



Assignment Project Exams Help

<https://powcoder.com>

Add WeChat powcoder

Paolo Turrini

🏠 www.dcs.warwick.ac.uk/~pturrini ✉ p.turrini@warwick.ac.uk

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder

The plan for today


Today we are going to define the notion of strategy, in a very basic form of extensive game.

- Actions, players, histories
- Strategies and winning conditions (in chess and beyond)

Then we are going to move to an abstract treatment of group strategies, which will be the prelude for the second part of the module instead, that dealing with cooperation.

We start without numbers, we are going to add them later on.

If you are interested in further results, Chapter 1 of:

 M. Maschler, E. Solan and S. Zamir
Game Theory.

What is a strategy?

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder

The blue circle is a lake. The black point is you, on a boat. The red point is a brute, who wants to catch you. The brute can't swim but moves quicker than you: if you travel a radius, the brute travels half a circumference. On land, you are quicker.

How do you escape?¹

¹Credit for this example goes to Chess GM Mihai Suba. □ ▶ ◀ ◻ ▶ ◀ ≡ ▶ ◀ ≡ ▶ ≡ ↺ ↻

What is a strategy?

Assignment Project Exam Help

"I will not spoil your own pleasure in discovering the solution. I want to emphasise that here you are not supposed to find a move but a strategy, that is a succession of moves, each one depending on the sum total of your own and your opponent's previous moves, and leading to a clear result."

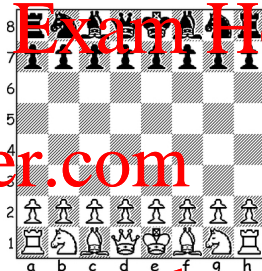
Mihai Suba,
Dynamic Chess Strategy, 1991



Mihai Suba

Assignment Project Exam Help

- Two-player
- Turn-based
- Pieces move according to specific rules
- Game ends when a player captures the opponent's king²



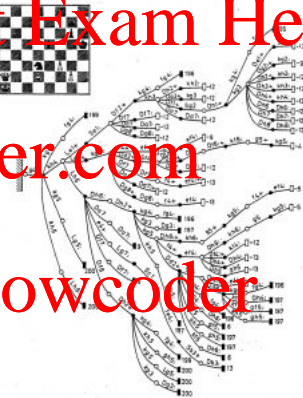
<https://powcoder.com>

Add WeChat powcoder

²This is pretty much the same as using checkmate as terminating condition. My educated guess (I haven't checked) is that this is a more ancient variant of the game, from which the modern version was born.

Assignment Project Exam Help

- The rules of the game (in particular captures and three-fold repetition) force the game to be finite
- Mathematically, it is a representable as a finite tree, where players (White and Black) take turns to move



Assignment Project Exam Help

- Our set of states is the set of all possible board positions. The starting state is the standard starting board position.
- Notice that a board position might be reached in different ways. We call these ways **histories**.

Question: is the set of histories finite?

Add WeChat powcoder

Assignment Project Exam Help

Definition (Chess Histories)

A **history** is a sequence (x_0, x_1, \dots, x_K) such that

- x_0 is the opening board position
- For each even integer k with $0 \leq k < K$, going from position x_k to position x_{k+1} can be accomplished by a single legal move by White.
- For each odd integer k with $1 \leq k < K$, going from position x_k to position x_{k+1} can be accomplished by a single legal move by Black.

Assignment Project Exam Help

Suppose we want to construct a computer program to play chess.

This program will have to take a decision at each history where its assigned role (White or Black) can move.

We call this decision a **strategy**.

Add WeChat powcoder

Assignment Project Exam Help

Definition (Strategies)

A **strategy for White** is a function σ_W that associates to each history (x_0, x_1, \dots, x_K) , with K even, a board position x_{K+1} reachable by White with a legal move.

A **strategy for Black** is a function σ_B that associates to each history (x_0, x_1, \dots, x_K) , with K odd, a board position x_{K+1} reachable by Black with a legal move.

A **play of the game** is a pair of strategies (σ_W, σ_B) .

Winning strategies

A play of the game ends either in:

- a victory for White
- a victory for Black
- a draw

Definition (Winning strategies)

A strategy for White is called **winning** if, no matter the strategy chosen by Black, it guarantees a win for White.

Formally:

σ_W is **winning for White** if and only if $\forall \sigma'_B, (\sigma_W, \sigma'_B)$ wins for White.

For Black, the definition of winning strategy is the same
(but with reversed names!)

Assignment Project Exam Help

Definition (At-least-drawing strategies)

A strategy for White is called **at-least-drawing** if, no matter the strategy chosen by Black, it guarantees a win or a draw for White.

Formally:

σ_W is **at-least-drawing for White** if and only if

$\forall \sigma'_B, (\sigma_W, \sigma'_B)$ wins for White or draws.

For Black, the definition of at-least-drawing strategy is the same (but with reversed names!)

<https://powcoder.com>

Add WeChat powcoder

Assignment Project Exam Help

Theorem (Zermelo (1913), von Neumann (1928))

In Chess, one of the following must be true:

- *White has a winning strategy*
- *Black has a winning strategy*
- *Both players have an at-least-drawing strategy*

https://powcoder.com

Add WeChat powcoder

Over 100 years and we still don't know which one of them is true!

