Trees

Class outline:

- Hog winners
- Trees

Hog winners

Hog strategy contest

hog-contest.cs61a.org

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At first, there was a 3-way tie for first:
Nishant Bhakar, Toby Worledge, Asrith Devalaraju & Aayush Gupta

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Nishant Bhakar, Toby Worledge, Asrith Devalaraju & Aayush Gupta

Then we fixed a bug:
1) Nishant Bhakar, 2) Toby Worledge, 3) Jiayin Lin & Roger Yu

Hog dice contest

dice.cs61a.org

Much ❤ for all the entries!

Place	Caption	Authors

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Place	Caption	Authors
Third	Super Piggy World	Taylor Moore

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Hog dice contest

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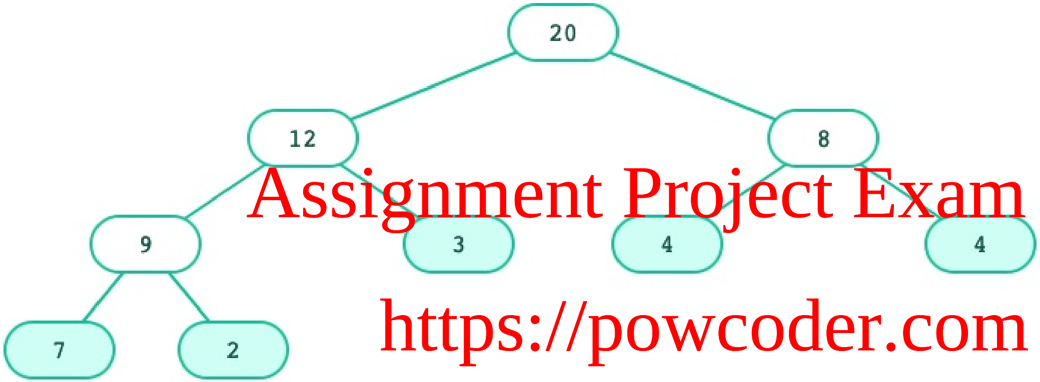
Much ❤ for all the entries!

Place	Caption	Authors
Third	Super Piggy World	Taylor Moore
Second	xlb piggies	Michelle Wu, Kevin Xu
First	based on our true story	Bella Lee, Dayeon Jang

Trees

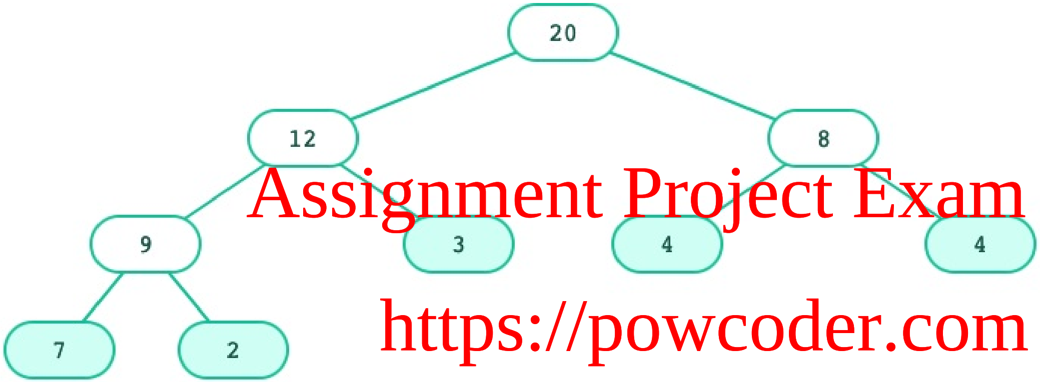
Trees



Recursive description

- A tree has a **root label** and a list of **branches**
- Each **branch** is itself a tree
- A tree with zero branches is called a **leaf**
- A tree starts at the **root**

Trees



Recursive description

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- A tree starts at the **root**

Relative description

- Each location in a tree is called a **node**
- Each node has a **label** that can be any value
- One node can be the **parent/child** of another
- The top node is the **root node**

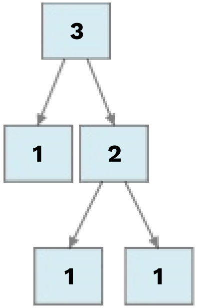
Trees: Data abstraction

We want this constructor and selectors:

`tree(label, branches)`	Returns a tree with root `label` and list of `branches`
`label(tree)`	Returns the root label of `tree`
`branches(tree)`	Returns the branches of `tree` (each a tree).
`is_leaf(tree)`	Returns true if `tree` is a leaf node.

