A Case Study
Using Bayesian MCMC Stochastic Loss Reserve Models

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Abstract

At the 2018 annual meeting of the Casualty Actuarial Society, Bob Wolf and Mary Frances Miller presented a loss reserve analysis[1] on real data (scaled to maintain anonymity). These data consisted of 16 x 16 paid and incurred loss triangles. Features of the data included.

- Rapid premium growth
- Change in claims philosophy?
- Underestimates of outstanding liability in previous years

Mr. Wolf provided me with an electronic copy of those data. The paper analyzes those data using Bayesian MCMC starting with models described in Meyers (2019). It ends up by making changes to these models suggested by various diagnostics.

[1] Session C-24 - Learning Lounge Case Study: Material Adverse Reserve Development? When is it just that stuff happens?

Editorial Notes

- I call this document a "paper" even though it is written in a presentation format. I proposed this format to the CAS editorial staff and they agreed to it as an experiment.
- I chose this format because much of what people, including myself, read these days is on a screen. I want to make it easy to navigate between text, tables and graphics.[2]
  - There are section titles on the sidebar. Clicking on a section title will take you directly to that section. There is also direct access the plots.
  - Advancing the pages with consecutive plots will make it easier to compare the plots.
- The discussion of the models will be at a fairly high level. To fill in the details, one will have to look at my monograph, Meyers (2019).

[2]Printing this paper is permitted.

Supplemental Data Files

- The following data files are included with this paper. If they are placed in the same directory as the file for this pdf document, you will be able to see them by clicking on the link.
- The loss triangles
  - LL_Paid_Triangle.csv
  - LL_Incurred_Triangle.csv
- Summary statistics for the posterior distributions
  - Posterior_Stats.xls

R/Stan Scripts for the Models in This Paper

- I use RStudio to run my scripts. When I click on one of the links below, the script comes up in RStudio ready to run.
- To run these scripts on your computer you will have to:
  1. Change the R "setwd" function to the same directory as this pdf.
  2. Install the "rstan", "loo" and "data.table" R packages.
- These R/Stan Scripts should be in the same directory as this pdf.
  - LL_CRC.R
  - LL_CSR_w.R
  - LL_CSR_c.R
  - LL_CSR_vc.R
  - LL_POS_vc.R

The Question to be Addressed

- As the next two pages show, taken from the "Learning Lounge" presentation, the opining actuary, underestimated the liability in the three prior years.
- The question posed by the presentation was — Is this simply a case of "bad stuff" that sometimes happens?
- In this paper I pose the question as — "Is there a loss reserve model that does a better job of predicting the "bad stuff?"
- To properly answer this question:
  - We need a model with features that allow us to predict the "bad stuff."
  - If such features cannot be identified, we need a stochastic model that provide a range of possible outcomes.

Page 4 from "Learning Lounge" Presentation

Run-off of Net Carried Loss and DCC (aka ALAE Reserves) ($000s)
Adequacy of Net reserves in Hindsight at Prior-Year Ends
Source- Derivations from using December 31, 2017 Schedule P Data

		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2014	$ 145,170	$ 158,865 9.4%	$ 166,370 14.6%	$ 182,100 25.4%
		$ -	$ -	$ -
Annual Change		$ 13,695	$ 7,505	$ 15,730
Cumulative Change		$ 13,695	$ 21,200	$ 36,930

		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2015	$ 207,945	$ 208,500 0.3%	$ 229,905 10.6%	
		$ -	$ -	
Annual Change		$ 555	$ 21,405	
Cumulative Change		$ 555	$ 21,960	

		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2016	$ 244,470	$ 272,095 11.3%		
		$ -		
Annual Change		$ 27,625		
Cumulative Change		$ 27,625		

		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2017	$ 306,365			
Annual Change				
Cumulative Change				

Actuarial Opinion- History

	Appointed Actuary	Carried Reserves	Appointed Actuary Central Estimate	Actuarial Opinion Summary Range			RMAD
2/31/2014	A	145,170	145,170	141,396	to	174,494	
		Reasonable	0.0%	-2.6%		20.2%	NO
2/31/2015	A	207,945	207,945	188,190	to	245,583	
		Reasonable	0.0%	-9.5%		18.1%	NO
2/31/2016	A	244,470	253,675	229,504	to	299,465	
		Reasonable	3.8%	-6.1%		22.5%	NO
2/31/2017	B	306,365	306,365	290,434	to	321,377	
		Reasonable	0.0%	-5.2%		4.9%	NO

Stochastic Loss Reserve Models

- Start with the model framework in Meyers (2019).

$$C_{wd} \sim \text{lognormal}(\mu_{wd}, \sigma_d)$$

- where:
  - w =Accident Year (AY), $w = 1, \ldots, W$
  - d =Development Year (DY), $d = 1, \ldots, D$
  - Also, let c =Calendar Year (CY), $c = w + d - 1$
- This paper will initially examine models where:

$$\mu_{wd} = \log(Premium_w) + logelr + \alpha_w + \beta_d \cdot Sp(t)$$

- The $Sp(t)$, i.e. the "Speedup", function specifies how the "development factors" change over the time, t, where t could be measured by accident year, or calendar year.
- This paper explores alternative $Sp(t)$ functions in an effort to find a model that makes better predictions of the ultimate losses.

Interpreting the Model Parameters

- To prevent overdetermining the model, set:

$$\alpha_1 \equiv 0 \text{ and } \beta_D \equiv 0$$

- Thus the expected ultimate loss, U_w for accident year w, is the mean of a lognormal distribution, i.e.

$$U_w \equiv Premium_w \cdot \exp(logelr + \alpha_w + \sigma_D^2/2) \qquad (1)$$

- If the reported losses are near ultimate, the parameter σ_D will be very small. Thus for $w = 1$ the ultimate loss is approximately equal to $Premium_1$ times the expected loss ratio, $\exp(logelr)$. The α_w parameters account for accident year differences in the loss ratio.

Interpreting the Model Parameters — Continued

- Note that since $\beta_D = 0$ the $Sp(t)$ does not *directly* affect the projected ultimate loss.
- However, the $Sp(t)$ indirectly affects the ultimate loss parameters, *logelr* and α_w, through the Bayesian MCMC fitting algorithm.
- Recall

$$C_{wd} \sim \text{lognormal}(\mu_{wd}, \sigma_d)$$

$$\mu_{wd} = \log(Premium_w) + logelr + \alpha_w + \beta_d \cdot Sp(t)$$

- Heuristically speaking, it is the entire sum of the terms in the expression for μ_{wd} that is "attracted" to C_{wd}. The values of $\beta_d \cdot Sp(t)$ will influence values of *logelr* and α_w.

Interpreting the Speedup Function, $Sp(t)$

- What will distinguish the models in this paper is the choice of the $Sp(t)$ function. Let's discuss its meaning.
- Recall that $\beta_D = 0$. If $Sp(1) > Sp(2) > \cdots$, then the product $\beta_d \cdot Sp(t)$ is moving closer to 0 as t increases.
- For paid losses, this means losses are being settled more quickly over time.
- For incurred losses, this means that losses are being recognized more quickly over time.
- The reverse is true if $Sp(1) < Sp(2) < \cdots$. That is, paid losses are being settled more slowly over time, and incurred losses are being recognized more slowly over time.

