
Algorithm 1 Power iteration method

Inputs: \hat{H} , z , ϵ_{abs} , ϵ_{rel} , ϵ_{buff} , j_{max}

Require: $\|z\|_2 > 0$

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1:  $\sigma \leftarrow \|z\|_2$  ▷ initialization
2: for  $j \leftarrow 1:j_{\text{max}}$  do
3:    $w \leftarrow \frac{1}{\sigma} \hat{H} z$ 
4:    $z \leftarrow \hat{H}^\top w$ 
5:    $\sigma^* \leftarrow \|z\|_2$ 
6:   if  $|\sigma^* - \sigma| \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max\{\sigma^*, \sigma\}$  then ▷ stopping criterion
7:     break
8:   else if  $j < j_{\text{max}}$  then
9:      $\sigma \leftarrow \sigma^*$ 
10:  end if
11: end for
12:  $\sigma \leftarrow (1 + \epsilon_{\text{buff}}) \sigma^*$  ▷ buffer the estimated maximum singular value
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Return: σ $\approx \max \text{spec } \hat{H}^\top \hat{H} = \sigma_{\max}(\hat{H}^\top \hat{H}) = \|\hat{H}\|_2^2$

Algorithm 2 Customized power iteration method

Inputs: $\hat{A}_{[1:N-1]}^-$, $\hat{A}_{[1:N-1]}^+$, $\hat{B}_{[1:N-1]}^-$, $\hat{B}_{[1:N-1]}^+$, $x_{[1:N]}$, $u_{[1:N]}$, $\phi_{[1:N-1]}$, $\psi_{[1:N-1]}$, ϵ_{abs} , ϵ_{rel} , ϵ_{buff} , j_{max}

Require: $\|x_{[1:N]}\|_2 > 0$, $\|u_{[1:N]}\|_2 > 0$, $\|\phi_{[1:N-1]}\|_2 > 0$, $\|\psi_{[1:N-1]}\|_2 > 0$

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1:  $\sigma \leftarrow 0$ 
2: for  $k \leftarrow 1:N-1$  do ▷ Algorithm 1, Line 1
3:    $\sigma \leftarrow \sigma + \|x_k\|_2^2 + \|u_k\|_2^2 + \|\phi_k\|_2^2 + \|\psi_k\|_2^2$ 
4: end for
5:  $\sigma \leftarrow \sigma + \|x_N\|_2^2 + \|u_N\|_2^2$ 
6:  $\sigma \leftarrow \sqrt{\sigma}$ 
7: for  $j \leftarrow 1:j_{\text{max}}$  do
8:   for  $k \leftarrow 1:N-1$  do ▷ Algorithm 1, Line 3
9:      $w_k \leftarrow \frac{1}{\sigma} (\hat{A}_k^- x_k + \hat{A}_k^+ x_{k+1} + \hat{B}_k^- u_k + \hat{B}_k^+ u_{k+1} + \phi_k - \psi_k)$ 
10:     $v_k \leftarrow \frac{1}{\sigma} (y_{k+1} - y_k)$  ▷  $y_k := [0_{1 \times (n_x-1)}, 1] x_k$ 
11:  end for
12:   $x_1 \leftarrow \hat{A}_1^{-\top} w_1 - \tilde{v}_1$  ▷  $\tilde{v}_k := [0_{1 \times (n_x-1)}, v_k]^\top$ 
13:   $u_1 \leftarrow \hat{B}_1^{-\top} w_1$ 
14:   $\phi_1 \leftarrow w_1$ 
15:   $\psi_1 \leftarrow -w_1$ 
16:  for  $k \leftarrow 2:N-1$  do ▷ Algorithm 1, Line 4
17:     $x_k \leftarrow \hat{A}_k^{-\top} w_k + \hat{A}_{k-1}^{+\top} w_{k-1} - \tilde{v}_k + \tilde{v}_{k-1}$ 
18:     $u_k \leftarrow \hat{B}_k^{-\top} w_k + \hat{B}_{k-1}^{+\top} w_{k-1}$ 
19:     $\phi_k \leftarrow w_k$ 
20:     $\psi_k \leftarrow -w_k$ 
21:  end for
22:   $x_N \leftarrow \hat{A}_{N-1}^{+\top} w_{N-1} + \tilde{v}_{N-1}$ 
23:   $u_N \leftarrow \hat{B}_{N-1}^{+\top} w_{N-1}$ 
24:   $\sigma^* \leftarrow 0$ 
25:  for  $k \leftarrow 1:N-1$  do ▷ Algorithm 1, Line 5
26:     $\sigma^* \leftarrow \sigma^* + \|x_k\|_2^2 + \|u_k\|_2^2 + \|\phi_k\|_2^2 + \|\psi_k\|_2^2$ 
27:  end for
28:   $\sigma^* \leftarrow \sigma + \|x_N\|_2^2 + \|u_N\|_2^2$ 
29:   $\sigma^* \leftarrow \sqrt{\sigma^*}$ 
30:  if  $|\sigma^* - \sigma| \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max\{\sigma^*, \sigma\}$  then ▷ stopping criterion
31:    break
32:  else if  $j < j_{\text{max}}$  then
33:     $\sigma \leftarrow \sigma^*$ 
34:  end if
35: end for
36:  $\sigma \leftarrow (1 + \epsilon_{\text{buff}}) \sigma^*$  ▷ buffer the estimated maximum singular value
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Return: σ ▷ $\approx \max \text{spec } \hat{H}^\top \hat{H} = \sigma_{\text{max}}(\hat{H}^\top \hat{H}) = \|\hat{H}\|_2^2$

