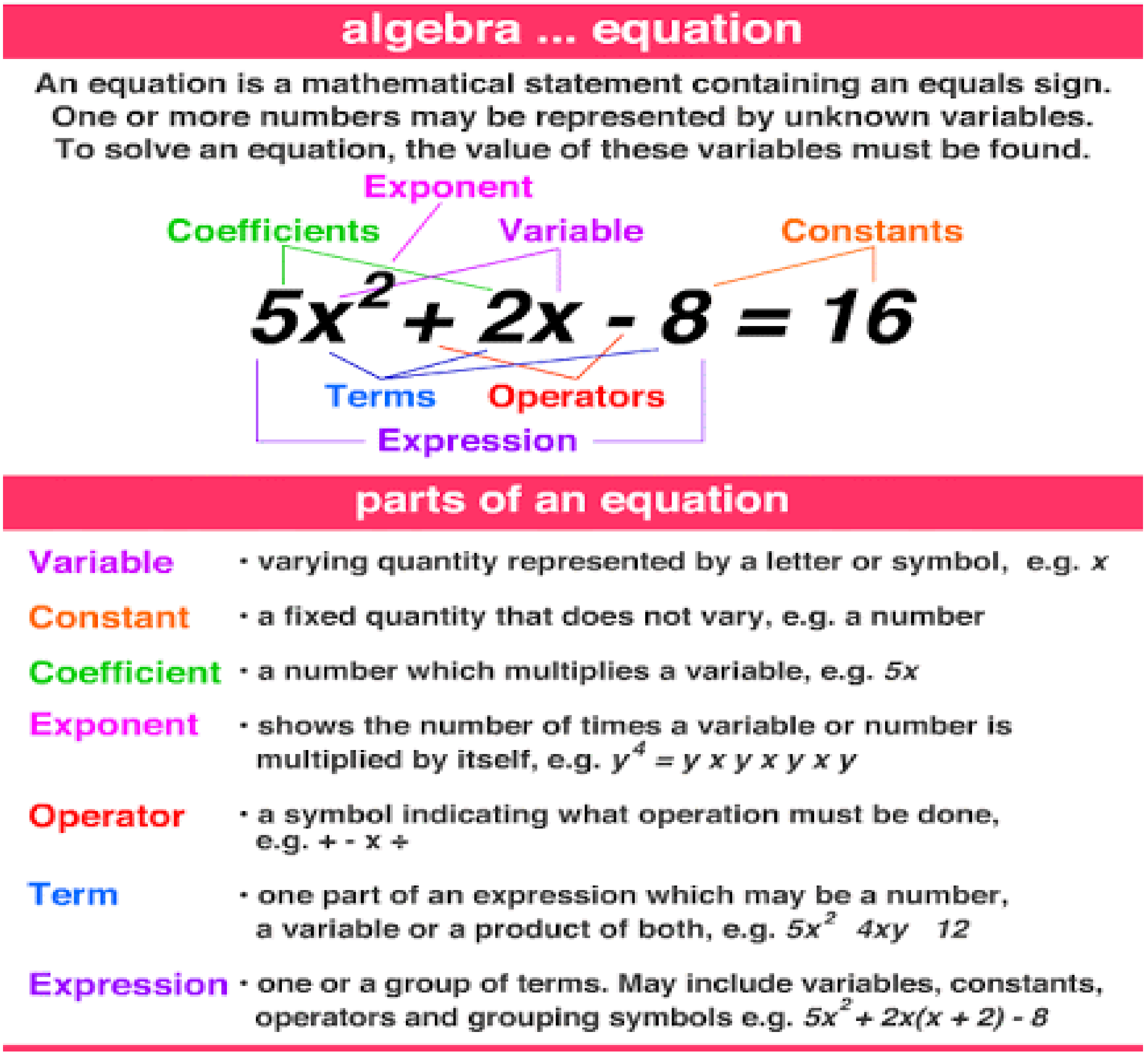
Algebra -1 (LINEAR & QUADRATIC)

