Algorithm 1 Power iteration method

Inputs: $\hat{H}$, z, ϵ_{abs}, ϵ_{rel}, ϵ_{buff}, $j_{\max}$

Require: $\|z\|_2 > 0$

$\sigma \leftarrow \|z\|_2$ ▷ initialization
for $j \leftarrow 1 \colon j_{\max}$ **do**
    $w \leftarrow \frac{1}{\sigma} \hat{H} z$
    $z \leftarrow \hat{H}^\top w$
    $\sigma^\star \leftarrow \|z\|_2$
    if $|\sigma^\star - \sigma| \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max\{\sigma^\star, \sigma\}$ **then** ▷ stopping criterion
        break
    else if $j < j_{\max}$ **then**
        $\sigma \leftarrow \sigma^\star$
    end if
end for
$\sigma \leftarrow (1 + \epsilon_{\text{buff}})\, \sigma^\star$ ▷ buffer the estimated maximum singular value

Return: σ ▷ $\approx \max \operatorname{spec} \hat{H}^\top \hat{H} = \sigma_{\max}(\hat{H}^\top \hat{H}) = \|\hat{H}\|_2^2$

Algorithm 2 Customized power iteration method

Inputs: $\hat{A}^-_{[1:N-1]}$, $\hat{A}^+_{[1:N-1]}$, $\hat{B}^-_{[1:N-1]}$, $\hat{B}^+_{[1:N-1]}$, $x_{[1:N]}$, $u_{[1:N]}$, $\phi_{[1:N-1]}$, $\psi_{[1:N-1]}$, ϵ_{abs}, ϵ_{rel}, ϵ_{buff}, $j_{\max}$

Require: $\left\|x_{[1:N]}\right\|_2 > 0,\ \left\|u_{[1:N]}\right\|_2 > 0,\ \left\|\phi_{[1:N-1]}\right\|_2 > 0,\ \left\|\psi_{[1:N-1]}\right\|_2 > 0$

$\sigma \leftarrow 0$
for $k \leftarrow 1\!:\!N-1$ **do** ▷ Algorithm 1, Line 1
  $\sigma \leftarrow \sigma + \|x_k\|_2^2 + \|u_k\|_2^2 + \|\phi_k\|_2^2 + \|\psi_k\|_2^2$
end for
$\sigma \leftarrow \sigma + \|x_N\|_2^2 + \|u_N\|_2^2$
$\sigma \leftarrow \sqrt{\sigma}$
for $j \leftarrow 1\!:\!j_{\max}$ **do**
  for $k \leftarrow 1\!:\!N-1$ **do** ▷ Algorithm 1, Line 3
    $w_k \leftarrow \frac{1}{\sigma}\left(\hat{A}^-_k\, x_k + \hat{A}^+_k\, x_{k+1} + \hat{B}^-_k u_k + \hat{B}^+_k u_{k+1} + \phi_k - \psi_k\right)$
    $v_k \leftarrow \frac{1}{\sigma}(y_{k+1} - y_k)$ ▷ $y_k := [0_{1\times(n_x-1)},\ 1]\, x_k$
  end for
  $x_1 \leftarrow \hat{A}^{-\top}_1 w_1 - \tilde{v}_1$ ▷ $\tilde{v}_k := [0_{1\times(n_x-1)}, v_k]^\top$
  $u_1 \leftarrow \hat{B}^{-\top}_1 w_1$
  $\phi_1 \leftarrow w_1$
  $\psi_1 \leftarrow -w_1$
  for $k \leftarrow 2\!:\!N-1$ **do** ▷ Algorithm 1, Line 4
    $x_k \leftarrow \hat{A}^{-\top}_k w_k + \hat{A}^{+\top}_{k-1} w_{k-1} - \tilde{v}_k + \tilde{v}_{k-1}$
    $u_k \leftarrow \hat{B}^{-\top}_k w_k + \hat{B}^{+\top}_{k-1} w_{k-1}$
    $\phi_k \leftarrow w_k$
    $\psi_k \leftarrow -w_k$
  end for
  $x_N \leftarrow \hat{A}^{+\top}_{N-1} w_{N-1} + \tilde{v}_{N-1}$
  $u_N \leftarrow \hat{B}^{+\top}_{N-1} w_{N-1}$
  $\sigma^\star \leftarrow 0$
  for $k \leftarrow 1\!:\!N-1$ **do** ▷ Algorithm 1, Line 5
    $\sigma^\star \leftarrow \sigma^\star + \|x_k\|_2^2 + \|u_k\|_2^2 + \|\phi_k\|_2^2 + \|\psi_k\|_2^2$
  end for
  $\sigma^\star \leftarrow \sigma + \|x_N\|_2^2 + \|u_N\|_2^2$
  $\sigma^\star \leftarrow \sqrt{\sigma^\star}$
  if $|\sigma^\star - \sigma| \le \epsilon_{\mathrm{abs}} + \epsilon_{\mathrm{rel}}\, \max\{\sigma^\star,\, \sigma\}$ **then** ▷ stopping criterion
    break
  else if $j < j_{\max}$ **then**
    $\sigma \leftarrow \sigma^\star$
  end if
end for
$\sigma \leftarrow (1 + \epsilon_{\mathrm{buff}})\, \sigma^\star$ ▷ buffer the estimated maximum singular value

