This time: Fuzzy Logic and Fuzzy Inference

- Why use fuzzy logic?
- Tipping example
- Fuzzy set theory
- Fuzzy inference

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What is fuzzy logic?

- A super set of Boolean logic

- Builds upon <u>fuzzy set theory</u>

- Graded truth. Truth values between True and False. Not everything is either/or, true/false, black/white, on/off etc.

- Grades of membership. Class of tall men, class of far cities, class of expensive things, etc.

- <u>Lotfi Zadeh</u>, UC/Berkely 1965. Introduced it to **model uncertainty in natural language**. *Tall, far, nice, large, hot, …*

- Reasoning using <u>linguistic terms</u>. Natural to express expert knowledge.
        If the weather is cold then wear warm clothing

Why use fuzzy logic?

Pros:

- Conceptually easy to understand w/ "natural" maths
- Tolerant of imprecise data
- Universal approximation: can model arbitrary nonlinear functions
- Intuitive
- Based on linguistic terms
- Convenient way to express expert and common sense knowledge

Cons:

- Not a cure-all
- Crisp/precise models can be more efficient and even convenient
- Other approaches might be formally verified to work

Tipping example

- **The Basic Tipping Problem:** Given a number between 0 and 10 that represents the quality of service at a restaurant what should the tip be?
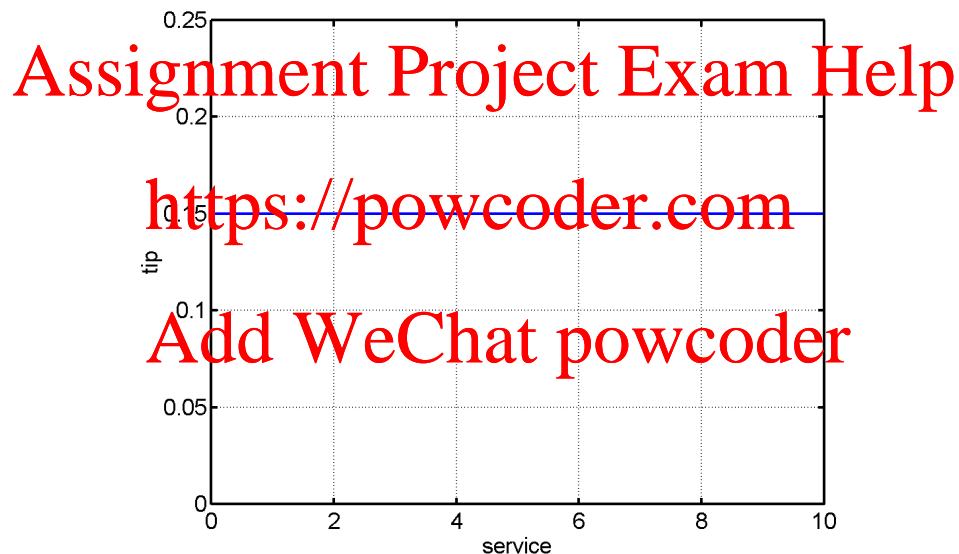
  Cultural footnote: An average tip for a meal in the U.S. is 15%, which may vary depending on the quality of the service provided.

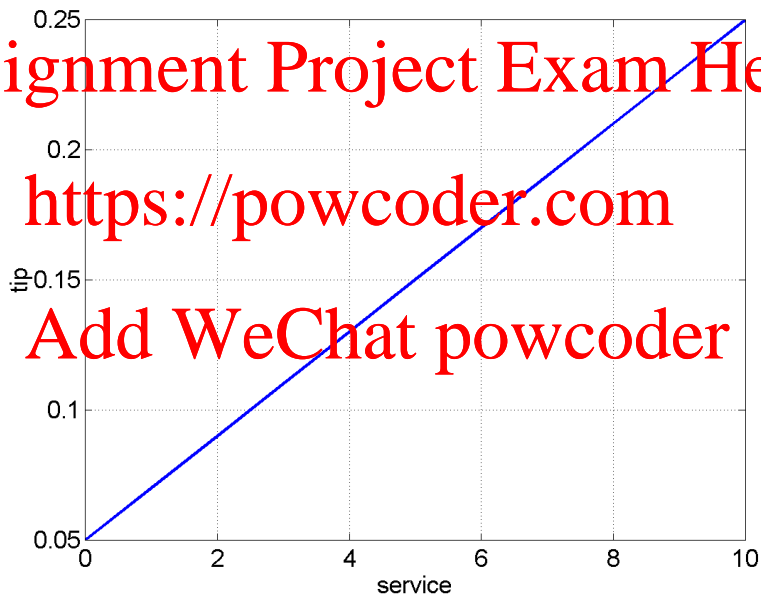# Tipping example: The non-fuzzy approach

- Tip = 15% of total bill

- What about quality of service?

Tipping example: The non-fuzzy approach

- Tip = linearly proportional to service from 5% to 25%
  tip = 0.20/10*service+0.05

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- What about quality of the food?

Tipping example: Extended

- **The Extended Tipping Problem:** Given a number between 0 and 10 that represents the quality of service <u>and the quality of the food</u>, at a restaurant, what should the tip be?
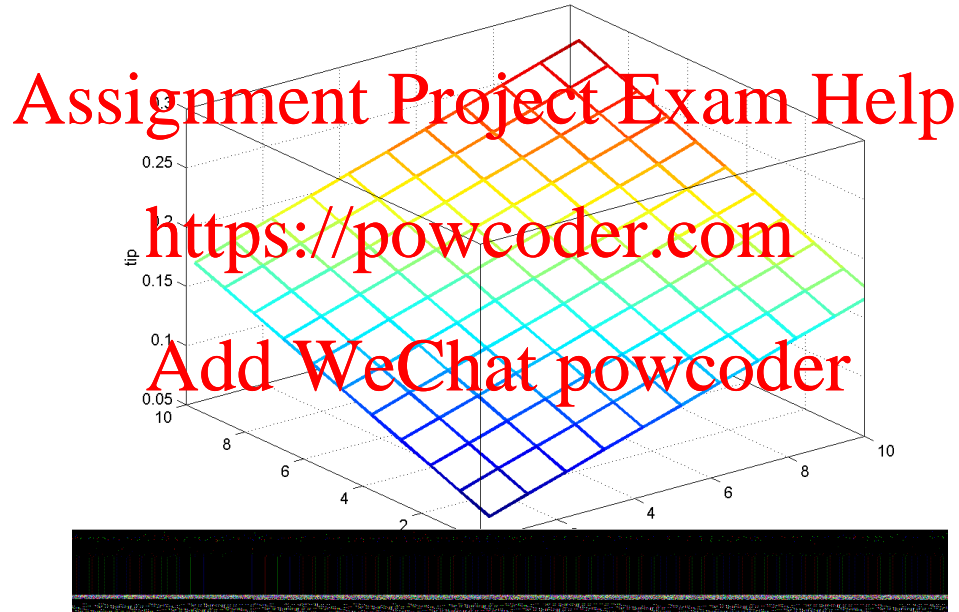
How will this affect our tipping formula?

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Tipping example: The non-fuzzy approach

- Tip = 0.20/20*(service+food)+0.05

- We want service to be more important than food quality. E.g., 80% for service and 20% for food.

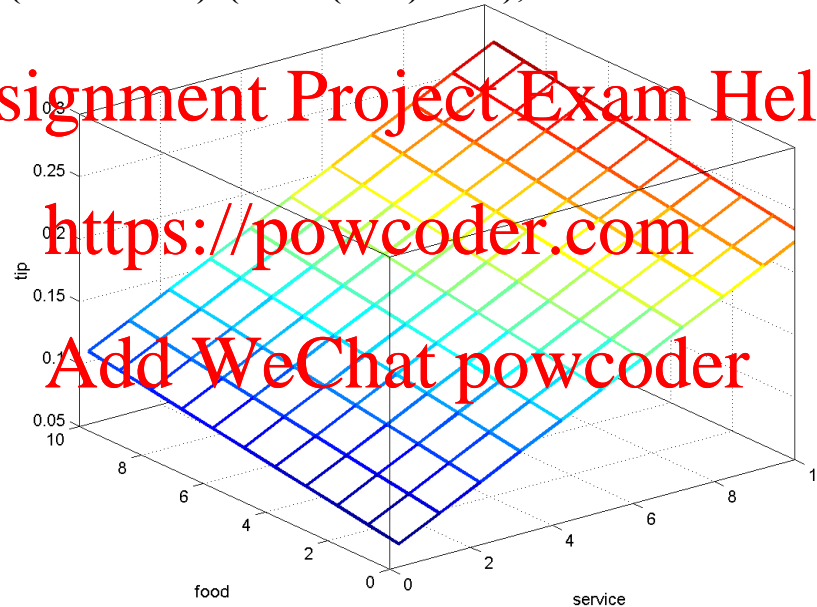# Tipping example: The non-fuzzy approach

- Tip = **servRatio*(.2/10*(service)+.05) +   servRatio = 80%**
  (1-servRatio)*(.2/10*(food)+0.05);

- Seems too linear. Want 15% tip in general and deviation only for exceptionally good or bad service.

