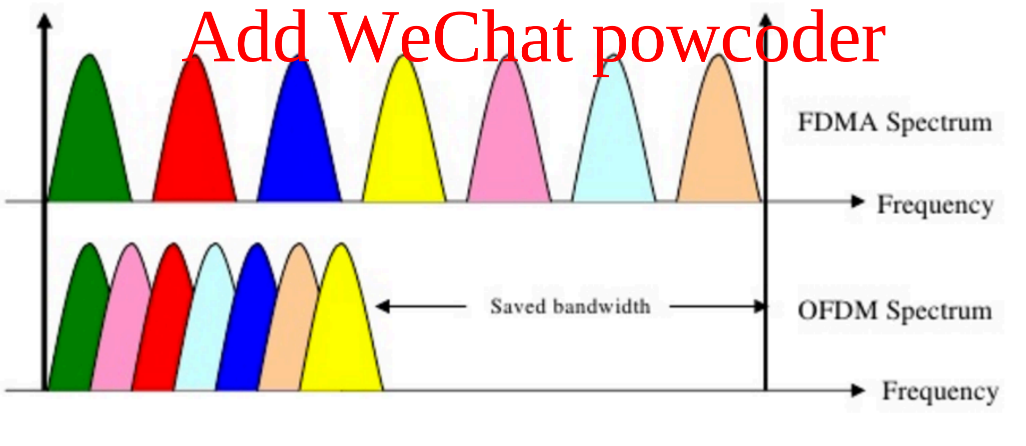
- Divide bandwidth into small chunks: "subcarriers"

But… so much waste!

OFDM

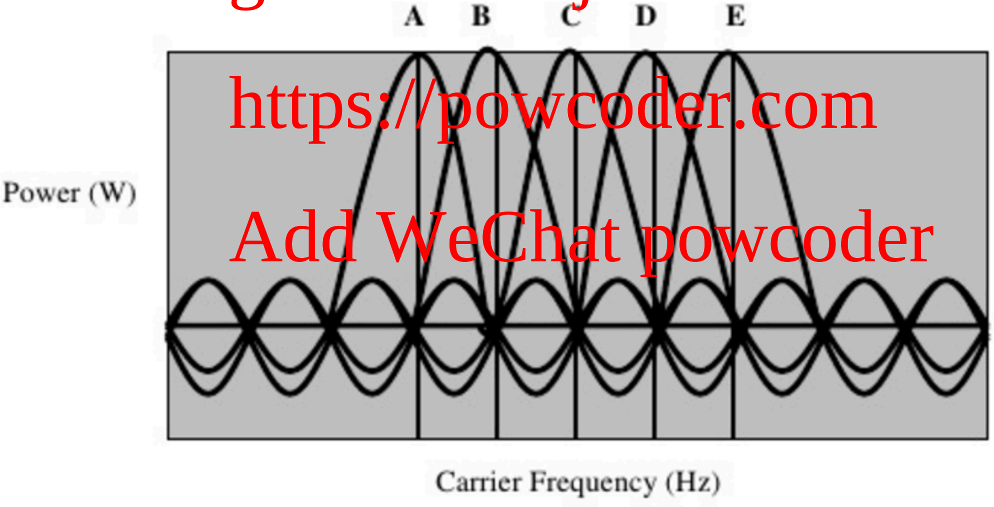
Orthogonal Frequency Division Multiplexing

- Get rid of guard bands by “orthogonal” frequency division



OFDM

Orthogonal Frequency Division Multiplexing



WiFi, LTE uses OFDM!

