This time: constraint satisfaction

- Constraint Satisfaction Problems (CSP)
- Backtracking search for CSPs
- Local search for CSPs

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Constraint satisfaction problems

Standard search problem:

- state is a “black box” – any data structure that supports successor function, heuristic function, and goal test

Assignment Project Exam Help

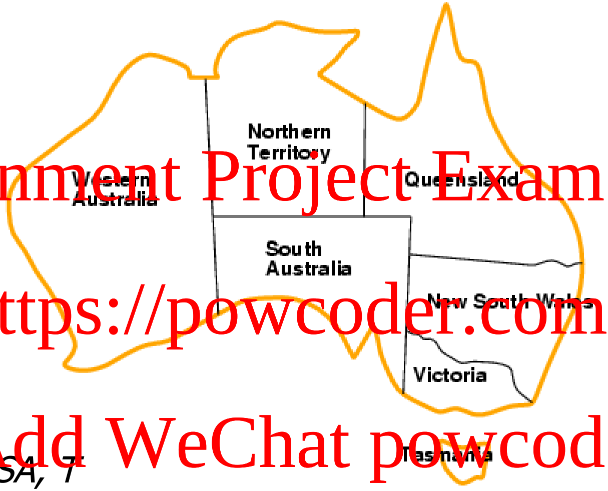
https://powcoder.com

Add WeChat powcoder

CSP:

- state is defined by variables Xi with values from domains Di
- goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Simple example of a formal representation language
- Allows useful general-purpose algorithms with more power than standard search algorithms

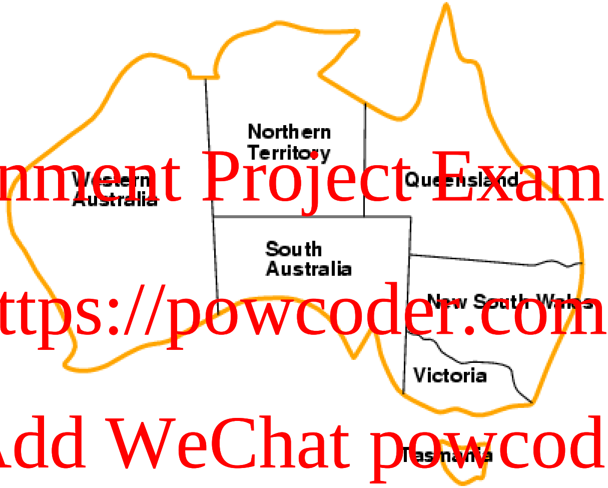
Example: map coloring problem



Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

- Variables: *WA, NT, Q, NSW, V, SA, T*
- Domains: D_i = {red, green, blue} (one for each variable)
- Constraints: C_i = <*scope*, *rel* > where *scope* is a tuple of variables and *rel* is the relation over the values of these variables
- E.g., here, adjacent regions must have different colors
  - e.g., WA ≠ NT, or (WA,NT) in {(red,green), (red,blue), (green,red), (green,blue), (blue,red), (blue,green)}

Example: map coloring problem



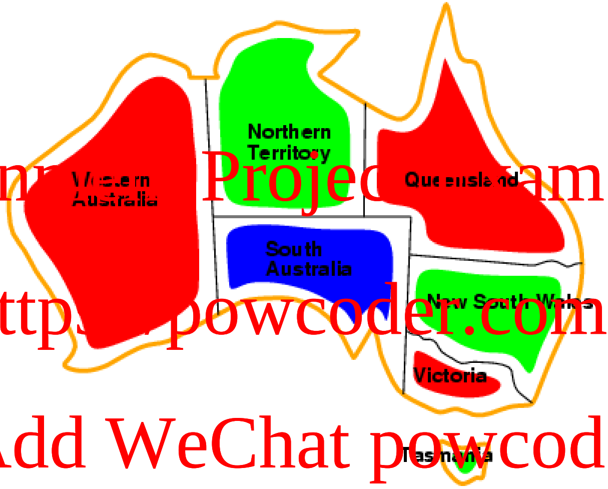
Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

- Assignment: values are given to some or all variables
- Consistent (legal) assignment: assigned values do not violate any constraint
- Complete assignment: every variable is assigned
- Solution to a CSP: a consistent and complete assignment