Tipping example: The non-fuzzy approach

```
if service < 3,
  tip(f+1,s+1) = servRatio*(.1/3*(s)+.05) + ...
                         (1-servRatio)*(.2/10*(f)
  +0.05);
elseif s < 7,
  tip(f+1,s+1) = servRatio*(.15) + ...
                         (1-servRatio)*(.2/10*(f)+0.05);
else,
  tip(f+1,s+1) = servRatio*(.1/3*(s-7)+.15) + ...
                         (1-servRatio)*(.2/10*(f)+0.05);
end;
```
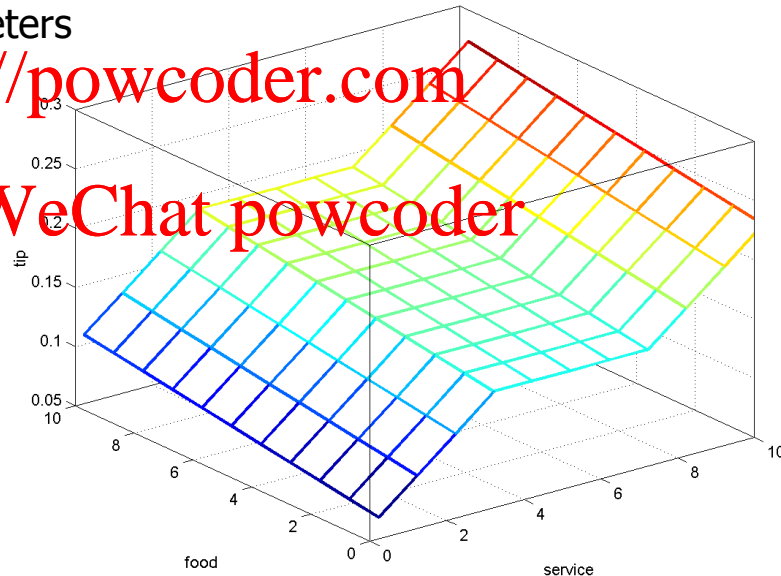
Tipping example: The non-fuzzy approach

Nice plot <u>but</u>
- 'Complicated' function
- Not easy to modify
- Not intuitive
- Many hard-coded parameters
- Not easy to understand

Tipping problem: the fuzzy approach

What we want to express is:

1. *If service is poor then tip is cheap*
2. *If service is good the tip is average*
3. *If service is excellent then tip is generous*
4. *If food is rancid then tip is cheap*
5. *If food is delicious then tip is generous*

or

6. *If service is poor or the food is rancid then tip is cheap*
7. *If service is good then tip is average*
8. *If service is excellent or food is delicious then tip is generous*

We have just defined the rules for a fuzzy logic system.

Decision function generated using the 3 rules.
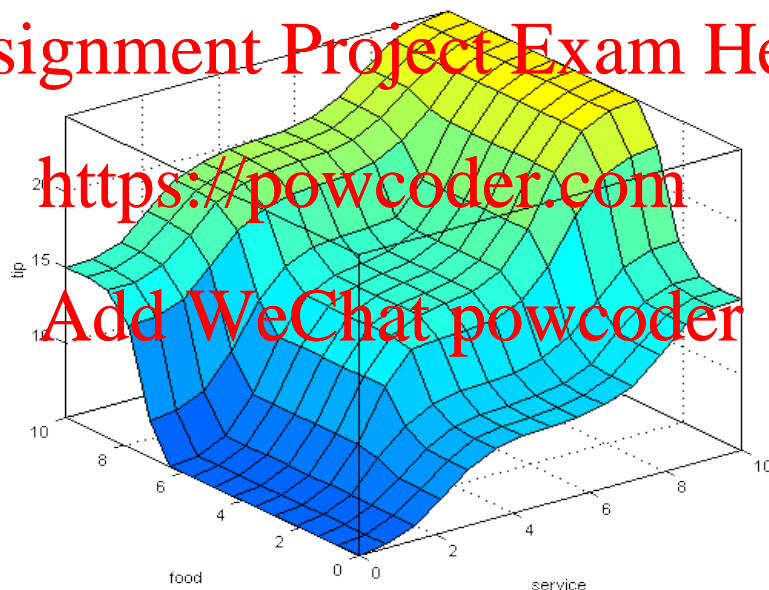
Tipping problem: fuzzy solution

- Before we have a fuzzy solution we need to find out

a) how to define terms such as *poor, delicious, cheap, generous etc.*
b) how to combine terms using AND, OR and other connectives
c) how to combine all the rules into one final output

Fuzzy sets

- **Boolean/Crisp set A** is a mapping for the elements of S to the set {0, 1}, i.e., A: S → {0, 1}

- *Characteristic function:*

$$\mu_A(x) = \begin{cases} 1 & \text{if x is an element of set A} \\ 0 & \text{if x is not an element of set A} \end{cases}$$

- **Fuzzy set F** is a mapping for the elements of S to the interval [0, 1], i.e., F: S → [0, 1]

- Characteristic function: $0 \leq \mu_F(x) \leq 1$

- 1 means full membership, 0 means no membership and anything in between, e.g., 0.5 is called **graded membership**

Example: Crisp set Tall

- Fuzzy sets and concepts are commonly used in natural language

  *John is **tall***
  *Dan is **smart***
  *Alex is **happy***
  *The class is **hot***

- E.g., the crisp set ***Tall*** can be defined as $\{x \mid \text{height } x > 1.8 \text{ meters}\}$
  But what about a person with a height = 1.79 meters?
  What about 1.78 meters?
  …
  What about 1.52 meters?

Example: Fuzzy set Tall

- In a fuzzy set a person with a height of 1.8 meters would be considered tall to a **high degree** A person with a height of 1.7 meters would be considered tall to a lesser degree etc.

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- The function can change for basketball players, Danes, women, children etc.

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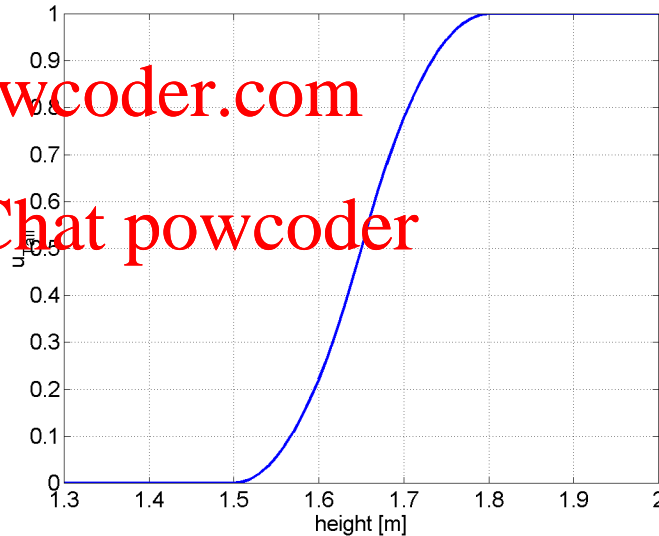
Membership functions: S-function

- The S-function can be used to define fuzzy sets
- $S(x, a, b, c) =$
  - 0 for $x \leq a$
  - $2(x\text{-}a/c\text{-}a)^2$ for $a \leq x \leq b$
  - $1 - 2(x\text{-}c/c\text{-}a)^2$ for $b \leq x \leq c$
  - 1 for $x \geq c$
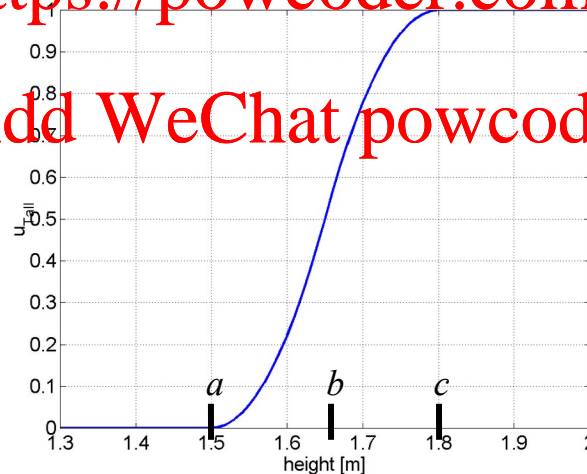
Membership functions: Π–Function

- Π(x, a, b) =
  - S(x, b-a, b-$a/2$, b)   for $x \leq b$
  - 1 – S(x, b, b+$a/2$, a+b)   for x b

E.g., *close* (to a)

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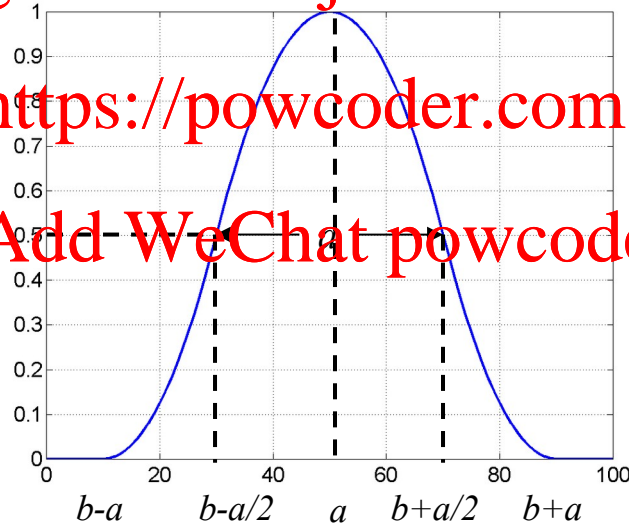
Simple membership functions

- Piecewise linear: triangular etc.
- Easier to represent and calculate $\Rightarrow$ saves computation

