Agent-Based Systems

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Risks and Decisions

Knowing what to expect

The plan for today

- Probabilities: basics
- Bayes' rule and conditional independence
- Back to the Wumpus World

The book

Stuart Russell and Peter Norvig
Artificial Intelligence: a modern approach
Chapters 13-14

You are back from a holiday on an exotic island, and your doctor has bad news and good news. The bad news is that you've been diagnosed a serious disease and the test is 99% accurate. The good news is that the disease is very rare (1 in 10.000 get it).

How worried should you be?

Probability basics

Begin with a set Ω—the sample space
e.g., 6 possible rolls of a dice

$w \in \Omega$ is a sample point/possible world/atomic event

A probability space or probability model is a sample space Ω with an assignment $P(w)$ for every $w \in \Omega$ s.t.

$0 \leqslant P(w) \leqslant 1$

$\sum_w P(w) = 1$

e.g., $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$.

An event A is any subset of Ω

$$P(A) = \sum_{\{w \in A\}} P(w)$$

E.g., $P(\text{dice roll} < 4) = P(1) + P(2) + P(3) = 1/6 + 1/6 + 1/6 = 1/2$

A random variable is a function from sample points to some range, e.g., $\mathbb{R}$, $[0,1]$, $\{true, false\}$ ...

e.g., $Odd(1) = true$.

P induces a probability distribution for any random variable X:

$$P(X = x_i) = \sum_{\{w : X(w) = x_i\}} P(w)$$

e.g., $P(Odd = true) = P(1) + P(3) + P(5) = 1/6 + 1/6 + 1/6 = 1/2$

Definition of conditional probability:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)} \text{ if } P(b) \neq 0$$

Product rule gives an alternative formulation:

$P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$ but then...

Theorem (Bayes' Rule)

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

Useful for assessing causal probability from diagnostic probability:

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

E.g., let c be cold, s be sore throat:

$$P(c|s) = \frac{P(s|c)P(c)}{P(s)} = \frac{0.9 \times 0.001}{0.005} = 0.18$$

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We might not know the prior probability of the evidence $P(S)$
In this case...

- we compute the posterior probability for each value of the query variable ($c, \neg c$)
- and then normalise by a normalisation constant α

$$\mathbf{P}(C|s) = \alpha \langle P(s|c)P(c), P(s|\neg c)P(\neg c)\rangle$$

Bayes' rule with random variables

Theorem (Bayes' rule with random variables)

$$\mathbf{P}(X|Y) = \alpha\,\mathbf{P}(Y|X)\mathbf{P}(X)$$

Holiday solved

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$P(d|p)$

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Notice:the posterior probability of disease is still very small!

A and B are independent iff

$\mathbf{P}(A|B) = \mathbf{P}(A)$ or $\mathbf{P}(B|A) = \mathbf{P}(B)$ or $\mathbf{P}(A, B) = \mathbf{P}(A)\mathbf{P}(B)$

$$
\begin{aligned}
& P(cavity) \\
= \; & P(cavity|\text{Weather}) \\
= \; & P(cavity|\text{CRIdives})
\end{aligned}
$$

Combining evidence

Start with the joint distribution:

	toothache		$\neg$ *toothache*	
	catch	$\neg$ *catch*	*catch*	$\neg$ *catch*
cavity	.108	.012	.072	.008
$\neg$ *cavity*	.016	.064	.144	.576

$\mathbf{P}(Cavity|toothache \wedge catch) =$

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$\mathbf{P}(Cavity \mid toothache \wedge catch) = \alpha\langle 0.108, 0.016\rangle = \langle 0.871, 0.129\rangle$

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	catch	$\neg$ *catch*	*catch*	$\neg$ *catch*
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- It doesn't scale up to a large number of variables
- Can we simplify?

We can't use absolute independence:

- Toothache and Catch are **not** independent: If the probe catches in the tooth then it is likely the tooth has a cavity, which means that toothache is likely too.