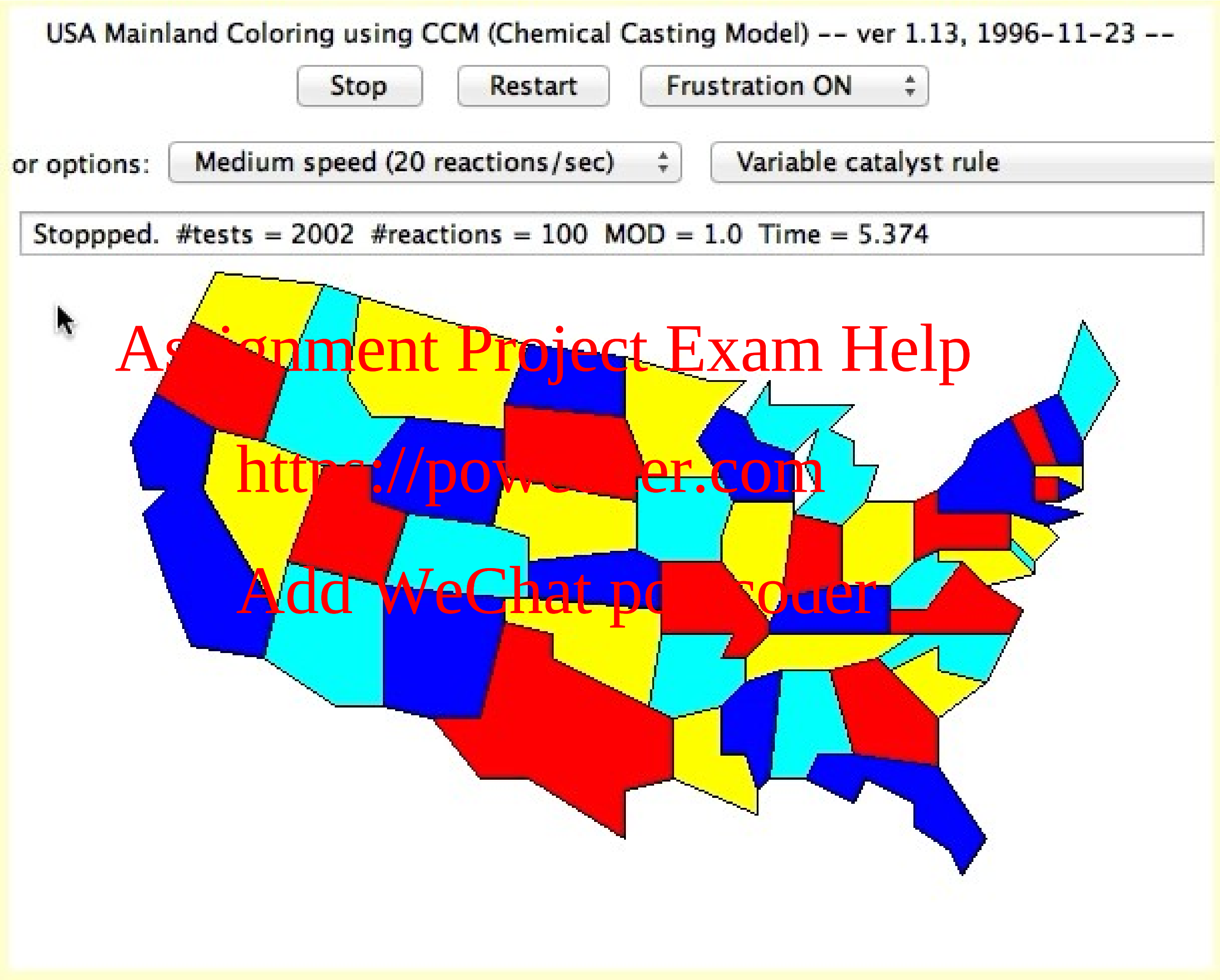
Example: map coloring problem



Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

- Solutions are complete and consistent assignments,
- e.g., WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green

Demo



USA Mainland Coloring using CCM (Chemical Casting Model) -- ver 1.13, 1996-11-23 --

Frustration ON

or options: Medium speed (20 reactions/sec)   Variable catalyst rule

Stoppped. #tests = 2002 #reactions = 100 MOD = 1.0 Time = 5.374

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

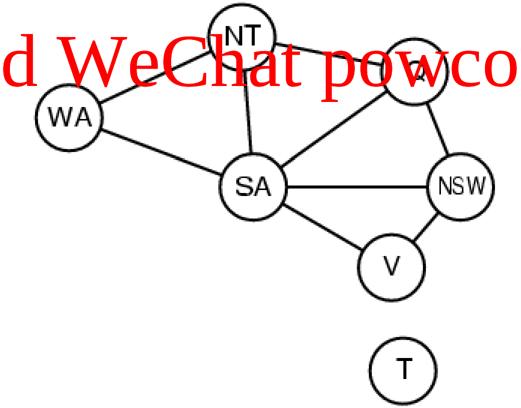
Constraint graph

- Binary CSP: each constraint relates two variables
- Constraint graph: nodes are variables, arcs are constraints

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder



Varieties of CSPs

- Discrete variables
  - finite domains:
    - *n* variables, domain size *d* → $O(d^n)$ complete assignments
    - e.g., Boolean CSPs, incl. Boolean satisfiability (NP-complete)
  - infinite domains:
    - integers, strings, etc.
    - e.g., job scheduling, variables are start/end days for each job
    - need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$
- Continuous variables
  - e.g., start/end times for Hubble Space Telescope observations
  - linear constraints solvable in polynomial time by linear programming

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Varieties of constraints

- Unary constraints involve a single variable,
  - e.g., SA ≠ green

Assignment Project Exam Help

- Binary constraints involve pairs of variables,
  - e.g., SA ≠ WA

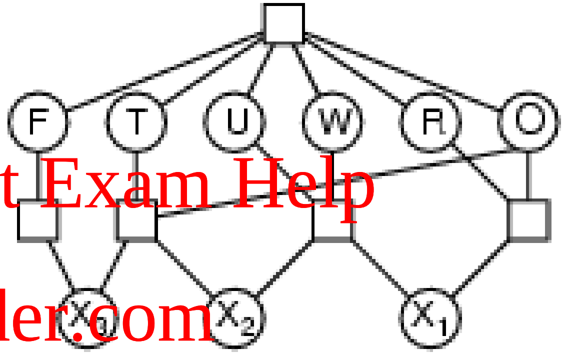
https://powcoder.com

Add WeChat powcoder

- Higher-order (sometimes called global) constraints involve 3 or more variables,
  - e.g., cryptarithmetic column constraints

Example: cryptarithmetic

```
  T W O
+ T W O
F O U R
```



Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

- Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$
- Domains: $\{0,1,2,3,4,5,6,7,8,9\}$
- Constraints:
  - *Alldiff (F,T,U,W,R,O)*
  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F,\ T \neq 0,\ F \neq 0$

Constraint hypergraph
Circles: nodes for variable
Squares: hypernodes for n-ary constraints

Real-world CSPs

- Assignment problems
  - e.g., who teaches what class

Assignment Project Exam Help

- Timetabling problems
  - e.g., which class is offered when and where?

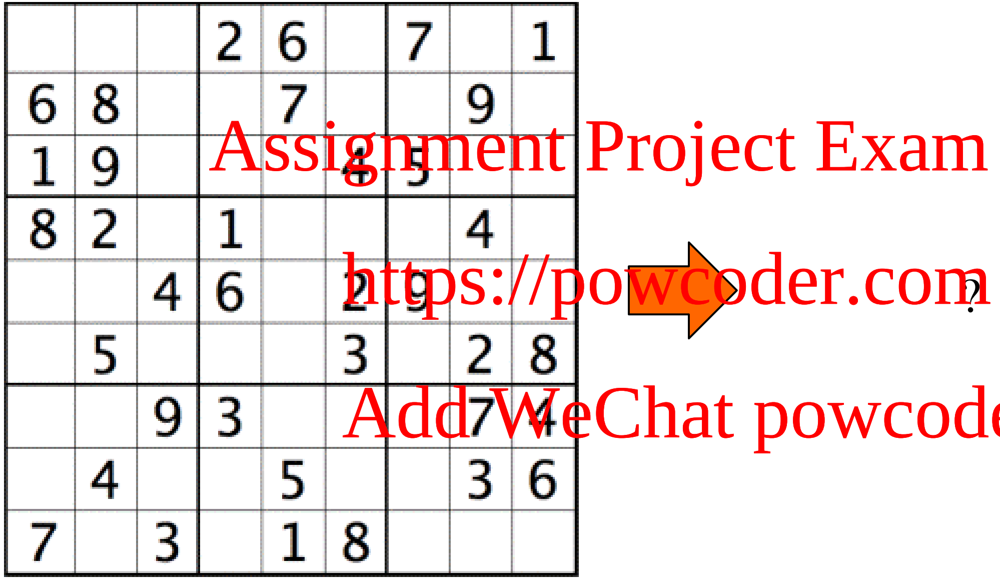
https://powcoder.com

- Transportation scheduling

Add WeChat powcoder

- Factory scheduling
- Notice that many real-world problems involve real-valued variables

Example: sudoku



			2	6		7		1
6	8			7			9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Variables: each square (81 variables)
Domains: [1 .. 9]
Constraints: each column, each row, and each of the nine 3×3 sub-grids that compose the grid contain all of the digits from 1 to 9