Algorithm 3 PIPG

Inputs: $q, H, h, \mathbb{D}, \bar{z}, \lambda, \sigma, \rho, \epsilon_{\text{abs}}, \epsilon_{\text{rel}}, j_{\text{check}}, j_{\text{max}},$

z^*, w^*

▷ warm start

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1:  $\zeta^1 \leftarrow z^*$  ▷ initialize primal variable
2:  $\eta^1 \leftarrow w^*$  ▷ initialize dual variable
3:  $\alpha \leftarrow \frac{2}{\lambda + \sqrt{\lambda^2 + 4\sigma}}$  ▷ step-size
4: for  $j \leftarrow 1:j_{\text{max}}$  do
5:    $z^{j+1} = \pi_{\mathbb{D}}[\zeta^j - \alpha(\lambda \zeta^j + q + H^\top \eta^j) + \bar{z}] - \bar{z}$  ▷ projected gradient step
6:    $w^{j+1} = \eta^j + \alpha(H(2z^{j+1} - \zeta^j) - h)$  ▷ PI feedback of affine equality constraint violation
7:    $\zeta^{j+1} = (1 - \rho)\zeta^j + \rho z^{j+1}$  ▷ extrapolate primal variable
8:    $\eta^{j+1} = (1 - \rho)\eta^j + \rho w^{j+1}$  ▷ extrapolate dual variable
9:   if  $j \bmod j_{\text{check}} = 0$  then ▷ check stopping criterion every  $j_{\text{check}}$  iterations
10:    if  $\|z^{j+1} - z^j\|_\infty \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max\{\|z^{j+1}\|_\infty, \|z^j\|_\infty\}$  and
11:     $\|w^{j+1} - w^j\|_\infty \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max\{\|w^{j+1}\|_\infty, \|w^j\|_\infty\}$  then ▷ stopping criterion
12:      break
13:    end if
14:  end if
15: end for
16:  $z^* \leftarrow z^{j+1}$  ▷ update primal variable
17:  $w^* \leftarrow w^{j+1}$  ▷ update dual variable
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Return: z^*, w^*

Algorithm 4 PIPG_{custom}

Inputs: $q_{x[1:N]}, q_{u[1:N]}, q_{\phi[1:N-1]}, q_{\psi[1:N-1]}, \hat{A}_{[1:N-1]}^-, \hat{A}_{[1:N-1]}^+, \hat{B}_{[1:N-1]}^-, \hat{B}_{[1:N-1]}^+, \hat{d}_{[2:N]}, \hat{x}_{[1:N]}, \hat{u}_{[1:N]}, \hat{\mathbb{D}}_{x[1:N]}, \hat{\mathbb{D}}_{u[1:N]}, \mathbb{D}_{\phi[1:N-1]}, \mathbb{D}_{\psi[1:N-1]}, \epsilon, \lambda, \sigma, \omega, \rho, \epsilon_{\text{abs}}, \epsilon_{\text{rel}}, j_{\text{check}}, j_{\text{max}}, \Delta \hat{x}_{[1:N]}^*, \Delta \hat{u}_{[1:N]}^*, \phi_{[1:N-1]}^*, \psi_{[1:N-1]}^*, w_{[1:N-1]}^*, v_{[1:N-1]}^*$ ▷ warm start

