Agent Based Systems

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The plan for today

We have seen a way of modelling knowledge using set theory, both in single and multi agent situations

Today and tomorrow we look at how to model this from a point of view of a reasoning agent

- Knowledge Representation:
  - how to encode the state of the world in a logical language
  - worlds, actions, goals
  - expressivity versus complexity
- We focus on one agent only! (MAS languages: similar pattern)

KR is one of the most active sub-fields of AI, producing many useful results in many areas (e.g., automated medical diagnosis)

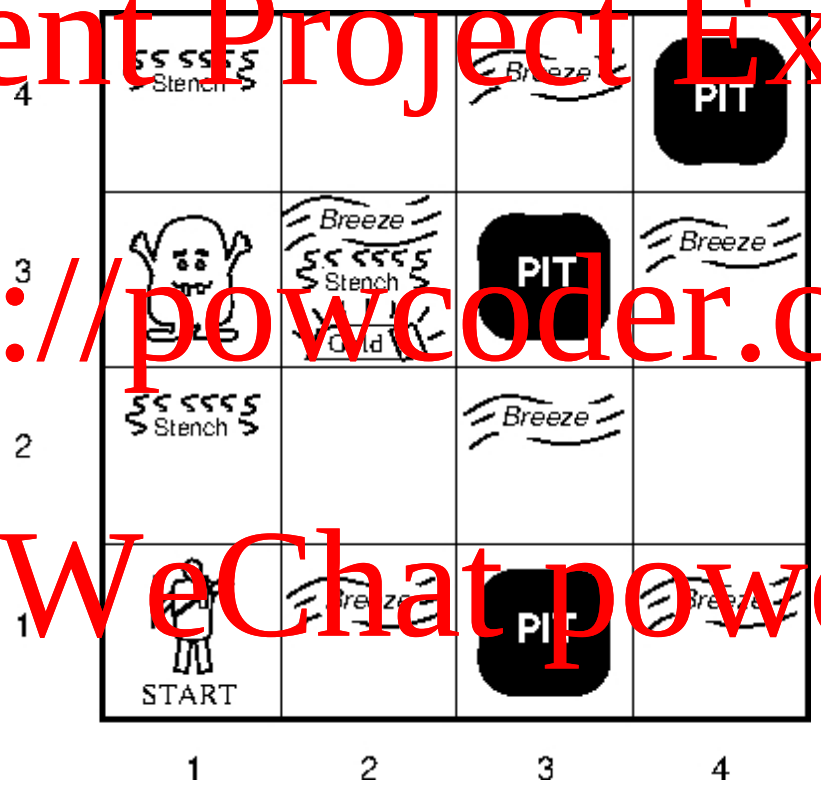
Knowledge Representation

An uncertain world

The main reference

- Stuart Russell and Peter Norvig
  Artificial Intelligence: a modern approach
  Chapters 7-9

The Wumpus World



The Wumpus World

Sensors Breeze, Glitter, Smell

Actuators Up, Down, Left, Right, Grab, Release, Shoot, Climb

Rewards 1000 escaping with gold, -1000 dying, -10 using arrow, -1 walking

Environment

- Squares adjacent to Wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter iff gold is in the same square
- Shooting kills Wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square

Knowledge base

- A set of sentences representing what the agent thinks about the world.
  - 'I am in [2,1]'
  - 'I am out of arrows'
  - 'I smell Wumpus'
  - 'I'd better not go forward'
- We interpret it as what the agent **knows**, but it works just fine for what the agent **believes**.