Example: sudoku

			2	6		7		1
6	8			7			9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

4	3	5	2	6	9	7	8	1
6	8	2	5	7	1	4	9	3
1	9	7	8	3	4	5	6	2
8	2	6	1	9	5	3	4	7
3	7	4	6	8	2	9	1	5
9	5	1	7	4	3	6	2	8
5	1	9	3	2	6	8	7	4
2	4	8	9	5	7	1	3	6
7	6	3	4	1	8	2	5	9

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Variables: each square (81 variables)
Domains: [1 .. 9]
Constraints: each column, each row, and each of the nine 3×3 sub-grids that compose the grid contain all of the digits from 1 to 9

Formulation as a search problem

Let's start with the straightforward approach, then fix it

States are defined by the values assigned so far

Assignment Project Exam Help

- Initial state: the empty assignment { }
- Successor function: assign a value to an unassigned variable that does not conflict with current assignment
- → fail if no legal assignments

https://powcoder.com

- Goal test: the current assignment is complete

Add WeChat powcoder

1. This is the same for all CSPs
2. Every solution appears at depth *n* with *n* variables → use depth-first search
3. Path is irrelevant, so can be discarded
4. b = (n - *l*)d at depth l, hence n! · d^n leaves

Backtracking search

- Variable assignments are commutative, i.e.,
- [WA = red then NT = green] same as [NT = green then WA = red]

Assignment Project Exam Help

- Only need to consider assignments to a single variable at each node
- → b = d and there are d^n leaves

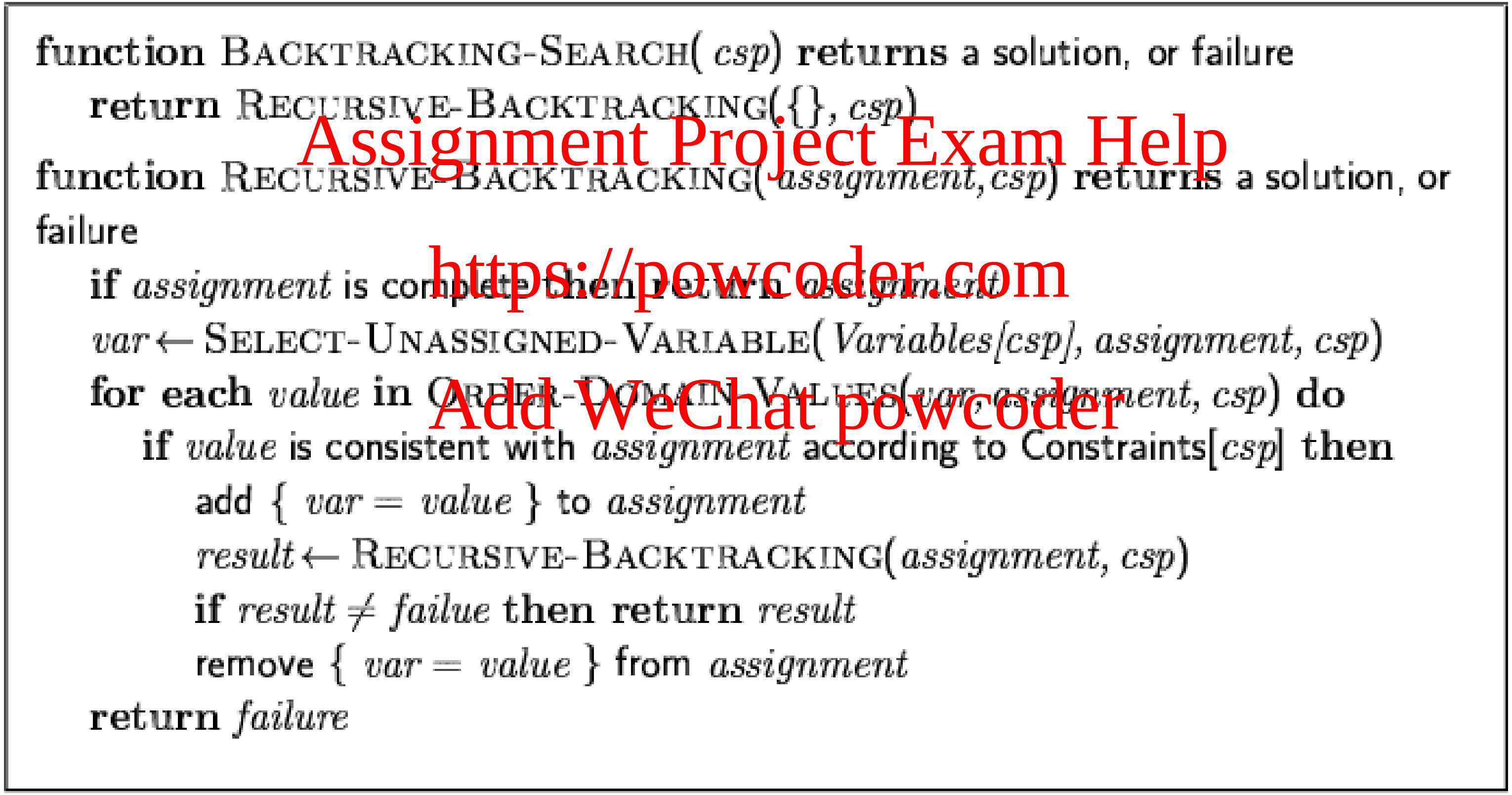
https://powcoder.com

- Depth-first search for CSPs with single-variable assignments is called backtracking search

Add WeChat powcoder

- Backtracking search is the basic uninformed algorithm for CSPs
- Can solve *n*-queens for *n* ≈ 25