1: $\Delta x_{\zeta[1:N]}^1 \leftarrow \Delta \hat{x}_{[1:N]}^*$ ▷ initialize primal variables
2: $\Delta u_{\zeta[1:N]}^1 \leftarrow \Delta \hat{u}_{[1:N]}^*$
3: $\phi_{\zeta[1:N-1]}^1 \leftarrow \phi_{[1:N-1]}^*$
4: $\psi_{\zeta[1:N-1]}^1 \leftarrow \psi_{[1:N-1]}^*$
5: $\eta_{[1:N-1]}^1 \leftarrow w_{[1:N-1]}^*$ ▷ initialize dual variables
6: $\gamma_{[1:N-1]}^1 \leftarrow v_{[1:N-1]}^*$
7: $\alpha \leftarrow \frac{2}{\lambda + \sqrt{\lambda^2 + 4\omega\sigma}}$ ▷ step-sizes
8: $\beta \leftarrow \omega\alpha$
9: **for** $j \leftarrow 1:j_{\text{max}}$ **do**
10: $\Delta \hat{x}_1^j \leftarrow \pi_{\hat{\mathbb{D}}_{x_1}} [\Delta x_{\zeta_1}^j - \alpha (\lambda \Delta x_{\zeta_1}^j + q_{x_1} + \hat{A}_1^{-\top} \eta_1^j - \tilde{\gamma}_1^j) + \hat{x}_1^j] - \hat{x}_1^j$ ▷ $\tilde{\gamma}_k := [0_{1 \times (n_x-1)}, \gamma_k]^\top$
11: $\Delta \hat{u}_1^j \leftarrow \pi_{\hat{\mathbb{D}}_{u_1}} [\Delta u_{\zeta_1}^j - \alpha (\lambda \Delta u_{\zeta_1}^j + q_{u_1} + \hat{B}_1^{-\top} \eta_1^j) + \hat{u}_1^j] - \hat{u}_1^j$
12: $\phi_1^j \leftarrow \pi_{\mathbb{D}_{\phi_1}} [\phi_{\zeta_1}^j - \alpha (\lambda \phi_{\zeta_1}^j + q_{\phi_1} + \eta_1^j)]$
13: $\psi_1^j \leftarrow \pi_{\mathbb{D}_{\psi_1}} [\psi_{\zeta_1}^j - \alpha (\lambda \psi_{\zeta_1}^j + q_{\psi_1} - \eta_1^j)]$
14: **for** $k \leftarrow 2:N-1$ **do** ▷ projected gradient step
15: $\Delta \hat{x}_k^{j+1} \leftarrow \pi_{\hat{\mathbb{D}}_{x_k}} [\Delta x_{\zeta_k}^j - \alpha (\lambda \Delta x_{\zeta_k}^j + q_{x_k} + \hat{A}_k^{-\top} \eta_k^j + \hat{A}_{k-1}^{+\top} \eta_{k-1}^j - \tilde{\gamma}_k^j + \tilde{\gamma}_{k-1}^j) + \hat{x}_k^j] - \hat{x}_k^j$
16: $\Delta \hat{u}_k^{j+1} \leftarrow \pi_{\hat{\mathbb{D}}_{u_k}} [\Delta u_{\zeta_k}^j - \alpha (\lambda \Delta u_{\zeta_k}^j + q_{u_k} + \hat{B}_k^{-\top} \eta_k^j + \hat{B}_{k-1}^{+\top} \eta_{k-1}^j) + \hat{u}_k^j] - \hat{u}_k^j$
17: $\phi_k^{j+1} \leftarrow \pi_{\mathbb{D}_{\phi_k}} [\phi_{\zeta_k}^j - \alpha (\lambda \phi_{\zeta_k}^j + q_{\phi_k} + \eta_k^j)]$
18: $\psi_k^{j+1} \leftarrow \pi_{\mathbb{D}_{\psi_k}} [\psi_{\zeta_k}^j - \alpha (\lambda \psi_{\zeta_k}^j + q_{\psi_k} - \eta_k^j)]$
19: **end for**
20: $\Delta \hat{x}_N^{j+1} \leftarrow \pi_{\hat{\mathbb{D}}_{x_N}} [\Delta x_{\zeta_N}^j - \alpha (\lambda \Delta x_{\zeta_N}^j + q_{x_N} + \hat{A}_{N-1}^{+\top} \eta_{N-1}^j + \tilde{\gamma}_{N-1}^j) + \hat{x}_N^j] - \hat{x}_N^j$
21: $\Delta \hat{u}_N^{j+1} \leftarrow \pi_{\hat{\mathbb{D}}_{u_N}} [\Delta u_{\zeta_N}^j - \alpha (\lambda \Delta u_{\zeta_N}^j + q_{u_N} + \hat{B}_{N-1}^{+\top} \eta_{N-1}^j) + \hat{u}_N^j] - \hat{u}_N^j$