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Fuzzy Sets

Membership Grade
μ

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1

Cold

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Mild

Warm

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0

30

60

°F
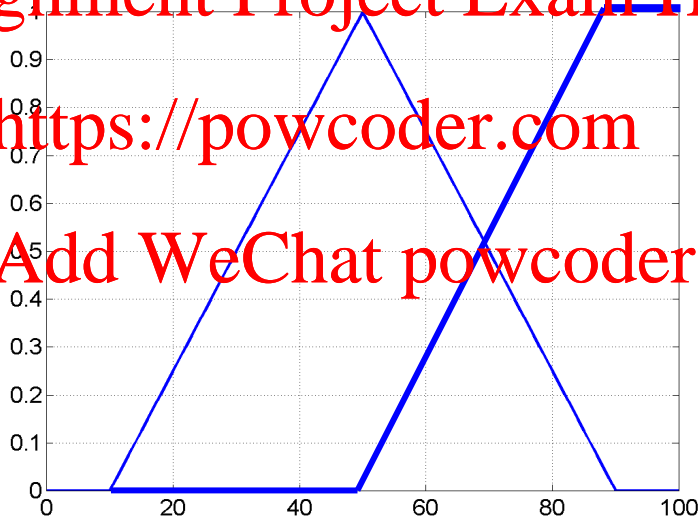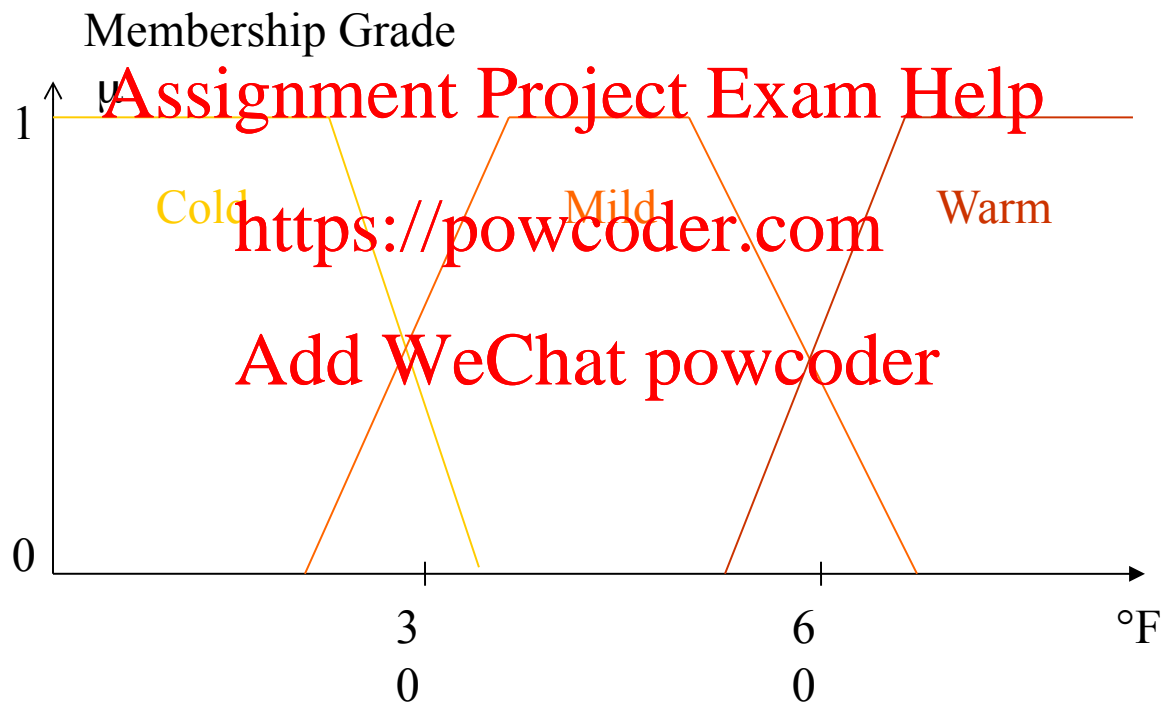
Observation

An observed temperature of 38 is cold with a belief of 0.14,
Mild with a belief of 0.85 and warm with a belief of 0

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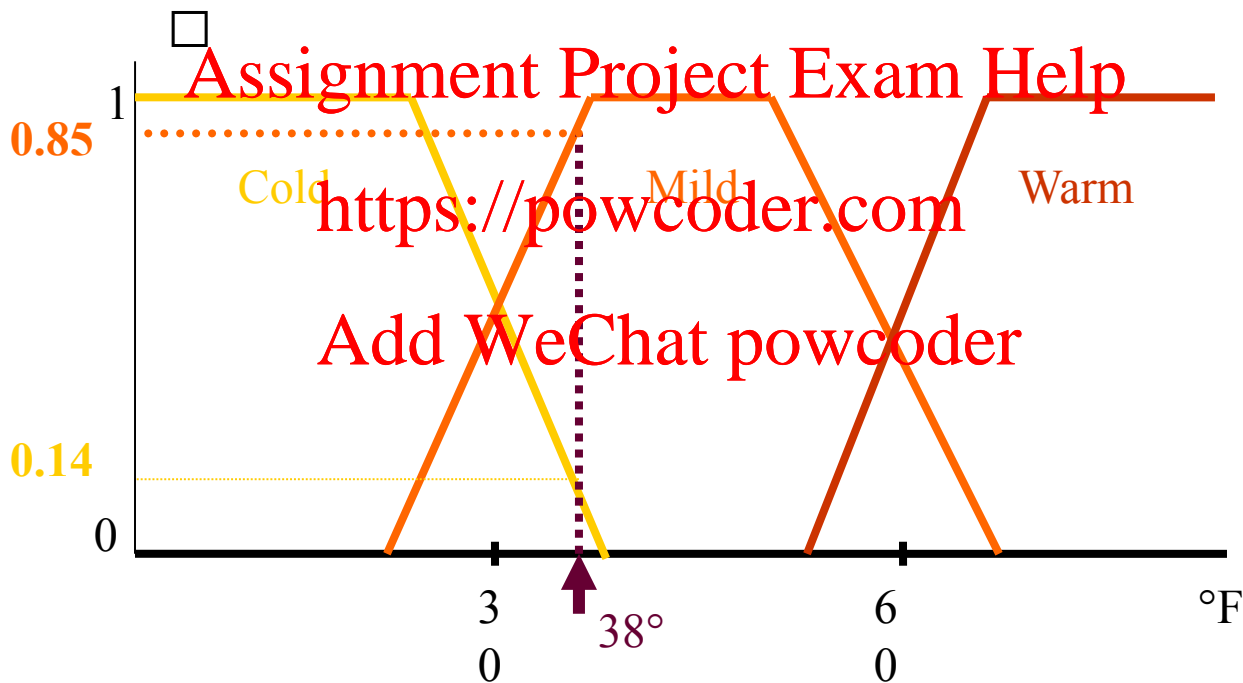
Other representations of fuzzy sets

- A finite set of elements:

  $F = \mu_1/x_1 + \mu_2/x_2 + \dots \mu_n/x_n$

  + means (Boolean) set union

- For example:

  TALL = {0/1.0, 0/1.2, 0/1.4, 0.2/1.6, 0.8/1.7, 1.0/1.8}

Fuzzy set operators

- Equality
  A = B
  $\mu_A(x) = \mu_B(x)$ for all $x \in X$
- Complement
  A'
  $\mu_{A'}(x) = 1 - \mu_A(x)$ for all $x \in X$
- Containment
  $A \subseteq B$
  $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$
- Union
  $A \cup B$
  $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$ for all $x \in X$
- Intersection
  $A \cap B$
  $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$ for all $x \in X$

Example fuzzy set operations

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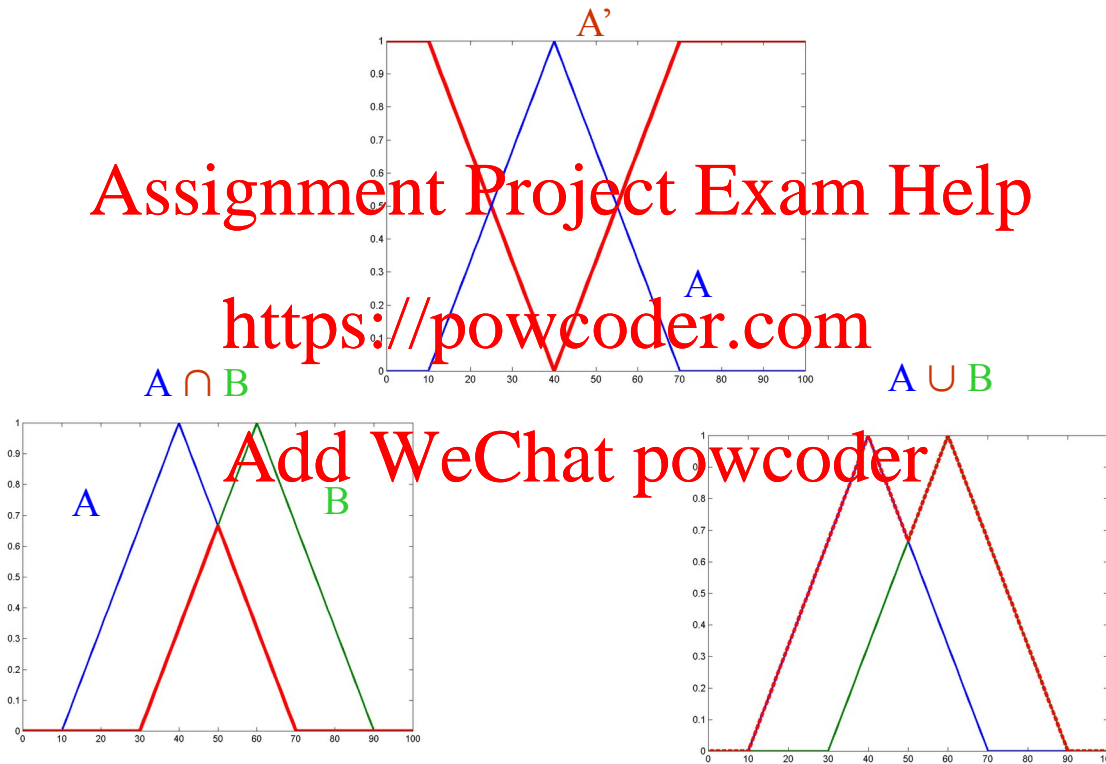
Linguistic Hedges