Backtracking search (note: textbook has a slightly more complex version)



```
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
    return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns a solution, or failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(Variables[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment according to Constraints[csp] then
            add { var = value } to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failue then return result
            remove { var = value } from assignment
    return failure
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

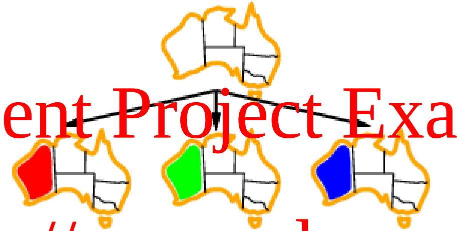
Backtracking example

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Backtracking example

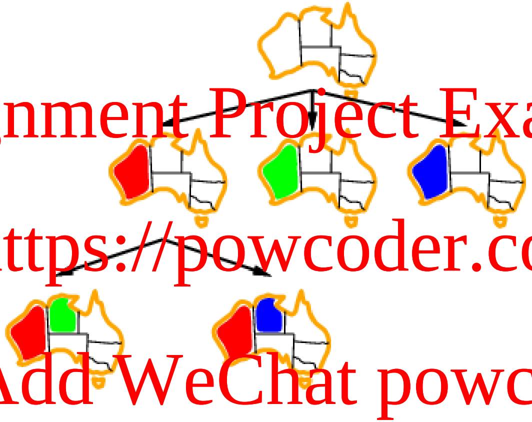


Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Backtracking example

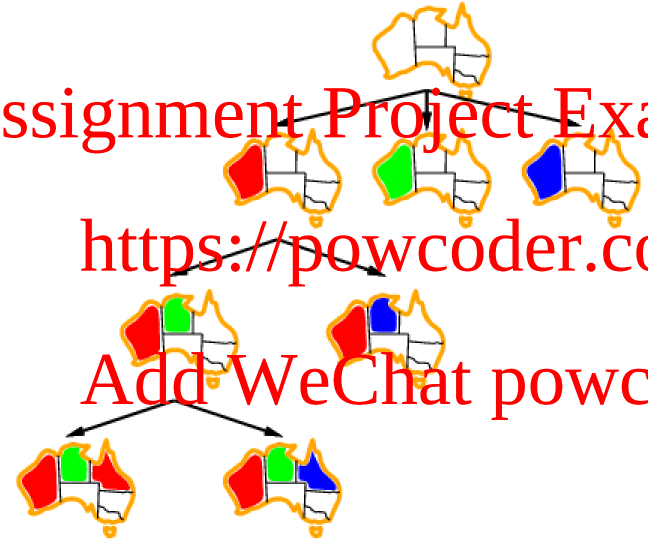


Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Backtracking example



Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Improving backtracking efficiency

- General-purpose methods can give huge gains in speed (like using heuristics in informed search):
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?

Assignment Project Exam Help

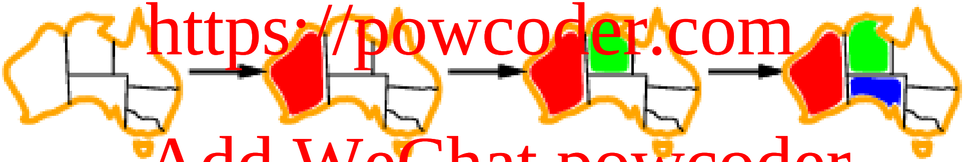
https://powcoder.com

Add WeChat powcoder

Most constrained variable

- Most constrained variable:
- choose the variable with the fewest legal values

Assignment Project Exam Help



https://powcoder.com

Add WeChat powcoder

- a.k.a. minimum remaining values (MRV) heuristic

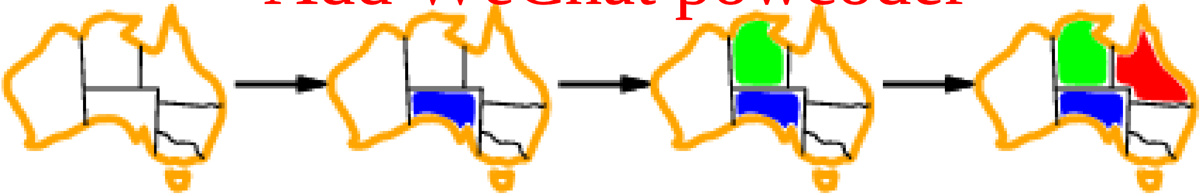
Most constraining variable

- Tie-breaker among most constrained variables
- Most constraining variable:
  - choose the variable with the most constraints on remaining variables

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder



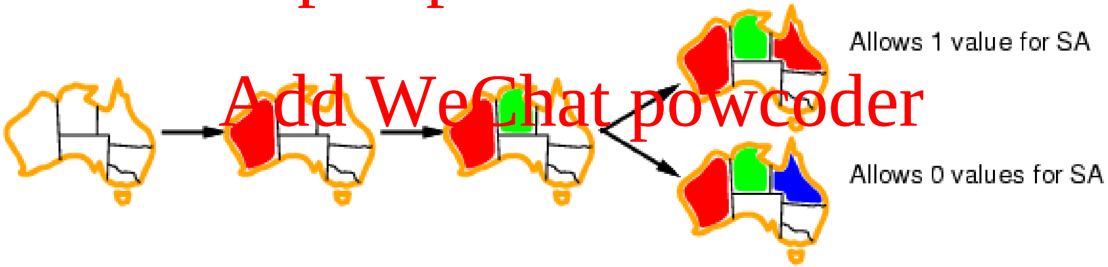
Least constraining value

- Given a variable, choose the least constraining value:
  - the one that rules out the fewest values in the remaining variables

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder



- Combining these heuristics makes 1000 queens feasible

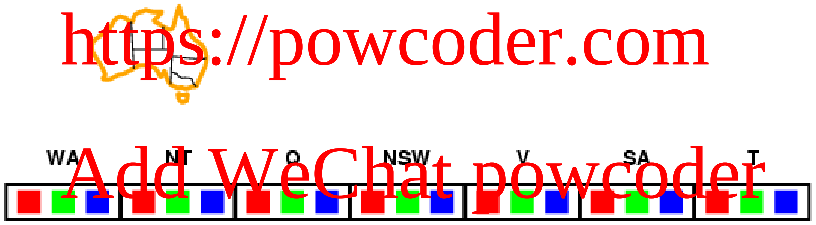
Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables (inference step)
  - Terminate search when any variable has no legal values

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder



Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables (inference step)
  - Terminate search when any variable has no legal values

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

WA	NT	Q	NSW	V	SA	T
🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦
🟥	🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟩🟦	🟥🟩🟦

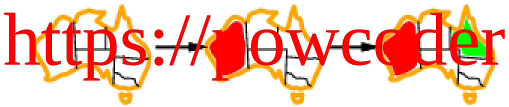
Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder



WA	NT	Q	NSW	V	SA	T
🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦
🟥	🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟩🟦	🟥🟩🟦
🟥	🟦	🟩	🟥🟦	🟥🟩🟦	🟦	🟥🟩🟦

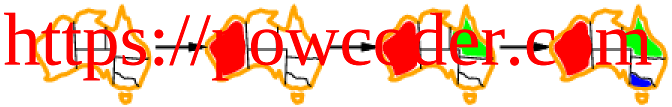
Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

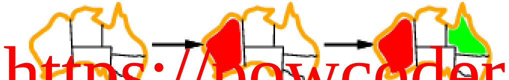


WA	NT	Q	NSW	V	SA	T
🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦
🟥	🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟩🟦	🟥🟩🟦