Models Considered in This Paper

- The CRC Model — $Sp(w) \equiv 1$
  - This model most closely resembles the standard actuarial models that do not allow the development patterns to change over time.
- The CSR-w Model — $Sp(w) = (1-\gamma)^{w-1}$
  - $\gamma > 0$ gives us a decreasing $Sp(w)$ as the accident year, w increases from 1 to W. $\gamma < 0$ gives us an increasing $Sp(w)$.
- The CSR-c Model — $Sp(c) = (1+\gamma)^{C-c}$
  - $\gamma < 0$ gives us a increasing $Sp(c)$ as the calendar year, c, increases from 1 to $C-1$. $\gamma > 0$ gives us a decreasing $Sp(c)$
- We refer to the γ parameter as the speedup rate. We call a negative speedup rate a slowdown.

Models Considered in This Paper - Continued

- The CSR-vc Model —

$$
\begin{aligned}
Sp(C) &= 1 \\
Sp(C-i) &= Sp(C-i+1)\cdot(1+\gamma_{C-i}) \\
&\quad \text{for } i = 1,\cdots,C-1
\end{aligned}
$$

  - This model allows the speedup rate to vary by calendar year.
- The first two models are described in Meyers (2019). The next two were developed during the research that led to this paper. As we shall see, analyses of the shortcomings of these models point to another model, the POS-vc model that I will describe below.

The Run ID

- The various model runs in this paper will be fit on a given set of calendar years of either the paid or incurred loss triangle.
- Each model run will have an identifier with three components.
  1. The model name
  2. The loss triangle used — either “P” or “I”
  3. The calendar year range.
- For example, the run id “CSR-vc P-7:16” means that the CSR-vc model was fit to the paid loss triangle using data from the calendar years from 7 to 16.

Invoking Bayesian MCMC

- As described in Meyers (2019), the Bayesian MCMC fitting algorithm produces 10,000 equally likely parameter sets[3] $\{logelr\}$, $\{\alpha_w\}_{w=1}^{W}$, $\{\beta_d\}_{d=1}^{D}$, $\{\gamma\}$ and $\{\sigma_d\}_{d=1}^{D}$.
- The R/Stan scripts for the five models are included with this paper.
  - LL_CRC.R
  - LL_CSR_w.R
  - LL_CSR_c.R
  - LL_CSR_vc.R
  - LL_POS_vc.R
- The scripts allow for the user to select which triangle (paid or incurred) , and the calendar years within each triangle to use in fitting the model.

[3] The presence of brackets $\{\cdot\}$ around a parameter will indicate that it is a sample of 10,000 values from the posterior distribution.

Parameter Summary Statistics

- The spreadsheet titled "Posterior_Stats.xls" contains the means and standard deviations of the 10,000 parameter sets for the models run in this paper.
- Some observations:
  - While somewhat volatile, values taken from the γ parameter sample are generally negative — indicating a slowdown in claim settlements.
  - The ranges of the $\{\alpha_w\}$ parameter samples are small in the earlier accident years where the reported losses are well known. But the ranges grow wider in the later accident years.
  - One would expect the ranges of the $\{\alpha_w\}$ parameter samples for the P-1:16 models to be smaller that those for the P-7:16 models, since there are more observations in the P-1:16 dataset. Instead the opposite is true.

Statistics of Interest

- With a sample of 10,000 parameter sets, one can use Equation 1 to to obtain a sample of 10,000 expected ultimate losses, $\{U_w\}$
- Define $\{U_{Tot}\} = \sum_{w=1}^{16}\{U_w\}$.
- Also of interest is a sample of 10,000 possible unpaid losses (ultimate loss less current paid loss), $\{R_c\}$, at calendar year c where:

$$R_c = \sum_{w=1}^{c} U_w - \sum_{d=1}^{c} C_{c+1-d,d} \qquad (2)$$

Statistics of Interest — Continued

- From the samples $\{R_c\}$ and $\{U_{Tot}\}$, we can calculate statistics of interest, such as:
  - Ultimate Loss $=$ mean$\{U_{Tot}\}$
  - Ultimate Standard Error $=$ standard deviation$\{U_{Tot}\}$
  - Reserve Low $=$ 2.5th percentile of $\{R_{16}\}$
  - Reserve $=$ mean$\{R_{16}\}$
  - Reserve High $=$ 97.5th percentile of $\{R_{16}\}$

Running the MCMC Models on P-1:16 Data

- The table below shows the calculations for the above statistics for each of the four models fit with the P-1:16 data.
- Note that the results vary significantly by model. To resolve these differences we need some model diagnostics — the $\widehat{elpd}_{loo}$ and the Standardized Residual Boxplots that we now turn to describing.

Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CRC P-1:16	1,147,142	38,750	118,780	187,214	272,284	222.13
CSR-w P-1:16	1,284,563	63,262	213,704	324,635	460,932	229.51
CSR-c P-1:16	1,328,029	69,501	248,047	368,101	519,552	230.03
CSR-vc P-1:16	1,251,066	80,223	159,958	291,137	473,496	238.60

The Expected Log Predictive Density $(\widehat{elpd}_{loo})$[4]

- For each observation, C_{wd} in the loss triangle:
  1. Remove that observation from the data.
  2. Fit the selected model to the data in the triangle that remains and obtain the parameter sets $\{\theta(-wd)\}$ (consisting of all the $\{\alpha_w\}$s, $\{\beta_d\}$s, etc.)
  3. Calculate the average likelihood, $p(C_{wd}|\{\theta(-wd)\})$ over all 10,000 parameter sets.
- Then

$$\widehat{elpd}_{loo} = \sum_{w,d} log(p(C_{wd}|\{\theta(-wd)\}))$$

- The "loo" term refers to the "leave one out" feature in bullet $\#1$ above.

[4] More details about this statistic are in Section 6 of Meyers (2019).

$\widehat{elpd}_{loo}$ — Continued

- A higher $\widehat{elpd}_{loo}$ indicates a better fit. By this measure, the CSR-vc model fits the P-1:16 data the best.
- Since the likelihoods are calculated on holdout data, there is no penalty for fitting models with a large number of parameters.

Standardized Residual Boxplots

- The models in this paper all assume a lognormal distribution with the parameters μ_{wd} and σ_d. Thus we expect that

$$\frac{log(C_{wd}) - \{\mu_{wd}\}}{\{\sigma_d\}}$$

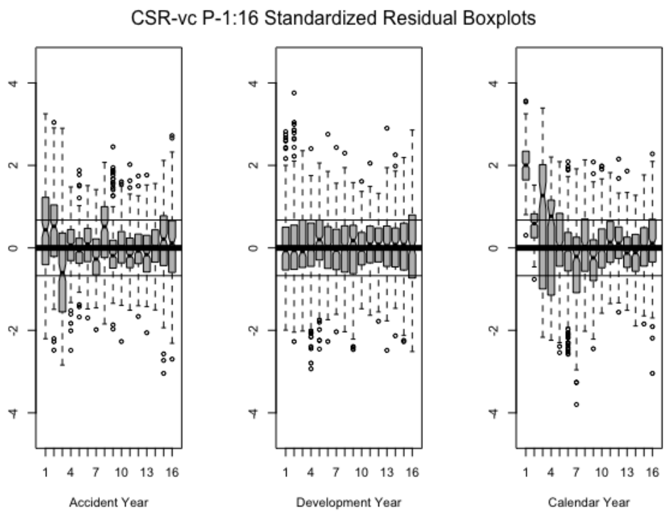
will have a normal(0,1) distribution.