Updating the knowledge base

- What we TELL the knowledge base
- What we ASK the knowledge base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function KB-Agent( percept) returns an action
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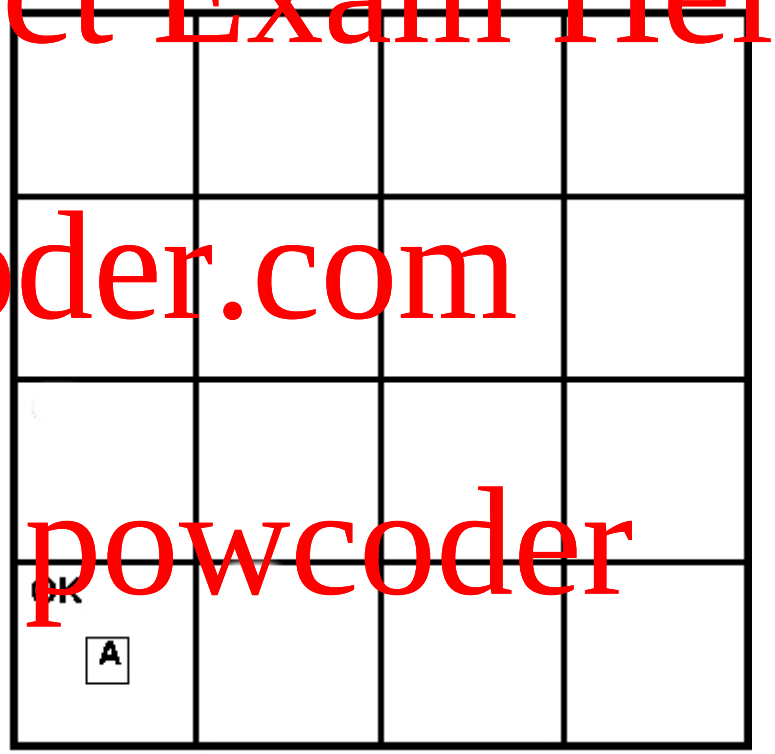
Rational explorations

- The starting state...

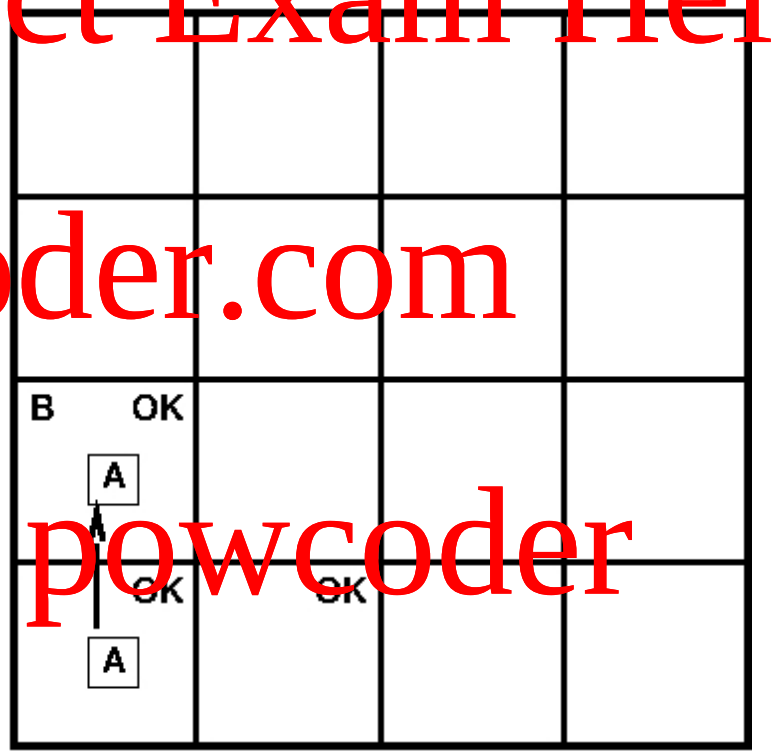


Rational explorations

- and what we know.