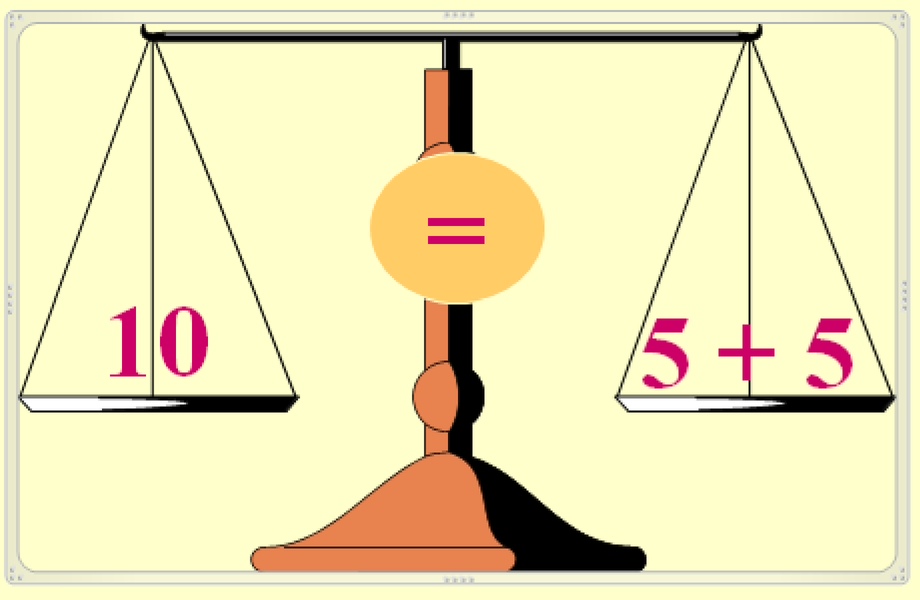


$8 + 4x = 12$

That equation says: what is on the left ($4x + 8$) is equal to what is on the right (12).

So an equation is like a statement "this equals that".

An Equation is like a balance scale. Everything must be equal on both sides.



Difference between Expression and Equation?

- An **Expression** is made up of **letters, numbers and operators** but has **NO EQUALS** sign – *expressions represent a number*

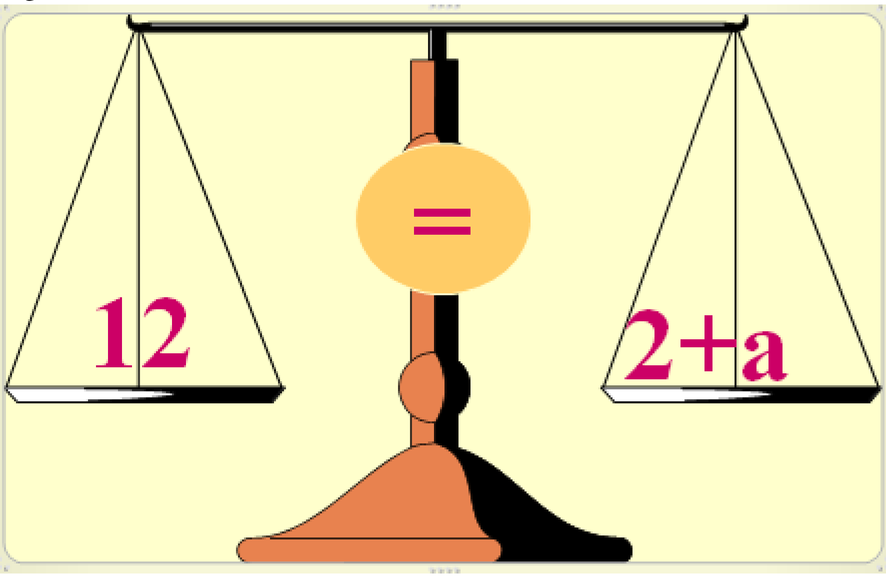
$$Examples\ of\ Expressions: \quad 2x+3, \quad a+2b+3c, \quad x^2+3x-4$$