- Modifying the meaning of a fuzzy set using hedges such as *very, more or less, slightly, etc.*

- *"Very F "= F ^{2}*
- *"More or less F "= F $^{1/2}$*
- *etc.*

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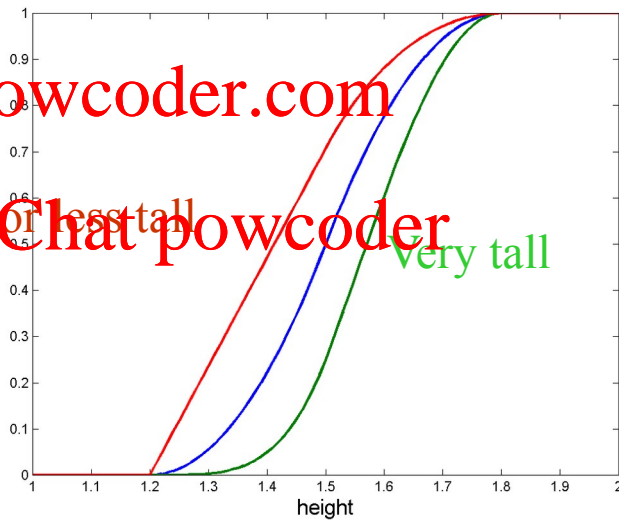
Fuzzy relations

- A fuzzy relation for N sets is defined as an extension of the crisp relation to include the membership grade.

$$R = \{\mu_R(x_1, x_2, \dots x_N)/(x_1, x_2, \dots x_N) | x_i \in X_i, i = 1, \dots N\}$$

which associates the membership grade, μ_R, of each tuple.

- E.g.

Friend = {0.9/(Manos, Nacho), 0.1/(Manos, Dan),
          0.8/(Alex, Mike), 0.3/(Alex, John)}

Fuzzy inference

- Fuzzy logical operations
- Fuzzy rules
- Fuzzification
- Implication
- Aggregation
- Defuzzification

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Sensor
Inputs

A to D
Converter

Fuzzifier

Inference
Engine

Defuzzifier

D to A
Converter

Controller
Outputs
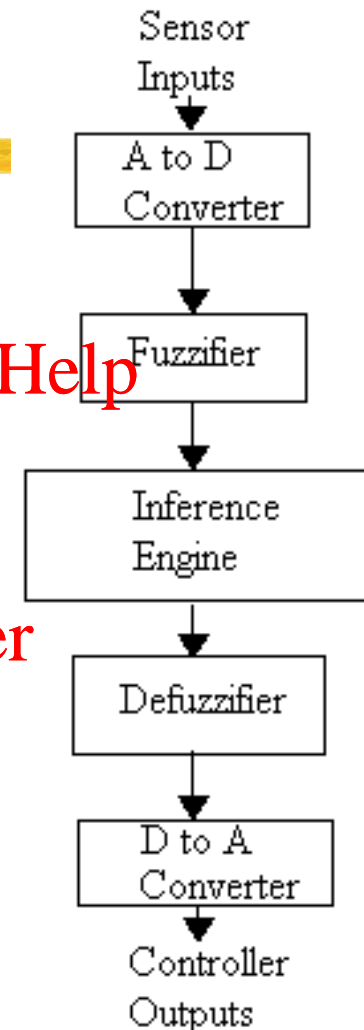
Fuzzy logical operations

- AND, OR, NOT, etc.

From the following truth tables it is seen that fuzzy logic is a superset of Boolean logic.

- **NOT** A   = A′   = $1 - \mu_A(x)$
- A **AND** B = $A \cap B = \min(\mu_A(x), \mu_B(x))$
- A **OR** B   = $A \cup B = \max(\mu_A(x), \mu_B(x))$

		min(A,B)
A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

		max(A,B)
A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

	1-A
A	not A
0	1
1	0

If-Then Rules

- Use fuzzy sets and fuzzy operators as the **subjects** and **verbs** of fuzzy logic to form rules.

if x is A then y is B

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where A and B are linguistic terms defined by fuzzy sets on the sets X and Y respectively.

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This reads

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if <u>x == A</u> then <u>y = B</u>

Evaluation of fuzzy rules

- In Boolean logic: p ⇒ q
  if p is true then q is true

- In fuzzy logic: p ⇒ q
  if p is true to some degree then q is true to some degree.

  0.5p => 0.5q   (partial premise implies partially)

- How?

1. fuzzify inputs 2. apply fuzzy operation (or = max) 3. apply implication method (min)

1. If **service** is **poor** or **food** is **rancid** then **tip = cheap**

poor rancid cheap

2. If **service** is **good** then **tip = average**

good rule 2 has no dependency on input 2 average

3. If **service** is **excellent** or **food** is **delicious** then **tip = generous**

excellent delicious generous

4. apply aggregation method (max)

service = 3
input 1

food = 8
input 2

Tip = 16.7 %
Result of defuzzification
(centroid)

result of aggregation

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Demo

```
                    Temperature
              DRY       MOIST      WET
         +-------+-------+-------+
   COOL  |  MED  | HIGH  | HIGH  |
         +-------+-------+-------+
Temp. WARM | LOW   |  MED  | HIGH  |
         +-------+-------+-------+
   HOT   |  MED  | HIGH  | HIGH  |
         +-------+-------+-------+
```

The above linguistic matrix can be reduced to
```
IF H = WET   THEN FS = HIGH
IF T = COOL AND H = DRY    THEN FS = MED
IF T = COOL AND H = MOIST  THEN FS = HIGH
IF T = WARM AND H = DRY    THEN FS = LOW
IF T = WARM AND H = MOIST  THEN FS = MED
IF T = HOT  AND H = DRY    THEN FS = MED
IF T = HOT  AND H = MOIST  THEN FS = HIGH
```

HUMIDITY (%)

DRY   MOIST   WET

CRISP FAN SPEED (rpm)

LOW   MED   HIGH

Fuzzy Fanspeed (rpm)

Temp (F) 72   Humidity (%) 45   Fanspeed (rpm) 613

TDN   TUP   HDN

Fuzzy Fan Controller
Decrease Temperature – TDN
Increase Temperature – TUP
Decrease Humidity – HDN
Increase Humidity – HUP