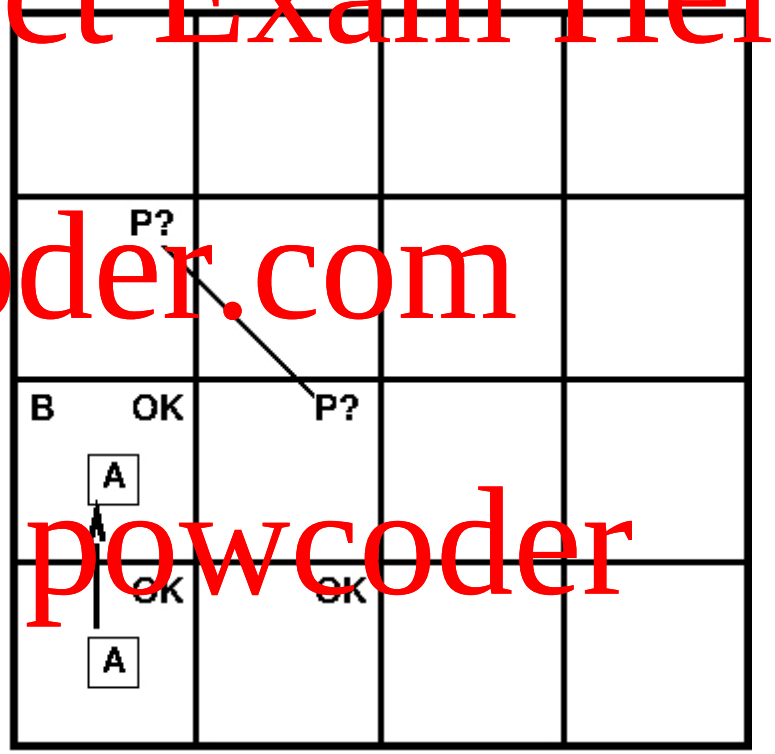
Rational explorations

- B stands for Breeze



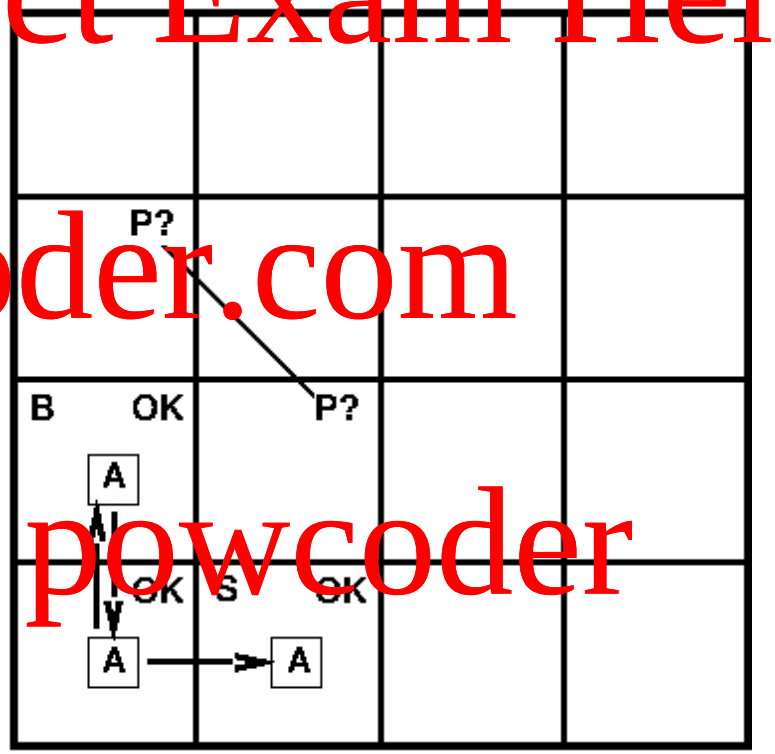
Rational explorations

- Where is the pit?
- We are ruling out one square!



