# Last time: Logic and Reasoning

- Knowledge Base (KB): contains a set of sentences expressed using a **knowledge representation language**
  - TELL: operator to add a sentence to the KB
  - ASK: to query the KB



- Logics are KRLs where conclusions can be drawn
  - Syntax
  - Semantics



- Entailment: KB |= a iff a is true in all worlds where KB is true



- Inference: KB |–$_i$ a = sentence a can be derived from KB using procedure $i$
  - Sound: whenever KB |–$_i$ a then KB |= a is true
  - Complete: whenever KB |= a then KB |–$_i$ a

# Last Time: Syntax of propositional logic

Propositional logic is the simplest logic-

The proposition symbols $P_1$, $P_2$ etc are sentences



If $S$ is a sentence, $\neg S$ is a sentence



If $S_1$ and $S_2$ is a sentence, $S_1 \wedge S_2$ is a sentence

If $S_1$ and $S_2$ is a sentence, $S_1 \vee S_2$ is a sentence



If $S_1$ and $S_2$ is a sentence, $S_1 \Rightarrow S_2$ is a sentence

If $S_1$ and $S_2$ is a sentence, $S_1 \Leftrightarrow S_2$ is a sentence

# Last Time: Semantics of Propositional logic

Each model specifies true/false for each proposition symbol

E.g. $\begin{array}{ccc} A & B & C \\ True & True & False \end{array}$



Rules for evaluating truth with respect to a model $m$:



$$\begin{array}{rlcccc}
\neg S & \text{is true iff} & S & \text{is false} & & \\
S_1 \wedge S_2 & \text{is true iff} & S_1 & \text{is true } \underline{\text{and}} & S_2 & \text{is true} \\
S_1 \vee S_2 & \text{is true iff} & S_1 & \text{is true } \underline{\text{or}} & S_2 & \text{is true} \\
S_1 \;\Rightarrow\; S_2 & \text{is true iff} & S_1 & \text{is false } \underline{\text{or}} & S_2 & \text{is true} \\
\text{i.e.,} & \text{is false iff} & S_1 & \text{is true } \underline{\text{and}} & S_2 & \text{is false} \\
S_1 \;\Leftrightarrow\; S_2 & \text{is true iff} & S_1 \Rightarrow S_2 & \text{is true } \underline{\text{and}} & S_2 \Rightarrow S_1 & \text{is true}
\end{array}$$

# Last Time: Inference rules for propositional logic

- **Modus Ponens** or **Implication-Elimination**: (From an implication and the premise of the implication, you can infer the conclusion.)

$$\frac{\alpha \Rightarrow \beta, \qquad \alpha}{\beta}$$

- **And-Elimination**: (From a conjunction, you can infer any of the conjuncts.)

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

- **And-Introduction**: (From a list of sentences, you can infer their conjunction.)

$$\frac{\alpha_1, \alpha_2, \ldots \quad \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$



- **Or-Introduction**: (From a sentence, you can infer its disjunction with anything else at all.)

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$



- **Double-Negation Elimination**: (From a doubly negated sentence, you can infer a positive sentence.)

$$\frac{\neg\neg\alpha}{\alpha}$$



- **Unit Resolution**: (From a disjunction, if one of the disjuncts is false, then you can infer the other one is true.)

$$\frac{\alpha \vee \beta, \qquad \neg\beta}{\alpha}$$

- **Resolution**: (This is the most difficult. Because $\beta$ cannot be both true and false, one of the other disjuncts must be true in one of the premises. Or equivalently, implication is transitive.)

$$\frac{\alpha \vee \beta, \qquad \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or equivalently} \quad \frac{\neg\alpha \Rightarrow \beta, \qquad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

# This time

- **First-order logic**
  - Syntax
  - Semantics
  - Wumpus world example

- **Ontology** (ont = ‘to be’; logica = ‘word’): kinds of things one can talk about in the language

## Why first-order logic?

- We saw that propositional logic is limited because it only makes the ontological commitment that the world consists of **facts**.