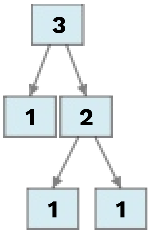

```
t = tree(3, [
          tree(1),
          tree(2, [
            tree(1),
            tree(1)
        ])])

label(t)   # 3
is_leaf(branches(t)[0])   # True
```


3
1
2
1
1

Tree: Our implementation

```
t = tree(3, [
          tree(1),
          tree(2, [
            tree(1),
            tree(1)
        ])])
```

Each tree is stored as a list where first element is label and subsequent elements are branches.

```
[3, [1], [2, [1], [1]]]
```

```
def tree(label, branches=[]):
        return [label] + list(branches)

def label(tree):
        return tree[0]
```

```
def branches(tree):
        return tree[1:]

def is_leaf(tree):
        return len(branches(tree)) == 0
```

Tree processing

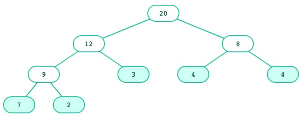
A tree is a recursive structure.

Each tree has:

- A label
- 0 or more branches, each a tree

Recursive structure implies recursive algorithm!

Tree processing: Counting leaves

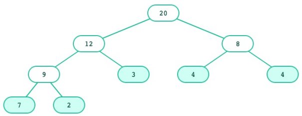



```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if

    else:
```

What's the base case? What's the recursive call?

Tree processing: Counting leaves




```
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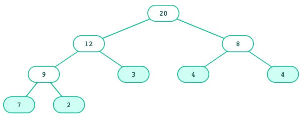
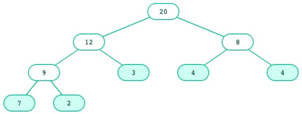



```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
```

What's the base case? What's the recursive call?

Tree processing: Counting leaves




```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
        leaves_under = 0
        for b in branches(t):
            leaves_under += count_leaves(b)
        return leaves_under
```

What's the base case? What's the recursive call?

Tree processing: Counting leaves

The `sum()` function sums up the items of an iterable.

```
sum([1, 1, 1, 1])    # 4
```

Tree processing: Counting leaves

The `sum()` function sums up the items of an iterable.

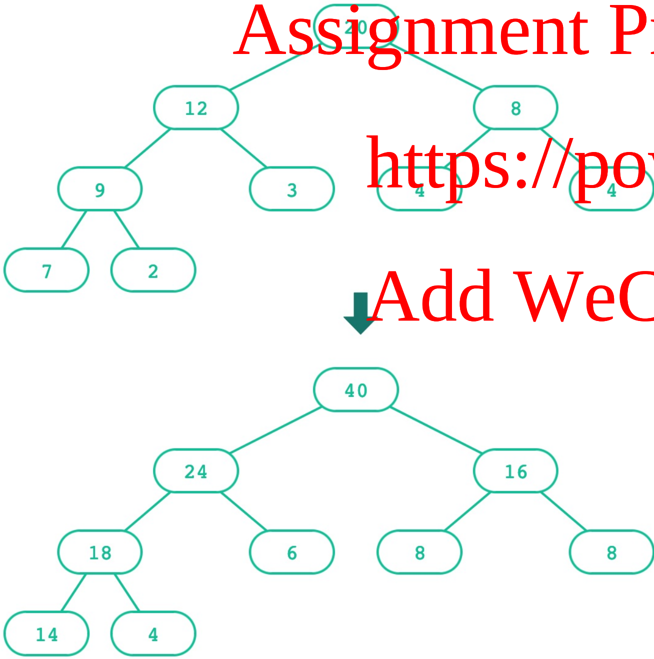
```
sum([1, 1, 1, 1])    # 4
```

That leads to this shorter function:


```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
        branch_counts = [count_leaves(b) for b in branches(t)]
        return sum(branch_counts)
```

Creating trees

A function that creates a tree from another tree is also often recursive.



Creating trees: Doubling labels

```
def double(t):
    """Returns a tree identical to T, but with all la
    if

    else:
```


What's the base case? What's the recursive call?

Creating trees: Doubling labels

```
def double(t):
    """Returns a tree identical to T, but with all la
    if is_leaf(t):

    else:
```


What's the base case? What's the recursive call?

Creating trees: Doubling labels

```
def double(t):
    """Returns a tree identical to T, but with all la
    if is_leaf(t):
        return tree(label(t) * 2)
    else:
```


What's the base case? What's the recursive call?

Creating trees: Doubling labels


```
def double(t):
    """Returns a tree identical to T, but with all la
    if is_leaf(t):
        return tree(label(t) * 2)
    else:
        return tree(label(t) * 2,
                [double(b) for b in branches(t)])
```


What's the base case? What's the recursive call?

Creating trees: Doubling labels

A shorter solution:

