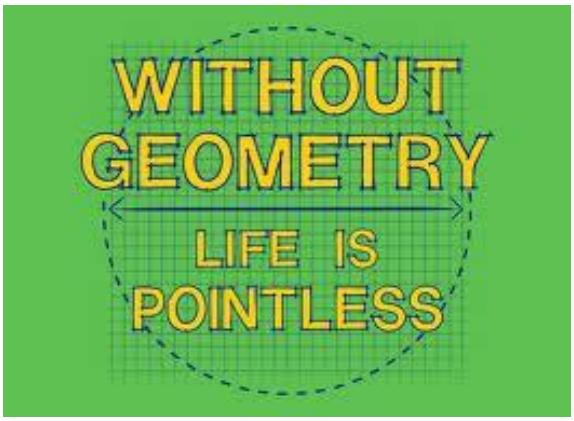
# Geometry -1




**Point:** A fine dot made by a sharp pencil, or the prick made by a fine pin on a sheet of paper is close to a point. It is dimensionless.

**Collinear points**: If points lie on the same line.

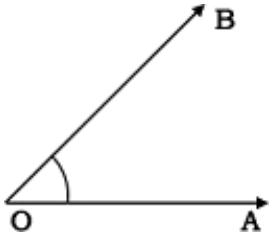
**Line:** Collection of infinite collinear points without any end point.

**Line-Segment:** Part of a line with 2 end points.

**Ray**: Part of a line with 1 fixed point.

**Plane:** The surface of a smooth wall or table-top is close to a portion of plane. In short, a plane is a flat surface. It has length and width but no thickness. A plane extends infinitely in all directions.

**Angle:** Two rays with a common initial point form an angle.




0 <Acute angle < 90

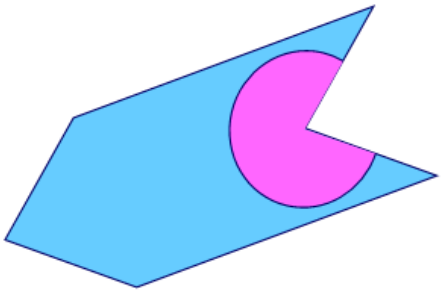
Right = 90

90< Obtuse < 180

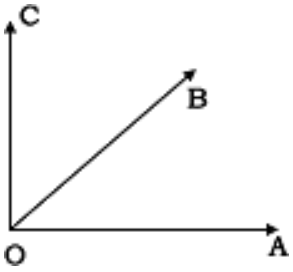
Straight line = 180

180< Reflex Angle< 360

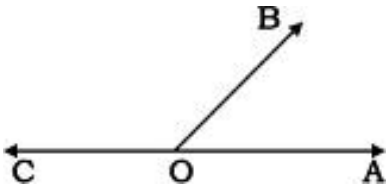
Complete Angle = 360



**Adjacent Angles:** In the given figure, ∠AOB and ∠BOC are adjacent angles.



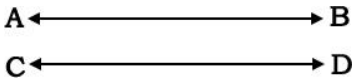
**Linear Pair:** In the given figure ∠ COB and ∠ AOB is a linear pair




Supplementary Angles : Sum of 2 angles is 180

Complementary Angles: Sum of 2 angles is 90.

**Parallel Lines:** Two distinct lines in the same plane are said to be parallel if they do not intersect at any point.



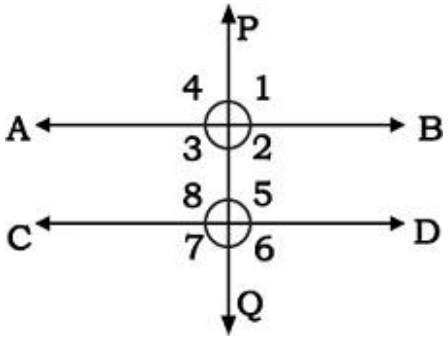

Or

If distance between 2 lines is same at all the points then lines are ||.