- An **Equation** has **an equals sign** – it says that two things are equal

$$Examples\ of\ Equations: \quad 2x+3=5 \quad x^2+3x-4=0$$

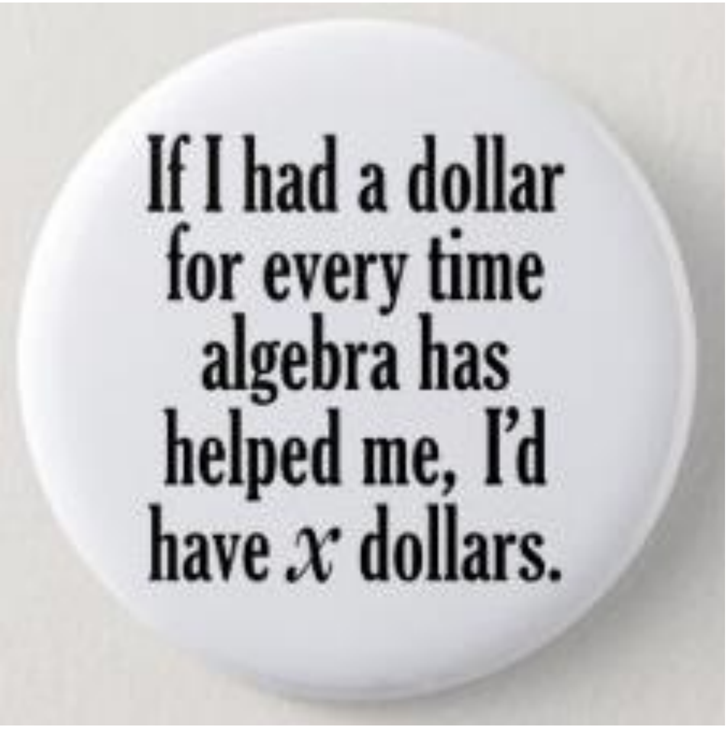
- An **Expression** can only be **SIMPLIFIED**
- An **Equation** can be **SOLVED** (x = something)

Q.



=
12
2+a

Find a?

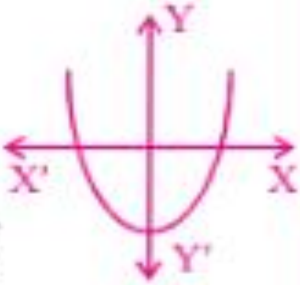


If I had a dollar for every time algebra has helped me, I'd have x dollars.

Degree of equation:
Highest power of variable when all the powers are whole numbers.

Linear equation:
$3x+5 = 10$
Quadratic Equation
$3x^2-7x+20 = 0$
Cubic equation
$5y^3+7y+29 = 0$

Geometrical Meaning of Zeroes of a Polynomial



Zero(es) of a polynomial is/are the x-coordinate of the point(s) where graph $y = f(x)$ intersects the x-axis.

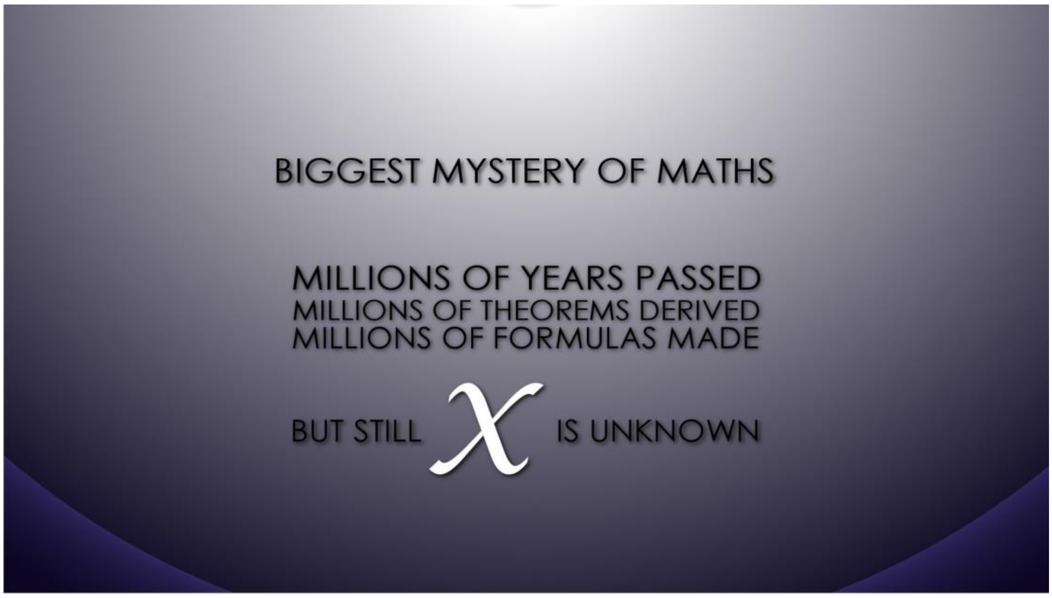
(i) **Linear polynomial:** Graph of linear polynomial is a straight line and has exactly one zero.

(ii) **Quadratic polynomial:** Graph of quadratic polynomial is always a parabola and this polynomial can have atmost two zeroes.

