CQF Exam One

June 2024 Cohort

Instructions

All questions must be attempted. Requested mathematical and full computational workings must be provided to obtain maximum credit. CQF material and books may be referred to but you should answer the questions in your own words. You are not allowed to discuss the exam work with anyone else.

Upload two files: E1_YOURNAME_REPORT.pdf and E1_YOURNAME_CODE.zip. ZIP file to include source code, signed declaration (if not in PDF). Use YOURNAME as registered on CQF Portal.

You must prepare PDF REPORT that integrates workings, numerical answers and plots in the order of questions. Guide to report preparation:

1. If you draft the report from Python notebook – remove unnecessary output and unused code, add maths with Markdown TeX. Save IPYNB file to HTML first, then print PDF.

2. If you draft from Word/LaTeX – add maths requested, numerical solutions and plots. Here you can insert code at your discretion, eg full/part code after each question or in an appendix.

3. Exam tasks give specific instructions, eg to organise results into a table or specific plot. Handwritten & scanned math workings are acceptable without loss in marks.

4. 'Code+output' Python printouts (absent maths, explanation, issues with plots) or Excel printouts will not be considered full reports.

Making operational sense of exam questions and coding is part of your individual exam work. Please make a good use of lectures, solutions and labs/tutorials. Tutor is unable to confirm formulae, discuss your numerical answers or provide hints beyond ones given.

Exam submissions and file names which do not follow these instructions will require extra processing time. Exam One computation in Excel is not recommended.

Marking Scheme: **Q1** 16% **Q2** 24% **Q3** 8% **Q4** 28% **Q5** 24%. Total is 100%.

Exam One June 2024. Optimal Portfolio Allocation

An investment universe of the following risky assets with a dependence structure (correlation) applies to all questions below as relevant:

Asset	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{w}$
A	0.05	0.07	w_1
B	0.07	0.28	w_2
C	0.15	0.25	w_3
D	0.22	0.31	w_4

$$\boldsymbol{Corr} = \begin{pmatrix} 1 & 0.4 & 0.3 & 0.3 \\ 0.4 & 1 & 0.27 & 0.42 \\ 0.3 & 0.27 & 1 & 0.5 \\ 0.3 & 0.42 & 0.5 & 1 \end{pmatrix}$$

Question 1. Global Minimum Variance portfolio is obtained subject to the budget constraint:

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w'}\,\boldsymbol{\Sigma}\,\boldsymbol{w} \qquad \text{s.t. } \boldsymbol{w'1} = 1$$

- Derive the analytical solution for optimal allocations $\boldsymbol{w^*}$. Provide full derivation workings.
- Compute optimal allocations (Global MV portfolio) for the given investment universe.

Question 2. Consider the optimization for a target return m. There is no risk-free asset.

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w'}\,\boldsymbol{\Sigma}\,\boldsymbol{w}$$

$$\begin{aligned} \boldsymbol{w'1} &= 1 \\ \boldsymbol{w'\mu} &= m \end{aligned}$$

- Compute correlation levels by stressing the matrix $\times 1$, $\times 1.3$, $\times 1.8$, subject to the upper limit 0.99 for each cross-asset correlation. Diagonal elements stay equal to one.

- Compute $\mathbf{w}^*$ and portfolio risk $\sigma_\Pi = \sqrt{\boldsymbol{w'\Sigma w}}$ for $m = 7\%$ for three levels of correlation given.

Hints: it is possible to compute this kind of optimal allocation via analytical formula. Negative and non-robust allocations (into $\pm$ 100s%) are possible, particularly for high correlation. Please do not reconfirm your numerical results via support.

Question 3. "Evaluating the P&L more frequently make it appear more risky than it actually is." Make the following computations to demonstrate this statement.

- Write down the formula for Sharpe Ratio and note that σ is scaled with time.
- Compute Daily, Monthly, and Quarterly Sharpe Ratio, for Annualised SR of 0.53. Hint: this is an abstract computation, not related to Questions 1 and 2.
- Convert each Sharpe Ratio into Loss Probability (daily, monthly, quarterly, annual), using

$$\Pr(\text{P\&L} < 0) = \Pr(x < -\text{SR}).$$

where x is a standard Normal random variable.

Exam One June 2024. Understanding Value-at-Risk

Assume you are an analyst concerned with how risky **NASDAQ-100** became over **S&P 500**. Perform the backtesting of Analytical VaR (99%/10day) on the data provided in .csv files.

Question 4. The quick guide is given below, but please refer to the tutorial and CQF material.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Compute the rolling standard deviation σ_t from 21 daily returns. Timescale of σ_t remains 'daily' regardless of how many returns are in the sample.
- To make a projection over 10 days, we use the additivity of variance $\sigma_{10D} = \sqrt{\sigma_t^2 \times 10}$.
- A breach occurs when the forward realised 10-day return is below the VaR_t quantity.

$$r_{10D,t+10} \; < \; \text{VaR}_{10D,t} \qquad\qquad \text{given both numbers are negative.}$$

VaR is fixed at time t and compared to the return from t to $t+10$, computed $\ln(S_{t+10}/S_t)$. Alternatively, you can compare to $\ln(S_{t+11}/S_{t+1})$ but state this assumption in your report upfront.

Prepare and present the following deliverables in your report:

(a) The count and percentage of VaR breaches.

(b) Provide a plot which identifies the breaches with crosses or other marks.

(c) Provide a list of breaches with columns [Date, ClosingPrice, LogReturn, VaR_10D, Ret_10D]. Hint: you need to have True/False breach indicator column in Python, and filter with '== True'.

(d) In your own words describe, was NASDAQ-100 more risky than S&P 500 during COVID pandemic news 2020-Feb to 2020-Mar? What about the subsequent market correction period in 2021-2022?

Question 5. Implement the backtest of $\text{VaR}_{10D,t}$ but now with the input of EWMA σ^2_{t+1} from the filtering formula below, instead of rolling std dev. Tutor will not reconfirm the formula/computation.

$$\sigma^2_{t+1\,|\,t} = \lambda\, \sigma^2_{t\,|\,t-1} + (1-\lambda)\, r_t^2$$

with $\lambda = 0.72$ value set to minimise out of sample forecasting error.

Hint: use the variance for the entire dataset to initialise the computation.

(a-c) Provide the same deliverables (a), (b) and (c) as in the previous Question.

(d) Briefly discuss the impact of λ on smoothness of EWMA-predicted volatility (3-4 lines).
Hint: you can discuss λ theoretically without recomputing EWMA-based backtest but, if you recompute for an extra illustration it is sufficient to do so for one market index only.

END OF EXAM