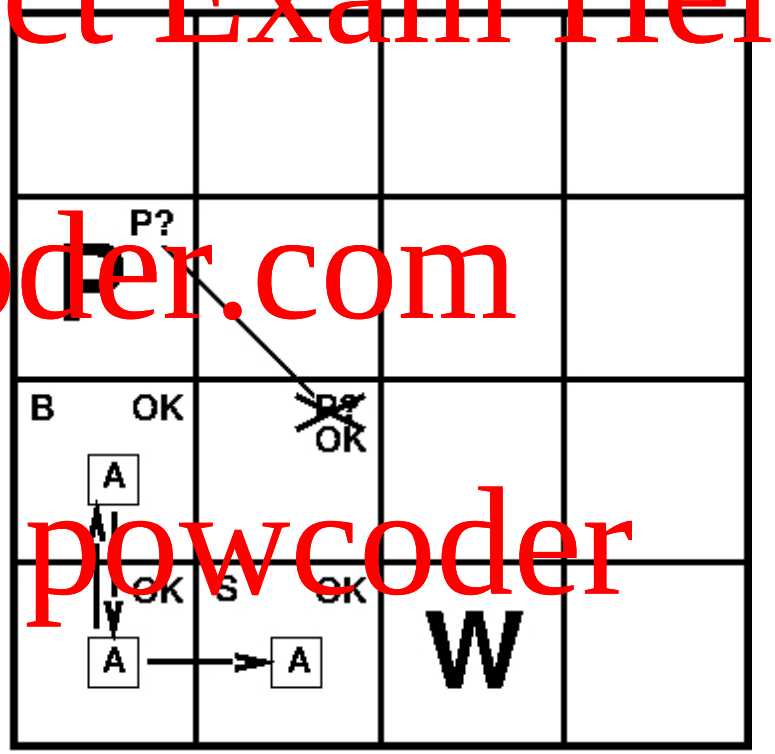
Rational explorations

- S stands for smell
- What do we know?



Rational explorations

- Logic is the key!



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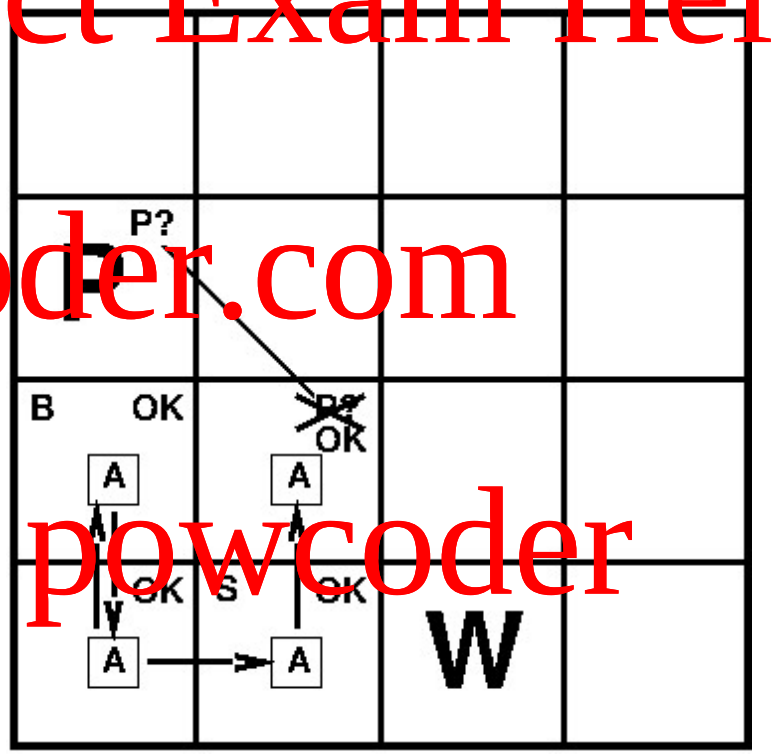
- The further we go the more we know