(iii) **Cubic polynomial:** Cubic polynomial can have atmost three zeroes.

Problem:

A number is 10 more than its triple. Find the number?



Soln: A number is 10 more than its triple. Find the number?

Let number is x

$x = 3x+10$

⇨ $x-3x = 10$

⇨ $x = -5$

Problem

A, B & C have total ₹240 with them. A gave some amount to each of B & C such that he doubles their amount and now left with ₹40. Find the initial amount with A?

Soln:

	A	B+C
Initial		
Final	40	200

	A	B+C
Initial	140	200/2=100
Final	40	200

x+2y =20 & 3x+6y = 60

Find the number of solutions?

Soln:

Infinite solutions since both the equations are same.

2x-5y = 70 & 4x-10y = 100

Find the number of solutions.

Soln:

2x-5y = 70 ---- (1)

2(2x-5y) = 100

⇨ 2x-5y = 50 ----- (2)

Both the equations are contradictory.

Therefore, no solution.

$$a_1x+b_1y+c_1 = 0$$
$$a_2x+b_2y+c_2 = 0$$

Consistent

System of equation has solution

- **Unique solution**

  $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ or } a_1b_2 - a_2b_1 = 0$$

  (Equation represents intersecting lines)

- **Infinite solution**

  $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

  (Equation represents coincident lines)

Inconsistent

System of equation has no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(Equation represents parallel disjoint lines)

Solving a system by substitution

Given:

$$y = x - 4$$
$$x + y = 10$$

$$y = x - 4$$
$$x + y = 10$$

Substitution:

$$x + x - 4 = 10$$
$$2x - 4 = 10$$
$$2x = 14$$
$$x = 7$$

$$y = x - 4$$
$$y = 7 - 4$$
$$y = 3$$

Elimation method

$$\begin{cases} 3x + y = 10 \\ -4x - 2y = 2 \end{cases}$$

$$2 \times (3x + y = 10)$$
$$-4x - 2y = 2$$

$$6x + 2y = 20$$
$$-4x - 2y = 2$$

$$6x + \cancel{2y} = 20$$
$$-4x - \cancel{2y} = 2$$

$$2x = 22$$
$$x = 11$$

$$\boxed{\begin{aligned} 3(11) + y &= 10 \\ 33 + y &= 10 \\ y &= 10 - 33 \\ y &= -23 \end{aligned}}$$

Problem:
A said to B,"I was 5 years older than you are now, when you were born. Today, I am one and half times as old as I was then." Find the present age of B?

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Soln:

	A	B
Past Age	x+5	0
Present Age	1.5(x+5)	X

Q. There are 2 boxes containing certain number of packets. If 10 packets were moved from 1st to 2nd box then there will be same number of packets in both boxes. If 20 packets are moved from 2nd to 1st box then 1st box will have double the number of packets as the 2nd box. Determine the number of packets in the 1st box?

(A)40 (B) 60 (C) 80 (D) 100 (E) 120

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(A)40 (B) 60 (C) 80 (D) 100 (E) 120

Soln: Let number of marbles in packet 1 =x

Number of marbles in packet 2 = y

$x-10 = y+10$ ---(1)

$x+20 = 2(y-20)$ -----(2)

$x = 100$

62-26 = 36 = 9×4, 83-38 = 45 = 9×5 Difference of any 2 digit number & its reverse is multiple of 9.	62+26 = 88 = 8×11 21+12 = 33 = 3×11 Sum of any 2 digit number & its reverse is multiple of 11.

Problem:

Sum of a 2 digit number N and its reverse 2 digit number is a square.

Quantity A	Quantity B
Number of values N can take	6

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Soln: Let 2 digit number is xy = 10x+y

$10x+y + 10y+x = 11(x+y) = N^2$

$11(x+y) = 121$

⇨ $x+y = 11$

(2,9)

(3,8)

(4,7)

(5,6)

Total 8 pairs.

Answer is (A)

$3x+7y = 127$

Find the number of positive integral solutions?