```
def double(t):
    """Returns the number of leaf nodes in T."""
    return tree(label(t) * 2,
                [double(b) for b in branches(t)])
```


Explicit base cases aren't always necessary in the final code, but it's useful to think in terms of base case vs. recursive case when learning.

Exercise: Printing trees

```
def print_tree(t, indent=0):
    """Prints the labels of T with depth-based indent.
    >>> t = tree(3, [tree(1), tree(2, [tree(1), tree(1)])])
    >>> print(t)
    3
      1
      2
        1
        1
    """
```

Exercise: Printing trees (solution)

```
def print_tree(t, indent=0):
    """Prints the labels of T with depth-based indent.
    >>> t = tree(3, [tree(1), tree(2, [tree(1), tree(1)])])
    >>> print(t)
    3
      1
      2
        1
        1
    """
    print(indent * " " + label(t))
    for b in branches(t):
        print_tree(t, indent + 2)
```

Exercise: List of leaves

```
def leaves(t):
    """Return a list containing the leaf labels of T.
    >>> t = tree(20, [tree(12, [tree(9, [tree(7), tree(2)]), tree(
    >>> leaves(t)
    [7, 2, 3, 4, 4]
    """
```


Hint: If you sum a list of lists, you get a list containing the elements of those lists. The sum function takes a second argument, the starting value of the sum.


```
sum([ [1], [2, 3], [4] ], []) # [1, 2, 3, 4]
sum([ [1] ], []) # [1]
sum([ [[1]], [2] ], []) # [[1], 2]
```

Exercise: List of leaves (Solution)

```
def leaves(t):
    """Return a list containing the leaf labels of T.
    >>> t = tree(20, [tree(12, [tree(9, [tree(7), tree(2)]), tree(1
    >>> leaves(t)
    [7, 2, 3, 4]
    """
    if is_leaf(t):
        return [label(t)]
    else:
        leaf_labels = [leaves(b) for b in branches(t)]
        return sum(leaf_labels, [])
```

Exercise: Counting paths

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
```

Exercise: Counting paths (solution)