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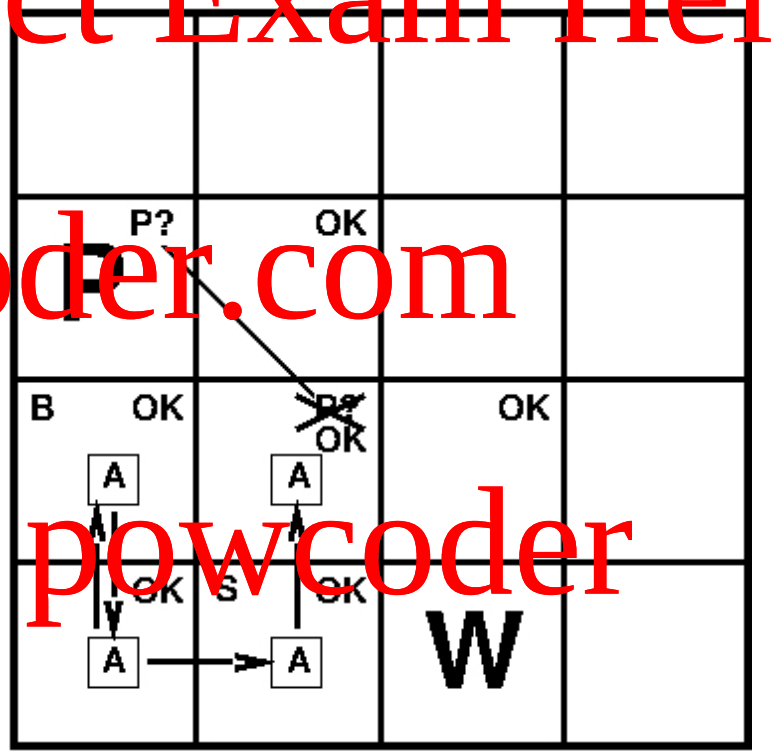
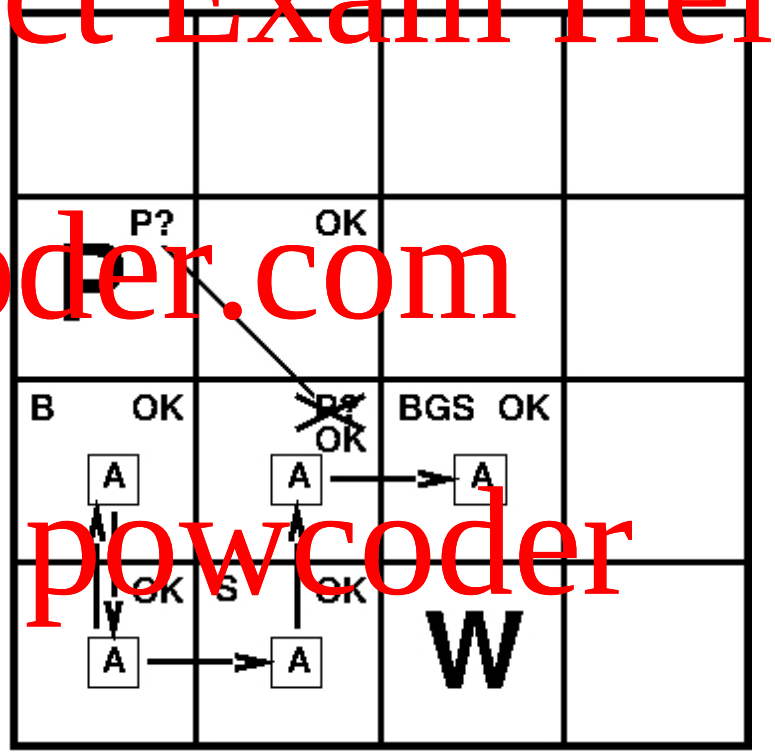


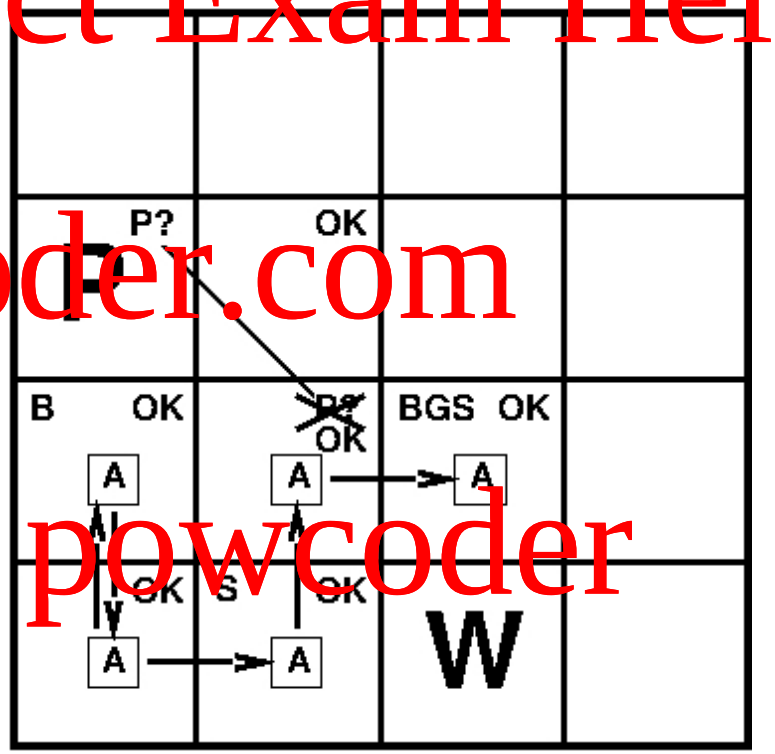
Rational explorations

- Gold!



Rational explorations

- We know the way out
- Game over



Representing the Wumpus World

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

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"Pits cause breezes in adjacent squares"

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"Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

"A square is breezy **if and only if** there is an adjacent pit"

Expressivity: at what cost?

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Propositional Logic lacks expressive power

- Massive increase of expressivity: we can use existential $\exists$ and universal $\forall$ quantifiers before sentences!
- But there are costs, e.g., decidability: we don't always have a way to establish whether a sentences is true or false.
- We will see how to exploit the gains while limiting the costs

Recall: Universal Instantiation

Every instantiation of a universally quantified sentence is entailed by it:

$$\frac{\forall v\ \alpha}{\alpha(\{v/g\})}$$

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$$King(Joffrey) \wedge Greedy(Joffrey) \Rightarrow Evil(Joffrey)$$
$$King(Aerys) \wedge Greedy(Aerys) \Rightarrow Evil(Aerys)$$
$$King(Father(Joffrey)) \wedge Greedy(Father(Joffrey)) \Rightarrow Evil(Father(Joffrey))$$
$$\vdots$$

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For any sentence α, variable v, and constant symbol k **that does not appear elsewhere in the knowledge base**:

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provided C_1 is a **new** constant symbol, called a Skolem constant

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EI can be applied once to **replace** the existential sentence;

the new KB is **not** equivalent to the old,

but is satisfiable iff the old KB was satisfiable

KB with FOL

- We can encode the KB at each particular time point using FOL


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  - Axioms from knowledge to knowledge, e.g., $\forall\, t \;\; AtGold(t) \wedge Action(Grab, t) \;\Rightarrow\; Holding(Gold, t+1)$

Perception $\forall s, b, t \;\; Percept([s, b, Glitter], t) \Rightarrow AtGold(t)$

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Decision-making $\forall t \;\; AtGold(t) \Rightarrow Action(Grab, t)$

Internal reflection $\forall t \;\; AtGold(t) \wedge \neg Holding(Gold, t) \Rightarrow Action(Grab, t)$,
do we have gold already? (notice we cannot observe if we are holding gold: we need to track it)

Representing the Wumpus World

Adjacent squares

$$\forall x, y, a, b \;\; Adjacent([x, y], [a, b]) \;\Leftrightarrow$$
$$(x = a \wedge (y = b - 1 \vee y = b + 1)) \vee (y = b \wedge (x = a - 1 \vee x = a + 1))$$

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"A square is breezy **if and only if** there is an adjacent pit"

$$\forall s \;\; , Breezy(s) \;\; \Leftrightarrow \;\; \exists r \, (Adjacent(r, s) \wedge Pit(r))$$

Representing the Wumpus World

- We can go on and describe plans, causal rules, etc.
- But let's do some reasoning now

'Joffrey Baratheon is a king'

'Jon Snow is a person'

Facts and knowledge bases

'Jon Snow is a king'

$Tell(KB, King(Joffrey))$

Telling and Asking

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Askvar returns a list of **substitutions**: $\{x/Joffrey\}, \{x/Jon\}$

Substitutions

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Given a sentence S and a substitution σ,

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$Askvar(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Unification

$$\forall x \;\; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$$

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$\theta = \{x/Joffrey, y/Joffrey\}$ works

UNIFY(α, β) returns θ if $\alpha\theta = \beta\theta$

Unification

$\textsc{Unify}(\alpha, \beta)$ returns θ if $\alpha\theta = \beta\theta$

p	q	θ
$Knows(Joffrey, x)$	$Knows(Joffrey, Sansa)$	
$Knows(Joffrey, x)$	$Knows(y, Sansa)$	
$Knows(Joffrey, x)$	$Knows(y, Mother(Joffrey))$	
$Knows(Jon, x)$	$Knows(x, Mother(Jon))$	

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$Knows(Joffrey, x)$	$Knows(y, Mother(Joffrey))$	$\{y/Joffrey, x/Mother(Joffrey)\}$
$Knows(Jon, x)$	$Knows(x, Mother(Jon))$	$fail$

Standardising apart

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Standardising apart eliminates overlap of variables, e.g.,

$Knows(z_{17}, Mother(Jon))$

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Definite clause:

disjunction of literals, **exactly** one of which positive

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$$\frac{p_1',\ p_2',\ \ldots,\ p_n',\ (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta}$$ where $p_i'\theta = p_i\theta$ for all i

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Assuming all variables are universally quantified...

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Assuming all variables are universally quantified...

p_1' is $King(Joffrey)$	p_1 is $King(x)$
p_2' is $Greedy(y)$	p_2 is $Greedy(x)$
θ is $\{x/Joffrey, y/Joffrey\}$	q is $Evil(x)$
$q\theta$ is $Evil(Joffrey)$	

Soundness of GMP

Need to show that

$$p_1', \ldots, p_n', \quad (p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models q\theta$$

provided that $p_i'\theta = p_i\theta$ for all i

Lemma: If φ is definite clause, then $\varphi \models \varphi\theta$ by Universal Instantiation.

1. $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models (p_1 \wedge \ldots \wedge p_n \Rightarrow q)\theta = (p_1\theta \wedge \ldots \wedge p_n\theta \Rightarrow q\theta)$
2. $p_1', \ldots, p_n \models p_1' \wedge \ldots \wedge p_n \models p_1'\theta \wedge \ldots \wedge p_n'\theta$
3. From 1 and 2, $q\theta$ follows by ordinary Modus Ponens

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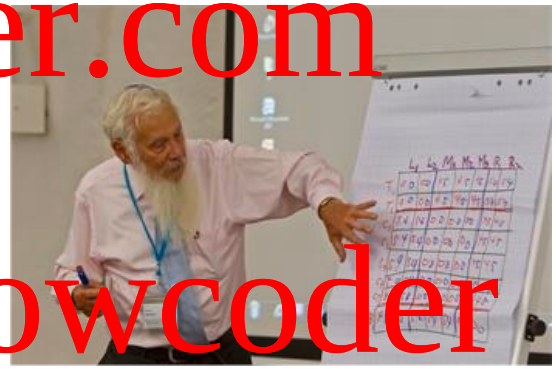

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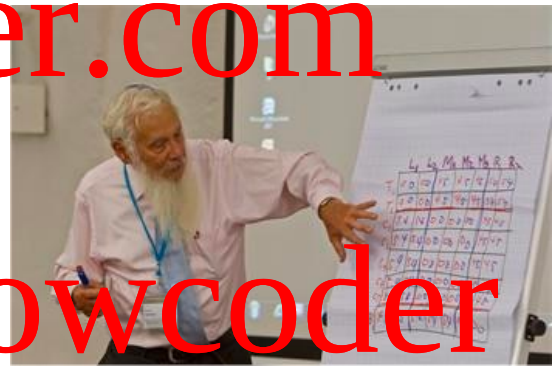

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