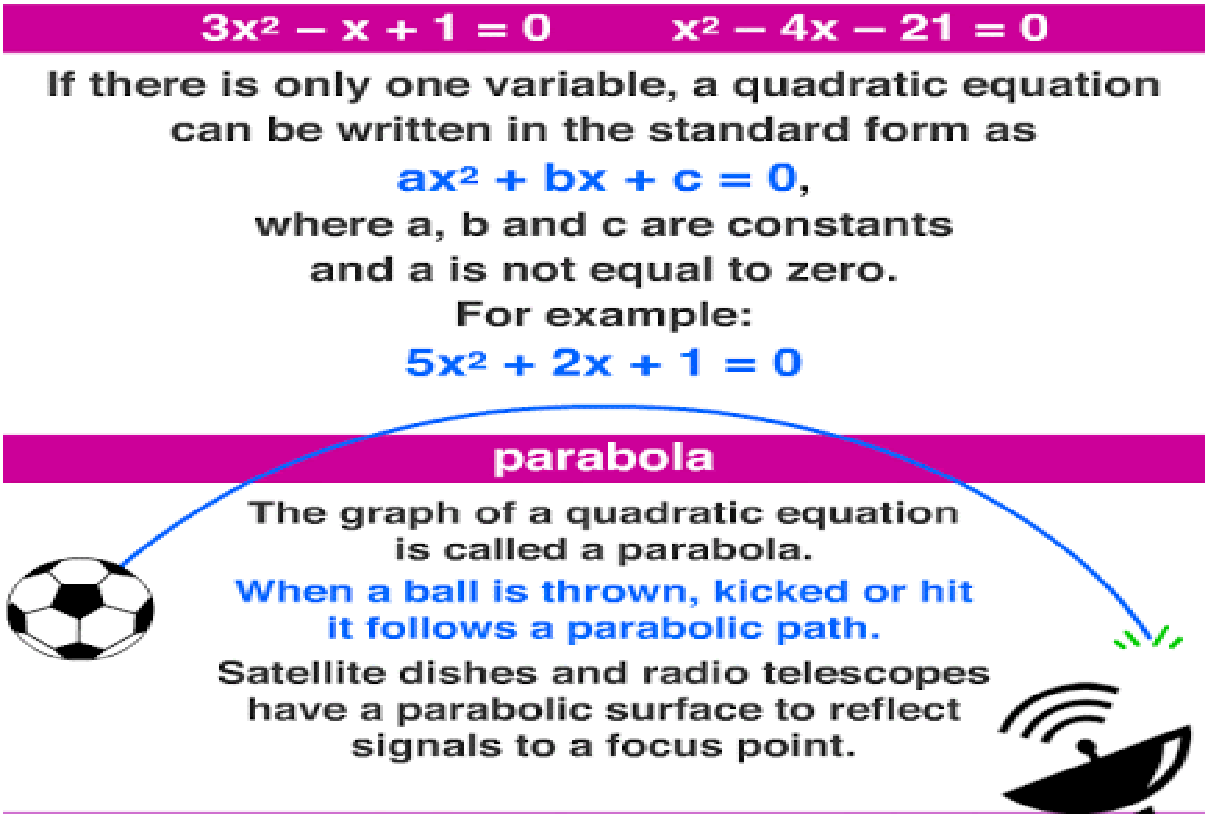
Soln: Find any 1 solution by hit & trial.

$x = 40$ & $y = 1$

Now decrease value of x by 7 and increase the value of y by 3.

Quadratic Equations

A quadratic equation is one in which the highest index of the variable is 2.



General quadratic equation is

$$ax^2+bx+c = 0$$

Please note that a is the coefficient of x^2, b is the coefficient of x and c is the constant term.

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Factor and Solve Quadratic Equations

$$ax^2+bx+c=0 \qquad\qquad 8x^2+2x-3=0$$

Find the two numbers that will make these equations true.

$$\square\times\square = ac \qquad\qquad \boxed{6}\times\boxed{-4}=-24$$

$$\square+\square = b \qquad\qquad \boxed{6}+\boxed{-4}=2$$

Put the two numbers in the equation, simplify and solve.

$$\frac{1}{a}\left(ax+\square\right)\left(ax+\square\right)=0 \qquad\qquad \frac{1}{8}\left(8x+\boxed{6}\right)\left(8x+\boxed{-4}\right)=0$$

$$\frac{1}{8}(2)(4x+3)(4)(2x-1)=0$$

$$(4x+3)(2x-1)=0$$

$$(4x+3)=0 \qquad\qquad (2x-1)=0$$

$$x=-\frac{3}{4} \qquad\qquad x=\frac{1}{2}$$

1. $x^2-12x+27=0$

2. $x^2+25x+150=0$

3. $x^2-8x-48 = 0$

4. $x^2-10x+20=0$

5.