MIMO

multiple input
multiple output

- Why so many antennas?

single in single out

Recap: SISO PHY

- Our discussion so far had single antenna transmitters and receivers

- "Single Input Single Output"

SISO: Channel Model

(Assuming narrowband)

$$y = hx + n$$

MIMO

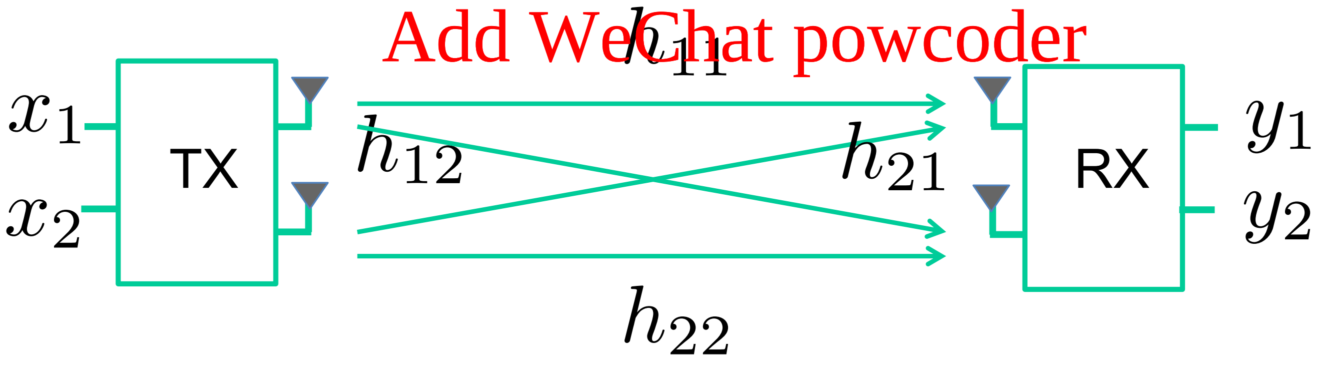
Multiple Input Multiple Output

- 2 x More antennas → 2 x More data

MIMO

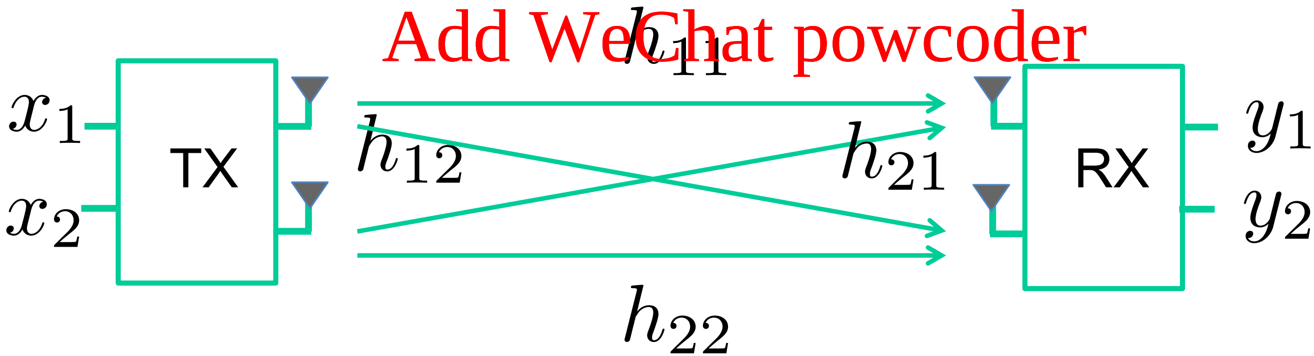
$$y_1 = h_{11}x_1 + h_{21}x_2$$

$$y_2 = h_{12}x_1 + h_{22}x_2$$



MIMO

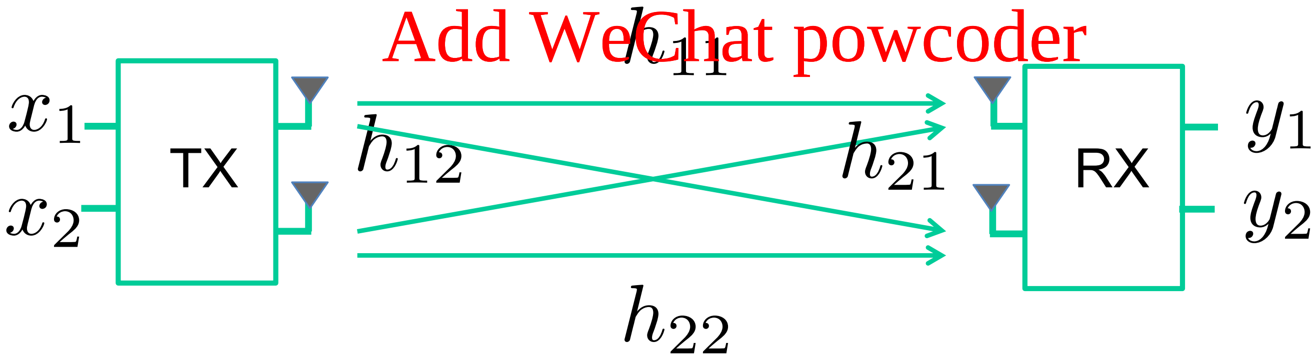
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



