Sequences

Class outline:

- Box+Pointer
- Slicing
- Recursive exercises
- Built-ins for iterables

Where to ask questions?

- **Zoom chat**: Good if you like getting responses from classmates or the lecture helper.
- **Zoom Q&A**: Good for asking questions that likely interest most students, and that should be answered in lecture.
- **Post-lecture OH**: Good for recapping a topic that went too fast. Or any questions!
- **Piazza thread**: Good for longer questions, tangential questions, or any unanswered questions.

Box + Pointer

Lists in environment diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element.

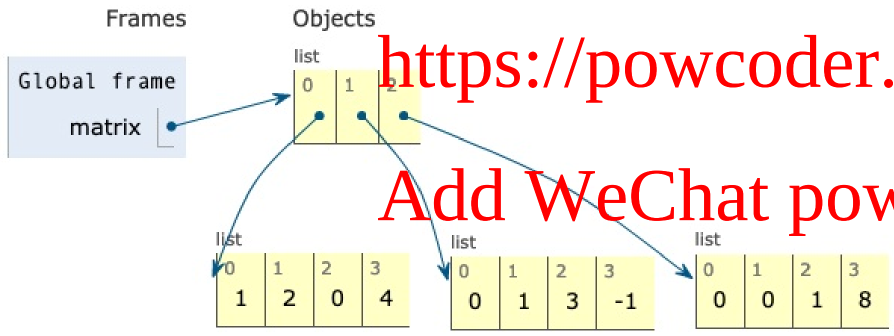
```
pair = [1, 2]
```

Try in PythonTutor.

Nested lists in environment diagrams

Each box either contains a primitive value or points to a compound value.

```
matrix = [ [1,2,0,4], [0,1,3,-1], [0,0,1,8] ]
```



Nested lists in environment diagrams

A very nested list:

```
worst_list = [ [1, 2],
               [],
               [ [3, False, None],
                 [4, lambda: 5]]]
```

View in PythonTutor

Slicing

Slicing syntax

Slicing a list creates a new list with a subsequence of the original list.

```
letters = ["A", "B", "C", "D", "E", "F"]
        #   0    1    2    3    4    5

sublist1 = letters[1:]
sublist2 = letters[1:4]
```

Slicing also works for strings.

```
compound_word = "cortaúñas"

word1 = compound_word[:5]
word2 = compound_word[5:]
```

Negatives indices and steps can also be specified.

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```

Negatives indices and steps can also be specified.

Copying whole lists

Slicing a whole list copies a list:

```
listA = [2, 3]
listB = listA

listC = listA[:]
listA[0] = 4
listB[1] = 5
```

`list()` creates a new list containing existing elements from any iterable:

```
listA = [2, 3]
listB = listA

listC = list(listA)
listA[0] = 4
listB[1] = 5
```

Try both in PythonTutor.

Python3 provides more ways in the copy module.

Recursion exercises

Recursively sum a list

Let's code this up recursively:

```
def sum_nums(nums):
    """Returns the sum of the numbers in NUMS.
    >>> sum_nums([6, 24, 1984])
    2014
    >>> sum_nums([-32, 0, 32])
    0
    """
```

Docstrings typically would not specify whether an approach was recursive or iterative, since that is an implementation detail.

However, we'll make it clear in assignments and exam questions.

Recursively sum a list (solution)

```
def sum_nums(nums):
"""Returns the sum of the numbers in NUMS.
>>> sum_nums([6, 24, 1984])
2014
>>> sum_nums([-32, 0, 32])
0
"""
if (nums == []):
    return 0
else:
    return nums[0] + sum_nums( nums[1:] )
```

When recursively processing lists, the base case is often the empty list and the recursive case is often all-but-the-first items.

Iteratively sum a range

Let's code this up iteratively:

```
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusiv
    >>> sum_up_to(5)
    15
    """
```

Iteratively sum a range (solution)

Using the `range` type:

```
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusi
    >>> sum_up_to(5)
    15
    """
    sum = 0
    for n in range(0, n+1):
        sum += n
    return sum
```

Remember that `range(start, end)` always ends right before `end`.

Recursively sum a range

Now try it recursively:

```
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusiv
    >>> sum_up_to(5)
    15
    """
```

Recursively sum a range (solution)

Now try it recursively:

```
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusiv
    >>> sum_up_to(5)
    15
    """
    if n == 1:
        return 1
    else:
        return n + sum_up_to(n-1)
```

Reversing a string

Recursively reversing a string

```
def reverse(s):
    """Returns a string with the letters of S
    in the inverse order.
    >>> reverse('ward')
    'draw'
    """
```

Breaking it down into subproblems:

```
reverse("ward") =
reverse("ard") =
reverse("rd") =
reverse("d") =
```

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Breaking it down into subproblems:

```
reverse("ward") = reverse("ard") + "w"
reverse("ard") = reverse("rd") + "a"
reverse("rd") = reverse("d") + "r"
reverse("d") =
```

