














**Note: We will start at 12:53 pm ET**

## Course Summary:

Date	Details	
Mon Feb 1, 2021	 <a href="#">18-441/741 Lecture 1</a>	12:50pm to 2:50pm
Wed Feb 3, 2021	 <a href="#">18-441/741 Lecture 2</a>	12:50pm to 2:50pm
Mon Feb 8, 2021	 <a href="#">18-441/741 Lecture 3</a>	12:50pm to 2:50pm
Wed Feb 10, 2021	 <a href="#">18-441/741 Lecture 4</a>	12:50pm to 2:50pm
Fri Feb 12, 2021	 <a href="#">18-441/741 Recitation 1 (Hybrid) -- Project-Intro -- Zoom / In-person (M-Z)</a>	12:50pm to 1:40pm
Sun Feb 14, 2021	 <a href="#">Quiz 1</a>	due by 11:59pm
→ Mon Feb 15, 2021	 <a href="#">18-441/741 Lecture 5</a>	12:50pm to 2:50pm
Wed Feb 17, 2021	 <a href="#">18-441/741 Lecture 6</a>	12:50pm to 2:50pm
Mon Feb 22, 2021	 <a href="#">18-441/741 Lecture 7</a>	12:50pm to 2:50pm
Wed Feb 24, 2021	 <a href="#">18-441/741 Lecture 8</a>	12:50pm to 2:50pm
Fri Feb 26, 2021	 <a href="#">18-441/741 Recitation 2 (Hybrid) -- Project 2 Intro -- Zoom / In-person (M-Z)</a>	12:50pm to 1:40pm
Sun Feb 28, 2021	 <a href="#">Quiz 2</a>	due by 11:59pm
	 <a href="#">Project 1</a>	due by 11:59pm

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# 18-441/741: Computer Networks

## Lecture 5: Physical Layer III

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# Physical Layer: Outline

- Digital networks
- Characterization of Communication Channels
- Fundamental Limits in Digital Transmission
- Modems and Digital Modulation
- Line Coding
- Error Detection and Correction
- Wired PHY 101 (if time permits)
- Wireless PHY 101

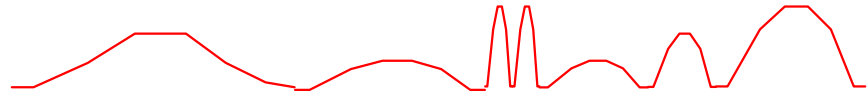
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# From Signals to Packets

Analog Signal



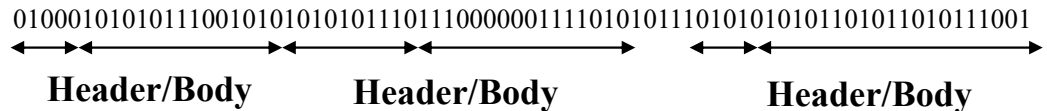
“Digital” Signal



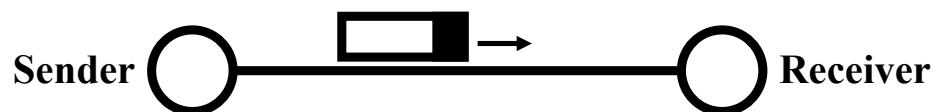
Bit Stream

0 0 1 0 1 1 1 0 0 0 1

Packets



Packet  
Transmission



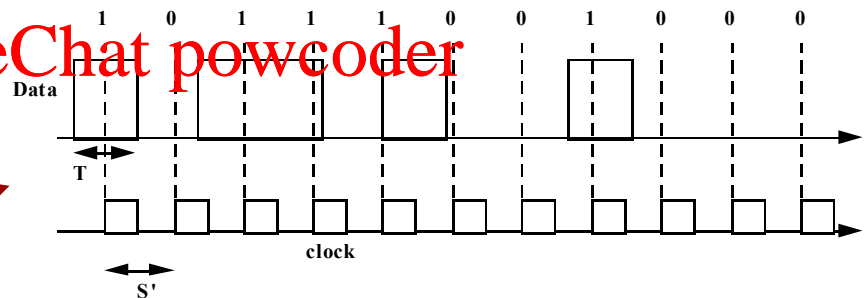
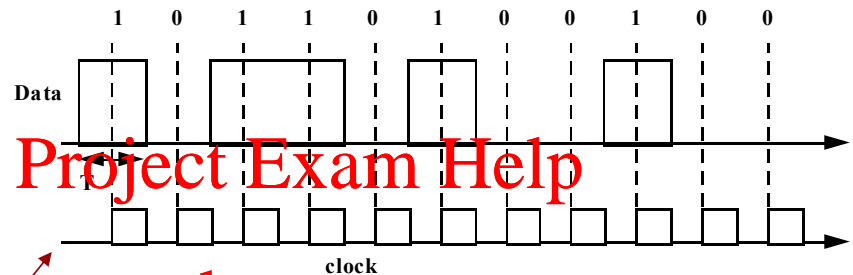
# Synchronization

- Synchronization of clocks in transmitters and receivers.

- clock drift causes a loss of synchronization

- Example: assume '1' and '0' are represented by V volts and 0 volts respectively

- Correct reception
  - Incorrect reception due to slow clock at the receiver



# Synchronization (cont')

- How to avoid a loss of synchronization?
  - Asynchronous transmission
  - Synchronous transmission

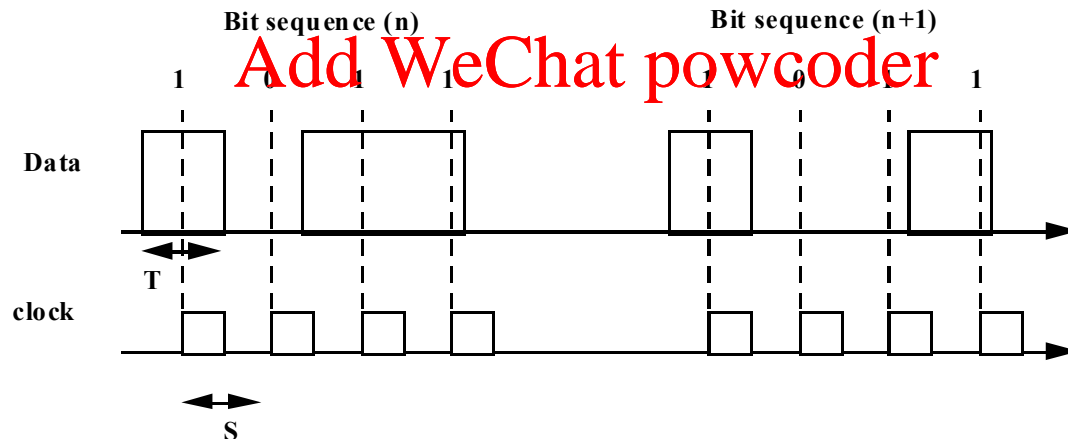
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# Asynchronous Transmission

- Avoids synchronization loss by
  - specifying a short maximum length for the bit sequences (so that clock doesn't drift much within sequence)
  - and resetting the clock in the beginning of each bit sequence (by using a 'start bit')
- Accuracy of the clock?