How do you solve?

MIMO

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boxed{\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix}^{-1}} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$



Estimating Channels

Preamble 1	Preamble 2	… Data …

h_{11} h_{21} $\longrightarrow$ Measure on Antenna 1

h_{12} h_{22} $\longrightarrow$ Measure on Antenna 2

Gains of MIMO

- 2 antennas → 2$\times$ data: $\begin{bmatrix} y_1 & y_2 \end{bmatrix}$

- n antennas → n$\times$ more data

<u>Assumption</u>: H is invertible

Quiz Question

Which of these has a gain (in Shannon Capacity) that is identical to that of doubling the number of antennas available on your wireless transmitter & receiver:

[A] Doubling Bandwidth

[B] Doubling Signal Power

[C] Doubling Noise Power

[D] Halving Noise Power

log $\frac{2S}{N}$

New Shannon Formula: C = n B log(1+SNR)

Outline - Wireless

- WiFi PHY
  - Wireless channel
  - OFDM
  - Multiple antennas (MIMO)

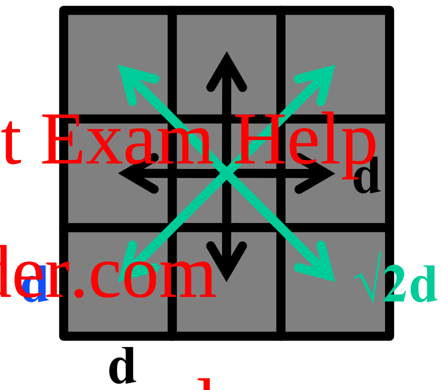
- Cellular Whirlwind (2G → 5G)

The Advent of Cellular Networks

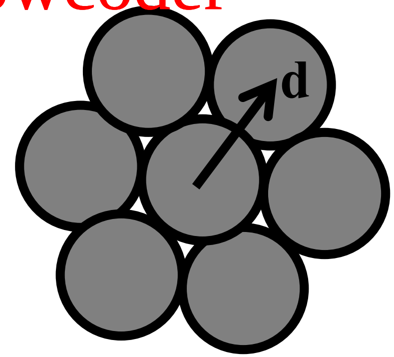
- Mobile radio telephone system was based on:
  - High power transmitter/receivers
  - Could support about 25 channels
  - in a radius of ~80 km
- To increase network capacity:
  - Multiple low-power transmitters (100W or less)
  - Small transmission radius -> area split in cells
  - Each cell with its own frequencies and base station
  - Adjacent cells use different frequencies
  - The same frequency can be reused at sufficient distance

Cellular Network Design Options

- Simplest layout
  - Adjacent antennas not equidistant – how do you handle users at the edge of the cell?