22: **for** $k \leftarrow 1:N-1$ **do** ▷ PI feedback of affine equality constraint violation
23: $w_k^{j+1} \leftarrow \eta_k^j + \beta (\hat{A}_k^- (2\Delta \hat{x}_k^{j+1} - \Delta x_{\zeta_k}^j) + \hat{A}_k^+ (2\Delta \hat{x}_{k+1}^{j+1} - \Delta x_{\zeta_{k+1}}^j) + \hat{B}_k^- (2\Delta \hat{u}_k^{j+1} - \Delta u_{\zeta_k}^j) + \hat{B}_k^+ (2\Delta \hat{u}_{k+1}^{j+1} - \Delta u_{\zeta_{k+1}}^j) + (2\phi_k^{j+1} - \phi_{\zeta_k}^j) + (2\psi_k^{j+1} - \psi_{\zeta_k}^j) + \hat{d}_{k+1})$
24: $v_k^{j+1} \leftarrow \max\{0, \gamma_k^j + \beta ((2\Delta \hat{y}_{k+1}^{j+1} - \Delta y_{\zeta_{k+1}}^j) - (2\Delta \hat{y}_k^{j+1} - \Delta y_{\zeta_k}^j) + (\hat{y}_{k+1} - \hat{y}_k) - \epsilon)\}$ ▷ $\square \hat{y}_k := [0_{1 \times (n_x-1)}, 1] \square \hat{x}_k$
25: **end for**
26: $\Delta x_{\zeta[1:N]}^{j+1} \leftarrow (1-\rho) \Delta x_{\zeta[1:N]}^j + \rho \Delta \hat{x}_{[1:N]}^{j+1}$ ▷ extrapolate primal variables
27: $\Delta u_{\zeta[1:N]}^{j+1} \leftarrow (1-\rho) \Delta u_{\zeta[1:N]}^j + \rho \Delta \hat{u}_{[1:N]}^{j+1}$
28: $\phi_{\zeta[1:N-1]}^{j+1} \leftarrow (1-\rho) \phi_{\zeta[1:N-1]}^j + \rho \phi_{[1:N-1]}^{j+1}$
29: $\psi_{\zeta[1:N-1]}^{j+1} \leftarrow (1-\rho) \psi_{\zeta[1:N-1]}^j + \rho \psi_{[1:N-1]}^{j+1}$
30: $\eta_{[1:N-1]}^{j+1} \leftarrow (1-\rho) \eta_{[1:N-1]}^j + \rho w_{[1:N-1]}^{j+1}$ ▷ extrapolate dual variables
31: $\gamma_{[1:N-1]}^{j+1} \leftarrow (1-\rho) \gamma_{[1:N-1]}^j + \rho v_{[1:N-1]}^{j+1}$
32: **if** $j \bmod j_{\text{check}} = 0$ **then** ▷ check stopping criterion every j_{check} iterations
33: $\text{TERMINATE} \leftarrow \text{STOPPING}_{\text{custom}}(\Delta \hat{x}_{[1:N]}^{j+1}, \Delta \hat{u}_{[1:N]}^{j+1}, \phi_{[1:N-1]}^{j+1}, \psi_{[1:N-1]}^{j+1}, w_{[1:N-1]}^{j+1}, v_{[1:N-1]}^{j+1}, \Delta \hat{x}_{[1:N]}^j, \Delta \hat{u}_{[1:N]}^j, \phi_{[1:N-1]}^j, \psi_{[1:N-1]}^j, w_{[1:N-1]}^j, v_{[1:N-1]}^j, \epsilon_{\text{abs}}, \epsilon_{\text{rel}})$
34: **if** $\text{TERMINATE} = \text{TRUE}$ **then** ▷ stopping criterion
35: **break**
36: **end if**
37: **end if**
38: **end for**
39: $\Delta \hat{x}_{[1:N]}^* \leftarrow \Delta \hat{x}_{[1:N]}^{j+1}$ ▷ update primal variables
40: $\Delta \hat{u}_{[1:N]}^* \leftarrow \Delta \hat{u}_{[1:N]}^{j+1}$
41: $\phi_{[1:N-1]}^* \leftarrow \phi_{[1:N-1]}^{j+1}$
42: $\psi_{[1:N-1]}^* \leftarrow \psi_{[1:N-1]}^{j+1}$
43: $w_{[1:N-1]}^* \leftarrow w_{[1:N-1]}^{j+1}$ ▷ update dual variables
44: $v_{[1:N-1]}^* \leftarrow v_{[1:N-1]}^{j+1}$
45: **Return:** $\Delta x_{[1:N]}^*, \Delta u_{[1:N]}^*, \phi_{[1:N-1]}^*, \psi_{[1:N-1]}^*, w_{[1:N-1]}^*, v_{[1:N-1]}^*$