- To test this graphically we split the residuals, in turn by accident year, development year and calendar year and plot a sample of size 200 in each "year" with the R "boxplot" function.

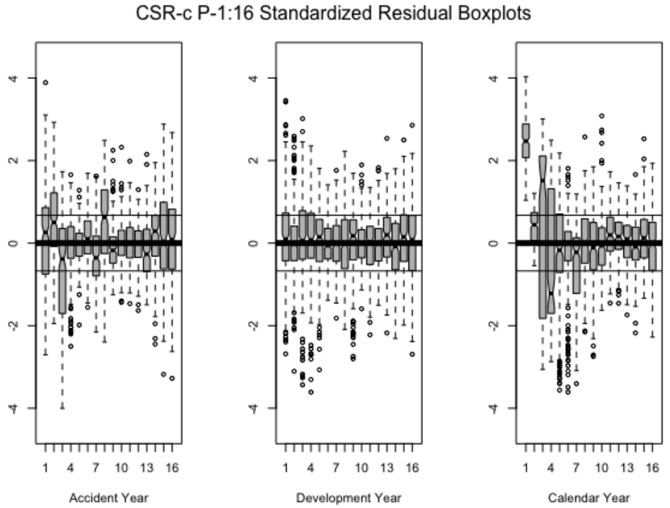
Expected Results with the R "boxplot" Function

- The following four pages contain the standardized residual Boxplots for the four models on the P-1:16 data.
- The gray bars correspond to the interquartile range. Ideally the bars should be centered on 0. The endpoints of those bars should be touching the black lines representing the interquartile range of the standard normal distribution.
- Most of the remaining residuals should be between $\pm$ 2. A few could be in the (-3,-2) or the (2,3) ranges. Very few should be outside the $\pm$ 3 range.
- Now flip through the next four pages to see how close the Boxplots are to the "ideal" Boxplot. I will give my take on the other side.

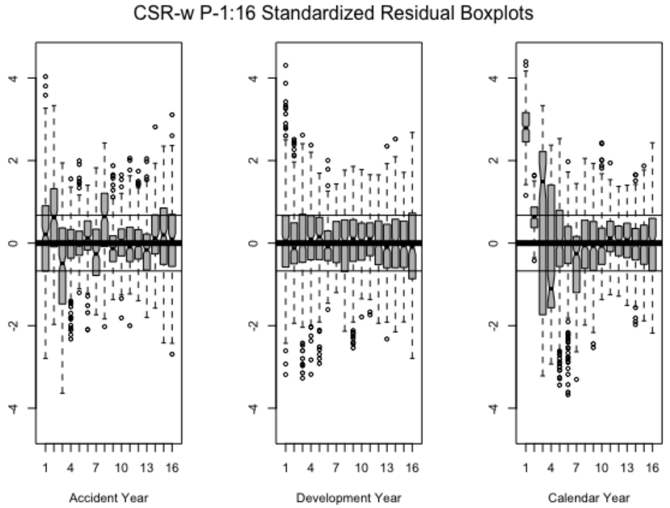
Boxplots P-1:16



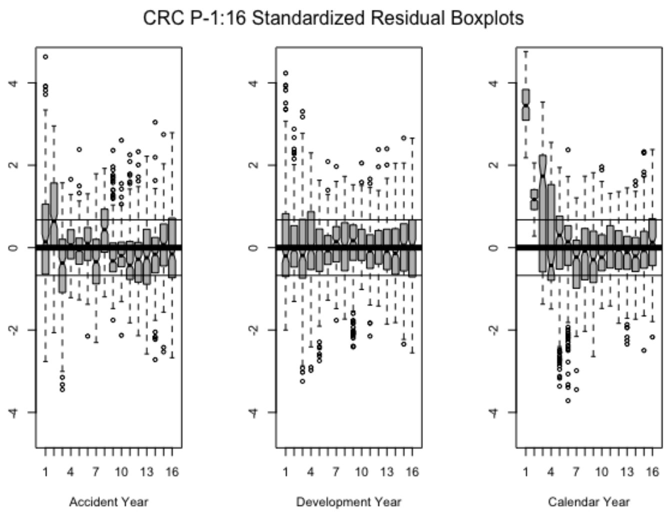
Boxplots P-1:16



Boxplots P-1:16



Boxplots P-1:16



P-1:16 Discussion

- The Boxplots of all four models:
  - Accident year — Not ideal
  - Development year — Pretty good
  - Calendar year — Bad for the early calendar years
- AY, DY and CY Boxplots get worse as you flip from CSR-vc $\Rightarrow$ CSR-c $\Rightarrow$ CSR-w $\Rightarrow$ CRC.
- CSR-vc had the best fit according to the $\widehat{elpd}_{loo}$ statistic.
- The CSR-vc appears to be the best model for P-1:16.
- The problem appears most prominently in the calendar year Boxplots. It appears that the calendar year changes in speedup rate are more complicated than assumed by the speedup function.

Choosing a Subset of the P-1:16 Data

- When we have a portion of the data that does not fit our current model we have two options.
  1. Find a modification to your model that fits all the data.
  2. Dropping that portion of the data that does not fit our current model.
- Lacking access to the claim handlers for these data, I elected to use the most recent 10 calendar years.
- Dropping older calendar years while fitting loss reserve models is a fairly common practice among actuaries

Running the MCMC Models on P-7:16 Data

Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CRC P-7:16	1,186,501	26,857	175,749	226,573	283,168	234.94
CSR-w P-7:16	1,233,188	46,782	185,870	273,260	369,290	235.05
CSR-c P-7:16	1,280,649	53,024	227,348	320,721	439,200	236.06
CSR-vc P-7:16	1,256,436	73,076	179,454	296,510	460,811	240.85

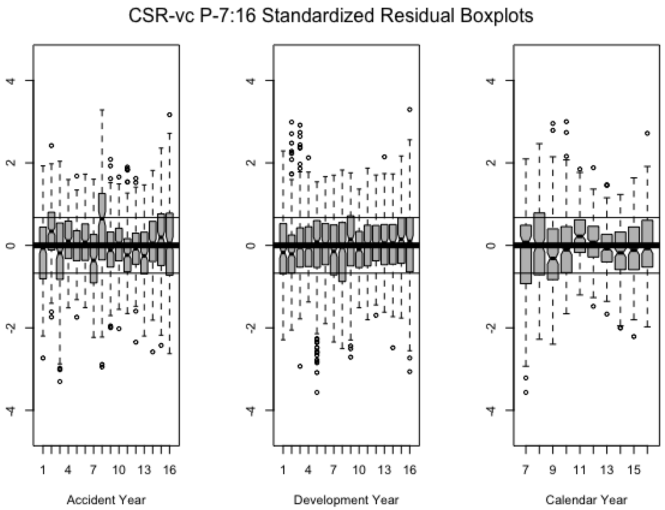
Some observations

- The CSR-vc model had the highest $\widehat{elpd}_{loo}$ statistic.
- The mean reserve estimates vary significantly by model.
- Look at the "Gamma" tab in "Posterior_Stats.xls".
    - The speedup parameter is -0.0156 for the CSR-w model, -0.0291 for the CSR-c model. For the CSR-vc model it starts as 0.0131 and moves down to fluctuate between the -0.012 to -0.035 range for the later calendar years.
    - A negative speedup parameter means a slowdown in claim settlements, and hence a higher predicted ultimate loss.