🟥	🟦	🟩	🟥🟦	🟥🟩🟦	🟦	🟥🟩🟦
🟥	🟦	🟩	🟥	🟦		🟥🟩🟦

Constraint propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

Assignment Project Exam Help



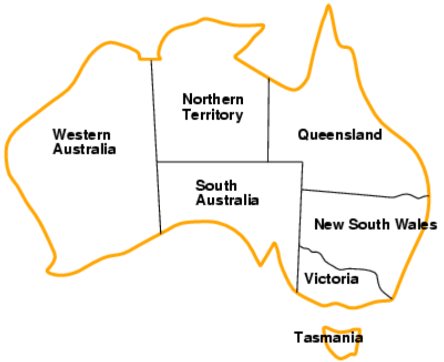
https://powcoder.com

WA	NT	Q	NSW	V	SA	T
🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦
🟥	🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟩 🟦	🟥 🟩 🟦
🟥	🟦	🟩	🟥 🟦	🟥 🟩 🟦	🟦	🟥 🟩 🟦

Add WeChat powcoder

- NT and SA cannot both be blue!

- Constraint propagation repeatedly enforces constraints locally



Node and Arc consistency

A single variable is node-consistent if all the values in its domain satisfy the variable’s unary constraints

Assignment Project Exam Help

A variable is arc-consistent if every value in its domain satisfies the binary constraints

https://powcoder.com

- i.e., Xi arc-consistent with Xj if for every value in Di there exists a value in Dj that satisfies the binary constraints on arc (Xi, Xj)

Add WeChat powcoder

A network is arc-consistent if every variable is arc-consistent with every other variable.

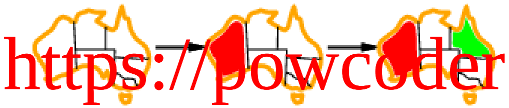
Arc-consistency algorithms: reduce domains of some variables to achieve network arc-consistency.

Arc consistency

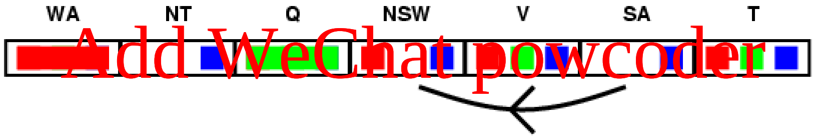
- Simplest form of propagation makes each arc consistent
- *X* → *Y* is consistent iff

Assignment Project Exam Help

for every value *x* of *X* there is some allowed *y*



https://powcoder.com



Add WeChat powcoder

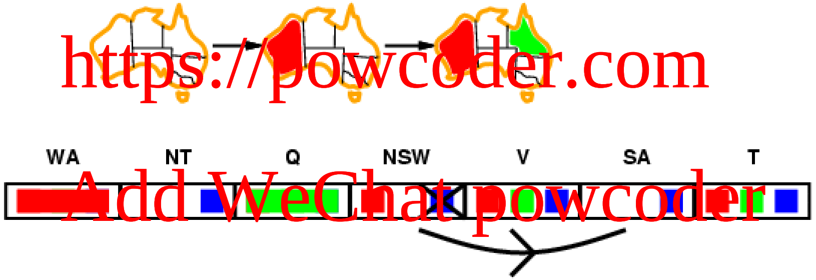
Arc consistency

- Simplest form of propagation makes each arc consistent
- *X* → *Y* is consistent iff

Assignment Project Exam Help

for every value *x* of *X* there is some allowed *y*

https://powcoder.com



Add WeChat powcoder

Arc consistency

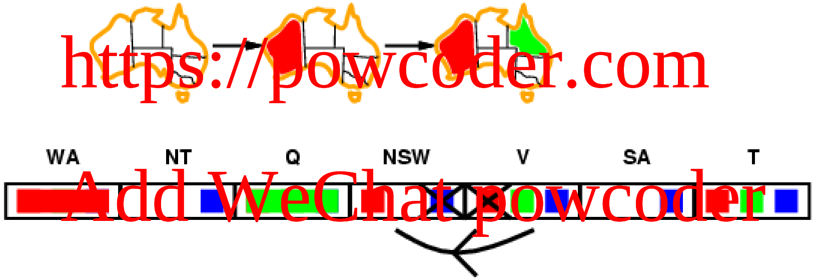
- Simplest form of propagation makes each arc consistent
- *X* → *Y* is consistent iff

for every value *x* of *X* there is some allowed *y*

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder



- If *X* loses a value, neighbors of *X* need to be rechecked

Arc consistency

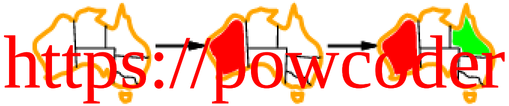
- Simplest form of propagation makes each arc consistent
- *X* → *Y* is consistent iff

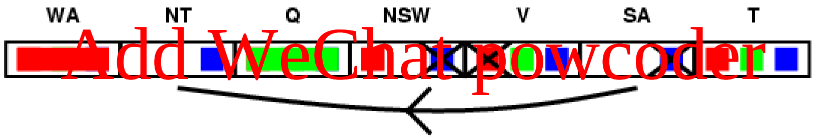
for every value *x* of *X* there is some allowed *y*

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder





- If *X* loses a value, neighbors of *X* need to be rechecked
- Arc consistency detects failure earlier than forward checking
- After running AC-3, either every arc is arc-consistent or some variable has empty domain, indicating the CSP cannot be solved.
- Can be run as a preprocessor or after each assignment

Arc consistency algorithm AC-3

- Start with a queue that contains all arcs

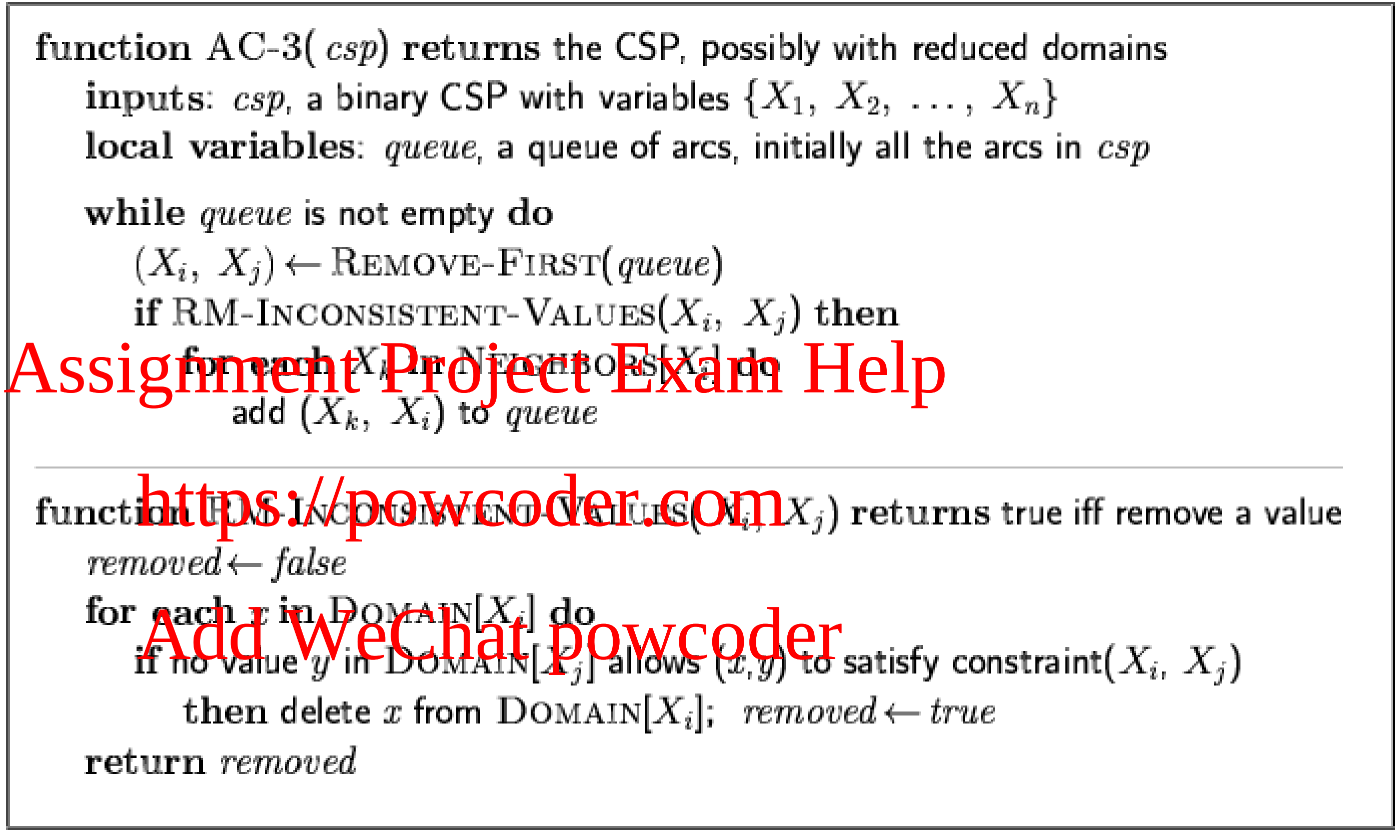
Assignment Project Exam Help

- Pop one arc (Xi, Xj) and make Xi arc-consistent with respect to Xj
  - If Di was not changed, continue to next arc,
  - Otherwise, Di was revised (domain was reduced), so need to check all arcs connected to Xi again: add all connected arcs (Xk, Xi) to the queue. (this is because the reduction in Di may yield further reductions in Dk)
  - If Di is revised to empty, then the CSP problem has no solution.

https://powcoder.com

Add WeChat powcoder

Arc consistency algorithm AC-3