Sum and product of the roots

From the general form of QE

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Forming QE from SoR PoR

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Therefore

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Problem: $x^2 + kx - 20 = 0$

Given equation has 2 real roots p & q such that p>q.

Quantity A	Quantity B
q	0

Soln: Answer is B

Forming the equation when the roots are given

If the roots of a quadratic equation are p and q, then the equation can be constructed as follows:

$(x-p)(x-q) = 0$

x^2-(Sum of the roots)x+Product of the roots = 0

Problem:

Form the quadratic equation with the roots 5 & -4.

Soln: Form the quadratic equation with the roots 5 & -4.

Quadratic equation with roots 5 & -4 is

$(x - 5)(x - (-4)) = 0$

⇨ $(x-5)(x+4) = 0$

Discriminant or Determinant and Nature of the Roots:

Quadratic formula is: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

The expression under the radical (root) sign i.e. (b^2-4ac) is called the Determinant of the equation and is denoted as D.

Determinant	**Nature of Roots**
D < 0 i.e. D is negative	Roots are imaginary
D = 0	Roots are real and equal
D > 0	Roots are real and unequal

Problem:
Betaal is able to purchase x chocolates in ₹120. When price of chocolate increases by ₹1 then he is able to purchase 10 chocolates less than earlier in ₹120. Find x?

Soln:

$120 = 1\times120$

$= 2\times60$

$= 3\times40$

$= 4\times30$

$= 5\times24$

$= 6\times20$

$= 8\times15$

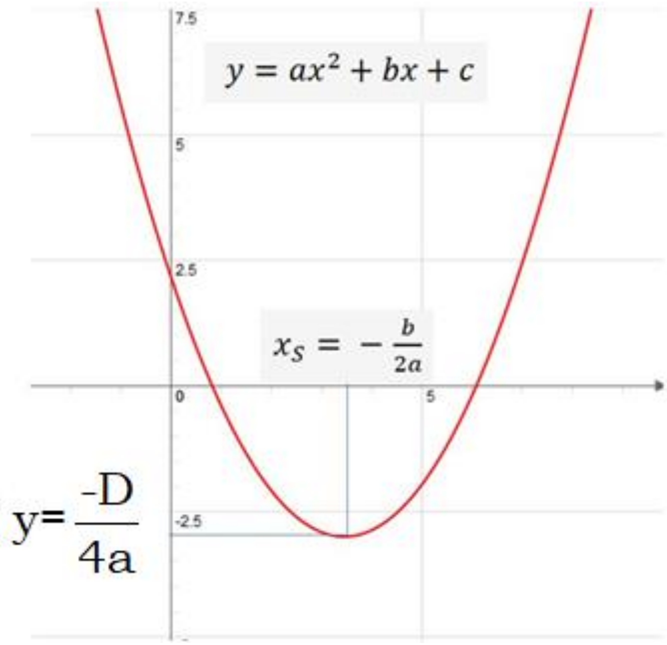
$= 10\times12$

Quadratic function

$y = f(x) = ax^2 + bx + c$

$f(2) = a \times 2^2 + b \times 2 + c$

If $f(5) = 0$ then $x=5$ is a root of the equation and $(x-5)$ is a factor of $f(x)$.



For $a>0$

Minimum value of $f(x) = \frac{-D}{4a}$

at $x = \frac{-b}{2a}$

For $a<0$

Maximum value of $f(x) = \frac{-D}{4a}$

at $x = \frac{-b}{2a}$

Problem:

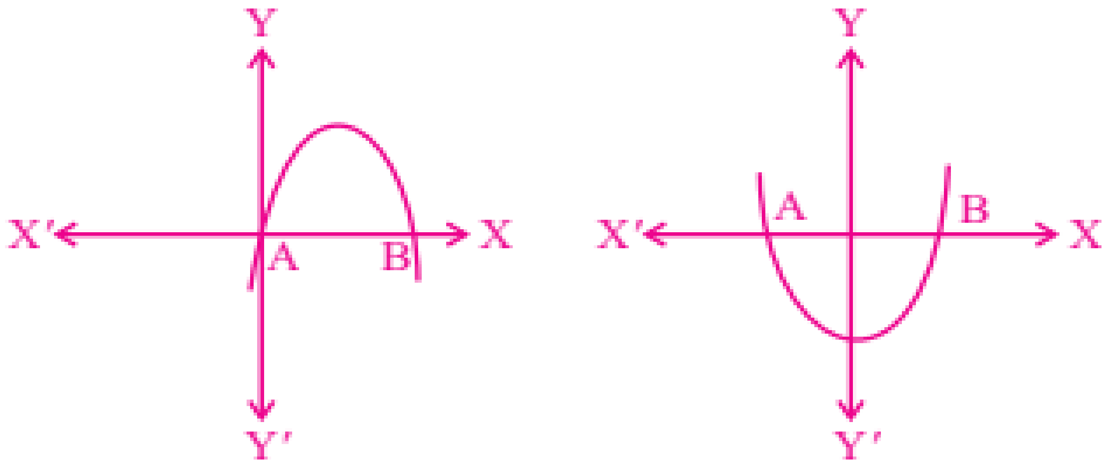
Find the minimum value of f(x), if $f(x) = 3+7x+x^2$