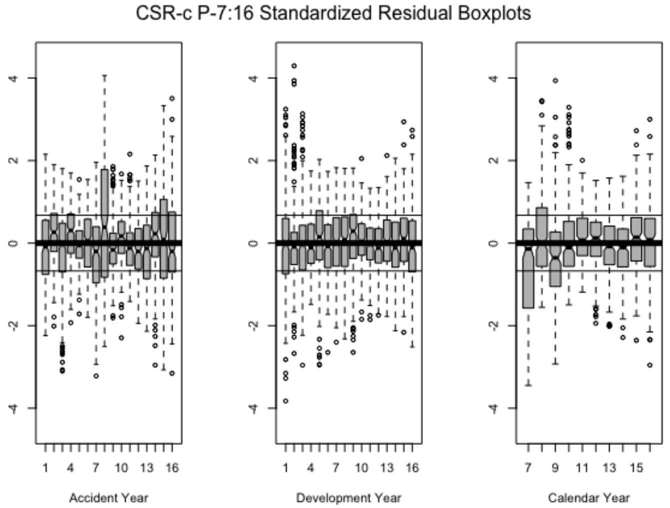
Observation on the P-1:16 and P-7:16 Datasets

- Looking at the CSR-vc model on each dataset:
  - The ultimate loss estimates were fairly close.
  - The range of the loss estimates are narrower for the P-7:16 data.
  - As the P-7:16 data has fewer observations, one should expect the reverse to be true.
- I attribute this reversal to model error with the P-1:16 data.
- Now scroll through the Boxplots for these models.

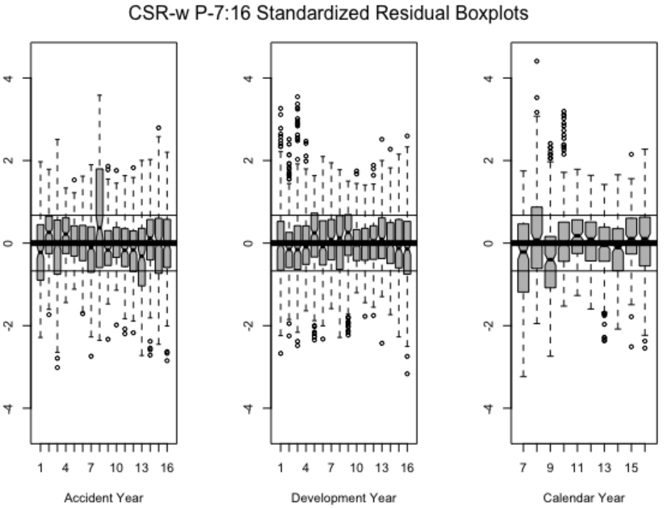
Boxplots P-7:16



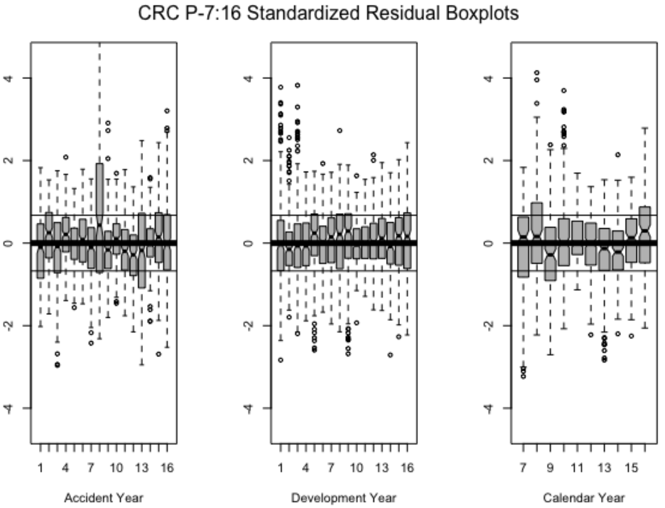
Boxplots P-7:16



Boxplots P-7:16



Boxplots P-7:16



P-7:16 Discussion

- I judge the CSR-vc model to have the best Boxplots.
    - The interquartile ranges are about the same and all pretty good.
    - The CSR-vc model has noticeably fewer outliers in the Boxplots, i.e. outside the ± 2 range.
- This combined with its having the highest $\widehat{elpd}_{loo}$ statistic make it the model of choice for the paid data.

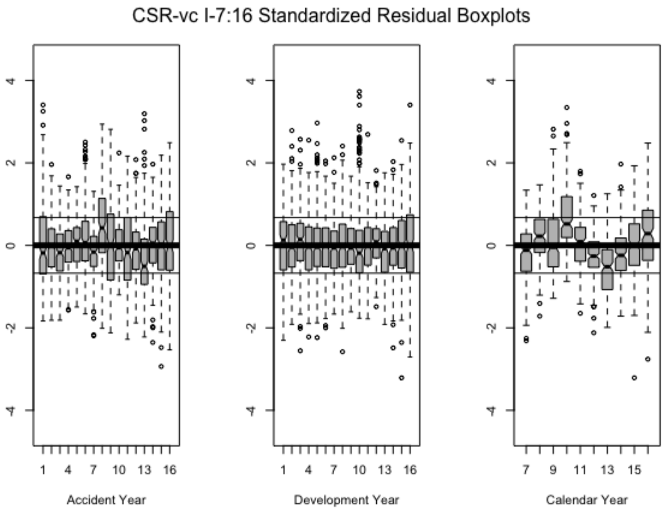
Running the MCMC Models on I-7:16 Data

Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CRC I-7:16	1,230,151	29,800	214,940	270,223	333,281	235.64
CSR-w I-7:16	1,193,518	37,085	167,331	233,590	313,557	232.50
CSR-c I-7:16	1,317,128	75,412	243,610	357,200	531,381	235.19
CSR-vc I-7:16	1,262,187	58,618	201,157	302,261	430,947	241.27

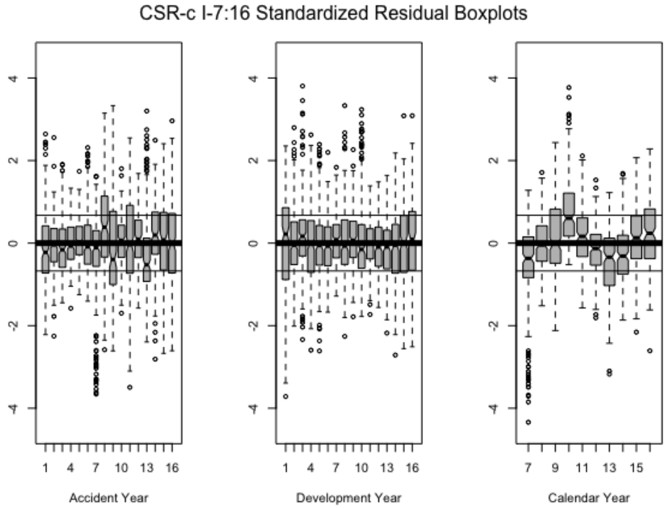
Some observations

- The CSR-vc model had the highest $\widehat{elpd}_{loo}$ statistic.
- The mean reserve varies significantly by model.
- Look at the "Gamma" tab in "Posterior_Stats.xls".
  - The speedup parameter is a *positive* 0.0375 for the CSR-w model, a *negative* 0.0675 for the CSR-c model. For the CSR-vc model it starts close to zero and moves up around the -0.02 to -0.04 range for the later calendar years.
  - A negative speedup parameter for incurred losses can also indicate a decreasing recognition of outstanding losses, and hence a higher predicted ultimate loss.
- Now scroll through the Boxplots for these models.