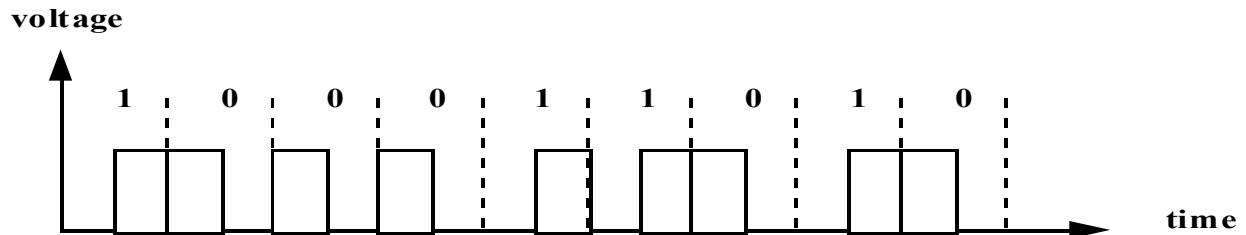
# Asynchronous transmission: ASCII code

- ASCII (American National Standard Code for Information Interchange) code
  - 7 bits to represent 128 letters, symbols, and control characters. (i.e. A=1000001, CR(Carriage Return)=‘0001101’ )
  - Asynchronous transmission sends sequences of 8 bits=one start bit + 7 ASCII bits.
  - some systems add one parity bit to make number of ‘1’ to be even number
  - i.e. ‘1100111’1 or ‘1010110’0



# Synchronous Transmission

- Overcomes the inefficiency of asynchronous transmission.
- Improves efficiency by transmitting longer sequences of bits, called packets (variable length). <https://powcoder.com>
- Requires extra information to indicate the end of the packet.



# Encoding

- Encoding converts a binary information sequence into a digital signal
  - Sender then uses the digital signal to modulate the signal in a way that the receiver can recognize
- Encoding can be done one bit at a time or in blocks of multiple bits called a symbol
  - Example: a symbol with 8 values means that 3 bits are sent in each time slot
- Transmission is synchronous, i.e., a clock is used to sample the signal.
  - Receiver's clock must be synchronized with the sender's clock

# Why Do We Need Encoding?

- To meet certain electrical constraints.
  - Many of them! See next slide
- Creates control symbols, besides regular data symbols.
  - E.g. start or end of frame, escape, ...
- Can do error detection or error correction
  - Some codes are illegal so receiver can detect certain classes of errors
  - Minor errors can be corrected by having multiple adjacent signals mapped to the same data symbol

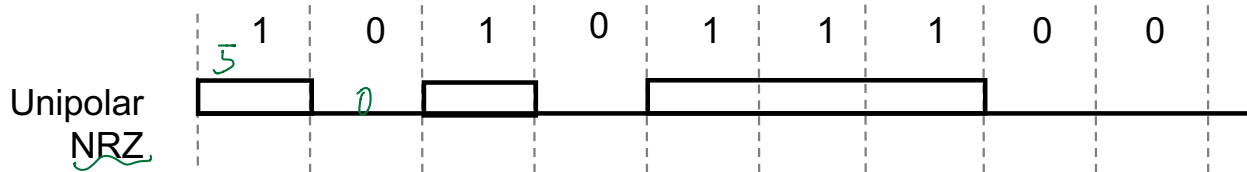
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can not appear in data

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# Line coding examples



*More power efficiency*

NRZ-inverted  
(differential encoding)

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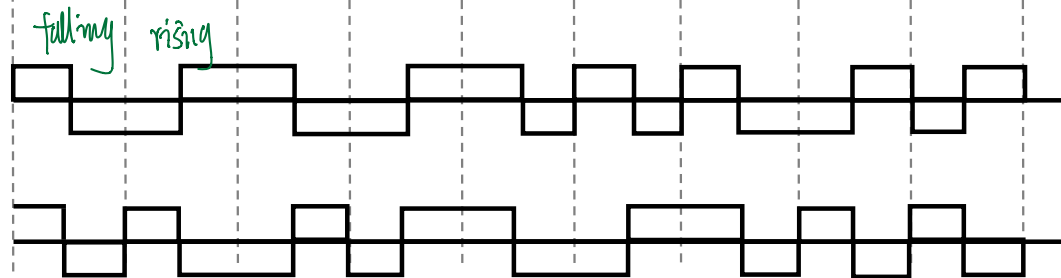
*0: don't change* *1: flip it*

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Bipolar encoding

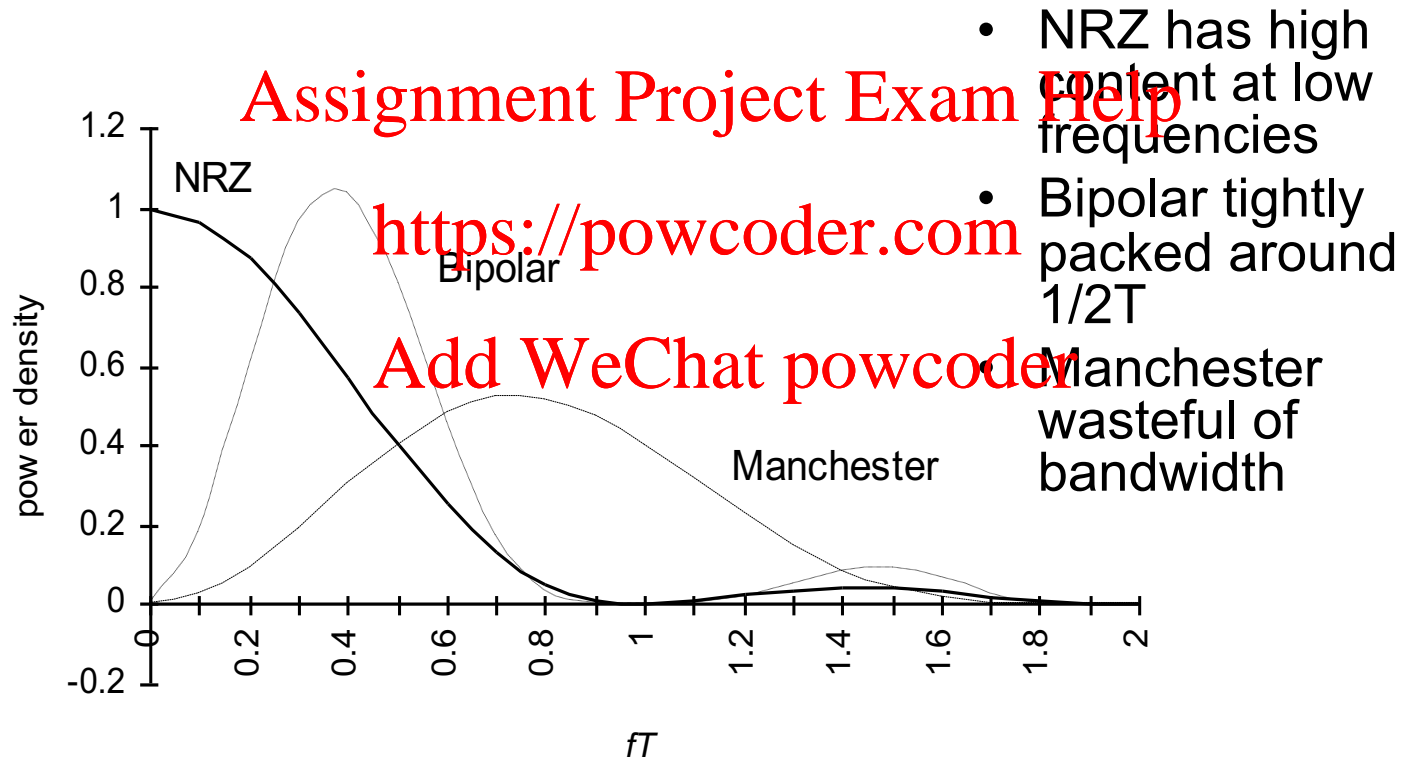
Manchester encoding

Differential Manchester encoding



# Spectrum of Line codes

- Assume 1's & 0's independent & equiprobable



# mB/nB Encoding

$$n \geq \sqrt{m}$$

- m data bits are coded as *symbols* of n line bits
- Example: FDDI uses 4B5B
  - 4 data bits for 5 line bits, so 100 Mbps uses 125 MHz.
  - Uses less frequency than Manchester encoding (1B2B)
- Each valid symbol has at least two 1s: get dense transitions.
- 16 data symbols, 8 control symbols
  - Data symbols: 4 data bits
  - Control symbols: idle, begin frame, etc.
- Also: 8B10B (Gigabit Ethernet, Fiber channel) and 64B66B code (10G Ethernet)