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = 1
    else:
        found = 0
    return found + sum([count_paths(b, total - label(t)) for b in branches(t)])
```

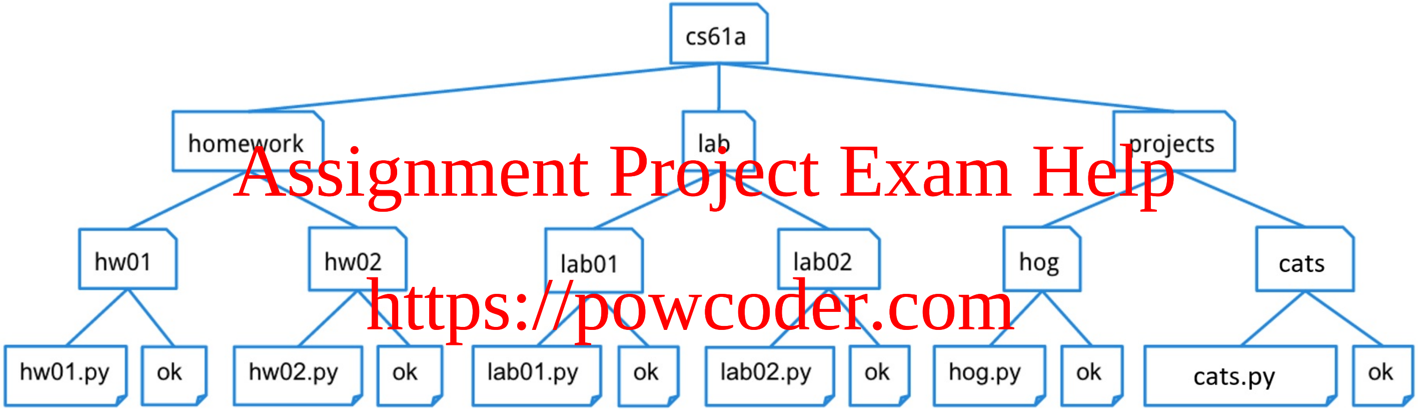
Tree: Layers of abstraction

Primitive Representation	`1` `2` `3` `"a"` `"b"` `"c"` `[..,..]`
Data abstraction	`tree()` `branches()` `label()` `is_leaf()`
User program	`double(t)` `count_leaves(t)`

Each layer only uses the layer above it.

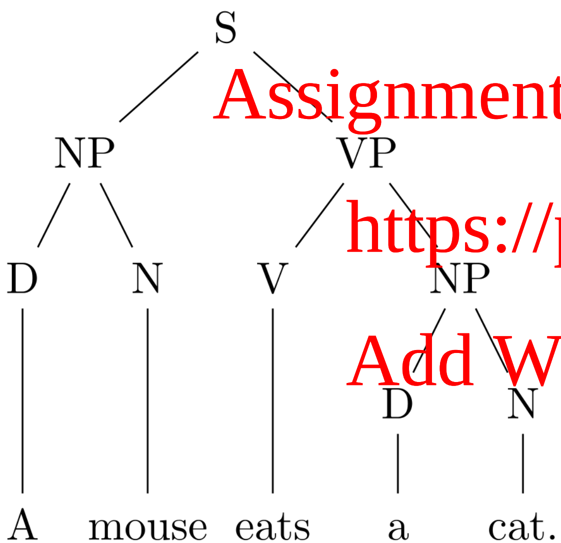
Trees, trees, everywhere!

Directory structures



Parse trees

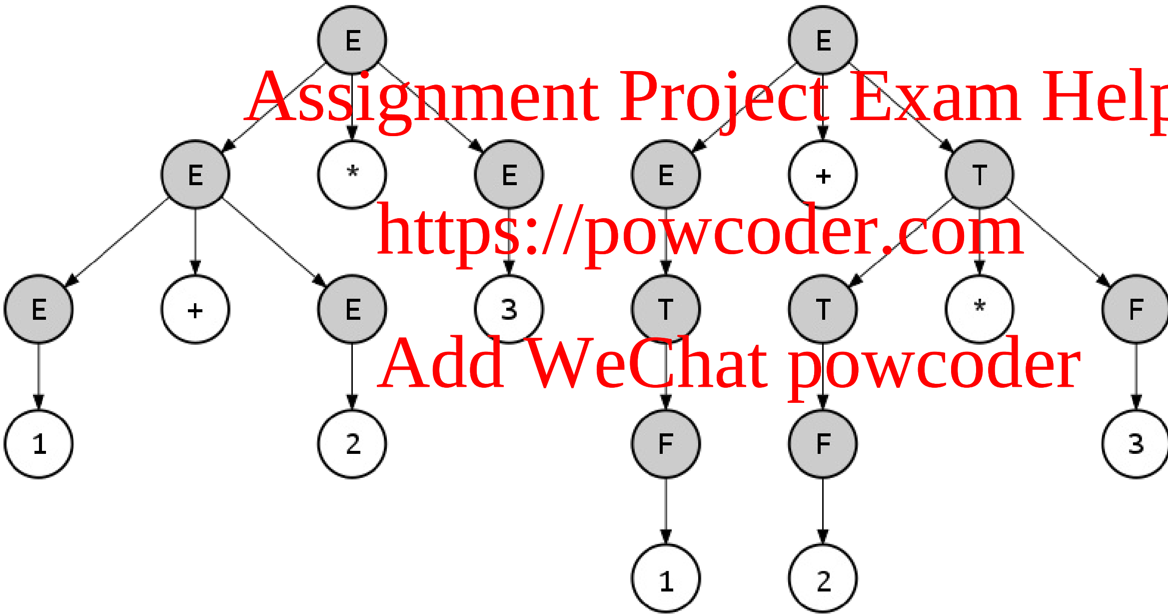
For natural languages...



Key: S = Sentence, NP = Noun phrase, D = Determiner, N = Noun, V = Verb, VP = Verb Phrase

Parse trees

For programming languages, too...

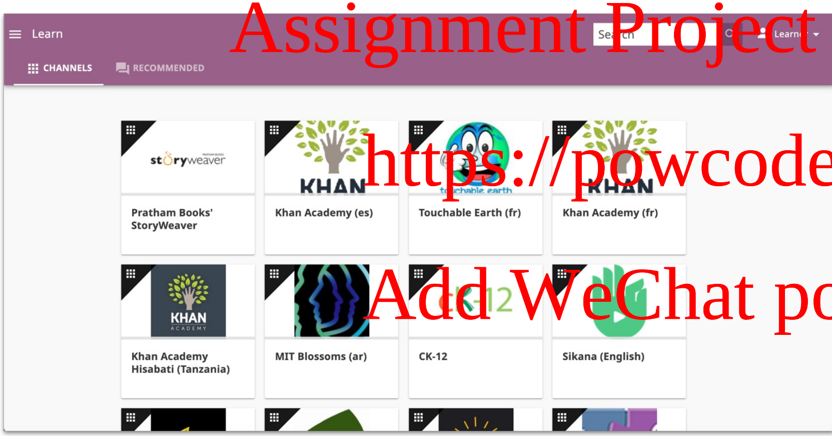


Key: E = expression

Python Project of The Day!

Kolibri

Kolibri: An open-source learning platform optimized for offline access.



Technologies used: Python, Django.
(Github repository)