- Difficult to represent even simple worlds like the Wumpus world;



e.g.,



"don't go forward if the Wumpus is in front of you" takes 64 rules

# First-order logic (FOL)

- Ontological commitments:
  - **Objects**: wheel, door, body, engine, seat, car, passenger, driver
  - **Relations**: Inside(car, passenger), Beside(driver, passenger)
  - **Functions**: ColorOf(car)
  - **Properties**: Color(car), IsOpen(door), IsOn(engine)



- Functions are relations with single value for each object

## Semantics

there is a correspondence between

- functions, which return values
- predicates, which are true or false





Function: father_of(Mary) = Bill

Predicate: father_of(Mary, Bill) [true or false]

# Examples:

- "One plus two equals three"
  Objects:
  Relations:
  Properties:
  Functions:



- "Squares neighboring the Wumpus are smelly"
  Objects:
  Relations:
  Properties:
  Functions:

# Examples:

- "One plus two equals three"

  Objects: one, two, three, one plus two

  Relations: equals

  Properties: --

  Functions: plus ("one plus two" is the name of the object obtained by applying function plus to one and two; three is another name for this object)

- "Squares neighboring the Wumpus are smelly"

  Objects: Wumpus, square

  Relations: neighboring

  Properties: smelly

  Functions: --

# FOL: Syntax of basic elements

- **Constant symbols:** 1, 5, A, B, USC, JPL, Alex, Manos, …
- **Predicate symbols:** >, Friend, Student, Colleague, …



- **Function symbols:** +, sqrt, SchoolOf, TeacherOf, ClassOf, …



- **Variables:** *x, y, z, next, first, last, …*



- **Connectives:** $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- **Quantifiers:** $\forall$, $\exists$
- **Equality:** $=$

## FOL: Atomic sentences

AtomicSentence → Predicate(Term, …) | Term = Term

Term → Function(Term, …) | Constant | Variable



- Examples:
  - SchoolOf(Manos)
  - Colleague(TeacherOf(Alex), TeacherOf(Manos))
  - >((+ x y), x)

## FOL: Complex sentences

Sentence → AtomicSentence
| Sentence Connective Sentence
| Quantifier Variable, ... Sentence
| ¬ Sentence
| (Sentence)



- Examples:
  - $S1 \wedge S2$, $S1 \vee S2$, $(S1 \wedge S2) \vee S3$, $S1 \Rightarrow S2$, $S1 \Leftrightarrow S3$
  - Colleague(Paolo, Maja) ⇒ Colleague(Maja, Paolo)
    Student(Alex, Paolo) ⇒ Teacher(Paolo, Alex)

# Semantics of atomic sentences

- Sentences in FOL are interpreted with respect to a model
- Model contains objects and relations among them
- Terms: refer to objects (e.g., Door, Alex, StudentOf(Paolo))
  - Constant symbols: refer to objects
  - Predicate symbols: refer to relations
  - Function symbols: refer to functional Relations



- An atomic sentence *predicate(term*$_1$*, …, term*$_n$*)* is **true** iff the relation referred to by *predicate* holds between the objects referred to by *term*$_1$*, …, term*$_n$

# Example model

- Objects: John, James, Marry, Alex, Dan, Joe, Anne, Rich

- Relation: sets of tuples of objects
  {<John, James>, <Marry, Alex>, <Marry, James>, ...}
  {<Dan, Joe>, <Anne, Marry>, <Marry, Joe>, ...}



- E.g.:
  Parent relation -- {<John, James>, <Marry, Alex>, <Marry, James>}



  then Parent(John, James) is true
  Parent(John, Marry) is false

# Quantifiers

- Expressing sentences about **collections** of objects without enumeration (naming individuals)



- E.g., All Trojans are clever



Someone in the class is sleeping



- Universal quantification (for all): $\forall$
- Existential quantification (there exists): $\exists$

## Universal quantification (for all): ∀

∀ *<variables> <sentence>*

- *"Every one in the cs561 class is smart"*:
  $\forall\ x\ \ \text{In}(cs561, x) \Rightarrow \text{Smart}(x)$



- **∀ P corresponds to the conjunction of instantiations of P**
  $(\text{In}(cs561, \text{Manos}) \Rightarrow \text{Smart}(\text{Manos})) \wedge$
  $(\text{In}(cs561, \text{Dan}) \Rightarrow \text{Smart}(\text{Dan})) \wedge$
  …
  $(\text{In}(cs561, \text{Bush}) \Rightarrow \text{Smart}(\text{Bush}))$

## Universal quantification (for all): $\forall$

- $\Rightarrow$ is a natural connective to use with $\forall$



- Common mistake: to use $\wedge$ in conjunction with $\forall$
  e.g: $\forall\, x \quad In(cs561,\, x) \wedge Smart(x)$
  means *"every one is in cs561 and everyone is smart"*