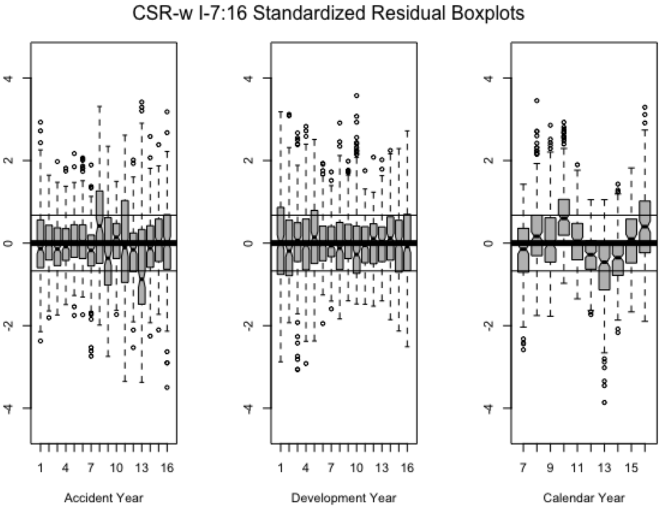
Plots I-7:16


CSR-vc I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1
4
7
10
13
16
Accident Year
Development Year
7
9
11
13
15
Calendar Year

Plots I-7:16

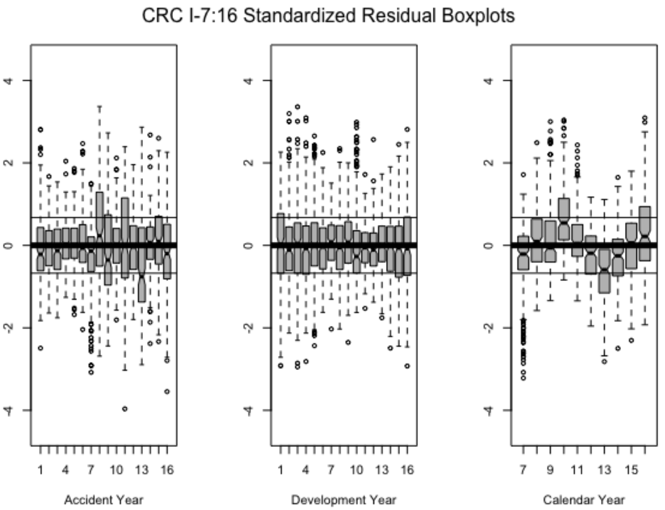

CSR-c I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1 4 7 10 13 16
Accident Year
Development Year
7 9 11 13 15
Calendar Year

Plots I-7:16



CSR-w I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1 4 7 10 13 16
Accident Year
1 4 7 10 13 16
Development Year
7 9 11 13 15
Calendar Year

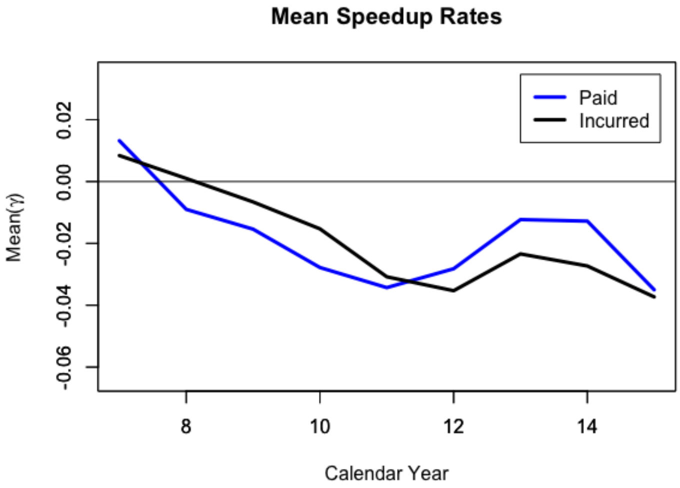
Plots I-7:16


CRC I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1
4
7
10
13
16
Accident Year
Development Year
7
9
11
13
15
Calendar Year

I-7:16 Discussion

- I judge that the CSR-vc model has the best Boxplots.
  - Slightly better by accident year and development year.
- The Boxplots by calendar year suggests that there as been a change in case reserving practices.
- The next page shows plots of the mean speedup rates, i.e. γ parameters, for the paid and the incurred models. One would expect to see the plots track closely with each other as a substantial portion of the incurred losses are already paid.
- But — As we can see from these plots, there is a noticeable difference between the plots. And moreover, they cross.
- This suggests that there should be separate $\{\gamma\}$ parameters for paid and outstanding losses.

Mean Speedup Rates for the CSR-vc P-7:16 and the CSR-vc I-7:16 Models



Integrated Paid and Outstanding (POS) Models

- This section proposes a model that simultaneously fits both paid and incurred losses.[5].
- This model has lognormal distributions for each of the paid and incurred losses.
  - The μ_{wd} parameter of the distribution for paid losses is the same as above.
  - The μ_{wd} of the incurred losses are equal to the sum of the μ_{wd} for the paid losses, plus a separate factor representing outstanding losses.
- More details on the next page.

[5] A more detailed discussion of fitting models simultaneously to paid and incurred is discussed in Section 9 of Meyers (2019)

The POS-vc Model

The prefixes P, I and OS denote “Paid”, “Incurred” and “Outstanding” respectively.

$$\begin{aligned} {}_P\mu_{wd} &= \log(Premium_w) + logelr + \alpha_w + {}_P\beta_d \cdot {}_PSp(c) \\ {}_I\mu_{wd} &= {}_P\mu_{wd} + {}_{OS}\beta_d \cdot {}_{OS}Sp(c) \end{aligned}$$

$${}_P\beta_D \equiv 0, \text{ and } {}_{OS}\beta_D \not\equiv 0$$

Where

$$\begin{aligned} {}_XSp(C) &= 1 \\ {}_XSp(C-i) &= Sp(C-i+1) \cdot (1 + {}_X\gamma_{C-i}) \\ &\quad \text{for } i = 1, \cdots, C-1 \text{ and } X = P \text{ or } OS \end{aligned}$$

Then

$$\begin{aligned} {}_PC_{wd} &\sim \text{lognormal}({}_P\mu_{wd}, {}_P\sigma_d) \\ {}_IC_{wd} &\sim \text{lognormal}({}_I\mu_{wd}, {}_I\sigma_d) \end{aligned}$$