$$f(x) = x^2 + 7x + 3$$

a = 1,b =7 & c= 3

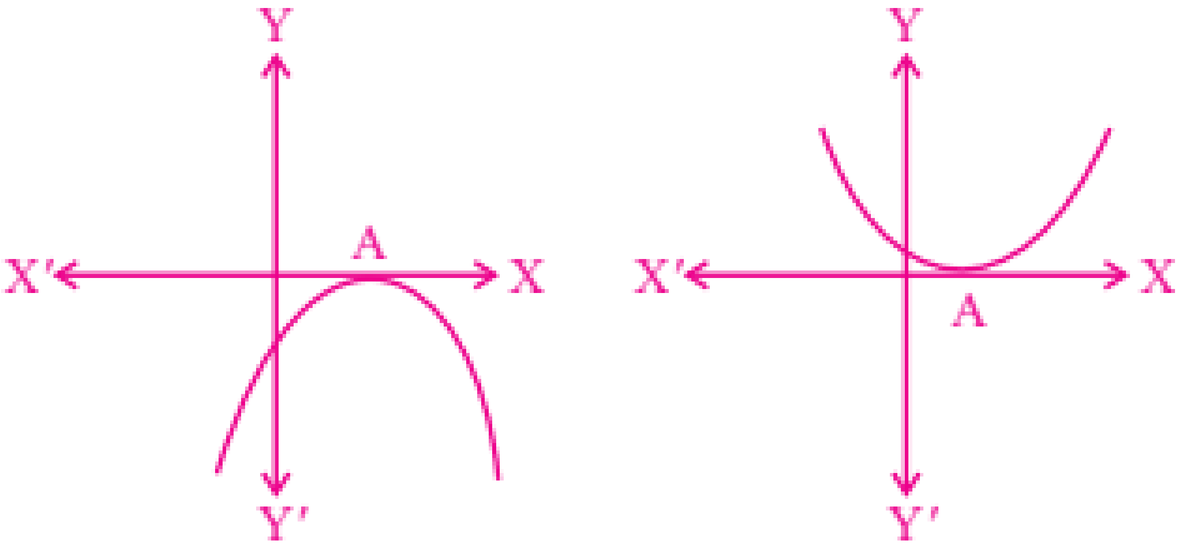
$$\text{Minimum value} = \frac{-D}{4a} = \frac{-(49-12)}{4} = \frac{-37}{4}$$

Case-I : If a quadratic polynomial $P(x) = ax^2 + bx + c$ has two zeroes, then its graph will intersect the x-axis at two distinct points A & B as shown in the figure.



D>0 or Real & distinct roots or parabola intersects x-axis at 2 distinct points.

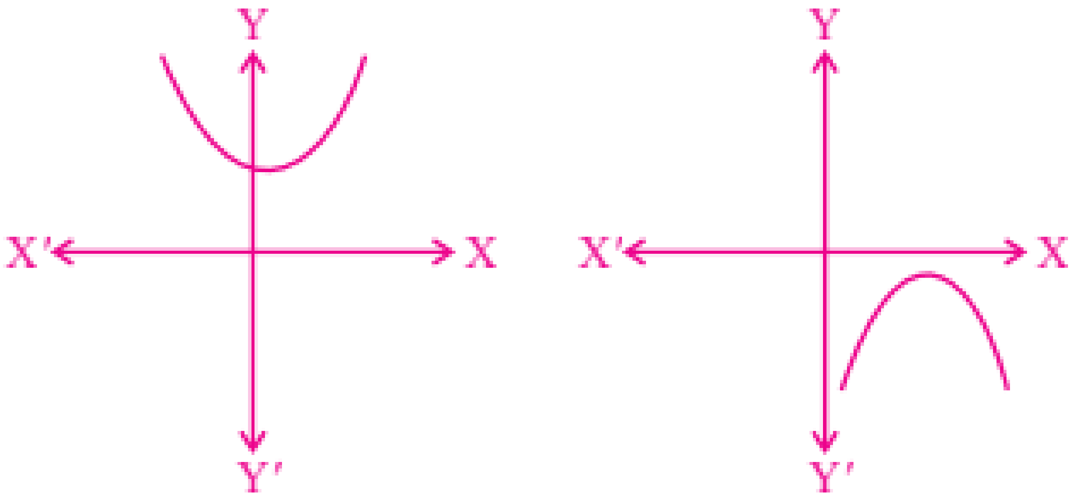
Case-II : If a quadratic polynomial $P(x) = ax^2 + bx + c$ has only one zero, then its graph will touch the x-axis at only one point A as shown in the figure.