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reverse("ward") = reverse("ard") + "w"
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reverse("rd") = reverse("d") + "r"
reverse("d") = "d"
```
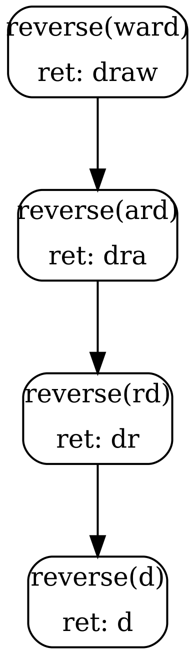

Recursively reversing a string (solution)

```
def reverse(s):
    """Returns a string with the letters of S
    in the inverse order.
    >>> reverse('ward')
    'draw'
    """
    if len(s) == 1:
        return s
    else:
        return reverse(s[1:]) + s[0]
```

When recursively processing strings, the base case is typically an empty string or single-character string, and the recursive case is often all-but-the-first characters.

Recursively reversing a string (visual)



< Prev > Next

Exercise: Reversing a number

```
def reverse(n):
    """Returns N with the digits reversed.
    >>> reverse_digits(123)
    321
    """
```

See walkthrough video here

Helper functions

If a recursive function needs to keep track of more state than the arguments of the original function, you may need a helper function.

```
def fUnKyCaSe(text):
    """Returns TEXT in fUnKyCaSe
    >>> fUnKyCaSe("wats up")
    'wAtS Up'
    """
```

Helper functions

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```
def fUnKyCaSe(text):
    """Returns TEXT in fUnKyCaSe
    >>> fUnKyCaSe("wats up")
    'wAtS Up'
    """

    def toggle_case(letter, should_up_case):
    return letter.upper() if should_up_case else letter.lower()

    def up_down(text, should_up_case):
        if len(text) == 1:
            return toggle_case(text, should_up_case)
        else:
            return toggle_case(text[0], should_up_case) + up_down(text[1:], not should_up

    return up_down(text, False)
```

Recursion on different data types

Data type	Base case condition	Current item	Recursive case argument

Numbers	`== 0`
	`== 1`
		`n % 10`	`n // 10`

Lists	`== []`	`L[0]`	`L[1:]`
			`L[:-1]`

Strings	`== ''`
	`len(S) == 1`
		`S[0]`	`S[1:]`
			`S[:-1]`

Built-in functions for iterables

Functions that process iterables

The following built-in functions work for sequence types (lists, strings, etc) and any other **iterable** data type.

Function	**Description**
`sum(iterable, start)`	Returns the sum of values in `iterable`, initializing sum to `start`
`all(iterable)`	Return `True` if all elements of `iterable` are true (or if `iterable` is empty)
`any(iterable)`	Return `True` if any element of `iterable` is true. Return `False` if `iterable` is empty.
`max(iterable, key=None)`	Return the max value in `iterable`
`min(iterable, key=None)`	Return the min value in `iterable`

Examples with sum/any/all

```
sum([73, 89, 74, 95], 0)  # 331
```

```
all([True, True, True, True])
any([False, False, False, True])

all([x < 5 for x in range(5)])

perfect_square = lambda x: x == round(x ** 0.5) ** 2
any([perfect_square(x) for x in range(50, 60)])
```

Examples with sum/any/all

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sum([73, 89, 74, 95], 0)  # 331
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```python
all([True, True, True, True])     # True
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Examples with max/min

```
max([73, 89, 74, 95])         # 95
max(["C+", "B+", "C", "A"])
max(range(10))
```

Examples with max/min

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max([73, 89, 74, 95])          # 95
max(["C+", "B+", "C", "A"])    # C+
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Examples with max/min

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max([73, 89, 74, 95])          # 95
max(["C+", "B+", "C", "A"])    # C+
max(range(10))                 # 9
```

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max([73, 89, 74, 95])          # 95
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```

A key function can decide how to compare each value:

```
coords = [ [37, -144], [-22, -115], [56, -163] ]
max(coords, key=lambda coord: coord[0])
min(coords, key=lambda coord: coord[0])
```

```
gymnasts = [ ["Brittany", 9.15, 9.4, 9.3, 9.2],
     ["Lea", 9, 8.8, 9.1, 9.5],
     ["Maya", 9.2, 8.7, 9.2, 8.8] ]
min(gymnasts, key=lambda scores: min(scores[1:]))
max(gymnasts, key=lambda scores: sum(scores[1:], 0))
```

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max(gymnasts, key=lambda scores: sum(scores[1:], 0)) # ["Brittany",
```
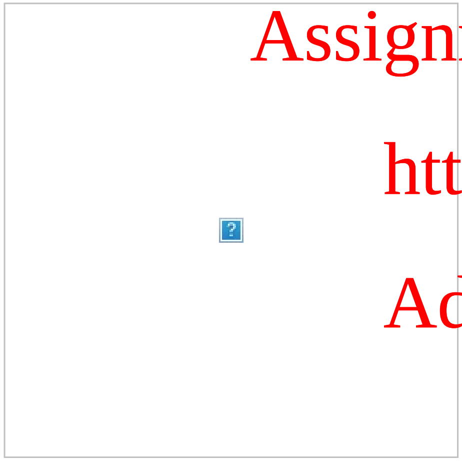

Python Project of The Day!

Sea Level Rise

Sea Level Rise, by Douwe Osinga: Visualize sea levels and population density on interactive maps.



Technologies used: Python (notebook) with PIL/numpy/Rasterio, HTML/CSS/JS with PanZoom
(Github repository)