Here line AB is parallel to line CD. This is denoted as AB||CD

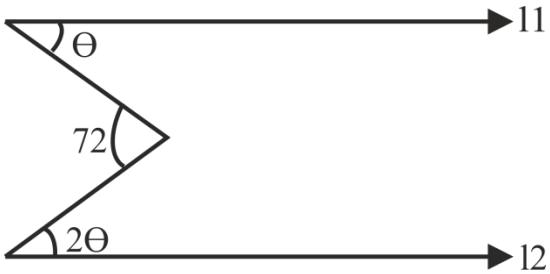
**Angles Made by a Transversal with Two Lines:**

**PQ is a transversal on two parallel lines AB and CD**

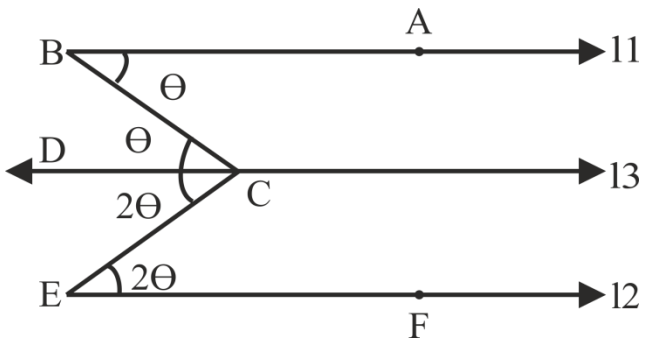



- $\angle 3 = \angle 5$ and $\angle 2 = \angle 8$ (Pairs of alternate interior angles)
- $\angle 1 = \angle 5$ , $\angle 2 = \angle 6$ , $\angle 4 = \angle 8$ and $\angle 3 = \angle 7$ (Pairs of corresponding angles)
- $\angle 2 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 8 = 180^\circ$ (Interior angles on the same side of the transversal)
- $\angle 1 = \angle 3$ , $\angle 2 = \angle 4$ , $\angle 6 = \angle 8$ and $\angle 5 = \angle 7$ (vertically opposite angles)

**Problem:**




Find **θ** if l1 || l2 ?




If we form a line $l_3 \parallel l_1 \parallel l_2$ then

$\angle ABC = \angle BCD = \theta$ and

$\angle DCE = \angle CEF = 2\theta$

[Alternate angles]

So, $3\theta = 72$

$\theta = 24°$

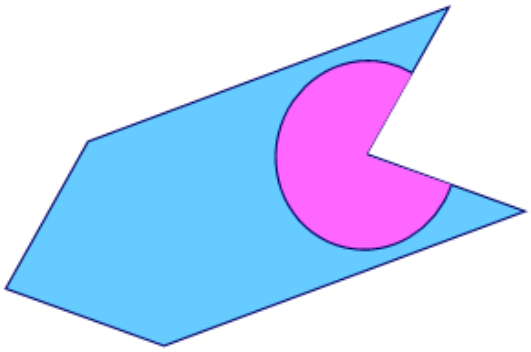
**Polygon:**

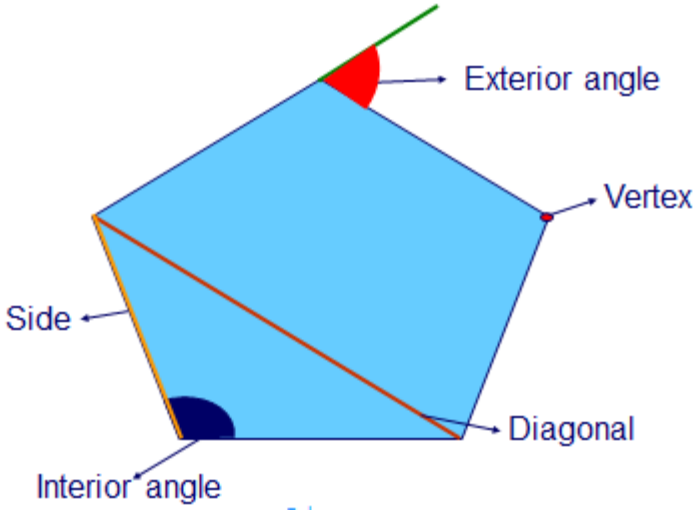
It is a closed plane figure bounded by some straight lines. Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Octagon, Nonagon and Decagon are polygons with 3, 4, 5, 6, 7, 8, 9, 10 sides respectively.

**Convex & Concave Polygon:**

A polygon, in which all the Interior angles are less than 180°, is a convex polygon.