## Existential quantification (there exists): ∃

∃ *<variables> <sentence>*

- *"Someone in the cs561 class is smart":*
  $\exists\, x \quad In(cs561, x) \wedge Smart(x)$



- **∃ P corresponds to the disjunction of instantiations of P**
  $In(cs561, Manos) \wedge Smart(Manos) \vee$
  $In(cs561, Dan) \wedge Smart(Dan) \vee$
  …
  $In(cs561, Bush) \wedge Smart(Bush)$

## Existential quantification (there exists): ∃

- $\wedge$ is a natural connective to use with ∃



- Common mistake: to use $\Rightarrow$ in conjunction with ∃
  e.g: $\exists\, x \quad \text{In}(cs561, x) \Rightarrow \text{Smart}(x)$
  is true if there is anyone that is not in cs561!
  (remember, false $\Rightarrow$ true is valid)

## Properties of quantifiers

$\forall x \ \forall y$ is the same as $\forall y \ \forall x$ (why??)

$\exists x \ \exists y$ is the same as $\exists y \ \exists x$ (why??)



$\exists x \ \forall y$ is not the same as $\forall y \ \exists x$

$\exists x \ \forall y \ Loves(x, y)$



"There is a person who loves everyone in the world"

$\forall y \ \exists x \ Loves(x, y)$



Not all by one person but each one at least by one

"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x \ Likes(x, IceCream) \qquad \neg\exists x \ \neg Likes(x, IceCream)$

Proof?

$\exists x \ Likes(x, Broccoli) \qquad \neg\forall x \ \neg Likes(x, Broccoli)$

## Proof

- In general we want to prove:

$\forall x \ P(x) <=> \neg \exists x \ \neg P(x)$





- $\forall x \ P(x) = \neg(\neg(\forall x \ P(x))) = \neg(\neg(P(x1) \wedge P(x2) \wedge \ldots \wedge P(xn)) ) = \neg(\neg P(x1) \vee \neg P(x2) \vee \ldots \vee \neg P(xn)) )$



- $\exists x \ \neg P(x) = \neg P(x1) \vee \neg P(x2) \vee \ldots \vee \neg P(xn)$

- $\neg \exists x \ \neg P(x) = \neg(\neg P(x1) \vee \neg P(x2) \vee \ldots \vee \neg P(xn))$

## Example sentences

- Brothers are siblings



- Sibling is transitive



- One’s mother is one’s sibling’s mother



- A first cousin is a child of a parent’s sibling

# Example sentences

- Brothers are siblings

  $\forall$ x, y  Brother(x, y) $\Rightarrow$ Sibling(x, y)



- Sibling is transitive

  $\forall$ x, y, z  Sibling(x, y) $\wedge$ Sibling(y, z) $\Rightarrow$ Sibling(x, z)



- One's mother is one's sibling's mother



  $\forall$ m, c, d  Mother(m, c) $\wedge$ Sibling(c, d) $\Rightarrow$ Mother(m, d)

- A first cousin is a child of a parent's sibling

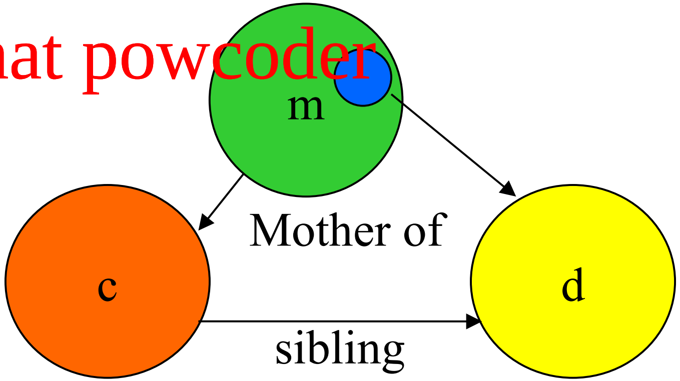
  $\forall$ c, d  FirstCousin(c, d) $\Leftrightarrow$
  $\exists$ p, ps  Parent(p, d) $\wedge$ Sibling(p, ps) $\wedge$ Child(c, ps)

# Example sentences

- One’s mother is one’s sibling’s mother
  $\forall$ m, c, d Mother(m, c) $\wedge$ Sibling(c, d) $\Rightarrow$ Mother(m, d)

- $\forall$ c, d **$\exists$m** Mother(m, c) $\wedge$ Sibling(c, d) $\Rightarrow$ Mother(m, d)





## Translating English to FOL

- Every gardener likes the sun.