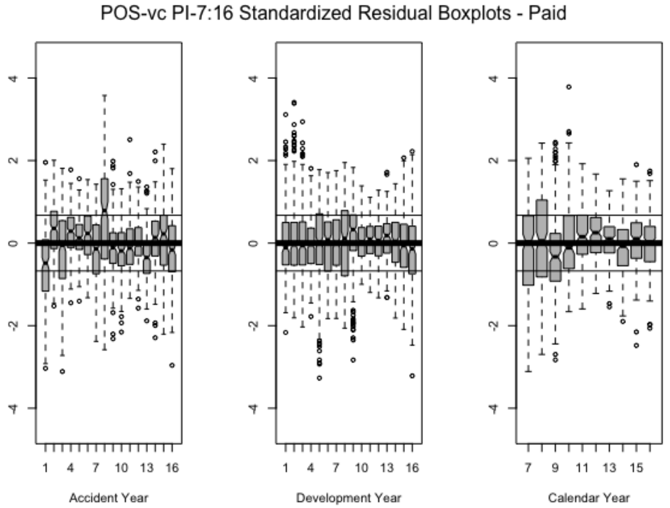
Running the POS Model on PI-7:16 Data

- Results for the comparable CSR model runs are also given.
- The script for this model is in LL_POS_vc.R

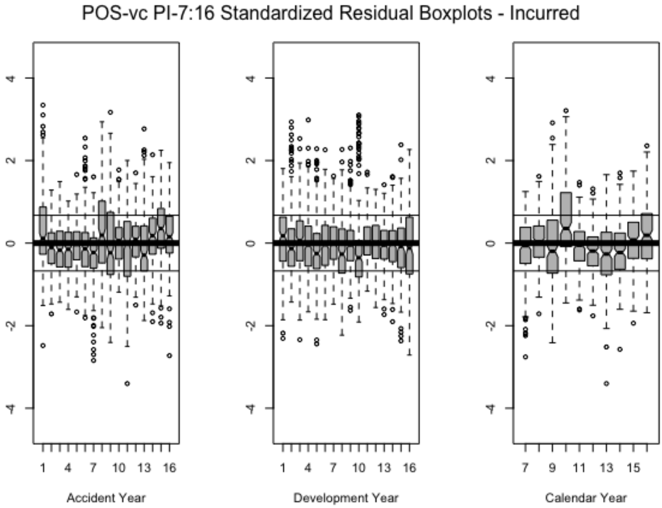
Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CSR-vc P-7:16	1,256,436	73,076	179,454	296,510	460,811	240.85
CSR-vc I-7:16	1,262,187	58,618	201,157	302,261	430,947	241.27
POS-vcp PI-7:16	1,262,897	58,400	205,683	302,969	432,516	251.85
POS-vci PI-7:16	1,262,897	58,400	205,691	302,969	432,529	255.56

- The $\widehat{elpd}_{loo}$ statistics are calculated separately on the paid and incurred data in the POS model. These statistics are significantly better for the POS-vc model than they are for the corresponding CSR-vc models.
- The standardized residual Boxplots are on the following three pages. Compared with the corresponding CSR-vc Boxplots:
  - The POS-vc plots look a bit worse for the paid losses.
  - They look a bit better for the incurred losses.
  - They look pretty good for the combined losses.

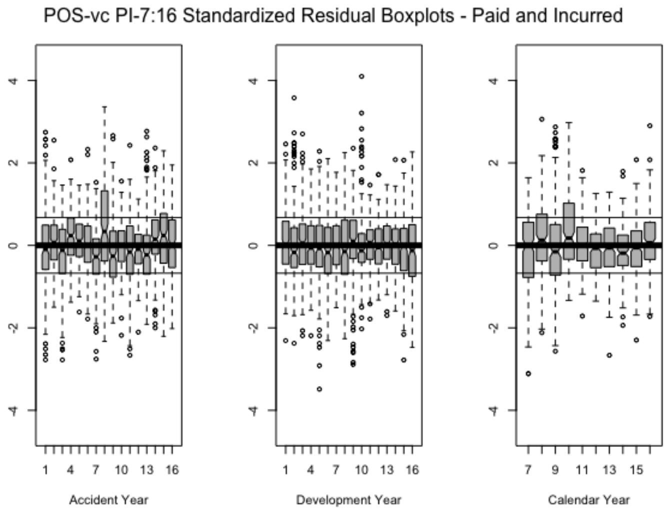
Boxplots for POS-vc Model



Boxplots for POS-vc Model



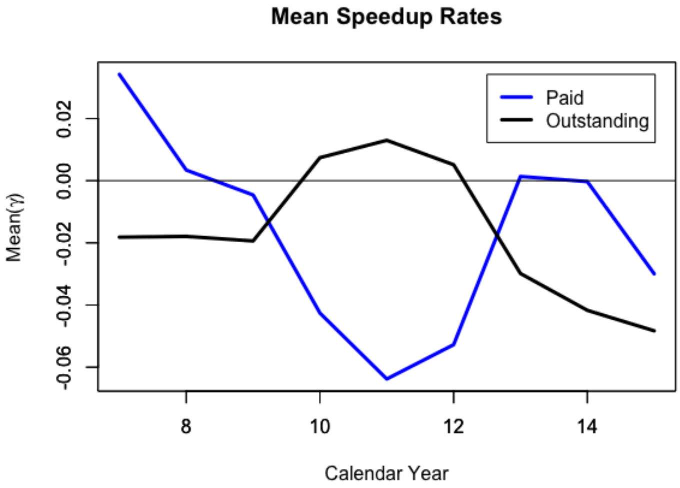
Boxplots for POS-vc Model



Claims Department Practices

- The following page has plots of the the mean paid claim speedup rate, mean$\{_P\gamma\}$, and the mean outstanding claim speedup rate, mean$\{_{OS}\gamma\}$.
- The claims department appears to be slowing down the paid claim settlements, while speeding up the recognition of outstanding claims, and vice versa.
- This observation should be discussed with the claims department.

Mean Speedup Rates for the POS-vc Model



Estimating Ultimate Losses

- Recall from Equation 1 that the ultimate expected loss for accident year w is equal to the expected value

$$Premium_w \cdot \mathrm{E}\{\exp(logelr + \alpha_w + {}_X\sigma_D^2/2)\}$$

where X can refer to either paid, P, or incurred, I, losses.

- For the POS-vc model the expected ultimate incurred loss is slightly more complicated.

$$Premium_w \cdot \mathrm{E}\{\exp(logelr + \alpha_w + {}_{OS}\beta_D +_I \sigma_D^2/2)\}$$

- After 16 years of development, the values of ${}_{OS}\beta_D$ and ${}_X\sigma_D$ are close to zero. So the paid and incurred loss estimates are very close to each other.
- The following three pages give the ultimate loss estimates by accident year for the CSR-vc and POS-vc models.