- But they are independent **given** the presence or the absence of cavity! Toothache depends on the state of the nerves in the tooth, catch depends on the dentist's skills, to which toothache is irrelevant

Conditional independence

1. $P(catch|toothache, cavity) = P(catch|cavity)$, the same independence holds if I haven't got a cavity:

2. $P(catch|toothache, \neg cavity) = P(catch|\neg cavity)$

Catch is conditionally independent of *Toothache* given *Cavity*:

Conditional independence contd.

Write out full joint distribution using chain rule:

$\mathbf{P}(Toothache, Catch, Cavity)$

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\end{aligned}$$

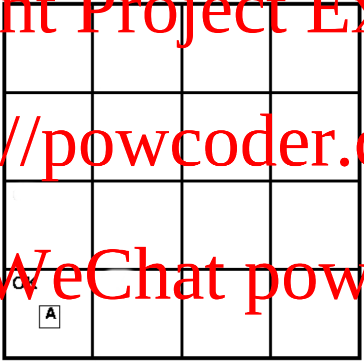
I.e., $2 + 2 + 1 = 5$ independent numbers (first and second steps remove two). Else 8-1=7. The gain is bigger the more the combinations.

In most cases, the use of conditional independence reduces the size of the representation of the joint distribution from exponential in n to linear in n.

Conditional independence is our most basic and robust form of knowledge about uncertain environments.

The Wumps World

The Wumpus World



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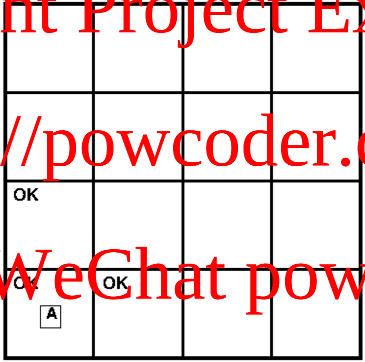

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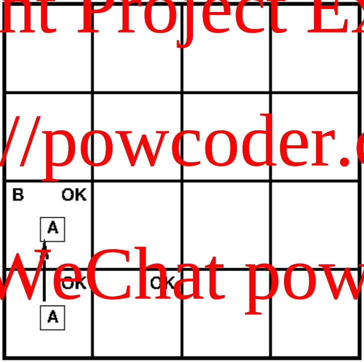

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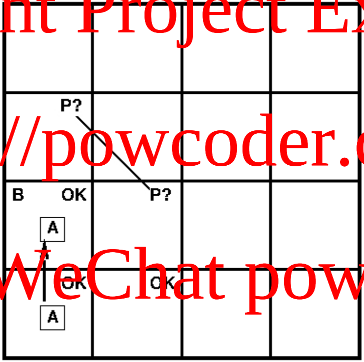

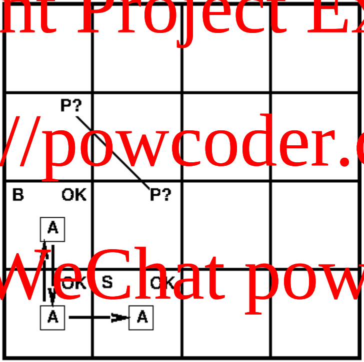
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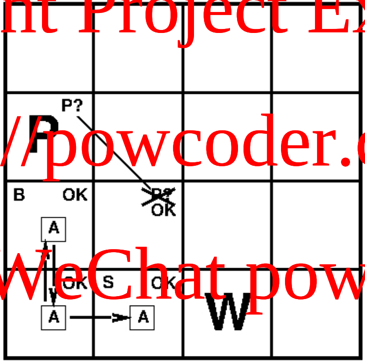

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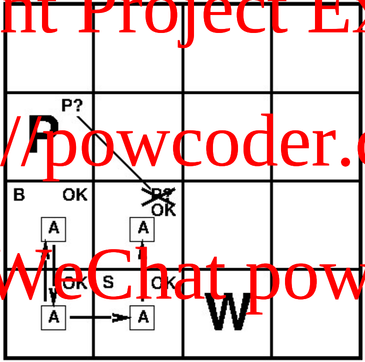