  `∀ x gardener(x) => likes(x,Sun)`



- You can fool some of the people all of the time.

  `∃ x ∀ t (person(x) ^ time(t)) => can-fool(x,t)`

# Translating English to FOL

- You can fool all of the people some of the time.

  `∀ x person(x) => ∃ t time(t) ^ can-fool(x,t)`



- All purple mushrooms are poisonous.

  `∀ x (mushroom(x) ^ purple(x)) => poisonous(x)`

## Caution with nested quantifiers

- $\forall x \exists y\ P(x,y)$ is the same as $\forall x\ (\exists y\ P(x,y))$ which means "for every x, it is true that there exists y such that P(x,y)"



- $\exists y \forall x\ P(x,y)$ is the same as $\exists y\ (\forall x\ P(x,y))$ which means "there exists a y, such that it is true that for every x P(x,y)"

# Translating English to FOL…

- No purple mushroom is poisonous.



$\neg(\exists x)$ purple(x) ^ mushroom(x) ^ poisonous(x)



or, equivalently,



$(\forall x)$ (mushroom(x) ^ purple(x)) => $\neg$poisonous(x)

# Translating English to FOL…

- There are exactly two purple mushrooms.



```
(∃ x)(∃ y) mushroom(x) ^ purple(x) ^
  mushroom(y) ^ purple(y) ^ ¬(x=y) ^ (∀ z)
   (mushroom(z) ^ purple(z)) => ((x=z) v (y=z))
```





- Deb is not tall.

```
¬tall(Deb)
```

# Translating English to FOL…

- X is above Y iff X is directly on top of Y or else there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.





```
(∀ x)(∀ y) above(x,y) <=> (on(x,y) v (∃ z)
   (on(x,z) ^ above(z,y)))
```

# Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object



E.g., $1 = 2$ and $\forall\, x \;\; \times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$2 = 2$ is valid



E.g., definition of (full) $Sibling$ in terms of $Parent$:

$$\forall\, x, y \;\; Sibling(x, y) \Leftrightarrow [\neg(x = y) \wedge \exists\, m, f \;\; \neg(m = f) \wedge Parent(m, x) \wedge Parent(f, x) \wedge Parent(m, y) \wedge Parent(f, y)]$$

## Higher-order logic?

- First-order logic allows us to quantify over objects (= the first-order entities that exist in the world).



- Higher-order logic also allows quantification over relations and functions.

  e.g., "two objects are equal iff all properties applied to them are equivalent":





  $\forall x,y \quad (x=y) \Leftrightarrow (\forall p,\ p(x) \Leftrightarrow p(y))$

- Higher-order logics are more expressive than first-order; however, so far we have little understanding on how to effectively reason with sentences in higher-order logic.

# Logical agents for the Wumpus world

Remember: generic knowledge-based agent:

```
function KB-AGENT(percept) returns an action
   static: KB, a knowledge base
           t, a counter, initially 0, indicating time

   TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
   action ← ASK(KB, MAKE-ACTION-QUERY(t))
   TELL(KB, MAKE-ACTION-SENTENCE(action, t))
   t ← t + 1
   return action
```



1. TELL KB what was perceived
   Uses a KRL to insert new sentences, representations of facts, into KB

2. ASK KB what to do.
   Uses logical reasoning to examine actions and select best.

# Using the FOL Knowledge Base

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$\text{TELL}(KB, Percept([Smell, Breeze, None], 5))$

$\text{ASK}(KB, \exists a \;\; Action(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?


Answer: $Yes$, $\{a/Shoot\}$ $\leftarrow$ <u>substitution</u> (binding list)

Set of solutions

Given a sentence $S$ and a substitution $\sigma$,
$S\sigma$ denotes the result of plugging $\sigma$ into $S$; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$\text{ASK}(\text{KB}, \text{S})$ returns some/all $\sigma$ such that $KB \models S\sigma$

## Wumpus world, FOL Knowledge Base



"Perception"

$\forall b, g, t \;\; Percept([Smell, b, g], t) \Rightarrow Smelt(t)$

$\forall s, b, t \;\; Percept([s, b, Glitter], t) \Rightarrow AtGold(t)$



Reflex: $\forall t \;\; AtGold(t) \Rightarrow Action(Grab, t)$



Reflex with internal state: do we have the gold already?