Return: σ ▷ $\approx \max \operatorname{spec} \hat{H}^\top \hat{H} = \sigma_{\max}(\hat{H}^\top \hat{H}) = \|\hat{H}\|_2^2$

Algorithm 3 PIPG

Inputs: q, H, h, $\mathbb{D}$, $\mathbb{K}^\circ$, $\bar{z}$, λ, σ, ρ, ϵ_{abs}, ϵ_{rel}, j_{check}, $j_{\max}$,
$z^\star$, $w^\star$ ▷ warm start

$\zeta^1 \leftarrow z^\star$ ▷ initialize primal variable
$\eta^1 \leftarrow w^\star$ ▷ initialize dual variable
$\alpha \leftarrow \frac{2}{\lambda+\sqrt{\lambda^2+4\omega\sigma}}$ ▷ step-sizes
$\beta \leftarrow \omega\alpha$
for $j \leftarrow 1\!:\!j_{\max}$ **do**
  $z^{j+1} = \pi_{\mathbb{D}}[\zeta^j - \alpha\,(\lambda\,\zeta^j + q + H^\top \eta^j) + \bar{z}] - \bar{z}$ ▷ projected gradient step
  $w^{j+1} = \pi_{\mathbb{K}^\circ}[\eta^j + \beta\,(H(2\,z^{j+1} - \zeta^j) - h)]$ ▷ PI feedback of affine equality constraint violation
  $\zeta^{j+1} = (1-\rho)\,\zeta^j + \rho\,z^{j+1}$ ▷ extrapolate primal variable
  $\eta^{j+1} = (1-\rho)\,\eta^j + \rho\,w^{j+1}$ ▷ extrapolate dual variable
  if $j \bmod j_{\text{check}} = 0$ **then** ▷ check stopping criterion every j_{check} iterations
    if $\left\|z^{j+1} - z^j\right\|_\infty \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}}\ \max\left\{\left\|z^{j+1}\right\|_\infty,\ \left\|z^j\right\|_\infty\right\}$ **and**
    $\left\|w^{j+1} - w^j\right\|_\infty \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}}\ \max\left\{\left\|w^{j+1}\right\|_\infty,\ \left\|w^j\right\|_\infty\right\}$ **then** ▷ stopping criterion
      break
    end if
  end if
end for
$z^\star \leftarrow z^{j+1}$ ▷ update primal variable
$w^\star \leftarrow w^{j+1}$ ▷ update dual variable

Return: $z^\star$, $w^\star$

Algorithm 4 PIPG$_{\text{custom}}$

Inputs: $q_{x_{[1:N]}}, q_{u_{[1:N]}}, q_{\phi_{[1:N-1]}}, q_{\psi_{[1:N-1]}},$
$\hat{A}^-_{[1:N-1]}, \hat{A}^+_{[1:N-1]}, \hat{B}^-_{[1:N-1]}, \hat{B}^+_{[1:N-1]}, \hat{d}_{[2:N]}, \hat{\bar{x}}_{[1:N]}, \hat{\bar{u}}_{[1:N]},$
$\hat{\mathbb{D}}_{x_{[1:N]}}, \hat{\mathbb{D}}_{u_{[1:N]}}, \mathbb{D}_{\phi_{[1:N-1]}}, \mathbb{D}_{\psi_{[1:N-1]}}, \varepsilon,$
$\lambda, \sigma, \omega, \rho, \epsilon_{\text{abs}}, \epsilon_{\text{rel}}, j_{\text{check}}, j_{\max},$
$\Delta\hat{x}^\star_{[1:N]}, \Delta\hat{u}^\star_{[1:N]}, \phi^\star_{[1:N-1]}, \psi^\star_{[1:N-1]}, w^\star_{[1:N-1]}, v^\star_{[1:N-1]}$ $\triangleright$ warm start