# 4B/5B Encoding

Data	Code	Data	Code
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

# Quiz Question

- The following are notable absentees in 4B/5B encoding.. Why?
  - 00000 ➤ Need transitions 16 transitions.
  - 00001 ➤ Need at least two ones
  - 11111 ➤ Need transitions
  - 10001 ➤ Control symbol! (start delimiter)



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# Error Control

- Channels introduce errors in digital communications
- Applications require certain reliability level
  - Data applications require error-free transfer
  - Voice & video applications tolerate some errors
- Error control may be needed to meet application requirement
- Error control ensures a data stream is transmitted to a certain level of accuracy despite errors
- Two basic approaches:
  - Error **detection** & retransmission (ARQ)
  - Forward error **correction** (FEC)

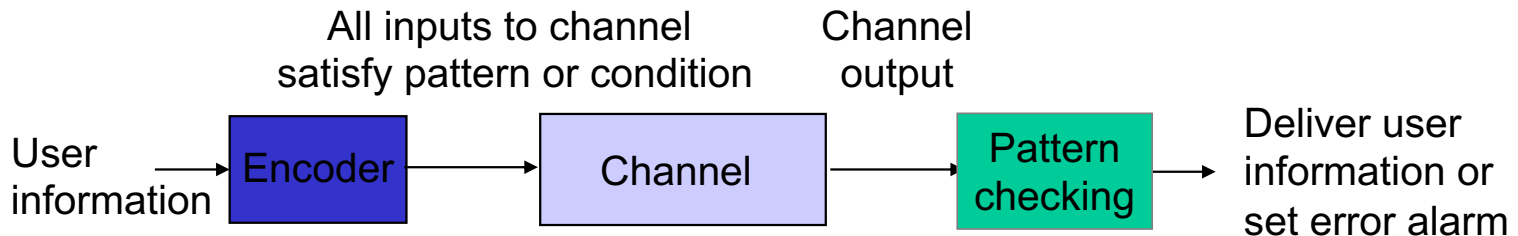
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# Key Idea

- All transmitted data blocks (“codewords”) are chosen so that they satisfy a pattern
- If received block doesn’t satisfy pattern, it is in error
- Redundancy: Only a subset of all possible blocks can be valid codewords
- Undetectable Error: When channel transforms a codeword into another valid codeword



# Single Parity Check

- Append an parity bit to  $k$  information bits

Info Bits:  $b_1, b_2, b_3, \dots, b_k$

Check Bit:  $b_{k+1} = b_1 + b_2 + b_3 + \dots + b_k \text{ modulo } 2$

Codeword:  $(b_1, b_2, b_3, \dots, b_k, b_{k+1})$

$b_s \begin{cases} \text{even} & 0 \\ \text{odd} & 1 \end{cases}$

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- All codewords have even # of 1s
- Receiver checks to see if # of 1s is even
  - All error patterns that create an odd # of 1 bits are detectable
  - All even-numbered error patterns are undetectable
- ASCII code is precisely such as code (7+1 bits)

# Quiz Question: Single Parity Code

- Information (7 bits): (0, 1, 0, 1, 1, 0, 0)
- Parity Bit? ("True"  $\rightarrow$  1, "False"  $\rightarrow$  0)

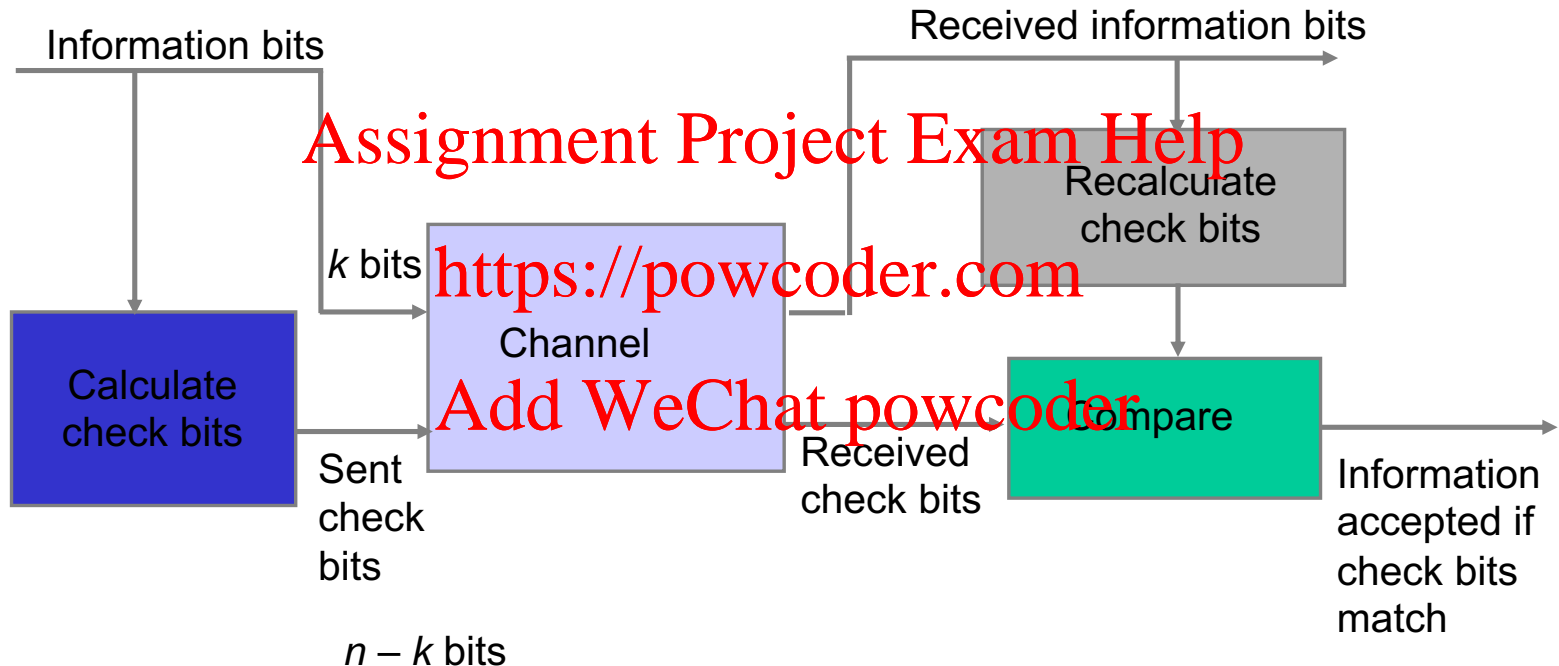
$$b_8 = 0 + 1 + 0 + 1 + 1 + 0 + 0 = 1$$

Codeword (8 bits): (0, 1, 0, 1, 1, 0, 0, 1)

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- If single error in bit 3? (0, 1, 1, 1, 1, 0, 0, 1)
  - # of 1's = 5, odd  $\Rightarrow$  Error detected
- If errors in bits 3 and 5? (0, 1, 1, 0, 0, 0, 0, 1)
  - # of 1's = 4, even  $\Rightarrow$  Error not detected

# Parity Checkbits & Error Detection



# How good is the single parity check code?

- *Redundancy*: Single parity check code adds 1 redundant bit per  $k$  information bits:

$$\text{overhead} = 1/(k+1)$$

- *Coverage*: all error patterns with odd # of errors can be detected
  - An error pattern is a binary  $(k+1)$ -tuple with 1's where errors occur and 0's elsewhere
  - Of  $2^{k+1}$  binary  $(k+1)$ -tuples,  $\frac{1}{2}$  are odd, so 50% of error patterns can be detected
- Is it possible to detect more errors if we add more check bits?
- Yes, with the right codes

# What if bit errors are random?

- Many transmission channels introduce bit errors at random, independently of each other, and with probability  $p$
- Some error patterns are more probable than others:

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$$P[100000000] = p(1-p)^8 = (1-p)^8 \left(\frac{p}{1-p}\right) \text{ and}$$

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$$P[110000000] = p^2(1-p)^6 = (1-p)^8 \left(\frac{p}{1-p}\right)^2$$

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- In any worthwhile channel  $p < 0.5$ , and so  $(p/(1-p)) < 1$
- It follows that patterns with 1 error are more likely than patterns with 2 errors and so forth
- What is the probability that an undetectable error pattern occurs?



# Single parity check code with random bit errors

- Undetectable error pattern if even # of bit errors:

$$P[\text{error detection failure}] = P[\text{undetectable error pattern}]$$

$$= P[\text{error patterns with even number of 1s}]$$

$$= \binom{n}{2} p^2 (1-p)^{n-2} + \binom{n}{4} p^4 (1-p)^{n-4} + \dots$$

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- Quiz! What's the probability for  $n=32$ ,  $p=10^{-3}$ ?

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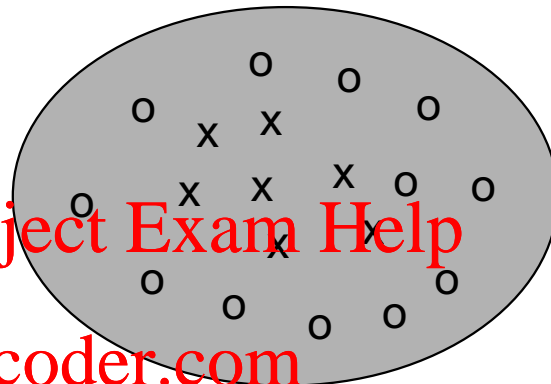
$$P[\text{undetectable error}] = \binom{32}{2} (10^{-3})^2 (1-10^{-3})^{30} + \binom{32}{4} (10^{-3})^4 (1-10^{-3})^{28}$$

$$\approx 496 (10^{-6}) + 35960 (10^{-12}) \approx 4.96 (10^{-4})$$

- For this example, roughly 1 in 2000 transmissions will result in an undetectable error

# What is a good code?

- Most channels will have relatively few bit errors
- Erroneous codewords transmitted over these channels will map to nearby n-tuples

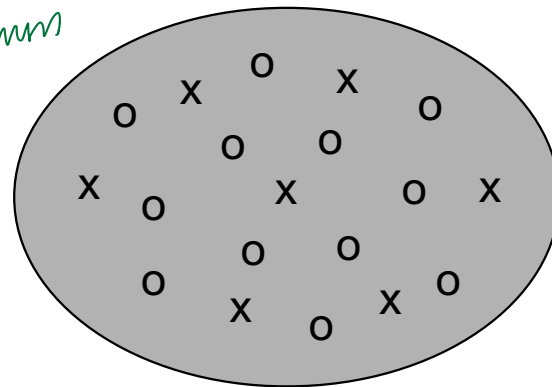


**Poor  
distance  
properties**

- If valid codewords are close to each other, then detection failures may occur

- Good codes should maximize separation between valid codewords

*the minimum*



**Good  
distance  
properties**

# Two-Dimensional Parity Check

- More parity bits to improve coverage
- Arrange information as columns
- Add single parity bit to each column
- Add a final “parity” column
- Used in early error control systems

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1	0	0	1	0	0	0	Last column consists of check bits for each row
0	1	0	0	0	0	1	
1	0	0	1	0	0	0	
1	1	0	1	1	0	0	
1	0	0	1	1	1	1	

Bottom row consists of  
check bit for each column

# Error-detecting capability

1	0	0	1	0	0
0	0	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

detect + correct  
One error

1	0	0	1	0	0
0	0	0	0	0	1
1	0	0	1	0	0
1	0	0	1	1	0
1	0	0	1	1	1

detect only  
Two errors

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1, 2, or 3 errors can always be detected; Not all patterns >4 errors can be detected

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1	0	0	1	0	0
0	0	0	1	0	1
1	0	0	1	0	0
1	0	0	1	1	0
1	0	0	1	1	1

detect only  
Three errors

1	0	0	1	0	0
0	0	0	1	0	1
1	0	0	1	0	0
1	0	0	1	0	0
1	0	0	1	1	1

Four errors

rectangle shaped error cannot be detected

Arrows indicate failed check bits

# Other Error Detection Codes

- Many applications require very low error rate
- Need codes that detect more number of errors
- Single parity check codes do not detect enough errors
- Two-dimensional codes require too many check bits
- The following error detecting codes are widely used in practice:
  - Internet Check Sums
  - CRC Polynomial Codes

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# Internet Checksum

- Several Internet protocols (e.g. IP, TCP, UDP) use check bits to detect errors in the **header**
- A checksum is calculated for header contents and included in a special field.
- Checksum is potentially recalculated at every router, so algorithm selected for ease of implementation in software
- Let header consist of  $L$ , 16-bit words,  
 $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_{L-1}$
- The algorithm appends a 16-bit checksum  $\mathbf{b}_L$

# Checksum Calculation

The checksum  $\mathbf{b}_L$  is calculated as follows:

- Treating each 16-bit word as an integer, find

$$\mathbf{x} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \dots + \mathbf{b}_{L-1} \text{ modulo } 2^{16}-1$$

- The checksum is then given by:

$$\mathbf{b}_L = -\mathbf{x} \text{ modulo } 2^{16}-1$$

Thus, the headers must satisfy the following **pattern** at the receiver:

$$\mathbf{0} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \dots + \mathbf{b}_{L-1} + \mathbf{b}_L \text{ modulo } 2^{16}-1$$

- The checksum calculation is carried out in software using one's complement arithmetic

# Internet Checksum Example

## Use Modulo Arithmetic

- Assume 4-bit words
- Use mod  $2^4 - 1 (= 15)$  arithmetic
- $\underline{b}_0 = 1100 = 12$
- $\underline{b}_1 = 1010 = 10$
- $\underline{b}_0 + \underline{b}_1 = 12 + 10 = 22 \equiv 7 \pmod{15}$
- $\underline{b}_2 = -7 = 8 \pmod{15}$
- Therefore
- $\underline{b}_2 = 1000$

## Use Binary Arithmetic

- Note  $16 \equiv 1 \pmod{15}$
- So:  $10000 \equiv 0001 \pmod{15}$

• leading bit wraps around

$$\begin{aligned}
 b_0 + b_1 &= 1100 + 1010 \\
 &= 10110 \\
 &= 10000 + 0110 \\
 &= 0001 + 0110 \\
 &= 0111 \\
 &= 7
 \end{aligned}$$

Take 1's complement

$$b_2 = -0111 = 1000$$



# Polynomial Codes

- Polynomials instead of vectors for codewords
- Polynomial arithmetic instead of check sums
- Implemented using shift-register circuits
- Also called cyclic redundancy check (CRC)
- Most data communications standards use polynomial codes for error detection
  - Have very simple hardware implementations
- Polynomial codes also basis for powerful error-correction methods

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# Binary Polynomial Arithmetic

- Binary vectors map to polynomials

$$(i_{k-1}, i_{k-2}, \dots, i_2, i_1, i_0) \rightarrow i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \dots + i_2x^2 + i_1x^1 + i_0$$

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Addition:

$$(x^7 + x^6 + 1) + (x^6 + x^5) = x^7 + x^6 + x^5 + x^5 + 1$$

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$$= x^7 + (1+1)x^6 + x^5 + 1$$

$$= x^7 + x^5 + 1 \quad \text{since } 1+1 = 0 \pmod{2}$$

Multiplication:

$$(x+1)(x^2 + x + 1) = x(x^2 + x + 1) + 1(x^2 + x + 1)$$

$$= (x^3 + x^2 + x) + (x^2 + x + 1)$$

$$= x^3 + 1$$

# Binary Polynomial Division

- Division with Decimal Numbers

$$\begin{array}{r}
 34 \leftarrow \text{quotient} \\
 35 \overline{) 1222} \leftarrow \text{dividend} \\
 \underline{105} \\
 172 \\
 \underline{140} \\
 32 \leftarrow \text{remainder}
 \end{array}$$

dividend = quotient x divisor + remainder

$1222 = 34 \times 35 + 32$

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- Polynomial Division

$$\begin{array}{r}
 x^3 + x^2 + x \leftarrow \text{divisor} \\
 x^3 + x + 1 \overline{) x^6 + x^5} \leftarrow \text{dividend} \\
 \underline{x^6 + \phantom{x^4} x^3} \\
 x^5 + x^4 + x^3 \\
 \underline{x^5 + \phantom{x^4} x^3 + x^2} \\
 x^4 + \phantom{x^3} x^2 \\
 \underline{x^4 + \phantom{x^3} x^2 + x} \\
 x \leftarrow \text{remainder}
 \end{array}$$

$x^3 + x^2 + x = q(x)$  quotient

$x = r(x)$  remainder

Handwritten polynomial division:

$$\begin{array}{r}
 x^3 + x^2 + x \\
 x^3 + x + 1 \overline{) x^6 + x^5} \\
 \underline{x^6 + \phantom{x^4} x^3} \\
 x^5 + x^4 + x^3 \\
 \underline{x^5 + \phantom{x^4} x^3 + x^2} \\
 x^4 + \phantom{x^3} x^2 \\
 \underline{x^4 + \phantom{x^3} x^2 + x} \\
 x
 \end{array}$$

Note: Degree of  $r(x)$  is less than degree of divisor

# Polynomial Coding

- k information bits define polynomial of degree k-1

$$i(x) = i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \dots + i_2x^2 + i_1x + i_0$$

- Code has binary generating polynomial of degree n-k

$$g(x) = x^{n-k} + g_{n-k-1}x^{n-k-1} + \dots + g_2x^2 + g_1x + 1$$

- Find remainder polynomial of at most degree n-k-1

$$\begin{array}{r}
 g(x) \overline{) x^{n-k} i(x)} \\
 \underline{\phantom{g(x)} x^{n-k} i(x)} \\
 r(x)
 \end{array}
 \quad
 \begin{array}{l}
 q(x) \\
 x^{n-k} i(x) = q(x)g(x) + r(x)
 \end{array}$$

- Define the codeword polynomial of degree n-1

$$\underbrace{b(x)}_{n \text{ bits}} = \underbrace{x^{n-k} i(x)}_{k \text{ bits}} + \underbrace{r(x)}_{n-k \text{ bits}}$$

is always  
a multiple of  
 $g(x)$

# Quiz Q: Find codeword if $k=4$ , $n-k=3$

And: Generator polynomial:  $g(x) = x^3 + x + 1$

Information:  $(1, 1, 0, 0)$        $i(x) = x^3 + x^2$

Encoding:  $x^3 i(x) = x^6 + x^5$

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# Quiz Q: Find codeword if $k=4$ , $n-k=3$

And: Generator polynomial:  $g(x) = x^3 + x + 1$

Information:  $(1, 1, 0, 0)$        $i(x) = x^3 + x^2$

Encoding:  $x^3 i(x) = x^6 + x^5$

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$x^3 + x + 1 \mid x^6 + x^5$   
 $x^6 + \quad x^4 + x^3$   
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$x^5 + x^4 + x^3$   
 $x^5 + \quad x^3 + x^2$   
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$x^4 + \quad x^2$   
 $x^4 + \quad x^2 + x$   
 $x$

# Quiz Q: Find codeword if $k=4$ , $n-k=3$

And: Generator polynomial:  $g(x) = x^3 + x + 1$  1001

Information:  $(1,1,0,0)$   $i(x) = x^3 + x^2$

Encoding:  $x^3 i(x) = x^6 + x^5$

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$$\begin{array}{r}
 x^3 + x^2 + x \\
 \hline
 x^3 + x + 1 \mid x^6 + x^5 \\
 \hline
 x^6 + \phantom{x^5} x^4 + x^3 \\
 \hline
 x^5 + x^4 + x^3 \\
 \hline
 x^5 + \phantom{x^4} x^3 + x^2 \\
 \hline
 x^4 + \phantom{x^3} x^2 \\
 \hline
 x^4 + \phantom{x^3} x^2 + x \\
 \hline
 x
 \end{array}
 \qquad
 \begin{array}{r}
 1110 \\
 \hline
 1011 \mid 110000 \\
 \hline
 1011 \\
 \hline
 1110 \\
 \hline
 1011 \\
 \hline
 1010 \\
 \hline
 1011 \\
 \hline
 010
 \end{array}$$

Transmitted codeword:

$$b(x) = x^6 + x^5 + x$$

$$\underline{b} = (1,1,0,0,0,1,0)$$

# The *Pattern* in Polynomial Coding

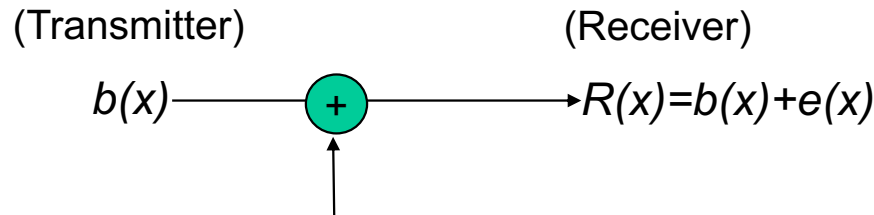
- All codewords satisfy the following **pattern**:

$$b(x) = x^{n-k} \cdot r(x) = q(x)g(x) + r(x) \quad \text{where } r(x) = q(x)g(x)$$

- All codewords are a multiple of  $g(x)$ !
- Receiver should divide received n-tuple by  $g(x)$  and check if remainder is zero
- If remainder is non-zero, then received n-tuple is not a codeword



# Undetectable error patterns



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- $e(x)$  has 1's in error locations & 0's elsewhere
- Receiver divides the received polynomial  $R(x)$  by  $g(x)$
- Undetectable error: If  $e(x)$  is a multiple of  $g(x)$ , that is,  $e(x)$  is a non-zero codeword, then

$$R(x) = b(x) + e(x) = q(x)g(x) + q'(x)g(x)$$

- *The set of undetectable error polynomials is the set of nonzero code polynomials*
- *Choose the generator polynomial so that selected error patterns can be detected.*

# Designing good polynomial codes

- Select generator polynomial so that likely error patterns are not multiples of  $g(x)$
- *Detecting Single Errors*
  - $e(x) = x^i$  for error in location  $i+1$
  - If  $g(x)$  has more than 1 term, it cannot divide  $x^i$
- *Detecting Double Errors*
  - $e(x) = x^i + x^j = x^i(x^{j-i} + 1)$  where  $j > i$
  - If  $g(x)$  has more than 1 term, it cannot divide  $x^i$
  - If  $g(x)$  is a *primitive* polynomial, it cannot divide  $x^m + 1$  for all  $m < 2^{n-k} - 1$  (Need to keep codeword length less than  $2^{n-k} - 1$ )
  - Primitive polynomials can be found by consulting coding theory books

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# Standard Generator Polynomials

CRC = cyclic redundancy check

- CRC-8:  $= x^8 + x^2 + x + 1$  ATM

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- CRC-16:  $= x^{16} + x^{15} + x^2 + 1$   
 $= (x+1)(x^{15} + x + 1)$  Bisync  
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- CCITT-16:  $= x^{16} + x^{12} + x^5 + 1$  HDLC, XMODEM, V.41

- CCITT-32: IEEE 802, DoD, V.42  
 $= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$

# Hamming Codes

- Class of error-correcting codes
- Capable of correcting all single-error patterns
- Provably optimal for 1-bit errors
- Very less redundancy, e.g. 1-bit error proof – adds  $O(\log n)$  bits of redundancy for  $n$  bit sequences

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# m=3 Hamming Code

- Information bits are  $b_1, b_2, b_3, b_4$
- Equations for parity checks  $b_5, b_6, b_7$

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$$b_5 = b_1 + b_3 + b_4$$

$$b_6 = b_1 + b_2 + b_4$$

$$b_7 = b_2 + b_3 + b_4$$

- There are  $2^4=16$  codewords
- $(0,0,0,0,0,0,0)$  is a codeword

# My "simple" proof of optimality

Assume you got the following 7 bit sequences and make the following checks:

$$b_5 = b_1 + b_3 + b_4$$

$$b_6 = b_1 + b_2 + b_4$$

$$b_7 = b_1 + b_2 + b_3 + b_4$$

Case	b <sub>1</sub> match	b <sub>6</sub> match	b <sub>7</sub> match
No error			
b <sub>1</sub> flipped			
b <sub>2</sub> flipped			
b <sub>3</sub> flipped			
b <sub>4</sub> flipped			
b <sub>5</sub> flipped			
b <sub>6</sub> flipped			
b <sub>7</sub> flipped			

# My "simple" proof of optimality

Assume you got the following 7 bit sequences and make the following checks:

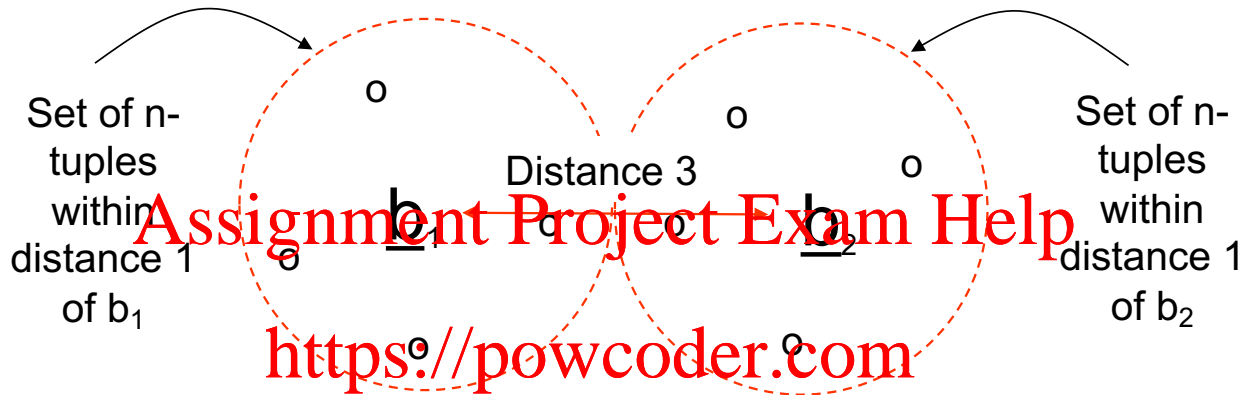
$$b_5 = b_1 + b_3 + b_4$$

$$b_6 = b_1 + b_2 + b_4$$

$$b_7 = b_1 + b_2 + b_3 + b_4$$

Case	b <sub>1</sub> match	b <sub>6</sub> match	b <sub>7</sub> match
No error	✓	✓	✓
b <sub>1</sub> flipped	X	X	✓
b <sub>2</sub> flipped	✓	X	X
b <sub>3</sub> flipped	X	✓	X
b <sub>4</sub> flipped	X	X	X
b <sub>5</sub> flipped	X	✓	✓
b <sub>6</sub> flipped	✓	X	✓
b <sub>7</sub> flipped	✓	✓	X

# Why is Hamming a “good code”?



- Two valid bit sequences have a minimum distance of 3 bit flips
- Spheres of distance 1 around each codeword do not overlap
- If a single error occurs, the resulting n-tuple will be in a unique sphere around the original codeword
- Thus, receiver can correct erroneous reception back to original codeword



# Physical Layer: Outline

- Digital networks
- Characterization of Communication Channels
- Fundamental Limits in Digital Transmission
- Line Coding
- Modems and Digital Modulation
- Error Detection and Correction
- **Wired PHY 101**
- Wireless PHY 101

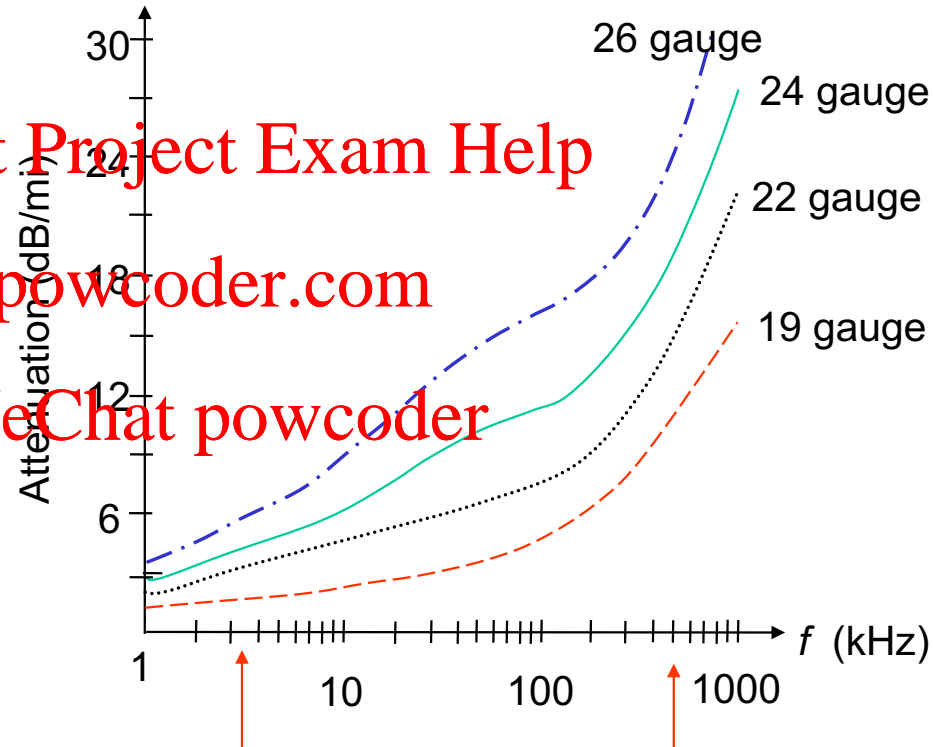
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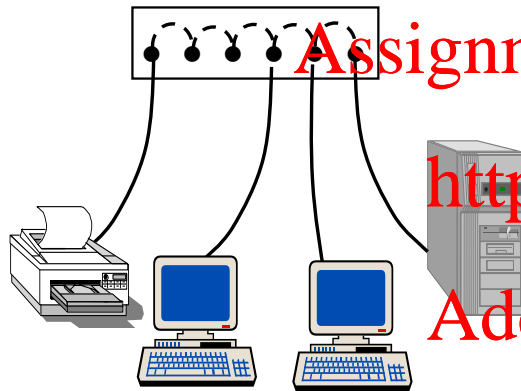
# Twisted Pair

- Two insulated copper wires arranged in a regular spiral pattern to minimize interference
- Various thicknesses, e.g. 0.016 inch (24 gauge)
- Low cost
- Telephone subscriber loop from customer to CO
- Old trunk plant connecting telephone COs
- Intra-building telephone from wiring closet to desktop



# Ethernet LANs

- Evolved from 10 → 100 → 1000 Mbps to now 10Gbps



- All use twisted pair in some form!

- 10BASE-T Ethernet

- 10 Mbps, Baseband, Twisted pair
- Two Cat3 pairs
- Manchester coding, 100 meters

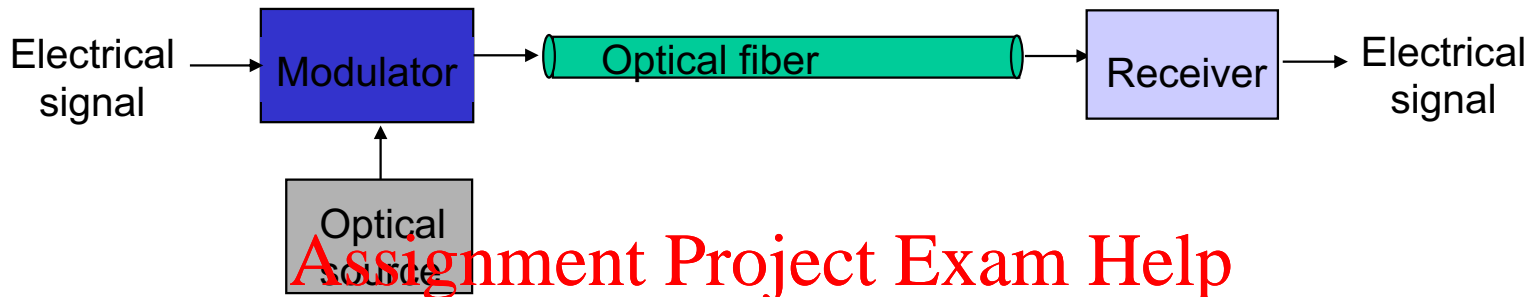
- 100BASE-T4 *Fast Ethernet*

- 100 Mbps, Baseband, Twisted pair
- Four Cat3 pairs
- Three pairs for one direction at-a-time
- 100/3 Mbps per pair;
- 3B6T line code, 100 meters

- 1000BASE-T

- 8b10b encoding, Four pairs

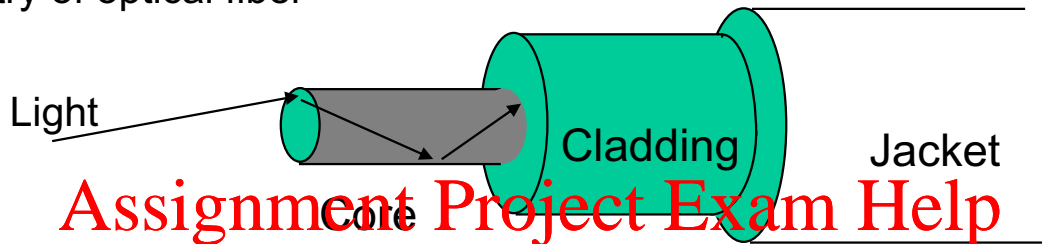
# Optical Fiber



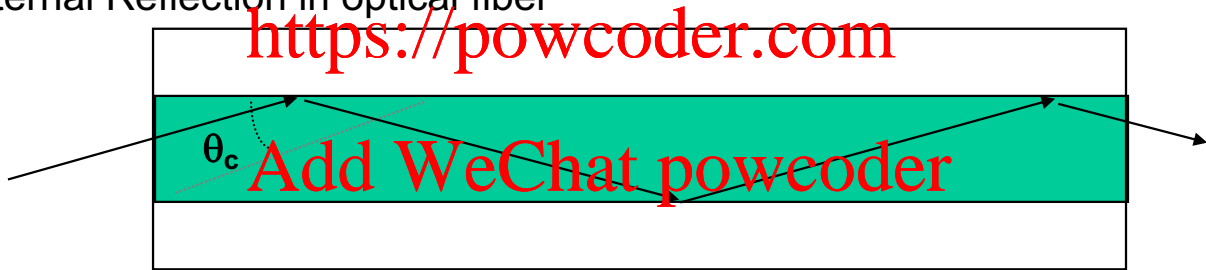
- Light sources (lasers, LEDs) generate pulses of light that are transmitted on optical fiber
  - Very long distances (>1000 km)
  - Very high speeds (>40 Gbps/wavelength)
  - Nearly error-free (BER of  $10^{-15}$ )
- Profound influence on network architecture
  - Dominates long distance transmission
  - Distance less of a cost factor in communications
  - Plentiful bandwidth for new services

# Transmission in Optical Fiber

Geometry of optical fiber



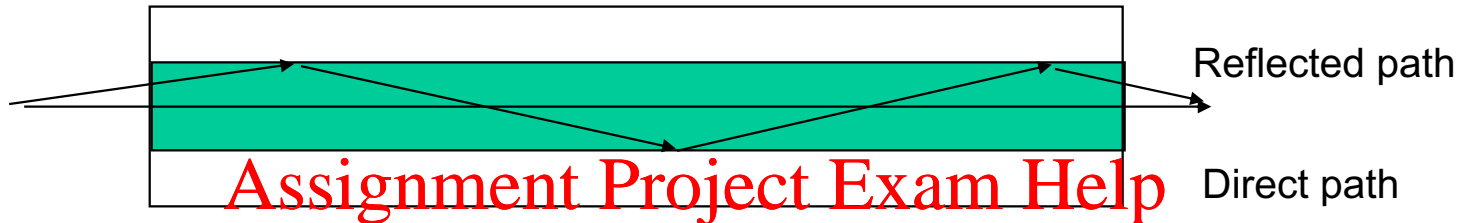
Total Internal Reflection in optical fiber



- Very fine glass cylindrical core surrounded by concentric layer of glass (cladding)
- Core has higher index of refraction than cladding
- Light rays incident at less than critical angle  $\theta_c$  is completely reflected back into the core

# Multimode & Single-mode Fiber

Multimode fiber: multiple rays follow different paths



Single-mode fiber: only direct path propagates in fiber



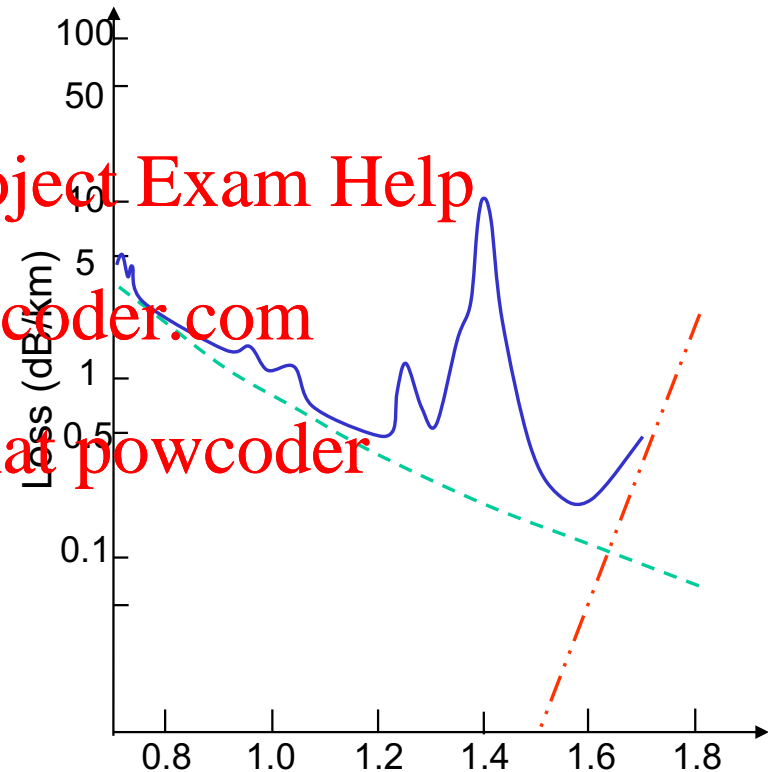
- Multi Mode: Thicker core, shorter reach
  - Rays on different paths interfere causing dispersion & limiting bit rate
- Single Mode: Very thin core supports only one mode (path)
  - More expensive lasers, but achieves very high speeds

# Huge Available Bandwidth

- Optical range from  $\lambda_1$  to  $\lambda_1 + \Delta\lambda$  contains bandwidth

$$B = f_1 - f_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_1 + \Delta\lambda}$$

$$= \frac{v}{\lambda_1} \left\{ \frac{\Delta\lambda / \lambda_1}{1 + \Delta\lambda / \lambda_1} \right\} \approx \frac{v\Delta\lambda}{\lambda_1}$$



# Quiz Question

How much optical fiber bandwidth is available between:

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$\lambda_1 = 1450 \text{ nm}$  and  $\lambda_1 + \Delta\lambda = 1650 \text{ nm}$ :

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Answer:

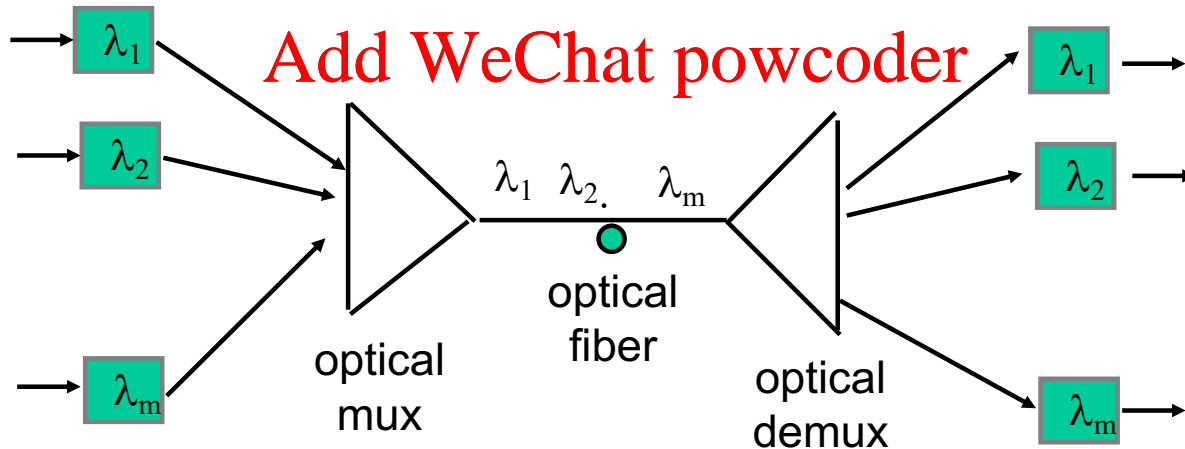
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$$B = \frac{2(10^8 \text{ m/s})(100 \text{ nm})}{(1450 \text{ nm})^2} \approx 19 \text{ THz}$$



# Wavelength-Division Multiplexing

- Different wavelengths carry separate signals
- Multiplex into shared optical fiber
- Each wavelength like a separate circuit
- A single fiber can carry 160 wavelengths, 10 Gbps per wavelength, 1.6 Tbps!



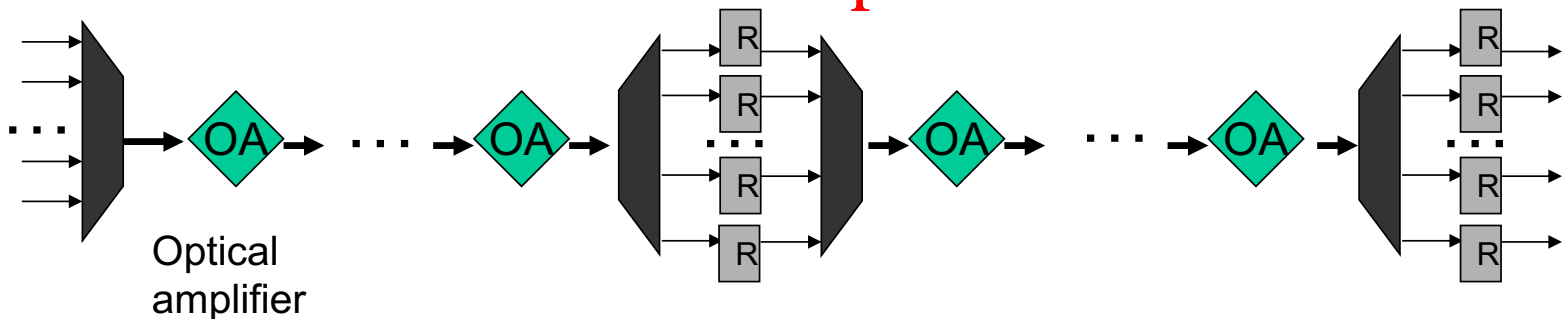
# How Do We Extend Range

- Use combinations of optical amplifiers and regenerators

- More amplifiers than regenerators (Why?)

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