D =0 or Real & equal roots or 1 real root or unique roots or

Parabola touches x-axis at only one point or f(x) is perfect square

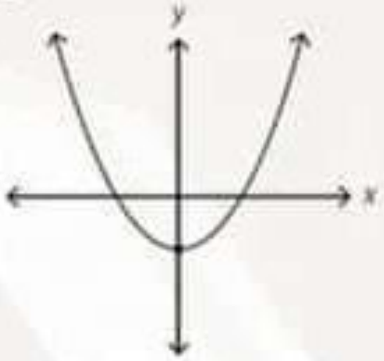
Case-III: If a quadratic polynomial $P(x) = ax^2 + bx + c$ has no zero, then its graph will not intersect /touch the x-axis at any point as shown in the figure.



D<0 or non-real roots or parabola neither touches nor intersects x-axis.

Problem:

Which of the following could be the equation of the figure above?



(A) $y = x - 2$

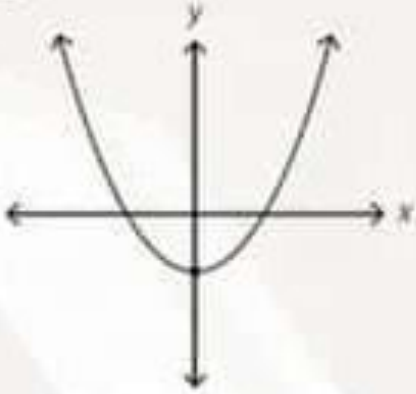
(B) $y = x^2 - x$

(C) $y = x^2 - 2$

(D) $y^2 = x^2$

(E) $y = x^3 - 2$

Which of the following could be the equation of the figure above?



(A) $y = x - 2$

(B) $y = x^2 - x$

(C) $y = x^2 - 2$

(D) $y^2 = x^2$

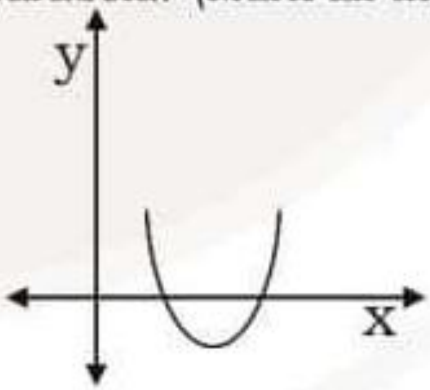
(E) $y = x^3 - 2$

Soln: Equation of parabola is a quadratic equation and therefore options A, D & E are eliminated.

Parabola intersects negative y-axis implies when $x = 0$ then y should be negative.

Out of B & C only C satisfies the condition.

Which of the following could be true about the given parabola? (Mark all the correct answers)?



(A) Sum of the roots is +ve

(B) Equation has Unique roots.

(C) D < 0

(D) Y co-ordinate of vertex of parabola is negative.

(E) a > 0

Q. X birds are seated on certain number of branches of a tree. If 16 birds are to be seated on each branch then 8 are left to be seated but if 20 birds are to be seated on each branch, 2 branches are left empty. Find X?

Q. Free notebooks were distributed equally among children of a class. The number of notebooks each child got was one-eighth of the number of children. Had the number of children been half, each child would have got 16 notebooks. Total how many notebooks were there?

Q. Tom is four times as old as Jerry. In x years Tom will be three times as old as Jerry. How old is Jerry, in terms of x?

(A) 2x (B) 3x (C) 4x (D) 8x (E) 12x