Algorithm 5 Customized stopping criterion evaluation:

$$\text{STOPPING}_{\text{custom}}(\Delta\hat{x}_{[1:N]}^{j+1}, \Delta\hat{u}_{[1:N]}^{j+1}, \phi_{[1:N-1]}^{j+1}, \psi_{[1:N-1]}^{j+1}, w_{[1:N-1]}^{j+1}, v_{[1:N-1]}^{j+1}, \\ \Delta\hat{x}_{[1:N]}^j, \Delta\hat{u}_{[1:N]}^j, \phi_{[1:N-1]}^j, \psi_{[1:N-1]}^j, w_{[1:N-1]}^j, v_{[1:N-1]}^j, \epsilon_{\text{abs}}, \epsilon_{\text{rel}})$$

Inputs: $\Delta\hat{x}_{[1:N]}^{j+1}, \Delta\hat{u}_{[1:N]}^{j+1}, \phi_{[1:N-1]}^{j+1}, \psi_{[1:N-1]}^{j+1}, w_{[1:N-1]}^{j+1},$

$\Delta\hat{x}_{[1:N]}^j, \Delta\hat{u}_{[1:N]}^j, \phi_{[1:N-1]}^j, \psi_{[1:N-1]}^j, w_{[1:N-1]}^j, \epsilon_{\text{abs}}, \epsilon_{\text{rel}}$

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1:  $z_{\infty}^{j+1} \leftarrow \max \left\{ \|\Delta\hat{x}_{[1:N]}^{j+1}\|_{\infty}, \|\Delta\hat{u}_{[1:N]}^{j+1}\|_{\infty}, \|\phi_{[1:N-1]}^{j+1}\|_{\infty}, \|\psi_{[1:N-1]}^{j+1}\|_{\infty} \right\}$ 
2:  $z_{\infty}^j \leftarrow \max \left\{ \|\Delta\hat{x}_{[1:N]}^j\|_{\infty}, \|\Delta\hat{u}_{[1:N]}^j\|_{\infty}, \|\phi_{[1:N-1]}^j\|_{\infty}, \|\psi_{[1:N-1]}^j\|_{\infty} \right\}$ 
3:  $z_{\infty}^{\Delta j} \leftarrow \max \left\{ \|\Delta\hat{x}_{[1:N]}^{j+1} - \Delta\hat{x}_{[1:N]}^j\|_{\infty}, \|\Delta\hat{u}_{[1:N]}^{j+1} - \Delta\hat{u}_{[1:N]}^j\|_{\infty}, \|\phi_{[1:N-1]}^{j+1} - \phi_{[1:N-1]}^j\|_{\infty}, \|\psi_{[1:N-1]}^{j+1} - \psi_{[1:N-1]}^j\|_{\infty} \right\}$ 
4:  $r_{\infty}^{j+1} \leftarrow \max \left\{ \|w_{[1:N-1]}^{j+1}\|_{\infty}, \|v_{[1:N-1]}^{j+1}\|_{\infty} \right\}$ 
5:  $r_{\infty}^j \leftarrow \max \left\{ \|w_{[1:N-1]}^j\|_{\infty}, \|v_{[1:N-1]}^j\|_{\infty} \right\}$ 
6:  $r_{\infty}^{\Delta j} \leftarrow \max \left\{ \|w_{[1:N-1]}^{j+1} - w_{[1:N-1]}^j\|_{\infty}, \|v_{[1:N-1]}^{j+1} - v_{[1:N-1]}^j\|_{\infty} \right\}$ 
7: if  $z_{\infty}^{\Delta j} \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max \{z_{\infty}^{j+1}, z_{\infty}^j\}$  and  $r_{\infty}^{\Delta j} \leq \epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max \{r_{\infty}^{j+1}, r_{\infty}^j\}$  then
8:   TERMINATE  $\leftarrow$  TRUE
9: else
10:   TERMINATE  $\leftarrow$  FALSE
11: end if
```

Return: TERMINATE