Accident Year Exhibit for CSR-vc P-7:16

AY	Premium	Estimate	SE	CV
2002	13,750	7,035	36	0.0051
2003	28,052	11,172	89	0.0080
2004	44,853	27,882	237	0.0085
2005	70,507	42,229	397	0.0094
2006	80,285	45,451	459	0.0101
2007	96,286	58,149	659	0.0113
2008	130,481	66,126	817	0.0124
2009	142,059	49,960	715	0.0143
2010	131,024	70,952	1,150	0.0162
2011	131,870	89,695	1,702	0.0190
2012	122,125	83,745	2,025	0.0242
2013	125,456	88,474	2,794	0.0316
2014	201,129	105,300	4,505	0.0428
2015	271,351	148,458	10,143	0.0683
2016	297,237	180,482	22,320	0.1237
2017	292,035	181,328	51,519	0.2841
Total	2,178,500	1,256,436	73,076	0.0582

Accident Year Exhibit for CSR-vc I-7:16

AY	Premium	Estimate	SE	CV
2002	13,750	7,037	35	0.0050
2003	28,052	11,056	84	0.0076
2004	44,853	27,643	231	0.0084
2005	70,507	41,556	376	0.0090
2006	80,285	44,764	439	0.0098
2007	96,286	57,033	619	0.0109
2008	130,481	65,057	765	0.0118
2009	142,059	49,100	658	0.0134
2010	131,024	70,228	1,078	0.0154
2011	131,870	89,487	1,542	0.0172
2012	122,125	82,593	1,863	0.0226
2013	125,456	87,515	2,526	0.0289
2014	201,129	105,847	4,581	0.0433
2015	271,351	148,245	9,905	0.0668
2016	297,237	184,843	21,850	0.1182
2017	292,035	190,185	46,733	0.2457
Total	2,178,500	1,262,187	58,618	0.0464

Accident Year Exhibit for POS-vc PI-7:16

AY	Premium	Estimate(p)	Estimate(i)	SE(p)	SE(i)	CV(p)	CV(i)
2002	13,750	7,038	7,038	34	34	0.0048	0.0048
2003	28,052	11,117	11,117	68	68	0.0061	0.0061
2004	44,853	27,779	27,779	184	184	0.0066	0.0066
2005	70,507	41,907	41,907	299	299	0.0071	0.0071
2006	80,285	45,135	45,135	347	347	0.0077	0.0077
2007	96,286	57,636	57,636	489	489	0.0085	0.0085
2008	130,481	65,630	65,630	603	603	0.0092	0.0092
2009	142,059	49,658	49,658	513	513	0.0103	0.0103
2010	131,024	70,692	70,692	792	792	0.0112	0.0112
2011	131,870	89,930	89,930	1,175	1,175	0.0131	0.0131
2012	122,125	83,456	83,456	1,373	1,373	0.0165	0.0165
2013	125,456	88,619	88,619	1,909	1,909	0.0215	0.0215
2014	201,129	106,701	106,701	3,372	3,372	0.0316	0.0316
2015	271,351	149,816	149,816	7,690	7,690	0.0513	0.0513
2016	297,237	183,299	183,299	17,254	17,254	0.0941	0.0941
2017	292,035	184,484	184,484	37,469	37,469	0.2031	0.2031
Total	2,178,500	1,262,897	1,262,897	58,400	58,400	0.0462	0.0462

Predictive Distribution of Loss Reserve Liability

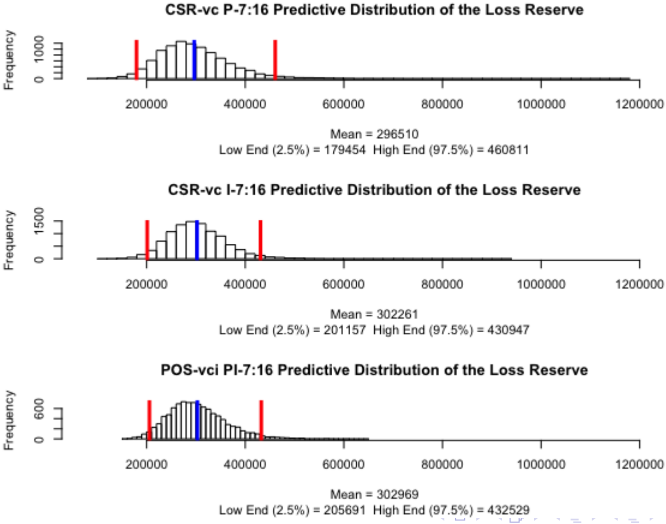
- Following Equation 2, a sample of the predictive distribution of the outstanding losses is given by:

$$\{_X R_C\} = \sum_{w=1}^{C} \{_X U_w\} - \sum_{d=1}^{C} C_{c+1-d,d}$$

where X =CSR-vc P-7:16, CSR-vc I-7:16 or POS-vc 7:16.

- Histograms of the predictive distributions for these models are given in the next page.
- Note that the POS-vc model reduces the range of ultimate estimates, by a lot for paid losses, and by a little for incurred losses.

Predictive Distribution of Loss Reserve Liability



Hindsight Reserves

- A "Hindsight Reserve" is a reserve for an earlier calendar year calculated with parameters derived from current data.
- In the notation of this paper, let's define:

$$\text{Reserve}(CY, CY_{Data}) \equiv \text{Mean}[\{R_{CY}\}]$$

where the parameters $\{logelr\}$, $\{\alpha_w\}$ and $\{\sigma_D\}$ were calculated from a loss triangle compiled in the year CY_{Data}.

- If $CY < CY_{Data}$, Reserve(CY, CY_{Data}) is called a hindsight reserve.
- If $CY = CY_{Data}$, Reserve(CY, CY_{Data}) is the original posted reserve.

Hindsight Reserves — Continued

- The next three pages compare hindsight reserves with the originally posted reserves using the following models.
  - POS-vc on P-7:c The results for I-7:c are very similar.
  - CSR-vc on P-7:c as the best model for paid data.
  - CSR-vc on I-7:c to demonstrate the effect of the poor fit along the calendar year dimension.
  - CRC on P-7:c is the model that ignores any changes in the claim speedup rate.
- Notation for the following tables
  - The original posted reserves are in **bold**.
  - Note — The relationship between the nominal calendar year, CY, and the calendar year index, c is $CY = c + 2001$. CY_{Data} for the dataset P-7:c is equal to $c + 2001$.

Hindsight Reserves for POS-vc P-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	164,503	156,036	143,788	**122,639**
2015	209,566	206,311	**201,284**	
2016	255,896	**284,712**		
2017	**299,989**			

Changes from Original to Latest Hindsight
2014 $+34.1\%$
2015 $+4.1\%$
2016 -10.1%

Hindsight Reserves for CSR-vc P-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	165,380	163,306	161,387	**157,006**
2015	209,387	215,340	**223,681**	
2016	253,559	**297,939**		
2017	**296,510**			

Changes from Original to Latest Hindsight

2014 +5.3%

2015 −6.4%

2016 −14.9%

Hindsight Reserves for CSR-vc I-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	158,126	140,872	123,894	**110,558**
2015	201,920	182,147	**176,653**	
2016	250,453	**247,591**		
2017	**302,261**			

Changes from Original to Latest Hindsight
2014 +43.0%
2015 +14.2%
2016 +1.2%

Hindsight Reserves for CRC P-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	156,230	144,382	130,750	**123,818**
2015	191,356	177,752	**158,059**	
2016	220,532	**212,310**		
2017	**226,575**			

Changes from Original to Latest Hindsight

2014 +26.2%

2015 +21.2%

2016 +3.9%

Discussion of Hindsight Results[6]

- Let's examine the difference between the original and hindsight reserves more closely. What affects future hindsight reserves?
- It follows from our lognormal assumption that for a given calendar year c:

$$\log(C_{wd}) = \log(Premium_w) + \mathrm{E}[\{logelr + \alpha_w + \beta_d \cdot Sp(c)\}]$$

- Suppose for calendar year $\mathscr{C} > C$ we obtain a new set of parameters for the model and obtain for a w and d in calendar year $\mathscr{C}$:

$$\log(C_{wd}) = \log(Premium_w) + \mathrm{E}[\{logelr' + \alpha'_w + \beta'_d\}]$$

[6] This analysis applies to the CSR-vc model. For the POS-vc model, the derivation has more terms, but follows the same logic.