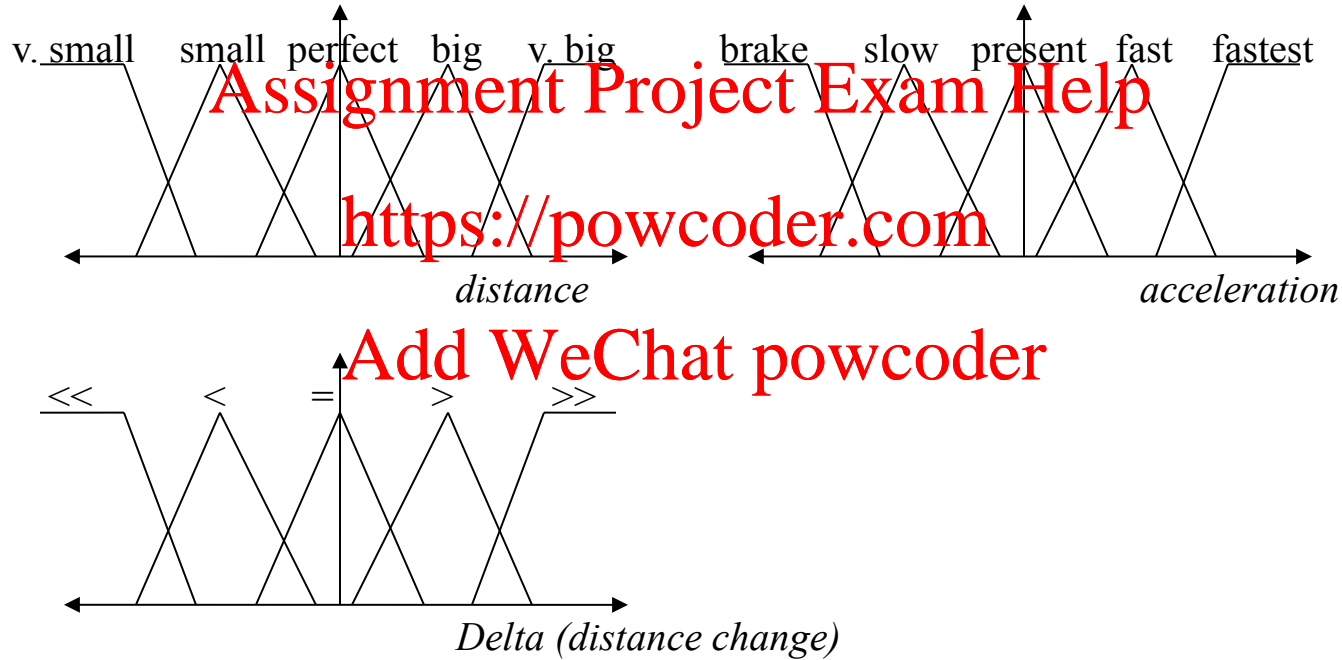Applet Created by Jeff Orr – May 1999
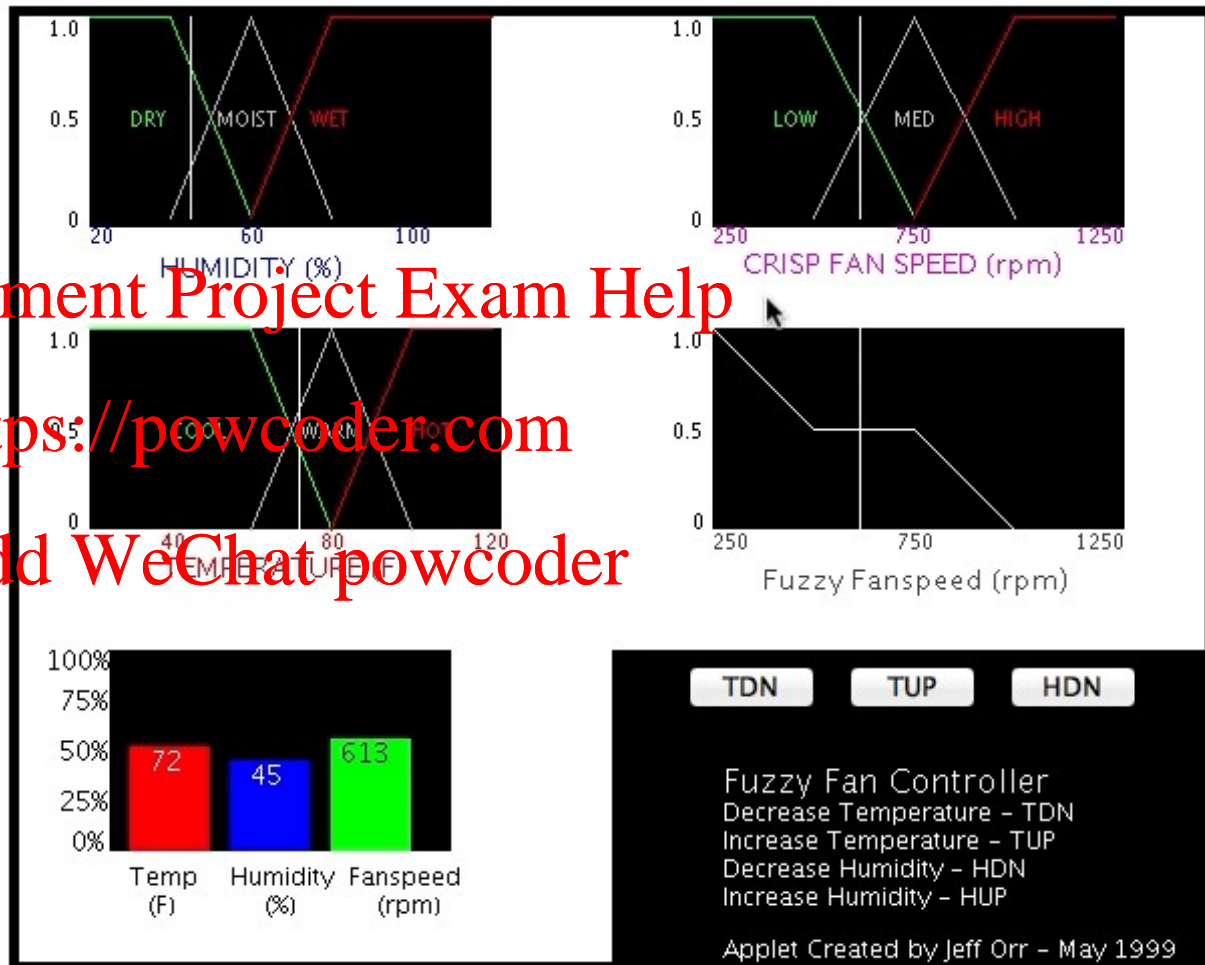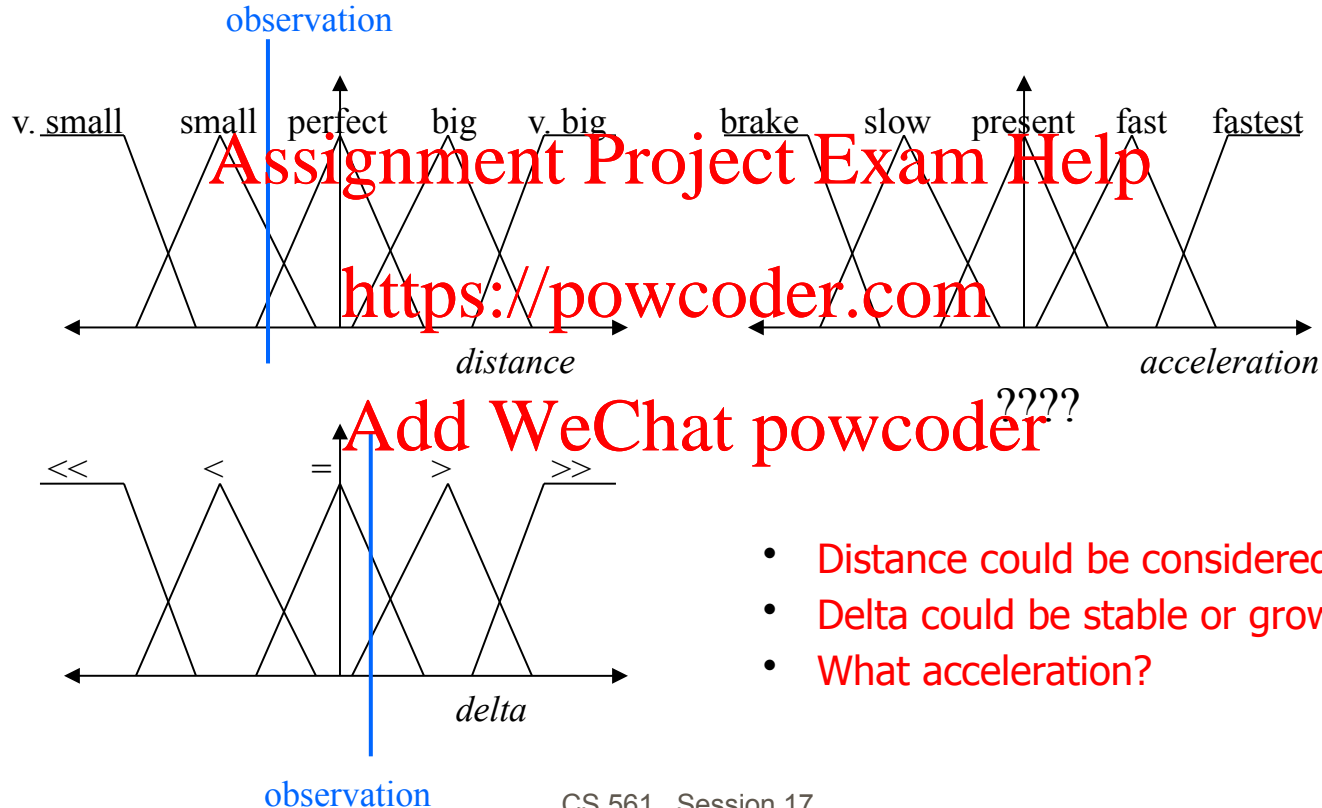
Fuzzy Rules

- Example: "If our distance to the car in front is small, and the distance is decreasing slowly, then decelerate quite hard."
  - Fuzzy variables in blue
  - Fuzzy sets in red

- *QUESTION:* Given the distance and the change in the distance, what acceleration should we select?