On the other hand, if at least one angle of a polygon is more than 180˚, then it is a concave polygon.






I. Sum of all interior angles = (n-2)×180

II. Sum of all exterior angles = 360˚

**Diagonal**: line segment joining any 2 non- adjacent vertices.

Total no. of diagonals = $\frac{n(n-3)}{2}$

**Regular Polygon :**

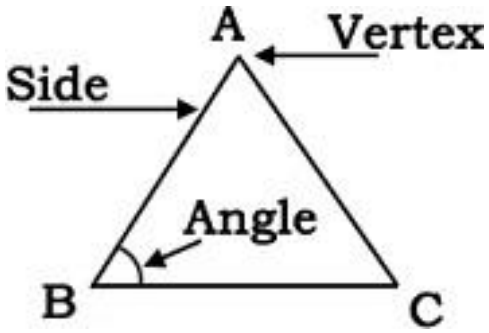
A polygon having all sides equal and all angles equal is called a regular polygon.

Each exterior angle = $\frac{360}{n}$

Each interior angle = 180˚ - (exterior angle)

**Find the each internal and each external angle of a regular octagon?**

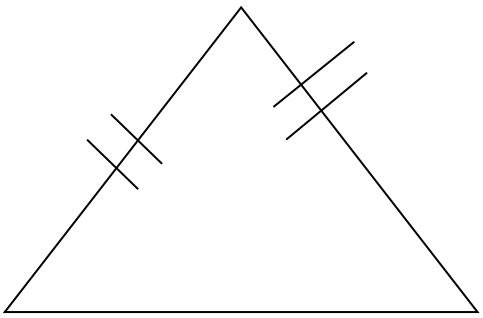
**Triangle:**



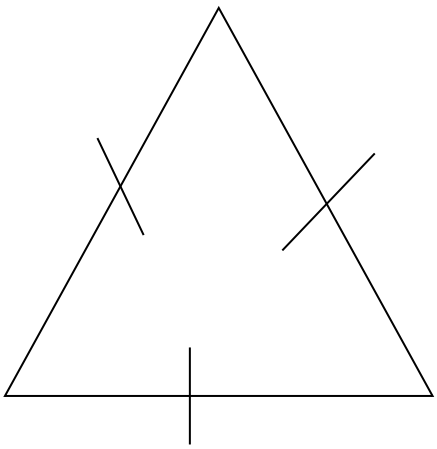
## Classification of Triangles

## I. According to sides:

1. A triangle having no two sides equal is called a scalene triangle.
2. A triangle having two sides equal is called an isosceles triangle.



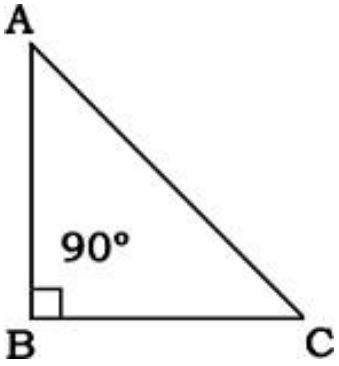
3. A triangle having all sides equal is called an equilateral triangle.



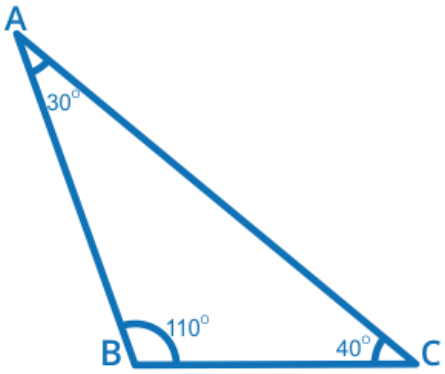
## II. According to Angles:

1. A triangle with 3 acute angles is acute triangles.

2. A triangle one of whose angle is a right angle is called a right-angled triangle.



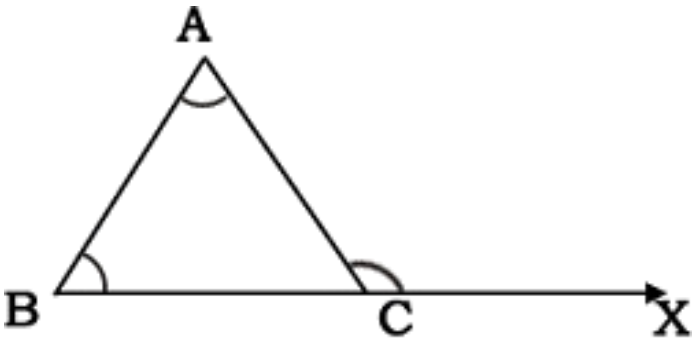

3. A triangle one of whose angle is obtuse is called an obtuse angled triangle or simply an obtuse triangle.



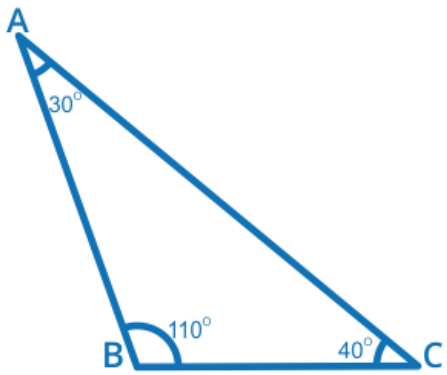
**Properties of a Triangle:**

1. The sum of three angles of a triangle is 180˚.

2. In a triangle, an exterior angle equals the sum of the two interior opposite angles. For example in the given figure ∠ACX = ∠ABC + ∠BAC.



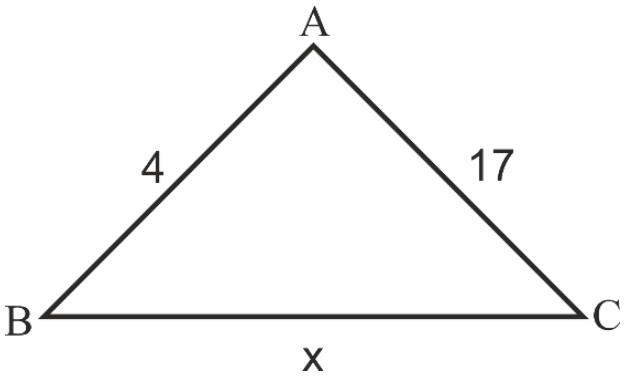

3. Side opposite to the greatest angle is the longest side.



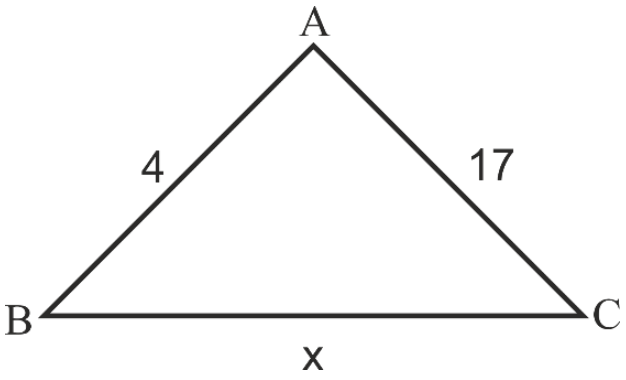
4. The sum of any two sides of a triangle is greater than the third side. So in the above figure
   AB + BC > AC, AB + AC > BC, BC + AC > AB

   The difference of any two sides of a triangle is less than the third side. Hence, |AB - BC| < AC.

**Problem:**




| Quantity A | Quantity B |
|---|---|
| No. of values that x can take | 20 |




For a triangle

$4 + 17 > x \Rightarrow x < 21$

$4 + x > 17 \Rightarrow x > 13$

Therefore, $13 < x < 21$

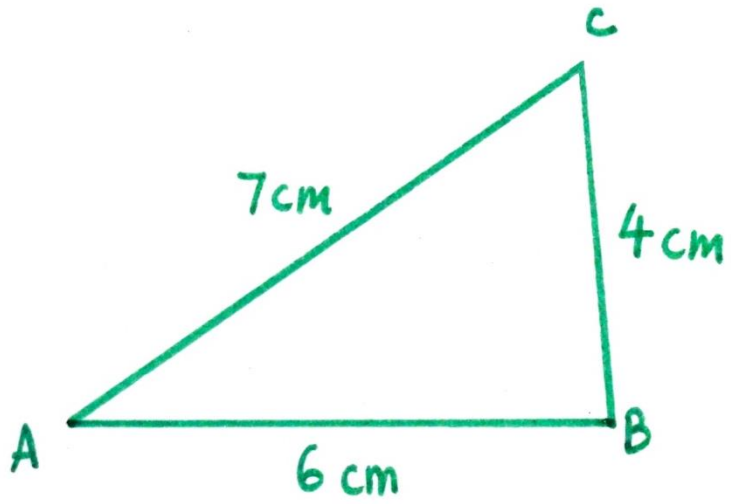
x can take any value between 13 & 21.