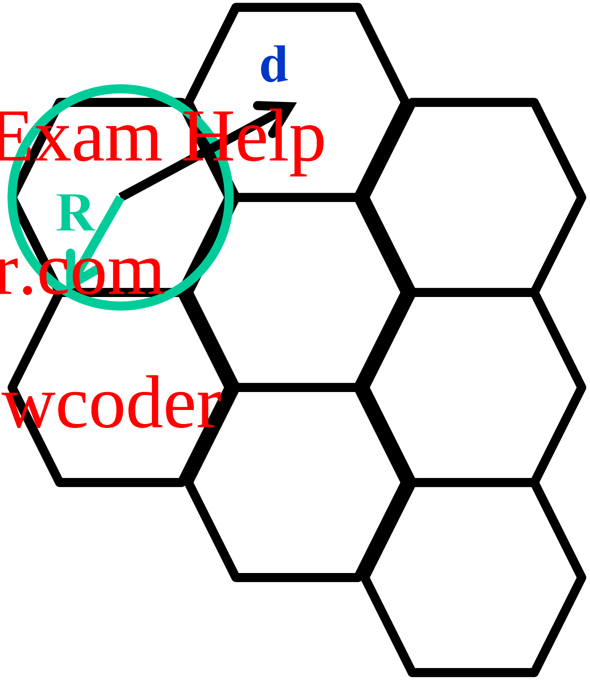


- Ideal layout
  - But we know signals travel whatever way they feel like



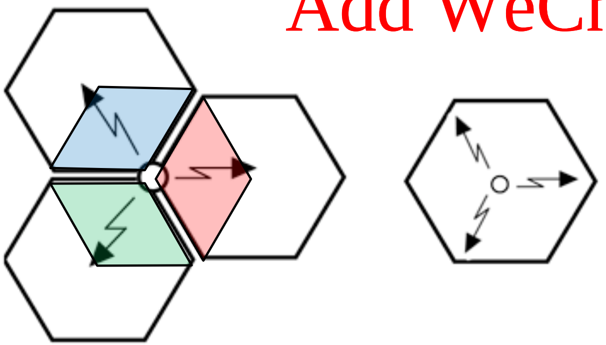
The Hexagonal Pattern

- A hexagon pattern can provide equidistant access to neighboring cell towers
  - Used as the basis for planning
  - d = √3R
- In practice, variations from ideal due to topological reasons
  - Signal propagation
  - Tower placement



Cell sectoring

- Cell divided into wedge shaped sectors
- 3-6 sectors per cell, each with own channels
- Use of directional antennas
- Even more messy with small + big cells!



Right! ☺ Wrong! ☹

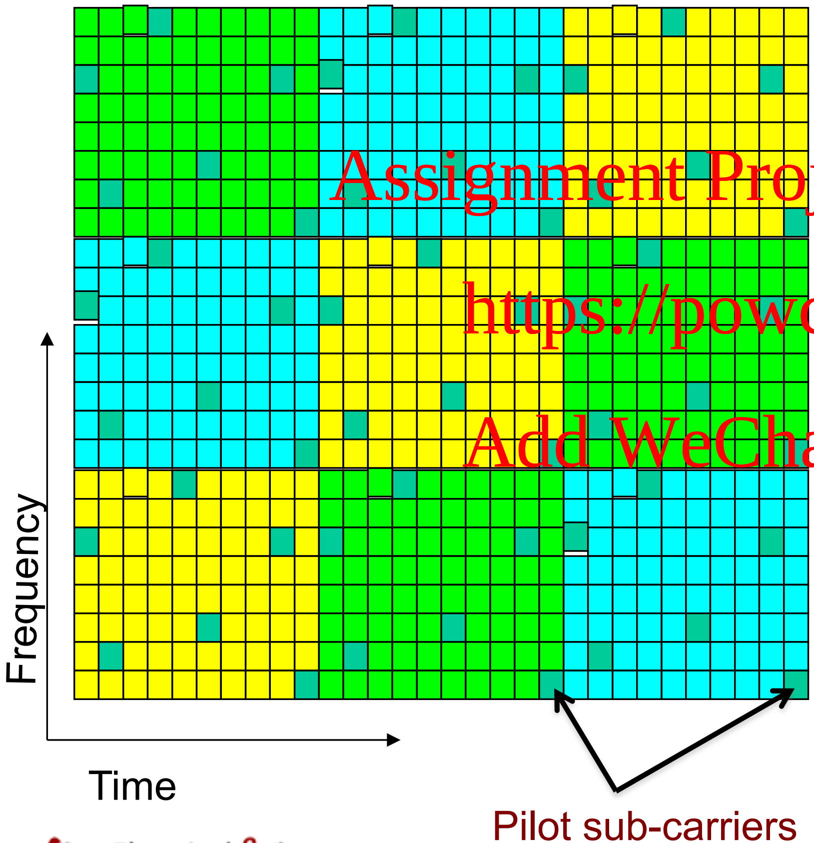
Cellular Standards

- 1G systems: analog voice
  - Not unlike a wired voice line (without the wire)
- 2G systems: digital voice
  - Many standards
  - Example: GSM - FDMA/TDMA, most widely deployed, 200 countries, a billion people
- 2.5G systems: voice and data channels
  - Example: GPRS - evolved from GSM, packet-switched, 170 kbps (30-70 in practice)