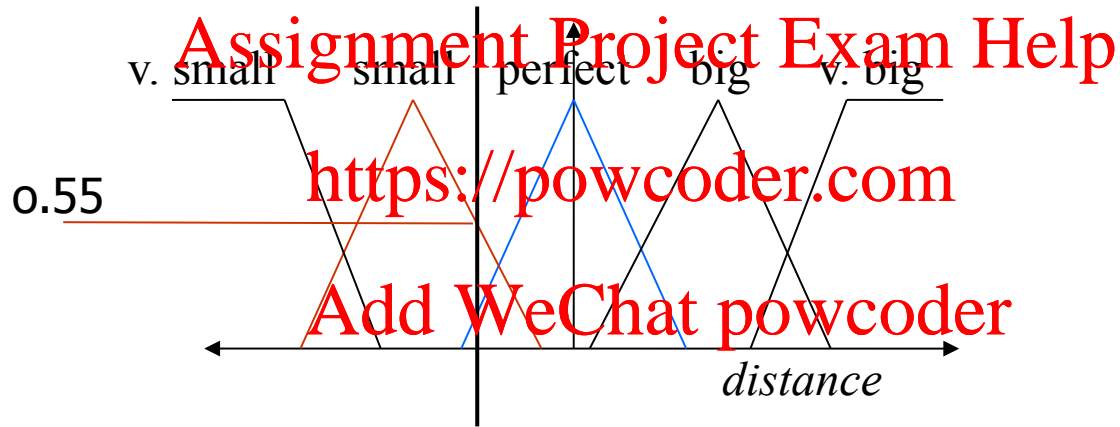v. small   small perfect   big   v. big   brake   slow   present fast   fastest

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distance   *acceleration*

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<<   <   =   >   >>

Delta (distance change)

Fuzzification: Instance

observation

v. small   small   perfect   big   v. big   brake   slow   present   fast   fastest

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distance   *acceleration*

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<< · < · = · > · >>

delta

- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

observation

Let's start with just one premise: fuzzification

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v. small   small   perfect   big   v. big

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0.55

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distance

IF distance is Small THEN Slow Down

Rule Evaluation

small

o.55
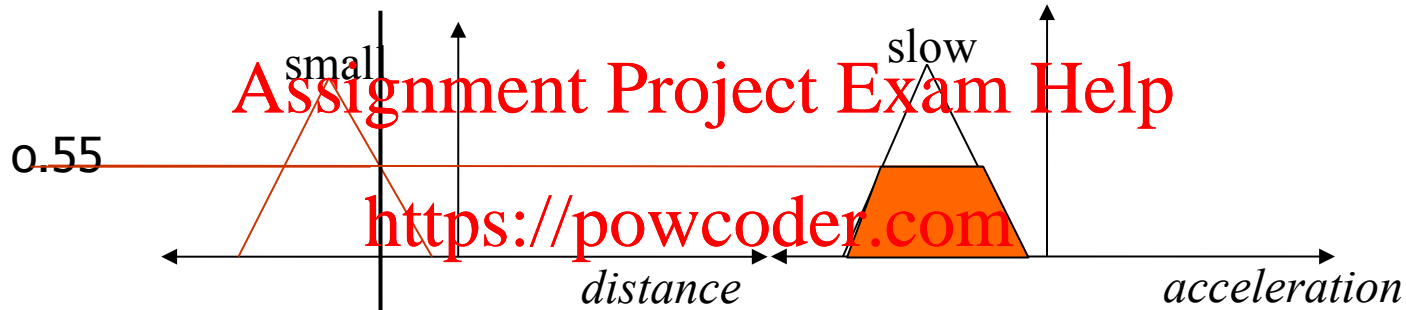
distance

brake slow present fast fastest

acceleration
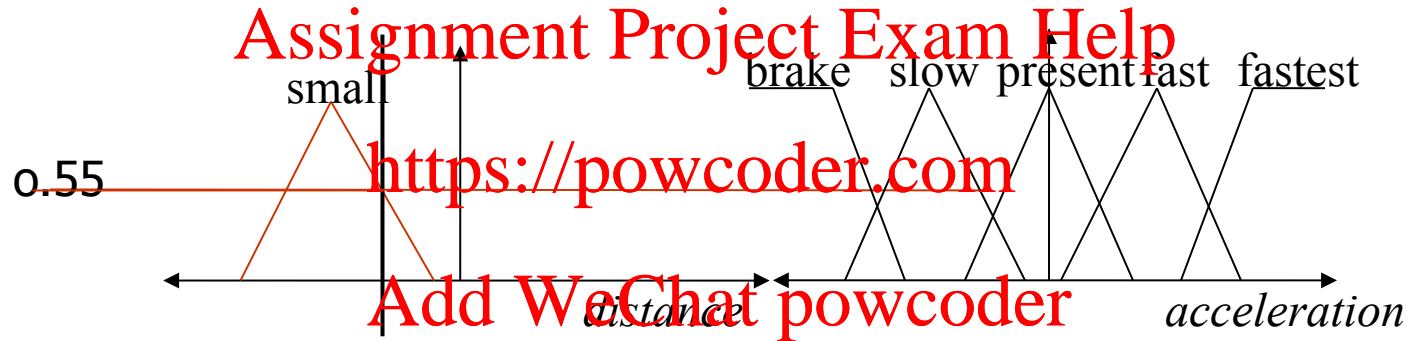
If distance is small, then slow down.
Question: What is the weight on slow down?

Rule Evaluation

Clipping approach (others are possible):

Clip the fuzzy set for "slow" (the consequent) at the height given by our belief in the premises (0.55)
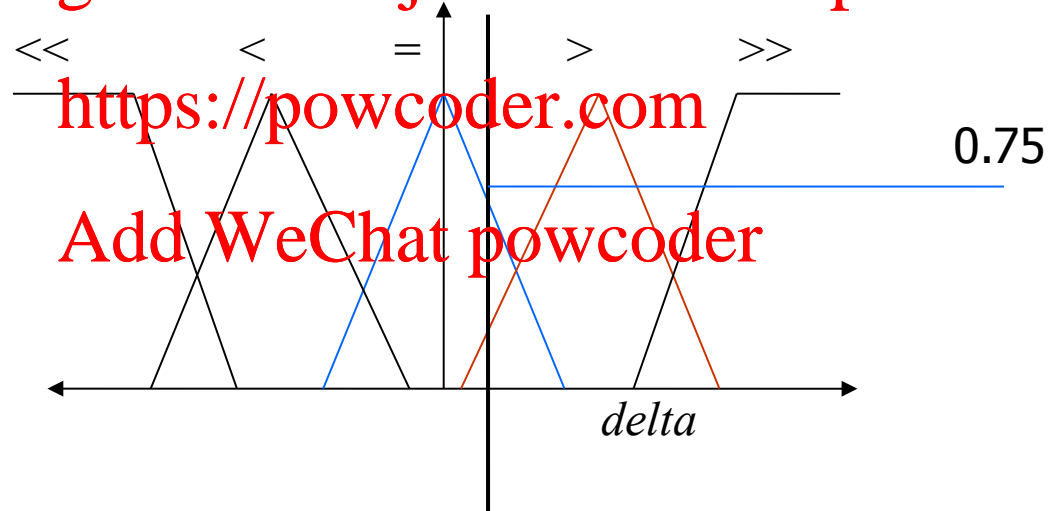
We will then consider the clipped AREA (orange) when making our final decision

Rationale: if belief in premises is low, clipped area will be very small
But if belief is high it will be close to the whole unclipped area
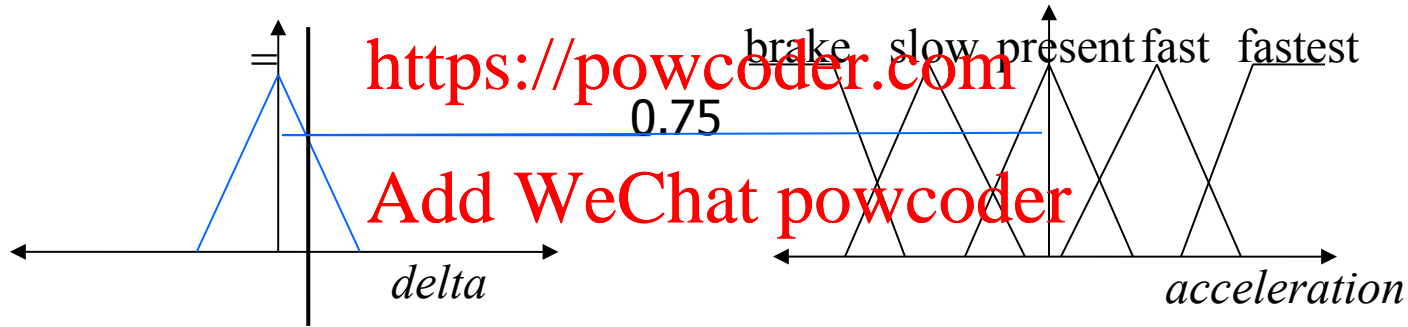
Another rule to evaluate: fuzzification

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<<   <   =   >   >>

0.75

delta

IF change in distance is = THEN keep present acceleration

Rule Evaluation

brake   slow present fast   fastest

0.75

delta

acceleration

IF change in distance is = THEN keep present acceleration

Rule Evaluation

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0.75

present

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delta
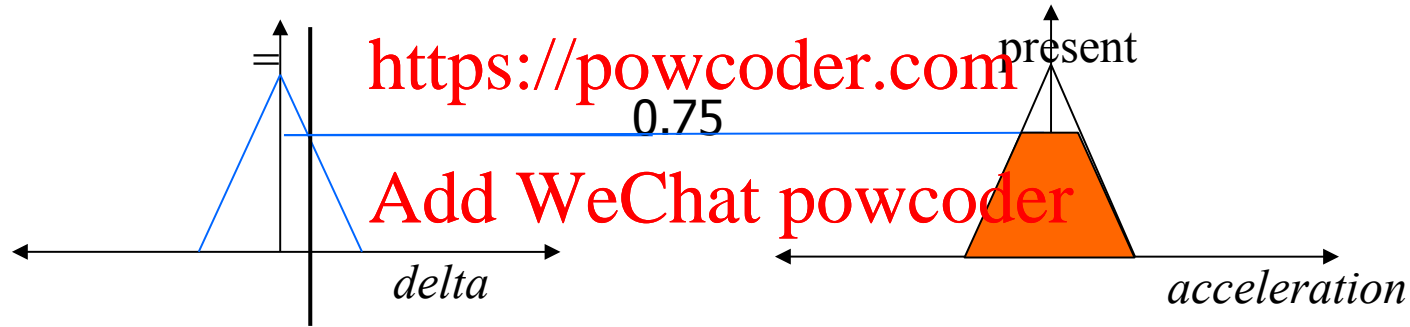
acceleration