$\Delta x^1_{\zeta_{[1:N]}} \leftarrow \Delta\hat{x}^\star_{[1:N]}$ $\triangleright$ initialize primal variables
$\Delta u^1_{\zeta_{[1:N]}} \leftarrow \Delta\hat{u}^\star_{[1:N]}$
$\phi^1_{\zeta_{[1:N-1]}} \leftarrow \phi^\star_{[1:N-1]}$
$\psi^1_{\zeta_{[1:N-1]}} \leftarrow \psi^\star_{[1:N-1]}$
$\eta^1_{[1:N-1]} \leftarrow w^\star_{[1:N-1]}$ $\triangleright$ initialize dual variables
$\gamma^1_{[1:N-1]} \leftarrow v^\star_{[1:N-1]}$
$\alpha \leftarrow \frac{2}{\lambda+\sqrt{\lambda^2+4\omega\sigma}}$ $\triangleright$ step-sizes
$\beta \leftarrow \omega\alpha$
for $j \leftarrow 1\!:\!j_{\max}$ **do**
  $\Delta\hat{x}^j_1 \leftarrow \pi_{\hat{\mathbb{D}}_{x_1}}[\Delta x^j_{\zeta_1} - \alpha\,(\lambda\,\Delta x^j_{\zeta_1} + q_{x_1} + \hat{A}_1^{-\top}\eta^j_1 - \tilde{\gamma}^j_1) + \hat{\bar{x}}^j_1] - \hat{\bar{x}}^j_1$ $\triangleright$ $\tilde{\gamma}_k := [0_{1\times(n_x-1)}, \gamma_k]^\top$
  $\Delta\hat{u}^j_1 \leftarrow \pi_{\hat{\mathbb{D}}_{u_1}}[\Delta u^j_{\zeta_1} - \alpha\,(\lambda\,\Delta u^j_{\zeta_1} + q_{u_1} + \hat{B}_1^{-\top}\eta^j_1) + \hat{\bar{u}}^j_1] - \hat{\bar{u}}^j_1$
  for $k \leftarrow 2\!:\!N-1$ **do** $\triangleright$ projected gradient step
    $\Delta\hat{x}^{j+1}_k \leftarrow \pi_{\hat{\mathbb{D}}_{x_k}}[\Delta x^j_{\zeta_k} - \alpha\,(\lambda\,\Delta x^j_{\zeta_k} + q_{x_k} + \hat{A}_k^{-\top}\eta^j_k + \hat{A}_{k-1}^{+\top}\eta^j_{k-1} - \tilde{\gamma}^j_k + \tilde{\gamma}^j_{k-1}) + \hat{\bar{x}}^j_k] - \hat{\bar{x}}^j_k$
    $\Delta\hat{u}^{j+1}_k \leftarrow \pi_{\hat{\mathbb{D}}_{u_k}}[\Delta u^j_{\zeta_k} - \alpha\,(\lambda\,\Delta u^j_{\zeta_k} + q_{u_k} + \hat{B}_k^{-\top}\eta^j_k + \hat{B}_{k-1}^{+\top}\eta^j_{k-1}) + \hat{\bar{u}}^j_k] - \hat{\bar{u}}^j_k$
  end for
  $\Delta\hat{x}^{j+1}_N \leftarrow \pi_{\hat{\mathbb{D}}_{x_N}}[\Delta x^j_{\zeta_N} - \alpha\,(\lambda\,\Delta x^j_{\zeta_N} + q_{x_N} + \hat{A}_{N-1}^{+\top}\eta^j_{N-1} + \tilde{\gamma}^j_{N-1}) + \hat{\bar{x}}^j_N] - \hat{\bar{x}}^j_N$
  $\Delta\hat{u}^{j+1}_N \leftarrow \pi_{\hat{\mathbb{D}}_{u_N}}[\Delta u^j_{\zeta_N} - \alpha\,(\lambda\,\Delta u^j_{\zeta_N} + q_{u_N} + \hat{B}_{N-1}^{+\top}\eta^j_{N-1}) + \hat{\bar{u}}^j_N] - \hat{\bar{u}}^j_N$
  $\phi^j_{[1:N-1]} \leftarrow \pi_{\mathbb{D}_{\phi_{[1:N-1]}}}[\phi^j_{\zeta_{[1:N-1]}} - \alpha\,(q_{\phi_{[1:N-1]}} + \eta^j_{[1:N-1]})]$
  $\psi^j_{[1:N-1]} \leftarrow \pi_{\mathbb{D}_{\psi_{[1:N-1]}}}[\psi^j_{\zeta_{[1:N-1]}} - \alpha\,(q_{\psi_{[1:N-1]}} - \eta^j_{[1:N-1]})]$
  for $k \leftarrow 1 : N-1$ **do** $\triangleright$ PI feedback of affine equality constraint violation