So, answer is A.

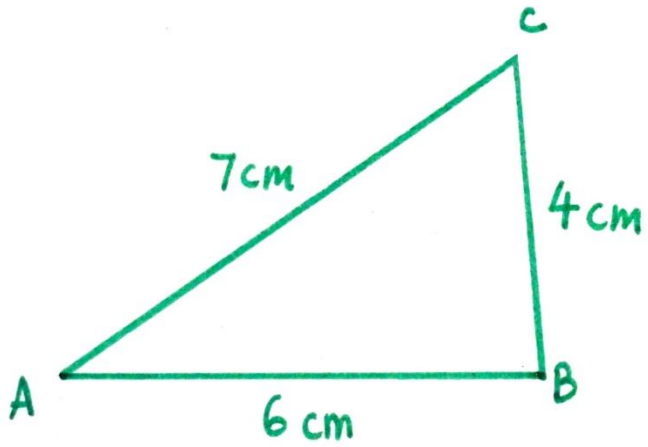
Note: - If x is an integer then x can take only 7 values from 14 to 20. Then answer is B.

5. Let a is the longest side of the triangle then
    1. If $a^2 > b^2 + c^2$ then triangle is obtuse.
    2. If $a^2 < (b^2 + c^2)$ then triangle is acute.

Problem:




**What is the type of the triangle based on the angles?**



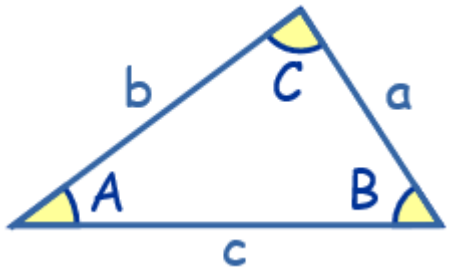

$7^2 < 4^2 + 6^2$

Therefore acute triangle.

**Sin rule:**

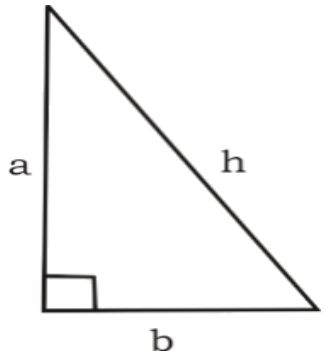
$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC} = 2R$$

**Cosine Rule**:




$$CosA = \frac{b^2 + c^2 - a^2}{2bc}$$

## Right Angle Triangles




$$\text{Area} = \frac{1}{2} \times ab$$

## Pythagoras Theorem

$$a^2 + b^2 = h^2$$

(3, 4, 5) ,(5, 12, 13),(7, 24, 25),(8, 15, 17),(9,40,41),(11,60,61)

$2 \times (3, 4, 5) = (6, 8, 10)$

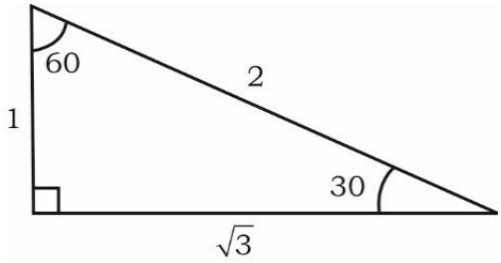
$3 \times (3, 4, 5) = (9, 12, 15)$

$4 \times (3, 4, 5) = (12, 16, 20)$

$2 \times (5, 12, 13) = (10, 24, 26)$

**30-60-90 triangle**

In a 30-60-90 triangle, the ratio of the lengths of the sides opposite to 30, 60 and 90 degrees is $1:\sqrt{3}:2$. This is also memorized as side opposite to 30 degrees is half the hypotenuse and side opposite to 6 degrees is $\frac{\sqrt{3}}{2}$ times the hypotenuse.