```
function AC-3( $csp$) returns the CSP, possibly with reduced domains
    inputs: $csp$, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
    local variables: $queue$, a queue of arcs, initially all the arcs in $csp$

    while $queue$ is not empty do
        $(X_i, X_j) \leftarrow$ REMOVE-FIRST($queue$)
        if RM-INCONSISTENT-VALUES($X_i$, $X_j$) then
            for each $X_k$ in NEIGHBORS[$X_i$] do
                add $(X_k, X_i)$ to $queue$
```

```
function RM-INCONSISTENT-VALUES( $X_i$, $X_j$) returns true iff remove a value
    $removed \leftarrow false$
    for each $x$ in DOMAIN[$X_i$] do
        if no value $y$ in DOMAIN[$X_j$] allows $(x,y)$ to satisfy constraint($X_i$, $X_j$)
            then delete $x$ from DOMAIN[$X_i$];   $removed \leftarrow true$
    return $removed$
```

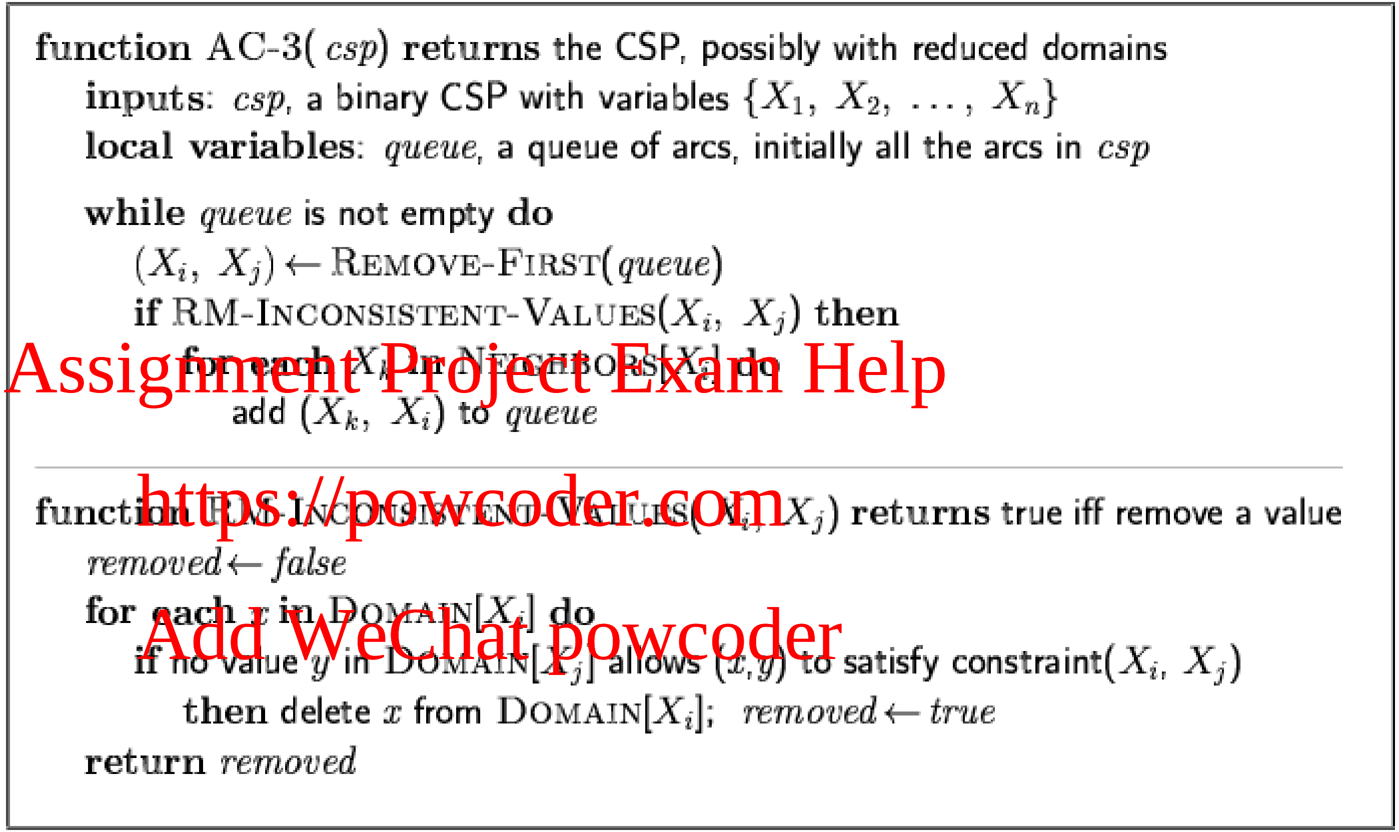
Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

- Time complexity: ?   (n variables, d values)

Arc consistency algorithm AC-3