Cellular Standards

- 3G: voice (circuit-switched) and data (packet-switched)
  - Several standards
  - Uses Code Division Multiple Access (CDMA) – UMTS
- 4G: 10 Mbps and up, seamless mobility between different cellular technologies
  - LTE the dominating technology
  - Packet switched (took them so long!)
- 5G: mm-wave, more bandwidth, massive MIMO

LTE in a Nutshell: Essentially OFDM



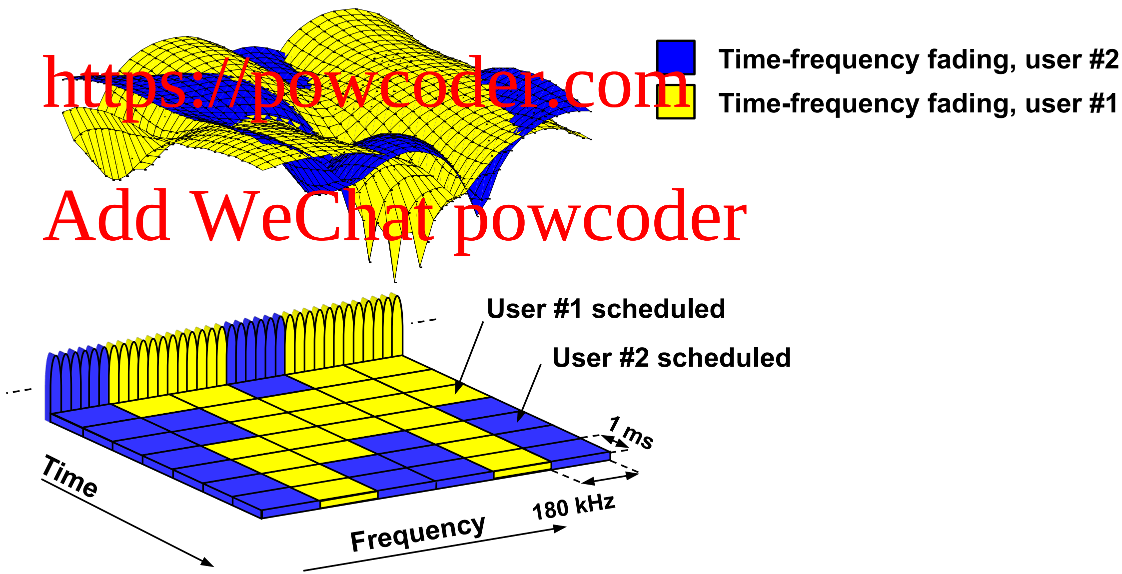
- Each color represents a user
- Each user is assigned a frequency-time tile which consists of pilot sub-carriers and data sub-carriers
- Block hopping of each user's tile for frequency diversity

Electrical & Computer ENGINEERING

Courtesy: Harish Vishwanath

LTE in a Nutshell: Or rather, OFDM-A!

- Call a chunk of subcarrier-time “resource blocks”
- Assign each user a chunk of resource blocks coordinated by the cell tower



Courtesy: Zoltán Turányi

5G in one slide(!)

- LTE bandwidths (in US) ~ 10-20 MHz
- 5G plays three games to increase based on C = n B log(1+S(I)NR)
  - Increase n: Massive MIMO
  - Increase B (option 1): mm-wave frequencies
  - Increase B (option 2): buy more spectrum (costs $$)
  - Reduce I: smaller cells (femto cells)
- Only major change to PHY: allow subcarrier width to change (fixed in LTE), otherwise mostly same as LTE (still uses OFDMA, etc.)