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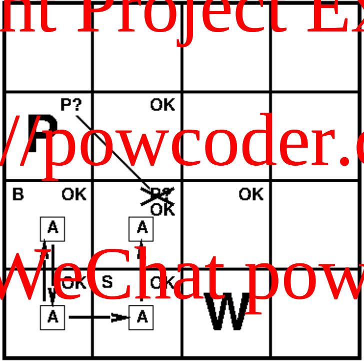

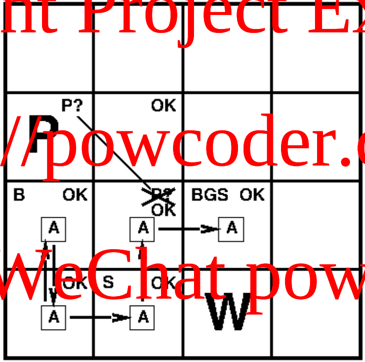
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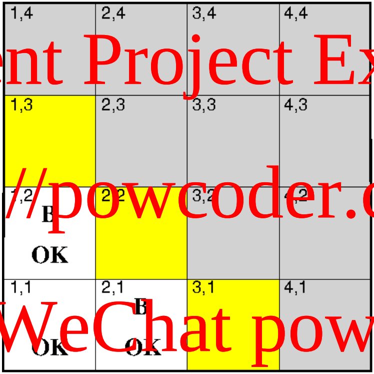

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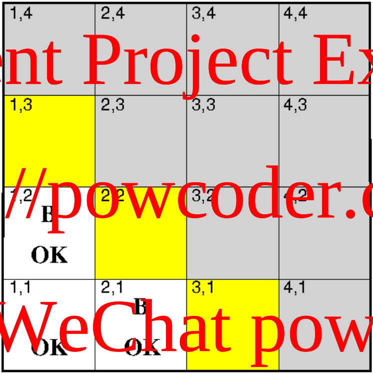
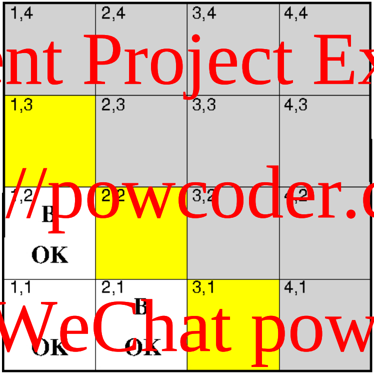


Assuming that pits can be in a square with a probability of 0.2...



Assuming that pits can be in a square with a probability of 0.2...

$P_{ij} = true$ iff $[i, j]$ contains a pit

$B_{ij} = true$ iff $[i, j]$ is breezy

Specifying the probability model

Include only $B_{1,1}, B_{1,2}, B_{2,1}$ in the probability model!

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(Do it this way to get $P(Effect|Cause)$.)

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$$\mathbf{P}(P_{1,1}, \ldots, P_{4,4}) = \prod_{i,j=1,1}^{4,4} \mathbf{P}(P_{i,j}) = 0.2^n \times 0.8^{16-n}$$

for n pits.

Observations and query

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Define $Unexplored = P_{ij}$s other than $P_{1,3}$ and $Explored$

Complexity

For inference by enumeration, we have

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And now?

Using conditional independence

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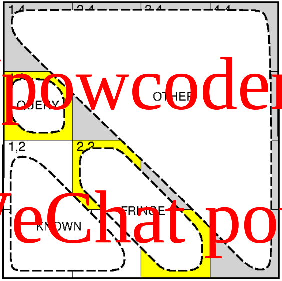

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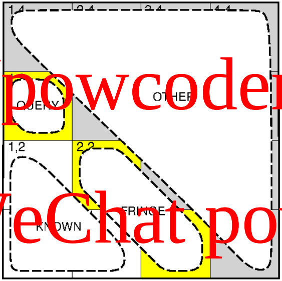

Define *Unexplored* $= Fringe \cup Other$

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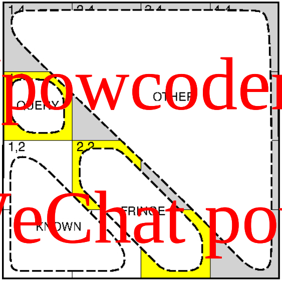