Explaining Hindsight Changes - The Math

- Since $\log(C_{wd})$ should be the same for models fit in each calendar year c and $\mathscr{C}$ we can combine the two equations on the last page to get:

$$\mathrm{E}[\{logelr + \alpha_w - logelr' - \alpha'_w\}] = \mathrm{E}[\{\beta'_d - \beta_d \cdot Sp(c)\}]$$

- If

$$\mathrm{E}[\{\beta_d \cdot Sp(c)\}] < \mathrm{E}[\{\beta'_d\}]$$

then we expect

$$\mathrm{E}[\{logelr + \alpha_w\}] > \mathrm{E}[\{logelr' + \alpha'_w\}]$$

which (according to Equation 1), will nudge the estimate of the hindsight reserve higher for accident year w.

Projecting Next Year's Hindsight Reserve

- Suppose, after a conversation with the claims department, we expect next year's speedup rate to be equal to γ'_C.
- Given current data, how would this change our reserve estimate?
- Keeping the same structure of the model:

$$\begin{aligned}\{\beta_d\} &= \{\beta'_d\} \cdot (1 + \gamma_C) \\ \{Sp'(c)\} &= \{Sp(c)\} \cdot (1 + \gamma_C)\end{aligned}$$

Since $\mathrm{E}[C_{wd}]$ is unchanged, this implies that

$$\begin{aligned}\{\beta_d \cdot Sp(c)\} &= \{\beta'_d \cdot Sp'(c)\} \text{ and} \\ \mathrm{E}[\{logelr + \alpha_w\}] &= \mathrm{E}[\{logelr' + \alpha'_w]\})\end{aligned}$$

Projecting Next Year's Hindsight Reserve

- Thus this projection does not change the reserve estimate!
- If there is a difference between the hindsight reserve and the current estimated reserve, it is due to the new data that changes the parameters in ways *unanticipated* by the model.
- For the paid data, the current and the hindsight reserve estimates are relatively close. This encourages confidence in the model for paid data.
- But for the incurred data, the current and hindsight estimates for the CSR and POS models are noticeably different.
- The POS model helps the incurred data somewhat, but I still think there is something different in the incurred data.

The Question Addressed by This Paper

- In prior years the original opining actuary underestimated the loss reserve liability.
- Was this a case of "bad stuff" that sometimes happens?
- Or was it the case that there is a loss reserve model that does a better job of predicting the "bad stuff?"
- The Learning Lounge presentation mentioned a number of red flags, e.g. declining paid to current ultimate and declining incurred to current ultimate ratios, and slowdown in claim settlement due to rapid premium growth.
- In looking at the "Actuarial Opinion - History" slide in the introduction, it appears that the opining actuary and the Learning Lounge presenters recognized by 2016 and 2017, that earlier reserve estimates were understated because of the slowdown in the claim settlements. To my way of thinking, this means that they needed a better model.

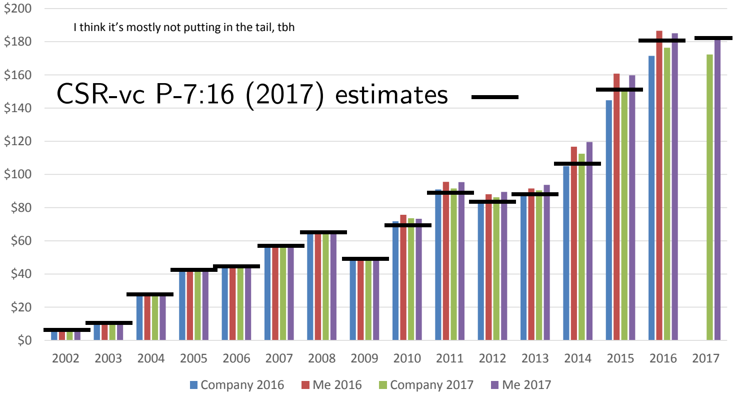
A Proposal for the "Bad Stuff" i

- This paper proposes the "vc" models that explicitly recognize changes in the claim speedup rate by calendar year.
- The CRS-vc model works well with paid losses, but not very well with incurred losses.
- POS-vc model obtains a better fit with the incurred losses, there were still shortcomings identified in the hindsight analyses.

Comparison with More Traditional Approaches

- In the right hands, traditional actuarial judgment can be pretty good.
- The next page compares the ultimate estimates by accident year obtained by Mary Frances, the company actuary and the CSR-vc P-7:16 model. The CSR-vc model estimates are close, but in general, are a bit lower than the estimates in the Learning Lounge presentation.

Page 62 from “Learning Lounge” Presentation



My Loss Reserving Philosophy

- I view loss reserving as a dialogue between an actuarial department and its corresponding claims department. One way this dialogue might work is as follows.
  1. In talking with the claims department, the actuaries try to find out how the claims adjustment process works.
  2. They then formulate a model that describe the claims adjustment process. Then test the model thoroughly.
  3. If testing reveals unexpected differences between the model and the data, repeat Steps 1-2 above as necessary.
- Advantages of using Bayesian MCMC for model building
  1. Flexibility in model building — If you can code the likelihood function, you can run the model.
  2. Bayesian models are transparent and reproducible. Your judgments are made explicit in your choice of models and prior distributions.
  3. Bayesian models provide output that can be used for calculating risk margins. See Section 11 of Meyers (2019).

Final Comments

- The flexibility of Bayesian MCMC models was very helpful in this exercise. It enabled me to to easily explore beyond my existing collection of models.
- Over time, I expect that I, and others, will add to our collection of such models in the future.
- I want to thank Bob and Mary Frances for making these data available to the public. It was interesting to see how well the estimates derived from a Bayesian MCMC model tracked with the estimates from experienced reserving actuaries. I was glad for my model and for the actuarial profession, to see that the estimates were reasonably close.
- Generally speaking, I am willing to try fitting a Bayesian MCMC model to any real loss triangle that is, or can be made, publicly available.

References

- This paper refers extensively to Meyers (2019). This also applies to the references in that monograph.
- I want to make a call out to Ben Zehnwirth, who for years has been insisting on a calendar year model for loss reserving. See, for example, Barnett and Zehnwirth (2000).
- This paper introduces a calendar year effect to my collection of Bayesian MCMC models. My reason for not doing this before is that until now, I had not figured out how to make sense out of a calendar year model applied to cumulative loss data. My attempts to come up with a satisfactory model for incremental loss data have failed.