```
function AC-3( $csp$ ) returns the CSP, possibly with reduced domains
   inputs: $csp$, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
   local variables: $queue$, a queue of arcs, initially all the arcs in $csp$

   while $queue$ is not empty do
      $(X_i, X_j) \leftarrow$ REMOVE-FIRST($queue$)
      if RM-INCONSISTENT-VALUES($X_i$, $X_j$) then
         for each $X_k$ in NEIGHBORS[$X_i$] do
            add $(X_k, X_i)$ to $queue$
```

```
function RM-INCONSISTENT-VALUES( $X_i$, $X_j$ ) returns true iff remove a value
   $removed \leftarrow false$
   for each $x$ in DOMAIN[$X_i$] do
      if no value $y$ in DOMAIN[$X_j$] allows $(x,y)$ to satisfy constraint($X_i$, $X_j$)
         then delete $x$ from DOMAIN[$X_i$];   $removed \leftarrow true$
   return $removed$
```

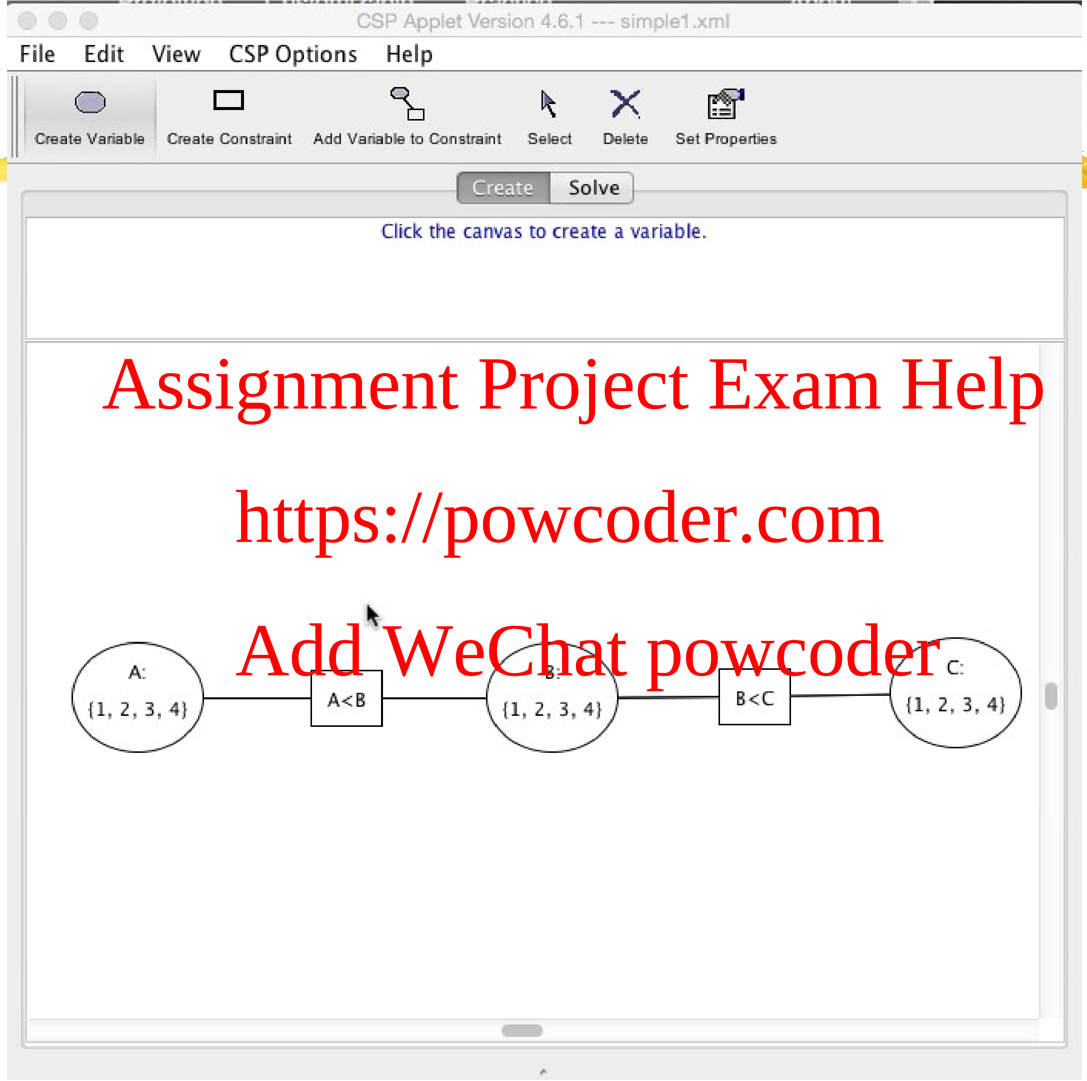
Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

- Time complexity: O(n^2d^3)   (n variables, d values)
- (each arc can be queued only d times, n^2 arcs (at most), checking one arc is O(d^2))

Demo



Click the canvas to create a variable.

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Local search for CSPs

- Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned
- To apply to CSPs:
  - allow states with unsatisfied constraints
  - operators reassign variable values

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

- Variable selection: randomly select any conflicted variable
- Value selection by min-conflicts heuristic:
  - choose value that violates the fewest constraints
  - i.e., hill-climb with $h(n)$ = total number of violated constraints

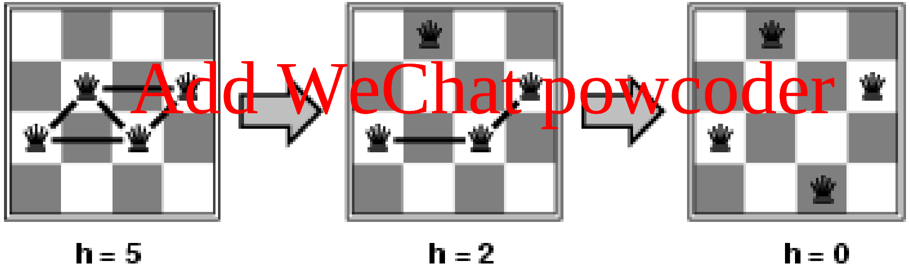
Example: 4-Queens

- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Actions: move queen in column
- Goal test: no attacks
- Evaluation: $h(n)$ = number of attacks

Assignment Project Exam Help