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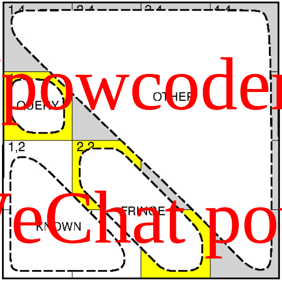

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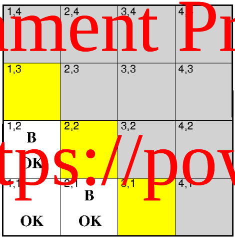
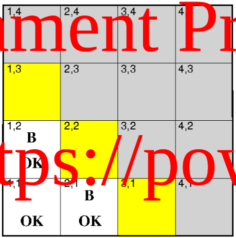
Manipulate query into a form where we can use this!

Using conditional independence



$\mathbf{P}(P_{1,3}|explored, b)$

Using conditional independence



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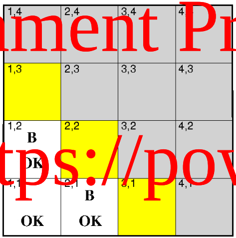
Using conditional independence



Inference by enumeration

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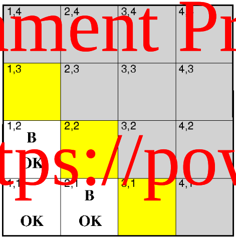

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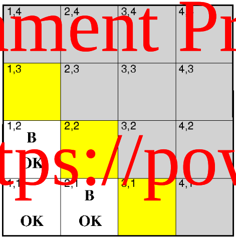
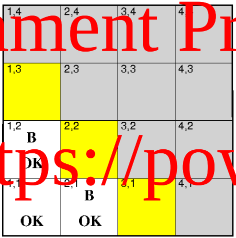


$$\alpha \sum_{unexplored} \mathbf{P}(P_{1,3}, unexplored, explored, b)$$

$$= \alpha \sum_{unexplored} \mathbf{P}(b | explored, P_{1,3}, unexplored) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, unexplored)$$

Using conditional independence

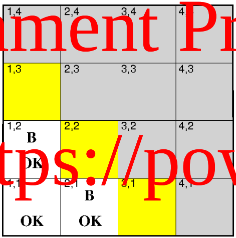


Product rule

$$\alpha \sum_{unexplored} \mathbf{P}(P_{1,3}, unexplored, explored, b)$$

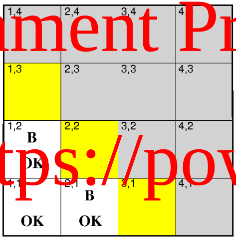
$$= \alpha \sum_{unexplored} \mathbf{P}(b|explored, P_{1,3}, unexplored) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, unexplored)$$

Using conditional independence



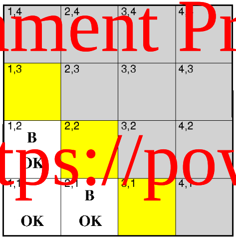
$$\alpha \sum_{unexplored} \mathbf{P}(b|P_{1,3}, unexplored, explored)\mathbf{P}(P_{1,3}, unexplored, explored)$$

Using conditional independence



$$\alpha \sum_{unexplored} \mathbf{P}(b|P_{1,3}, unexplored, explored)\mathbf{P}(P_{1,3}, unexplored, explored)$$

$$= \alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe, other)\times$$
$$\times\mathbf{P}(P_{1,3}, explored, fringe, other)$$

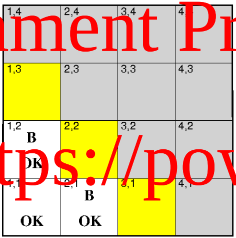


Distinguishing the unknown

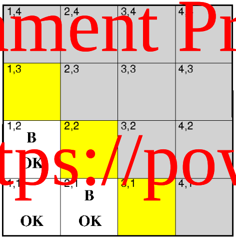
$$\alpha \sum_{unexplored} \mathbf{P}(b|P_{1,3}, unexplored, explored)\mathbf{P}(P_{1,3}, unexplored, explored)$$

$$= \alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe, other)\times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$