    $w^{j+1}_k \leftarrow \eta^j_k + \beta\,(\hat{A}^-_k\,(2\Delta\hat{x}^{j+1}_k - \Delta x^j_{\zeta_k}) + \hat{A}^+_k\,(2\Delta\hat{x}^{j+1}_{k+1} - \Delta x^j_{\zeta_{k+1}}) + \hat{B}^-_k\,(2\Delta\hat{u}^{j+1}_k - \Delta u^j_{\zeta_k}) + \hat{B}^+_k\,(2\Delta\hat{u}^{j+1}_{k+1} - \Delta u^j_{\zeta_{k+1}})$
    $+ (2\,\phi^{j+1}_k - \phi^j_{\zeta_k}) - (2\,\psi^{j+1}_k - \psi^j_{\zeta_k}) + \hat{d}_{k+1})$
    $v^{j+1}_k \leftarrow \max\{0,\, \gamma^j_k + \beta\,((2\Delta\hat{y}^{j+1}_{k+1} - \Delta y^j_{\zeta_{k+1}}) - (2\Delta\hat{y}^{j+1}_k - \Delta y^j_{\zeta_k}) + (\hat{\bar{y}}_{k+1} - \hat{\bar{y}}_k) - \varepsilon)\}$ $\triangleright$ $\square\overset{\diamond}{y}_k := [0_{1\times(n_x-1)},\, 1]\,\square\overset{\diamond}{x}_k$
  end for
  $\Delta x^{j+1}_{\zeta_{[1:N]}} \leftarrow (1-\rho)\,\Delta x^j_{\zeta_{[1:N]}} + \rho\,\Delta\hat{x}^{j+1}_{[1:N]}$ $\triangleright$ extrapolate primal variables
  $\Delta u^{j+1}_{\zeta_{[1:N]}} \leftarrow (1-\rho)\,\Delta u^j_{\zeta_{[1:N]}} + \rho\,\Delta\hat{u}^{j+1}_{[1:N]}$
  $\phi^{j+1}_{\zeta_{[1:N-1]}} \leftarrow (1-\rho)\,\phi^j_{\zeta_{[1:N-1]}} + \rho\,\phi^{j+1}_{[1:N-1]}$
  $\psi^{j+1}_{\zeta_{[1:N-1]}} \leftarrow (1-\rho)\,\psi^j_{\zeta_{[1:N-1]}} + \rho\,\psi^{j+1}_{[1:N-1]}$
  $\eta^{j+1}_{[1:N-1]} \leftarrow (1-\rho)\,\eta^j_{[1:N-1]} + \rho\,w^{j+1}_{[1:N-1]}$ $\triangleright$ extrapolate dual variables
  $\gamma^{j+1}_{[1:N-1]} \leftarrow (1-\rho)\,\gamma^j_{[1:N-1]} + \rho\,v^{j+1}_{[1:N-1]}$
  if $j \bmod j_{\text{check}} = 0$ **then** $\triangleright$ check stopping criterion every j_{check} iterations
    TERMINATE $\leftarrow$ STOPPING$_{\text{custom}}$$(\Delta\hat{x}^{j+1}_{[1:N]}, \Delta\hat{u}^{j+1}_{[1:N]}, \phi^{j+1}_{[1:N-1]}, \psi^{j+1}_{[1:N-1]}, w^{j+1}_{[1:N-1]}, v^{j+1}_{[1:N-1]}$
    $\Delta\hat{x}^j_{[1:N]}, \Delta\hat{u}^j_{[1:N]}, \phi^j_{[1:N-1]}, \psi^j_{[1:N-1]}, w^j_{[1:N-1]}, v^j_{[1:N-1]}\,\epsilon_{\text{abs}}, \epsilon_{\text{rel}})$
    if TERMINATE = TRUE **then** $\triangleright$ stopping criterion
      break
    end if
  end if
end for
$\Delta\hat{x}^\star_{[1:N]} \leftarrow \Delta\hat{x}^{j+1}_{[1:N]}$ $\triangleright$ update primal variables
$\Delta\hat{u}^\star_{[1:N]} \leftarrow \Delta\hat{u}^{j+1}_{[1:N]}$
$\phi^\star_{[1:N-1]} \leftarrow \phi^{j+1}_{[1:N-1]}$
$\psi^\star_{[1:N-1]} \leftarrow \psi^{j+1}_{[1:N-1]}$
$w^\star_{[1:N-1]} \leftarrow w^{j+1}_{[1:N-1]}$ $\triangleright$ update dual variables
$v^\star_{[1:N-1]} \leftarrow v^{j+1}_{[1:N-1]}$