$\forall t \;\; AtGold(t) \wedge \neg Holding(Gold, t) \Rightarrow Action(Grab, t)$

$Holding(Gold, t)$ cannot be observed

$\Rightarrow$ keeping track of change is essential

## Deducing hidden properties

Properties of locations:

$$\forall l, t \;\; At(Agent, l, t) \land Smelt(t) \;\Rightarrow\; Smelly(l)$$
$$\forall l, t \;\; At(Agent, l, t) \land Breeze(t) \;\Rightarrow\; Breezy(l)$$

Squares are breezy near a pit:



Diagnostic rule—infer cause from effect

$$\forall y \;\; Breezy(y) \;\Rightarrow\; \exists x \;\; Pit(x) \land Adjacent(x, y)$$



Causal rule—infer effect from cause

$$\forall x, y \;\; Pit(x) \land Adjacent(x, y) \;\Rightarrow\; Breezy(y)$$



Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy

Definition for the $Breezy$ predicate:

$$\forall y \;\; Breezy(y) \;\Leftrightarrow\; [\exists x \;\; Pit(x) \land Adjacent(x, y)]$$

## Situation calculus

Facts hold in situations, rather than eternally
E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

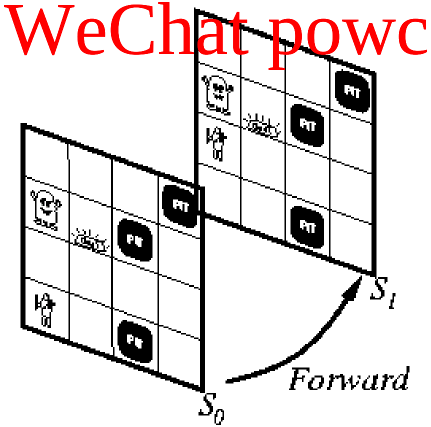
Situation calculus is one way to represent change in FOL:
Adds a situation argument to each non-eternal predicate
E.g., $Now$ in $Holding(Gold, Now)$ denotes a situation





Situations are connected by the $Result$ function
$Result(a, s)$ is the situation that results from doing $a$ is $s$





# Describing actions

"Effect" axiom—describe changes due to action

$\forall s \;\; AtGold(s) \Rightarrow Holding(Gold, Result(Grab, s))$

"Frame" axiom—describe non-changes due to action

$\forall s \;\; HaveArrow(s) \Rightarrow HaveArrow(Result(Grab, s))$



May result in too many frame axioms

<u>Frame problem:</u> find an elegant way to handle non-change

(a) representation—avoid frame axioms

(b) inference—avoid repeated "copy-overs" to keep track of state





<u>Qualification problem:</u> true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or ...

<u>Ramification problem:</u> real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, ...

# Describing actions (cont'd)

Successor-state axioms solve the representational frame problem

Each axiom is "about" a predicate (not an action per se):

P true afterwards $\Leftrightarrow$ [an action made P true

$\lor$ P true already and no action made P false]

For holding the gold:

$$\forall a, s \;\; Holding(Gold, Result(a, s)) \;\Leftrightarrow$$
$$[(a = Grab \land AtGold(s))$$
$$\lor\ (Holding(Gold, s) \land a \neq Release)]$$

# Planning

Initial condition in KB:

$At(Agent, [1,1], S_0)$

$At(Gold, [1,2], S_0)$



Query: $\textsc{Ask}(KB, \exists s \;\; Holding(Gold, s))$

i.e., in what situation will I be holding the gold?



Answer: $\{s / Result(Grab, Result(Forward, S_0))\}$

i.e., go forward and then grab the gold



This assumes that the agent is interested in plans starting at $S_0$ and that $S_0$ is the only situation described in the KB

# Generating action sequences

Represent plans as action sequences $[a_1, a_2, \ldots, a_n]$

$PlanResult(p, s)$ is the result of executing $p$ in $s$



Then the query $\text{ASK}(KB, \exists p \;\; Holding(Gold, PlanResult(p, S_0)))$
has the solution $\{p/[Forward, Grab]\}$



Definition of $PlanResult$ in terms of $Result$:

$\forall s \;\; PlanResult([\,], s) = s$ [ ] = empty plan



$\forall a, p, s \;\; PlanResult([a|p], s) = PlanResult(p, Result(a, s))$

Recursively continue until it gets to empty plan [ ]

Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner

## Summary

First-order logic:

- objects and relations are semantic primitives
- syntax: constants, functions, predicates, equality, quantifiers



Increased expressive power: sufficient to define wumpus world



Situation calculus:

- conventions for describing actions and change in FOL
- can formulate planning as inference on a situation calculus KB