Using conditional independence

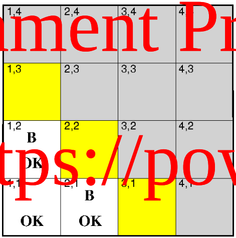


$\alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe, other) \times$
$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$



$$\alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe, other) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$

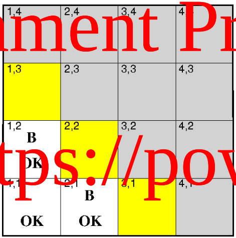
$$= \alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$



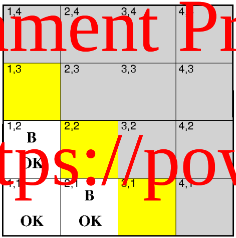
Conditional Independence

$$\alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe, other) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$

$$= \alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$

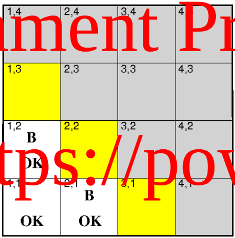


$$\alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$



$$\alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$

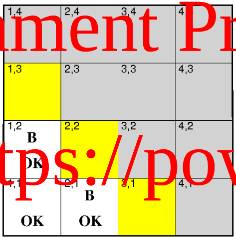
$$= \alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \sum_{other} \mathbf{P}(P_{1,3}, explored, fringe, other)$$



Pushing the sums inwards

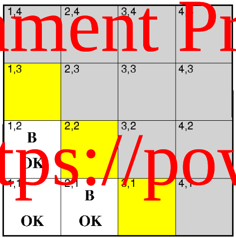
$$\alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \mathbf{P}(P_{1,3}, explored, fringe, other)$$

$$= \alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \sum_{other} \mathbf{P}(P_{1,3}, explored, fringe, other)$$



$$\alpha \sum_{fringe} \mathbf{P}(b | explored, P_{1,3}, fringe) \times \sum_{other} \mathbf{P}(P_{1,3}, explored, fringe, other)$$

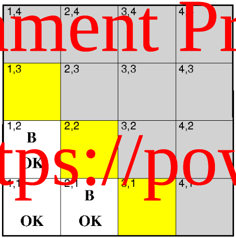
Using conditional independence



$$\alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times \sum_{other} \mathbf{P}(P_{1,3}, explored, fringe, other)$$

$$= \alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times \sum_{other} \mathbf{P}(P_{1,3}) P(explored) P(fringe) P(other)$$

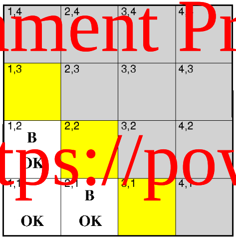
Using conditional independence



Independence

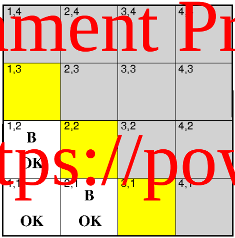
$$\alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times \sum_{other} \mathbf{P}(P_{1,3}, explored, fringe, other)$$

$$= \alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times \sum_{other} \mathbf{P}(P_{1,3}) P(explored) P(fringe) P(other)$$



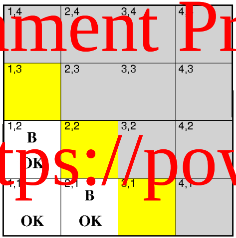
$$\alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \sum_{other} \mathbf{P}(P_{1,3}) P(explored) P(fringe) P(other)$$

Using conditional independence



$$\alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe)\times$$
$$\times \sum_{other} \mathbf{P}(P_{1,3}) P(explored) P(fringe) P(other)$$

$$= \alpha P(explored)\mathbf{P}(P_{1,3})\times$$
$$\times \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) P(fringe) \sum_{other} P(other)$$