Assignment Project Exam Help

Proof.

Let us first recall that the game is finite, i.e., there is a natural number K such that every play of the game concludes after at most $2K$ rounds, (K turns by White, K by Black).

<https://powcoder.com>

Add WeChat powcoder

Assignment Project Exam Help

Proof.

Let us first recall that the game is finite, i.e., there is a natural number K such that every play of the game concludes after at most $2K$ rounds, (K turns by White, K by Black).

Assume there are exactly $2K$ turns in every play of the game. Notice that if some plays are shorter, we can simply continue them by adding a “do nothing” move, preserving the result (so, for instance if we extend a play where White wins, we keep track of this).

Assignment Project Exam Help

For every k with $1 \leq k \leq K$ denote:

<https://powcoder.com>

Add WeChat powcoder

Assignment Project Exam Help

For every k with $1 \leq k \leq K$ denote:

- a_k the move implemented by White at their turn.
- b_k the move implemented by Black at their turn.

<https://powcoder.com>

Add WeChat powcoder

Assignment Project Exam Help

Proof.

For every k with $1 \leq k \leq K$ denote:

- a_k the move implemented by White at their turn.
- b_k the move implemented by Black at their turn.

Denote W the fact that White wins (after $2K$ turns), $\neg W$ the fact that White does not.

Add WeChat powcoder

Assignment Project Exam Help

For every k with $1 \leq k \leq K$ denote:

- a_k the move implemented by White at their turn.
- b_k the move implemented by Black at their turn.

Denote W the fact that White wins (after $2K$ turns), $\neg W$ the fact that White does not.

But then, the fact that White has a winning strategy can be written as:

$$\exists a_1 \forall b_1 \exists a_2 \forall b_2 \dots \exists a_K \forall b_K (W)$$

Proof.

So, the fact that White has not a winning strategy can be written as:

$$\neg \exists a_1 \forall b_1 \exists a_2 \forall b_2 \dots \exists a_K \forall b_K (W)$$

<https://powcoder.com>

Add WeChat powcoder

Proof.

So, the fact that White has not a winning strategy can be written as:

$$\neg \exists a_1 \forall b_1 \exists a_2 \forall b_2 \dots \exists a_K \forall b_K (W)$$

This, using first order logic, is equivalent to:

$$\forall a_1 \exists b_1 \forall a_2 \exists b_2 \dots \forall a_K \exists b_K (\neg W)$$

Add WeChat powcoder

Proof (3/3)

Proof.

So, the fact that White has not a winning strategy can be written as:

$$\neg \exists a_1 \forall b_1 \exists a_2 \forall b_2 \dots \exists a_K \forall b_K (W)$$

This, using first order logic, is equivalent to:

$$\forall a_1 \exists b_1 \forall a_2 \exists b_2 \dots \forall a_K \exists b_K (\neg W)$$

But this says that Black is guaranteed at least a draw!

Proof (3/3)

Proof.

So, the fact that White has not a winning strategy can be written as:

$$\neg \exists a_1 \forall b_1 \exists a_2 \forall b_2 \dots \exists a_K \forall b_K (W)$$

This, using first order logic, is equivalent to:

$$\forall a_1 \exists b_1 \forall a_2 \exists b_2 \dots \forall a_K \exists b_K (\neg W)$$

But this says that Black is guaranteed at least a draw!

We can do exactly the same for Black.

Proof.

So, the fact that White has not a winning strategy can be written as:

$$\neg \exists a_1 \forall b_1 \exists a_2 \forall b_2 \dots \exists a_K \forall b_K (W)$$

This, using first order logic, is equivalent to:

$$\forall a_1 \exists b_1 \forall a_2 \exists b_2 \dots \forall a_K \exists b_K (\neg W)$$

But this says that Black is guaranteed at least a draw!

We can do exactly the same for Black.

Therefore, one of the three alternatives must hold.



Assignment Project Exam Help

Solving a game = finding the **objectively** best continuation from the start.

Solving a position = solving the game starting from there

<https://powcoder.com>

AI is very good at some games. But is it perfect?

Add WeChat powcoder

Solving a game vs beating the World Champion

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder



Chess is **not** a solved game. Even if DeepBlue has beaten Kasparov.

Solving a game vs beating the World Champion

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder



Go is **not** a solved game. Even if AlphaGo has beaten Lee Sedol.

Solving a game vs beating the World Champion

Assignment Project Exam Help

Computers Solve Checkers—It's a Draw

King me! Top computer scientist proves perfect play leads to draw, recounts battle for world championship goes tangled

<https://powcoder.com>

By JR Minkel on July 19, 2007



READ THIS NEXT

Play checkers against the computer

Checkers **is** a solved game. It's a draw.

Assignment Project Exam Help

Chess is not solved, but some chess endgames are.

Chance it gets solved any soon?

<https://powcoder.com>

<https://arxiv.org/abs/1712.01815>

Add WeChat powcoder

Assignment Project Exam Help

In Chess, we might have a strategy to win or to draw.

But, if we don't know it, there is not so much we can do...

We are going to be talking about knowledge

(e.g., knowing that you can win vs. knowing how you can win)

Add WeChat powcoder