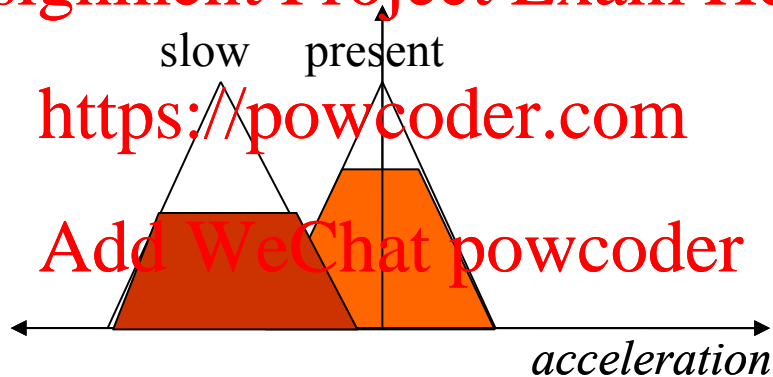IF change in distance is = THEN keep present acceleration

Rule Aggregation

How do we make a final decision? From each rule we have obtained a clipped area. But in the end we want a single number output: our desired acceleration
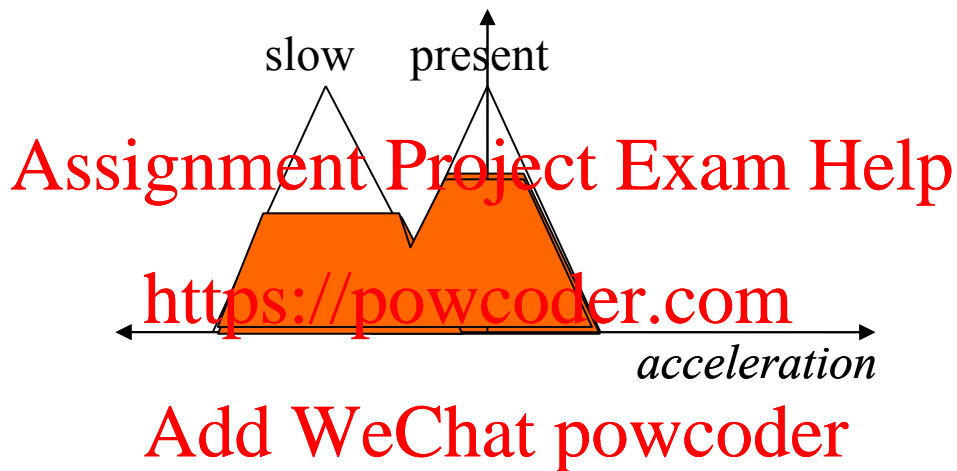
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slow   present

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acceleration

■ From distance

■ From delta (distance change)

Rule Aggregation

slow   present

acceleration

In the rule aggregation step, we merge all clipped areas into one (taking the union).

Intuition: rules for which we had a strong belief that their premises were satisfied will tend to "pull" that merged area towards their own central value, since their clipped areas will be large

Defuzzification

slow   present

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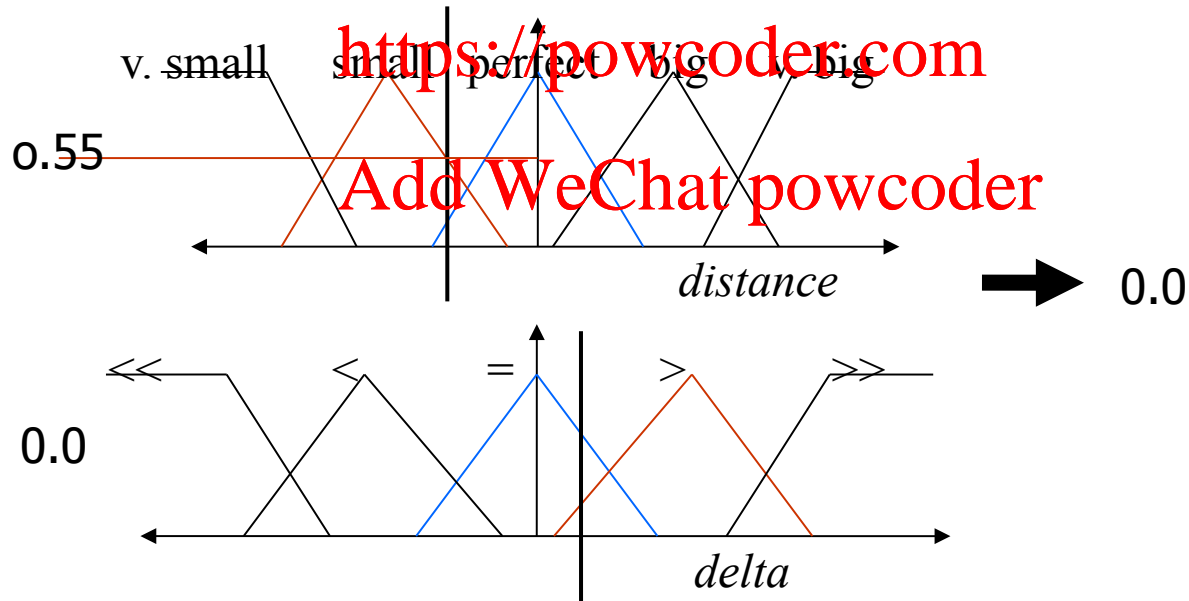
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-2.3m/s^2
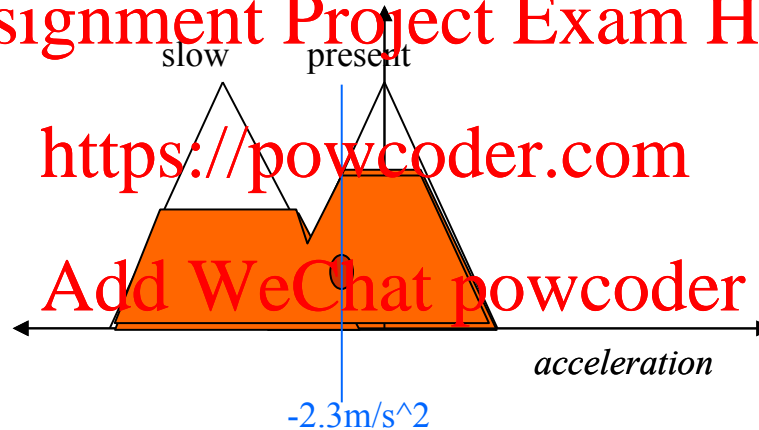
acceleration

In the last step, defuzzification, we return as our acceleration
Value the x coordinate of the center of mass of the merged area

AND/OR Example

- IF Distance Small **AND** change in distance (delta) negative THEN high deceleration

v. small   small   perfect   big   v. big

o.55

distance   ➡ 0.0

<<   <   =   >   >>

0.0

delta

AND/OR Example

- IF Distance Small **AND** change in distance (delta) = THEN slow deceleration

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v. small   small perfect   big   v. big

0.55

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distance ➡ 0.55

«   <   =   >   »