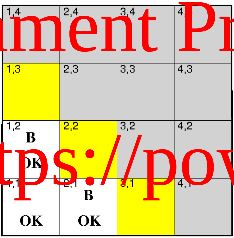
Reordering

and pushing sums inwards

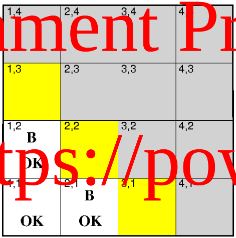
$$\alpha \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) \times$$
$$\times \sum_{other} \mathbf{P}(P_{1,3}) P(explored) P(fringe) P(other)$$

$$= \alpha\, P(explored) \mathbf{P}(P_{1,3}) \times$$
$$\times \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe) P(fringe) \sum_{other} P(other)$$

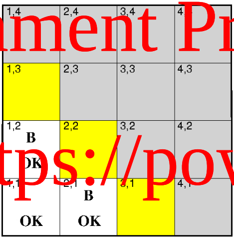
Using conditional independence



$$\alpha\, P(explored)\mathbf{P}(P_{1,3})\times$$
$$\times \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe)P(fringe) \sum_{other} P(other)$$



$$\alpha\, P(explored)\mathbf{P}(P_{1,3})\times$$
$$\times \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe)P(fringe)\sum_{other} P(other)$$
$$= \alpha'\, \mathbf{P}(P_{1,3})\sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe)P(fringe)$$

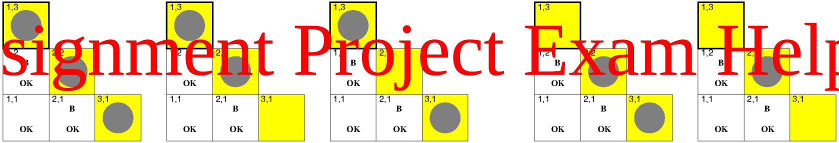


Simplifying

$$\alpha\, P(explored)\mathbf{P}(P_{1,3})\times$$
$$\times \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe)P(fringe) \sum_{other} P(other)$$

$$= \alpha'\, \mathbf{P}(P_{1,3}) \sum_{fringe} \mathbf{P}(b|explored, P_{1,3}, fringe)P(fringe)$$

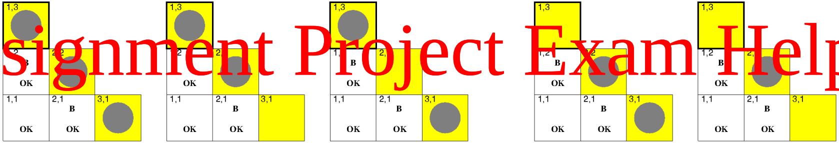
Using conditional independence contd.



0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16   0.8 x 0.2 = 0.16   0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16

$\mathbf{P}(b|explored, P_{1,3}, fringe)$

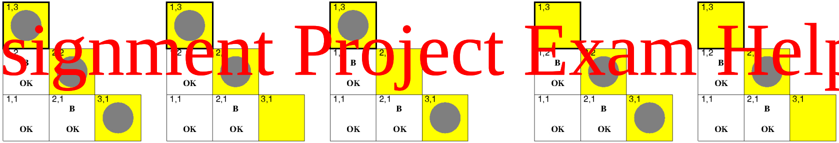
Using conditional independence contd.



0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16   0.8 x 0.2 = 0.16   0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16

$\mathbf{P}(b|explored, P_{1,3}, fringe)$

- $=1$ when the frontier is consistent with the observations
- $=0$ otherwise



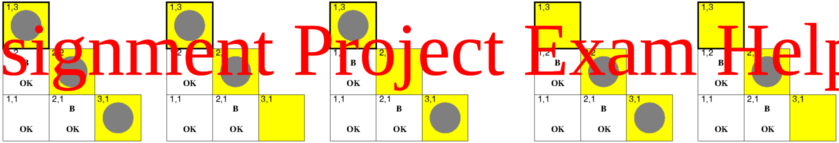
0.2 x 0.2 = 0.04　0.2 x 0.8 = 0.16　0.8 x 0.2 = 0.16　0.2 x 0.2 = 0.04　0.2 x 0.8 = 0.16

$\mathbf{P}(b|explored, P_{1,3}, fringe)$

- $= 1$ when the frontier is consistent with the observations
- $= 0$ otherwise

We can sum over the *possible configurations* for the frontier variables that are consistent with the known facts.