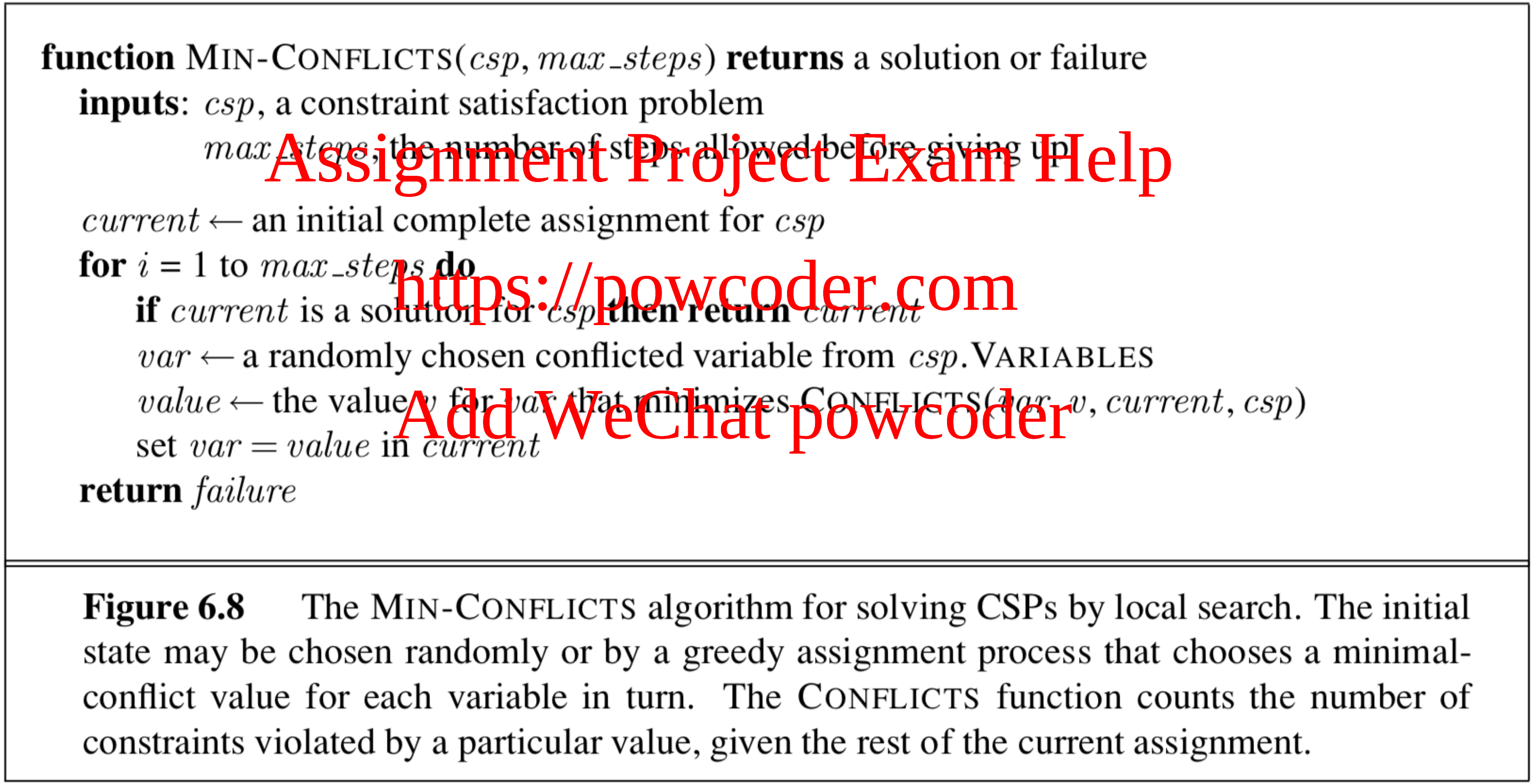
https://powcoder.com

Add WeChat powcoder



- Given random initial state, can solve *n*-queens in almost constant time for arbitrary *n* with high probability (e.g., *n* = 10,000,000)

Min-conflicts algorithm



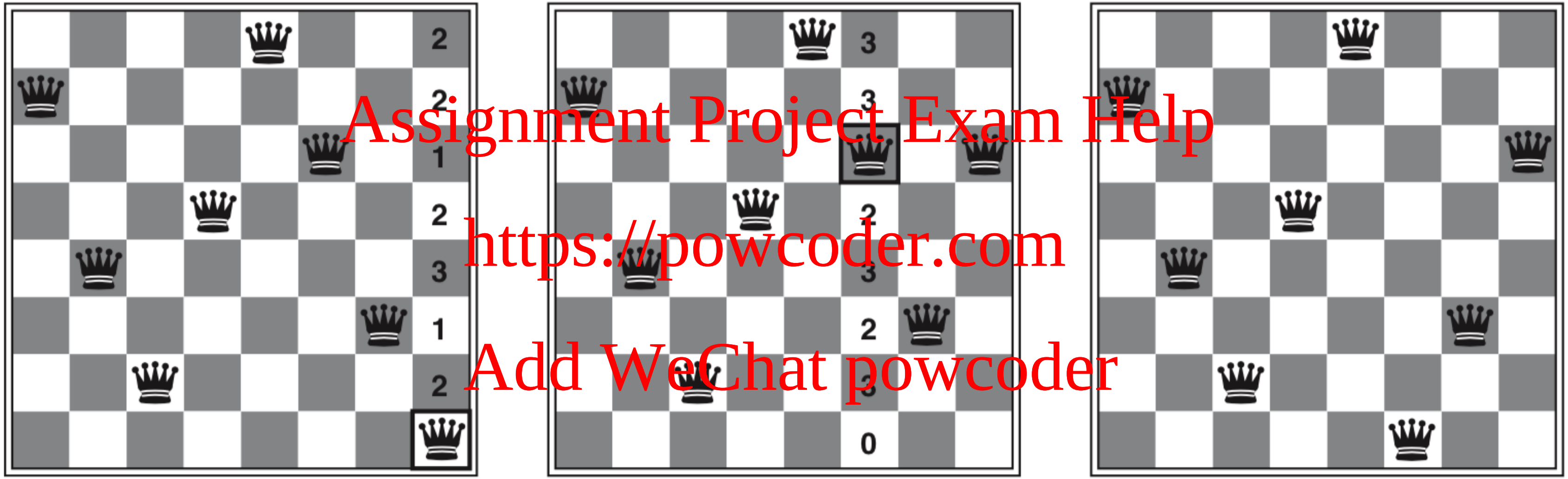
```
function MIN-CONFLICTS(csp, max_steps) returns a solution or failure
   inputs: csp, a constraint satisfaction problem
           max_steps, the number of steps allowed before giving up

   current ← an initial complete assignment for csp
   for i = 1 to max_steps do
      if current is a solution for csp then return current
      var ← a randomly chosen conflicted variable from csp.VARIABLES
      value ← the value v for var that minimizes CONFLICTS(var, v, current, csp)
      set var = value in current
   return failure
```

Figure 6.8 The MIN-CONFLICTS algorithm for solving CSPs by local search. The initial state may be chosen randomly or by a greedy assignment process that chooses a minimal-conflict value for each variable in turn. The CONFLICTS function counts the number of constraints violated by a particular value, given the rest of the current assignment.

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Example: N-Queens



Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Figure 6.9 A two-step solution using min-conflicts for an 8-queens problem. At each stage, a queen is chosen for reassignment in its column. The number of conflicts (in this case, the number of attacking queens) is shown in each square. The algorithm moves the queen to the min-conflicts square, breaking ties randomly.

Summary

- CSPs are a special kind of problem:
  - states defined by values of a fixed set of variables
  - goal test defined by constraints on variable values
- Backtracking = depth-first search with one variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder