Distribution and Network Models

Sanjay Dominik Jena

Master of Business Administration

ESG UQÀM

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Overview of the presentation

- Definitions
  - Network
  - Flow
  - General Optimization Model
- Applications
  - Multi-period planning
    - General principles
    - Production planning basic case
    - Production planning with general deliveries
  - Logistics and transportation
    - Transportation problem

Definitions

Network

Network :

Defined using a **graph**, which is a structure defined as a set of nodes for which some pairs of the nodes are connected via arcs.

- Arc (i, j) : where i = initial node and j = terminal node
  $(i, j) \Rightarrow$ emerges (leaves) node i and is incident to (arrives) at node j
- Arc : defines a specific relationship between two nodes
  Examples
  - route linking intersection i to j
  - assignment of employee i to task j
  - renting a vehicle i to a client j
  - etc.

Definitions

Network

FIGURE – Visualization of social network analysis

Definitions

Flow

Flow :

In a network, flow refers to the units (e.g., goods, materials, people, etc.) that move on the arcs following their specific direction.

- Associated with each arc (i, j)
  - x_{ij} = the number of units (i.e., quantity of flow) that move along the arc (i, j)
  - c_{ij} = unit cost for the flow moving along (i, j)
  - Bounds on the quantity of flow associated with x_{ij}
    - u_{ij} = maximum quantity of flow that can move along arc (i, j)
    - l_{ij} = minimum quantity of flow that can move along arc (i, j)

  Therefore

  $$l_{ij} \leq x_{ij} \leq u_{ij}$$

Definitions
Flow

Flow (cont'd) :

- Associated with each node i :
  - $b(i)$ = demand value associated with i.

    There are three possible cases with respect to the demand values :
    - $b(i) > 0 \Rightarrow$ node i defines an origin for the flow (i.e., flow enters the network at this node).
    - $b(i) < 0 \Rightarrow$ node i defines a destination for the flow (i.e., flow leaves the network at this node).
    - $b(i) = 0 \Rightarrow$ node i is a transshipment node (i.e., flow simply transits at this node and remains within the network).

Remark

If the values $b(i)$, for all nodes i, are integer, then solution to the network flow problem will also be integer (i.e., without the need to impose the integrality requirements).

Definitions

General Optimization Model

- **Decision variables**
  - x_{ij} = number of units of flow that transit on arc (i,j)
- **Objective Function**

min Total cost incurred to distribute the flow through the network

$$\updownarrow$$

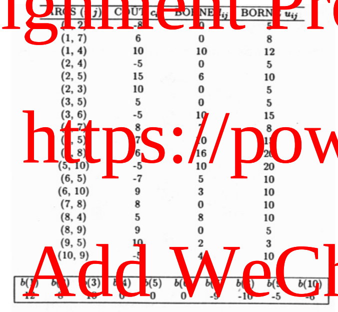
$$\min \sum_{\text{for all arcs in the network } (i,j)} c_{ij} x_{ij}$$

- **Subject to**
  - Flow conservation constraints at all nodes
    - For each node $i \rightarrow$ total flow on the arcs leaving i - total flow on the arcs arriving at $i = b(i)$
  - Bounds on the flow transiting through each arc
    - For each arc $(i,j) \rightarrow l_{ij} \leq x_{ij} \leq u_{ij}$

Definitions

General Optimization Model

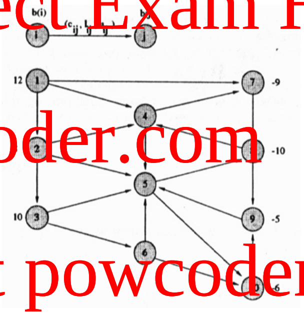
General Example :



ARCS (i, j)	COÛT c_{ij}	BORNE l_{ij}	BORNE u_{ij}
(, 2)	-8	[illegible]	5
(1, 7)	6	0	8
(1, 4)	10	10	12
(2, 4)	-5	0	5
(2, 5)	15	6	10
(2, 3)	10	0	5
(3, 5)	5	0	5
(3, 6)	-5	10	15
(, 7)	8	[illegible]	8
(, 5)	7	10	[illegible]
(, 8)	6	16	[illegible]
(5, 10)	-5	10	20
(6, 5)	-7	5	10
(6, 10)	9	3	10
(7, 8)	8	0	10
(8, 4)	5	8	10
(8, 9)	9	0	5
(9, 5)	10	2	3
(10, 9)	-5	4	10

$b(1)$	$b(2)$	$b(3)$	$b(4)$	$b(5)$	$b(6)$	$b(7)$	$b(8)$	$b(9)$	$b(10)$
12	8	10	0	0	0	-9	-10	-5	-6

(a) Costs, bounds, $b(i)$



(b) Network

FIGURE – Network flow problem

Applications

Multi-period planning

General Principles

Single period planning

Consists of decisional problems that occur for a single moment in a time horizon and that only considers the ressources available (supply) and state of the market (demand) for that particular moment.

Multi-period planning

Consists of decisional problems that occur over multiple moments in a time horizon and that explicitly take into account the dynamic by which available ressources and market conditions can evolve (i.e., change) through time.

Applications
Multi-period planning

General Principles (cont'd)

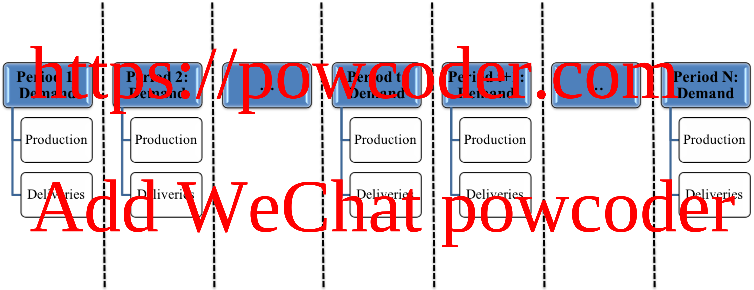


FIGURE – Single period planning process

Applications
Multi-period planning

General Principles (cont'd)

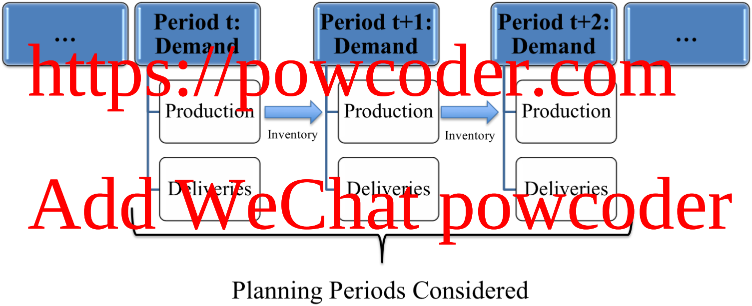


FIGURE – Multiple periods planning process

Applications

Multi-period planning

Production planning basic case : Pastissimo inc.

Pastissimo is an Italian company that specializes in the production of high-quality pasta for a variety of clients. The company has recently received an important order from one of its client, Hyper-Halli. Following this order, for the next 6 months, Pastissimo will deliver (in 1kg bag units) the spaghetti that is sold by Hyper-Halli as its own brand. Therefore at the end of each month, Pastissimo will deliver 4 tons of spaghetti to Hyper-Halli, which has agreed to pay 5.28$ per bag for these deliveries. The production of spaghetti requires the use of wheat. To ensure that enough wheat will be available, Pastissimo has negotiated a contract with a local producer. The details of the contract are provided in the following table :

Month	Price (in $/t.)	Minimum (in t.)	Maximum (in t.)
1	1000	4	6
2	975	3	4
3	1000	5	7
4	980	2	3
5	1020	4	7
6	1025	5	6

FIGURE – Contract with wheat producer

Applications
Multi-period planning

Production planning basic case : Pastissimo inc. (cont'd)

To store either the wheat that is bought from the producer or the spaghetti that is produced, Pastissimo has both a silo, where wheat can be stockpiled, and a warehouse, where the final products can be kept. At the beginning of month 1, the silo already has 2 tons of wheat and the company would like to keep the same amount at the end of the 6^{th} month. The silo can store up to 3 tons of wheat and the monthly storing cost is 20$/t. As for the store, its capacity is 1 ton of spaghetti and the monthly storing cost is 25$/t. To ensure that Pastissimo delivers the required amounts of spaghetti to Hyper-Halli for the next 6 months, the manager planned the production capacity and costs as follows :

Month	Production Capacity (in t.)	Production Costs (in $/t.)
1	6	160
2	5	150
3	4	150
4	4	160
5	4	175
6	3	165

FIGURE – Production capacity and costs

Applications

Multi-period planning

Production planning basic case : Pastissimo Inc. (cont'd)

Question

Pastissimo is interested in planning its operations to perform the order to Hyper-Halli for the next 6 months.

Therefore,

The company is looking to determine the following :

- Supply of wheat
- Inventory (wheat and spaghetti)
- Production levels

Applications

Multi-period planning

Production planning basic case : Pastissimo inc. (cont'd)

Network flow model

Nodes :

- Identify the Beginning and End of each month
  - $B_i, i = 1, \dots, 6$
  - $E_i, i = 1, \dots, 6$

Arcs :

- Supply of wheat : $\bullet \to B_i$
- Production : $B_i \to E_i$
- Storing wheat : $B_{i-1} \to B_i$
- Storing spaghettis : $E_{i-1} \to E_i$

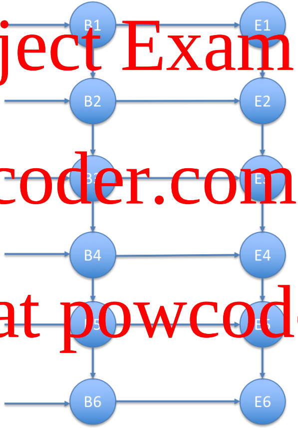
Applications

Multi-period planning

Arcs (i,j)	Costs c_{ij}	Bound l_{ij}	Bound u_{ij}
$(\bullet, B_1)$	1000	4	6
$(\bullet, B_2)$	975	3	4
$(\bullet, B_3)$	1000	5	[illegible]
$(\bullet, B_4)$	980	2	[illegible]
$(\bullet, B_5)$	1020	4	7
$(\bullet, B_6)$	1025	5	6
(B_1, E_1)	160	0	6
(B_2, E_2)	150	0	5
(B_3, E_3)	150	0	4
(B_4, E_4)	160	0	4
(B_5, E_5)	175	0	[illegible]
(B_6, E_6)	165	0	3
(B_1, B_2)	20	0	3
(B_2, B_3)	20	0	3
(B_3, B_4)	20	0	3
(B_4, B_5)	20	0	3
(B_5, B_6)	20	0	3
(E_1, E_2)	2[illegible]	0	1
(E_2, E_3)	25	0	1
(E_3, E_4)	25	0	1
(E_4, E_5)	25	0	1
(E_5, E_6)	25	0	1

$b(B_1)$	$b(B_2)$	$b(B_3)$	$b(B_4)$	$b(B_5)$	$b(B_6)$
2	0	0	0	0	-2
$b(E_1)$	$b(E_2)$	$b(E_3)$	$b(E_4)$	$b(E_5)$	$b(E_6)$
-4	-4	-4	-4	-4	-4

(a) Costs, bounds, $b(i)$



(b) Network

FIGURE – Network flow for Pastissimo

Applications

Multi-period planning

Decision variables

- $X_{\bullet B_i}$ = number of t. of wheat that are bought at the beginning of month i, where $i = 1, 2, \ldots, 6$
- $X_{B_i B_{i+1}}$ = number of t. of wheat that is stored in the silo from the beginning of month i to the beginning of month $i+1$, where $i = 1, 2, \ldots, 5$
- $X_{B_i E_i}$ = number of t. of spaghetti produced during the month i, where $i = 1, 2, \ldots, 6$
- $X_{E_i E_{i+1}}$ = number of t. of spaghetti that are stored in the warehouse from the end of month i to the end of month $i+1$, where $i = 1, 2, \ldots, 5$

Applications

Multi-period planning

Objective function

- $\min 1000X_{\bullet B_1} + 975X_{\bullet B_2} + \ldots + 20(X_{B_1B_2} + X_{B_2B_3} + \ldots + X_{B_5B_6}) + 160X_{B_1E_1} + 150X_{B_2E_2} + \ldots + 25(X_{E_1E_2} + X_{E_2E_3} + \ldots + X_{E_5E_6})$

Subject to

- Flow conservation constraints at all nodes

  For example
  - Node $B_4 \rightarrow X_{B_4B_5} + X_{B_4E_4} - X_{\bullet B_4} - X_{B_3B_4} = 0$
  - Node $E_2 \rightarrow X_{E_2E_3} - X_{B_2E_2} - X_{E_1E_2} = -4$
  - etc.

- Bounds on the arcs

  For example
  - $4 \leq X_{\bullet D_1} \leq 6$
  - $0 \leq X_{D_5F_5} \leq 4$
  - $0 \leq X_{F_2F_3} \leq 1$
  - etc.

Applications

Multi-period planning

Production planning with general deliveries

Assuming the Pastissimo renegotiates its distribution contract with Hyper-Halli, which now accepts advanced deliveries, or, late deliveries. Specifically, the new contract allows the following delivery options :

- Late deliveries by one month can be accepted by Hyper-Halli, provided that Pastissimo pays a fee of 35$/t. for all spaghetti that is delivered late.
- Advanced deliveries by one or two moths can be accepted by Hyper-Halli, provided that Pastissimo pays a fee of either 14$/t. or 17$/t. for all spaghetti that is delivered in advance by one and two months, respectively.

In addition, each time a bag of spaghetti is delivered to Hyper-Halli, Pastissimo pays a cost of 0.05$/kg. in transportation fees.

Question : How can the previous network flow model be adjusted to represent this new situation ?

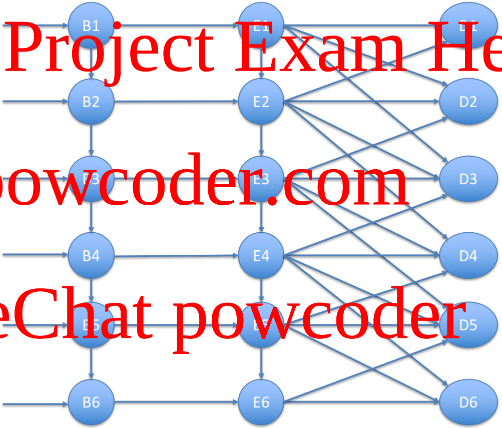
Applications

Multi-period planning

Arcs (i,j)	Costs c_{ij}	Bound l_{ij}	Bound u_{ij}
⋮	⋮	⋮	⋮
(E_1, D_1)	50	0	-
(E_1, D_2)	64	0	-
(E_1, D_3)	67	0	-
(E_2, D_1)	85	0	-
(E_2, D_2)	50	0	-
(E_2, D_3)	64	0	-
(E_2, D_4)	67	0	-
(E_3, D_2)	85	0	-
(E_3, D_3)	50	0	-
(E_3, D_4)	64	0	-
(E_3, D_5)	67	0	-
(E_4, D_3)	85	0	-
(E_4, D_4)	50	0	-
(E_4, D_5)	64	0	-
(E_4, D_6)	67	0	-
(E_5, D_4)	85	0	-
(E_5, D_5)	50	0	-
(E_5, D_6)	64	0	-
(E_6, D_5)	85	0	-
(E_6, D_6)	50	0	-

$b(B_1)$	$b(B_2)$	$b(B_3)$	$b(B_4)$	$b(B_5)$	$b(B_6)$
2	0	0	0	0	-2
$b(E_1)$	$b(E_2)$	$b(E_3)$	$b(E_4)$	$b(E_5)$	$b(E_6)$
0	0	0	0	0	0
$b(D_1)$	$b(D_2)$	$b(D_3)$	$b(D_4)$	$b(D_5)$	$b(D_6)$
-4	-4	-4	-4	-4	-4

(a) Costs, bounds, $b(i)$



(b) Network

FIGURE – Network flow for Pastissimo with generalized deliveries

Applications

Logistics and transportation

General Context

Transportation is at the heart of logistics operations and one of the major drivers of economic activities. People and goods need to be efficiently moved throughout the world in order for societies and economies to function and thrive.

Transportation Problem

Base case :

- Adopting the point of view of either the shipper or the receiver
- Specific detailed routes are not considered
- Service from origin-destination and the overall cost is important

Applications

Logistics and transportation

Transportation Problem (cont'd)

Consider the problem of a company who needs to supply its warehouses with finished products that are then distributed to clients. The products are produced at a series of plants and, at the end of each month, they are transported towards the different warehouses of the company.

For the next month, the company needs to perform the following operations :

Plants		
Chicago	Kansas City	Houston
120 u.	80 u.	80 u.
Warehouses		
New York	Atlanta	Los Angeles
150 u.	60 u.	70 u.

Applications

Logistics and transportation

Transportation Problem (cont'd)

The cost of shipping between cities is usually a function of, given a set of possible services, min distances between the cities $\times$ a tarif per unit.

Assuming that the following unit costs (i.e. \$/unit) apply :

	New York	Atlanta	Los Angeles
Chicago	8	6	5
Kansas City	15	12	10
Houston	3	10	9

Question : How can this problem be formulated as a network flow problem ?

Applications

Logistics and transportation

FIGURE – Illustration of the transportation problem

Applications

Logistics and transportation

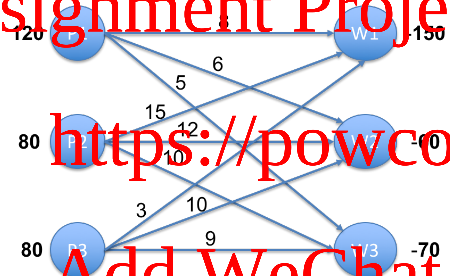
Defining the network

Nodes :

- 2 types of nodes (Plants and Warehouses) :
  - P_j, where $j = 1 \rightarrow$ Chicago, $j = 2 \rightarrow$ Kansas City, $j = 1 \rightarrow$ Houston
  - W_i, where $i = 1 \rightarrow$ New York, $i = 2 \rightarrow$ Atlanta, $i = 3 \rightarrow$ Los Angeles
- Arcs represent the transportation of units
  - Plants $\rightarrow$ Warehouses

Applications

Logistics and transportation



(a) Transportation problem network

$$\min z = 8x_{P1W1} + 6x_{P1W2} + 5x_{P1W3} + 15x_{P2W1} + 12x_{P2W2} + 10x_{P2W3} + 3x_{P3W1} + 10x_{P3W2} + 9x_{P3W3}$$

s.t.

$$x_{P1W1} + x_{P1W2} + x_{P1W3} = 120$$
$$x_{P2W1} + x_{P2W2} + x_{P2W3} = 80$$
$$x_{P3W1} + x_{P3W2} + x_{P3W3} = 80$$
$$-x_{P1W1} - x_{P2W1} - x_{P3W1} = -150$$
$$-x_{P1W2} - x_{P2W2} - x_{P3W2} = -60$$
$$-x_{P1W3} - x_{P2W3} - x_{P3W3} = -70$$
$$x_{ij} \geq 0$$

(b) Optimization model

FIGURE – Network flow - Transportation problem

Applications

Logistics and transportation

Assignment problem

The assignment problem is a special case of the transportation problem.

Definition

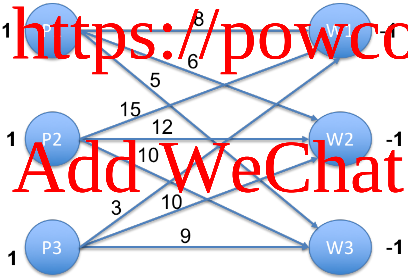
The problem instance has a number of agents and a number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task and exactly one task to each agent in such a way that the total cost of the assignment is minimized.

Applications

Logistics and transportation

Assignment problem (cont'd)

Assuming that in the previous context, the company was looking to assign a single production plant to a single warehouse to perform the necessary supply activities by simply considering the unit transportation costs.



(a) Assignment problem

$$
\begin{aligned}
\min z = {} & 8x_{P1W1} + 6x_{P1W2} + 5x_{P1W3} \\
& + 15x_{P2W1} + 12x_{P2W2} + 10x_{P2W3} \\
& + 3x_{P3W1} + 10x_{P3W2} + 9x_{P3W3} \\
\text{s.t.} & \\
& x_{P1W1} + x_{P1W2} + x_{P1W3} = 1 \\
& x_{P2W1} + x_{P2W2} + x_{P2W3} = 1 \\
& x_{P3W1} + x_{P3W2} + x_{P3W3} = 1 \\
& -x_{P1W1} - x_{P2W1} - x_{P3W1} = -1 \\
& -x_{P1W2} - x_{P2W2} - x_{P3W2} = -1 \\
& -x_{P1W3} - x_{P2W3} - x_{P3W3} = -1 \\
& x_{ij} \geq 0
\end{aligned}
$$

(b) Optimization model

FIGURE – Network flow - Assignment problem

Applications

Logistics and transportation

Assignment problem (cont'd)

Intercity truck transportation

Distances (km) :

	Loads						
Trucks	1 NY	2 NY	3 Dover	4 Paterson	5 Flemington	6 Easton	7 Newton
1 Scranton	229	229	139	176	146	116	125
2 Honesdale	212	212	114	155	153	123	91
3 Franklin	111	111	32	54	108	81	25
4 Edison	62	62	69	68	46	81	82
5 Princeton	92	92	84	95	33	88	89
6 Warwick	116	116	62	69	130	111	44
7 Newark	54	54	43	26	80	101	76

Question :

How should the company proceed to solve this transportation problem ?

Applications

Logistics and transportation

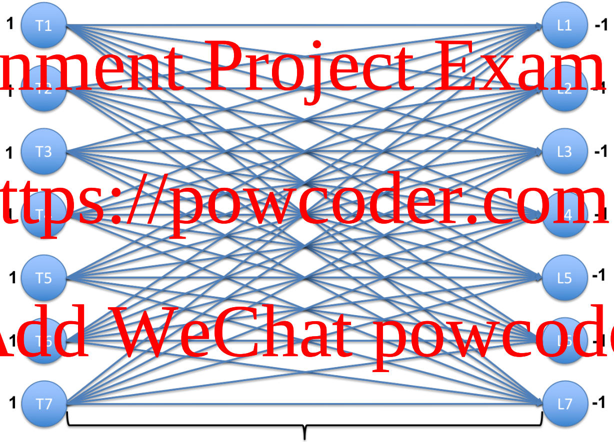


FIGURE – Network - Intercity truck transportation problem