

Question 11 Solution

COMP3121/9101 21T3 Final Exam

This document gives a model solution to question 11 of the final exam. Note that alternative solutions may exist.

1. You are given n intervals on an axis. The i th interval $[l_i, r_i)$ has integer endpoints $l_i < r_i$ and has a score of s_i . Your task is to select a set of disjoint intervals with maximum total score. Note that if intervals i and j satisfy $r_i = l_j$ then they are still disjoint.

Design an algorithm which solves this problem and runs in $O(n^2)$ time.

You must provide reasoning to justify the correctness and time complexity of your algorithm.

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The input consists of the positive integer n , as well as $2n$ integers $l_1, r_1, \dots, l_n, r_n$ and n positive real numbers s_1, \dots, s_n .

The output is the set of intervals chosen, organised in any format or data structure.

For example, if $n = 4$ and the intervals are:

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$l_1 = 1$	$r_1 = 3$	$s_1 = 2$
$l_2 = 1$	$r_2 = 3$	$s_2 = 1$
$l_3 = 2$	$r_3 = 4$	$s_3 = 4$
$l_4 = 3$	$r_4 = 5$	$s_4 = 3$

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then you should select only the first and fourth intervals, for a maximum total score of 5. Note that interval 3 is not disjoint with any other interval.

Sort the intervals by increasing order of endpoint r_i and relabel accordingly. Henceforth we assume $r_1 \leq \dots \leq r_n$.

We then proceed by dynamic programming.

Subproblems: for $0 \leq i \leq n$, let $P(i)$ be the problem of determining $\text{opt}(i)$, the maximum total score of a set of disjoint intervals where the last chosen interval is the i th, and $m(i)$, the second largest interval index in one such set. If the set consists of only one interval, then $m(i)$ will be zero.

Recurrence: for $1 \leq i \leq n$,

$$m(i) = \operatorname{argmax}_{j: r_j \leq l_i} \text{opt}(j)$$

and

$$\text{opt}(i) = s_i + \text{opt}(m(i)).$$

The solution for i must include interval i , so we extend the best solution with last chosen interval j finishing at or before the start of interval i .

Base case: $\text{opt}(0) = 0$ and $m(0)$ is undefined.

Order of computation: subproblem $P(i)$ depends only on earlier subproblems ($P(j)$, where $j < i$), so we can solve the subproblems in increasing order of i .

Final answer: The maximum total score is

$$\max_{1 \leq i \leq n} \text{opt}(i).$$

To recover the set which yields this score, we let

$$i^* = \operatorname{argmax}_{1 \leq i \leq n} \text{opt}(i),$$

and backtrack through the m array to obtain the set $\{i^*, m(i^*), m(m(i^*)), \dots\}$.

Time complexity: There are $O(n)$ subproblems, each solved in $O(n)$, and constructing the final answer also takes $O(n)$. Thus the overall time complexity is $O(n^2)$.

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