Return: $\Delta x^\star_{[1:N]}, \Delta u^\star_{[1:N]}, \phi^\star_{[1:N-1]}, \psi^\star_{[1:N-1]}, w^\star_{[1:N-1]}, v^\star_{[1:N-1]}$

Algorithm 5 Customized stopping criterion evaluation:

$$\textsc{stopping}_{\text{custom}}\big(\Delta\hat{x}^{j+1}_{[1:N]},\, \Delta\hat{u}^{j+1}_{[1:N]},\, \phi^{j+1}_{[1:N-1]},\, \psi^{j+1}_{[1:N-1]},\, w^{j+1}_{[1:N-1]},\, v^{j+1}_{[1:N-1]}$$
$$\Delta\hat{x}^{j}_{[1:N]},\, \Delta\hat{u}^{j}_{[1:N]},\, \phi^{j}_{[1:N-1]},\, \psi^{j}_{[1:N-1]},\, w^{j}_{[1:N-1]},\, v^{j}_{[1:N-1]},\, \epsilon_{\text{abs}},\, \epsilon_{\text{rel}}\big)$$

Inputs: $\Delta\hat{x}^{j+1}_{[1:N]}$, $\Delta\hat{u}^{j+1}_{[1:N]}$, $\phi^{j+1}_{[1:N-1]}$, $\psi^{j+1}_{[1:N-1]}$, $w^{j+1}_{[1:N-1]}$,
$\Delta\hat{x}^{j}_{[1:N]}$, $\Delta\hat{u}^{j}_{[1:N]}$, $\phi^{j}_{[1:N-1]}$, $\psi^{j}_{[1:N-1]}$, $w^{j}_{[1:N-1]}$, ϵ_{abs}, ϵ_{rel}

$z^{j+1}_{\infty} \leftarrow \max\Big\{\|\Delta\hat{x}^{j+1}_{[1:N]}\|_\infty,\, \|\Delta\hat{u}^{j+1}_{[1:N]}\|_\infty,\, \|\phi^{j+1}_{[1:N-1]}\|_\infty,\, \|\psi^{j+1}_{[1:N-1]}\|_\infty\Big\}$

$z^{j}_{\infty} \leftarrow \max\Big\{\|\Delta\hat{x}^{j}_{[1:N]}\|_\infty,\, \|\Delta\hat{u}^{j}_{[1:N]}\|_\infty,\, \|\phi^{j}_{[1:N-1]}\|_\infty,\, \|\psi^{j}_{[1:N-1]}\|_\infty\Big\}$

$z^{\Delta j}_{\infty} \leftarrow \max\Big\{\|\Delta\hat{x}^{j+1}_{[1:N]} - \Delta\hat{x}^{j}_{[1:N]}\|_\infty,\, \|\Delta\hat{u}^{j+1}_{[1:N]} - \Delta\hat{u}^{j}_{[1:N]}\|_\infty,\, \|\phi^{j+1}_{[1:N-1]} - \phi^{j}_{[1:N-1]}\|_\infty,\, \|\psi^{j+1}_{[1:N-1]} - \psi^{j}_{[1:N-1]}\|_\infty\Big\}$

$r^{j+1}_{\infty} \leftarrow \max\Big\{\|w^{j+1}_{[1:N-1]}\|_\infty,\, \|v^{j+1}_{[1:N-1]}\|_\infty\Big\}$

$r^{j}_{\infty} \leftarrow \max\Big\{\|w^{j}_{[1:N-1]}\|_\infty,\, \|v^{j}_{[1:N-1]}\|_\infty\Big\}$

$r^{\Delta j}_{\infty} \leftarrow \max\Big\{\|w^{j+1}_{[1:N-1]} - w^{j}_{[1:N-1]}\|_\infty,\, \|v^{j+1}_{[1:N-1]} - v^{j}_{[1:N-1]}\|_\infty\Big\}$

if $z^{\Delta j}_{\infty} \le \epsilon_{\text{abs}} + \epsilon_{\text{rel}}\, \max\Big\{z^{j+1}_{\infty},\, z^{j}_{\infty}\Big\}$ **and** $r^{\Delta j}_{\infty} \le \epsilon_{\text{abs}} + \epsilon_{\text{rel}}\, \max\Big\{r^{j+1}_{\infty},\, r^{j}_{\infty}\Big\}$ **then**

    TERMINATE $\leftarrow$ TRUE

else

    TERMINATE $\leftarrow$ FALSE

end if

Return: TERMINATE