0.75

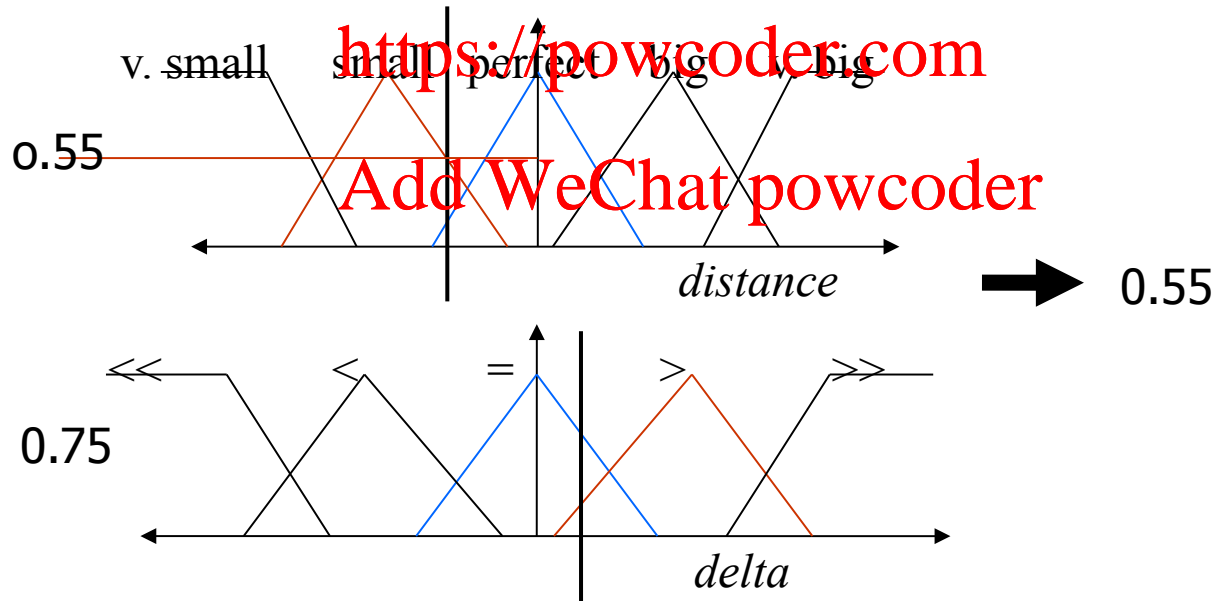*delta*

AND/OR Example

- IF Distance Small **AND** change in distance (delta) = THEN slow deceleration

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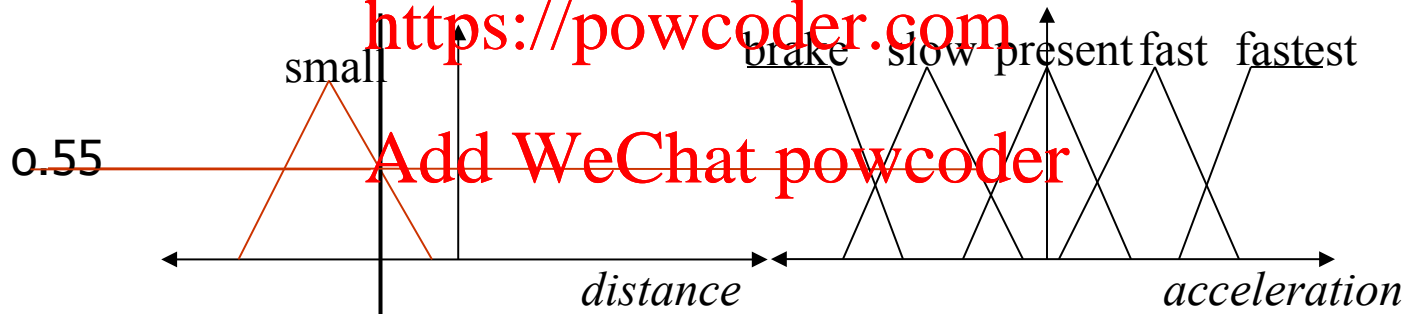
small
brake slow present fast fastest

o.55

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distance   *acceleration*
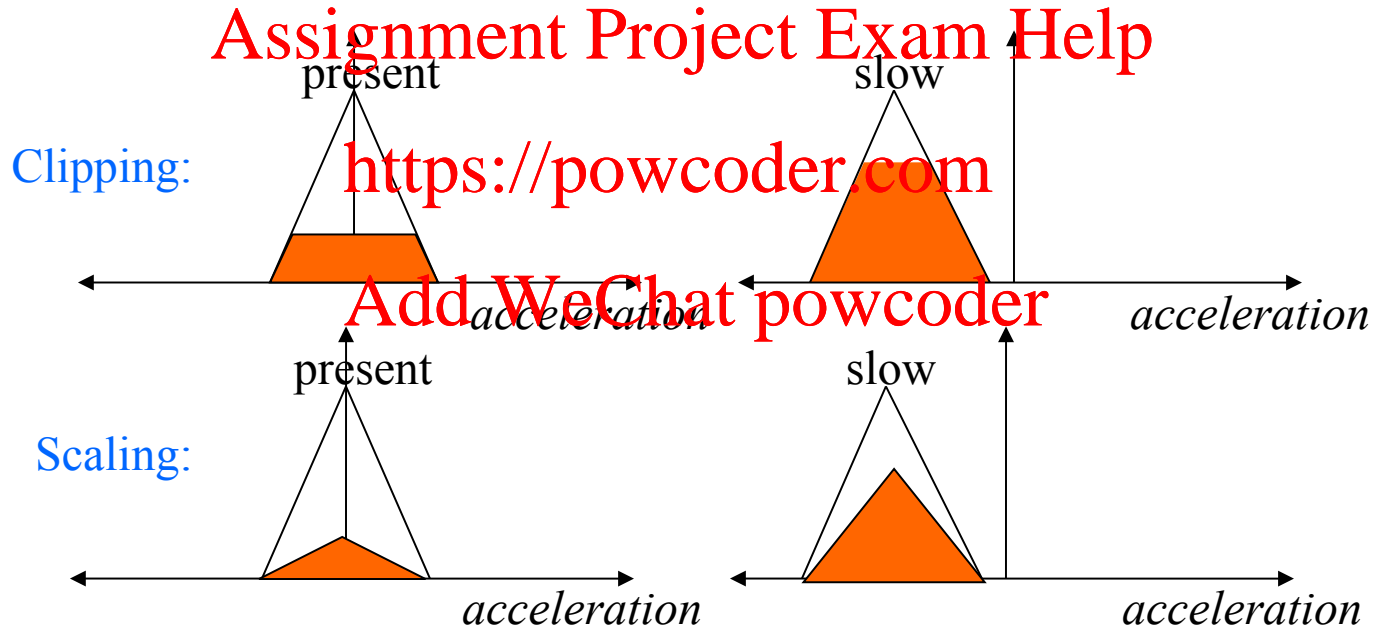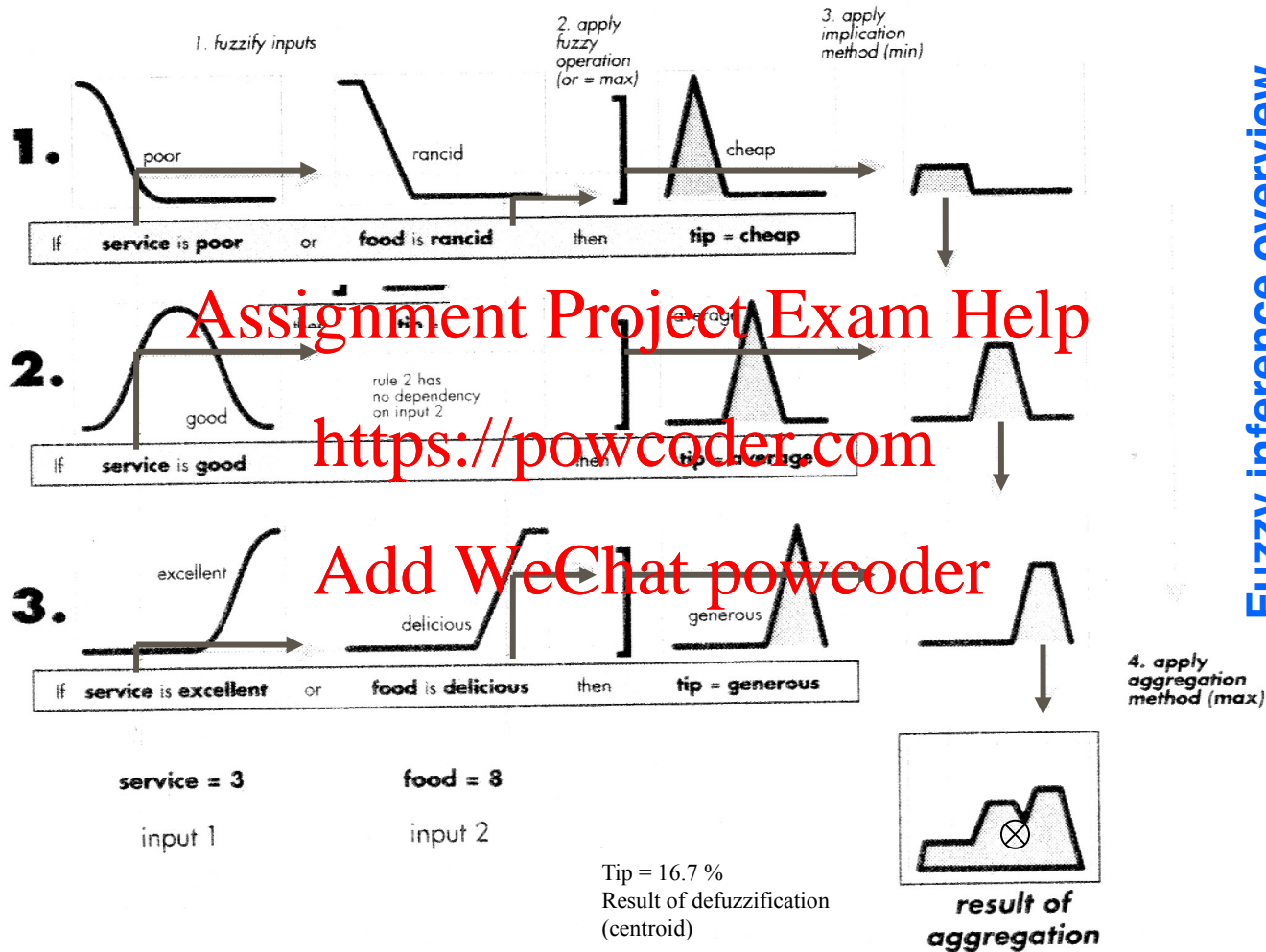
Scaling vs. Clipping

Instead of clipping, another approach is to scale the fuzzy set by the belief in the premises

1. fuzzify inputs

2. apply fuzzy operation (or = max)

3. apply implication method (min)

1.
poor → rancid → cheap

If **service** is **poor** or **food** is **rancid** then **tip = cheap**

2.
good
rule 2 has no dependency on input 2

If **service** is **good** then **tip = average**

3.
excellent → delicious → generous

If **service** is **excellent** or **food** is **delicious** then **tip = generous**

service = 3   **food = 8**

input 1   input 2

4. apply aggregation method (max)

Tip = 16.7 %
Result of defuzzification
(centroid)

result of aggregation

Fuzzy inference overview

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Limitations of fuzzy logic

- How to determine the membership functions? Usually requires fine-tuning of parameters

- Defuzzification can produce undesired results.

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Fuzzy tools and shells

- Matlab's Fuzzy Toolbox
- FuzzyClips
- Etc.

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