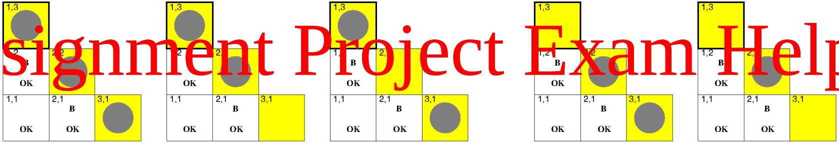
Using conditional independence contd.



0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16   0.8 x 0.2 = 0.16   0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16

$\mathbf{P}(P_{1,3}|explored, b) =$

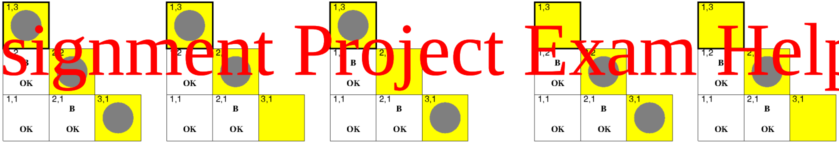
Using conditional independence contd.



0.2 x 0.2 = 0.04 | 0.2 x 0.8 = 0.16 | 0.8 x 0.2 = 0.16 | 0.2 x 0.2 = 0.04 | 0.2 x 0.8 = 0.16

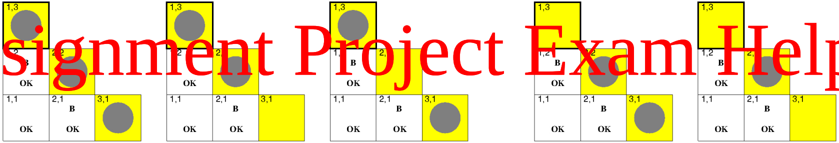
$$\mathbf{P}(P_{1,3}|explored, b) = \alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16) \rangle$$

Using conditional independence contd.



0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16   0.8 x 0.2 = 0.16   0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16

$$\mathbf{P}(P_{1,3}|explored, b) = \alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16)\rangle \approx \langle 0.31, 0.69\rangle$$

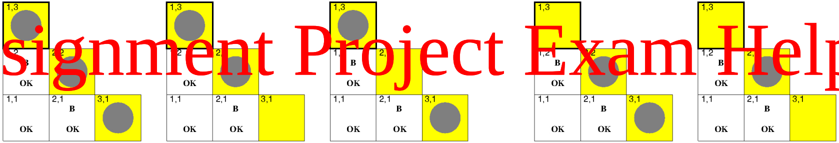


0.2 x 0.2 = 0.04 0.2 x 0.8 = 0.16 0.8 x 0.2 = 0.16 0.2 x 0.2 = 0.04 0.2 x 0.8 = 0.16

$\mathbf{P}(P_{1,3}|explored, b) =$
$\alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16)\rangle$
$\approx \langle 0.31, 0.69\rangle$

$\mathbf{P}(P_{2,2}|explored, b) \approx \langle 0.86, 0.14\rangle$

Using conditional independence contd.



0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16   0.8 x 0.2 = 0.16   0.2 x 0.2 = 0.04   0.2 x 0.8 = 0.16

$$\mathbf{P}(P_{1,3}|explored, b) = \alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16)\rangle \approx \langle 0.31, 0.69\rangle$$

$$\mathbf{P}(P_{2,2}|explored, b) \approx \langle 0.86, 0.14\rangle$$

What we have seen

- Probabilities and conditional probabilities
- Independence and conditional independence
- Estimating chances of possible outcomes

Coming next

- Combining chances and rewards
- Maximising the expected reward