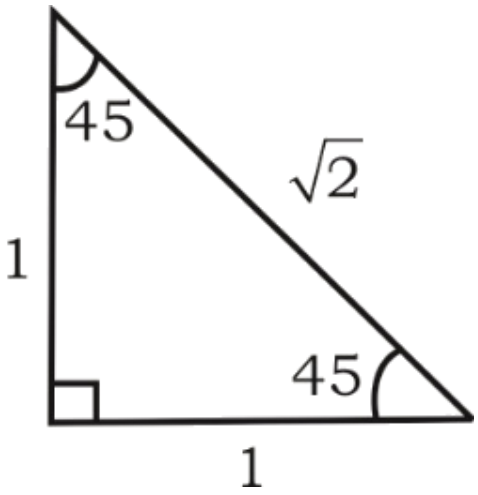



.

| **30°** | **60°** | **90°** |
|---|---|---|
| x | $\sqrt{3}$ x | 2x |

**Isosceles Right angle triangle, 45-45-90 triangle**

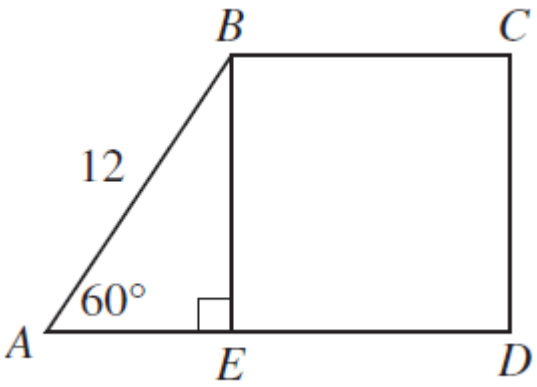
In an isosceles right angle triangle, if the lengths of the perpendicular sides is a, the hypotenuse is $\sqrt{2}a$. Accordingly if the hypotenuse is h, length of each perpendicular side is $\frac{h}{\sqrt{2}}$.



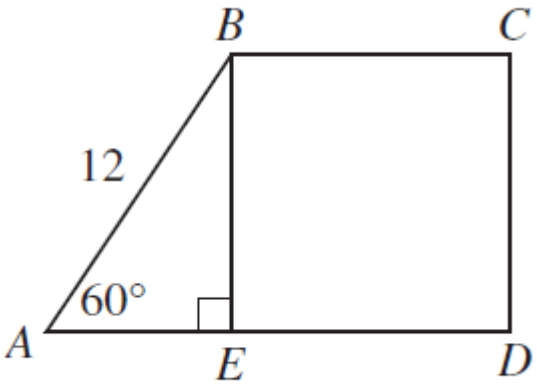

| **45°** | **45°** | **90°** |
|---|---|---|
| **a** | **a** | $\sqrt{2}$ a |

**Problem:**

In the figure below, BCDE is a square and AB = 12. What is the area of square BCDE?



In the figure below, BCDE is a square and AB = 12. What is the area of square BCDE?




In the figure below, BCDE is a square and AB = 12. What is the area of square BCDE?

$\angle ABE = 30$, Let $AE = x$

Then $BE = \sqrt{3}\ x$ and

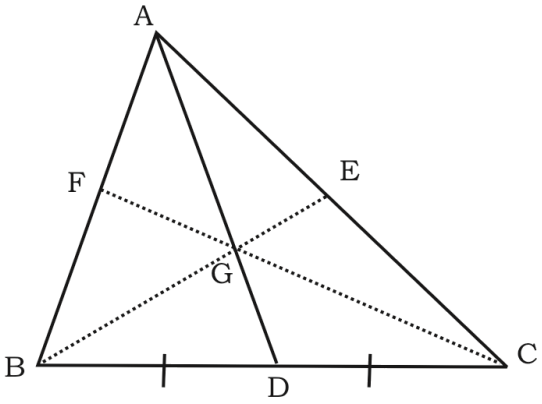
$AB = 2x = 12 \Rightarrow x = 6$

$BE = \sqrt{3}\ x = 6\sqrt{3}$

Area of square $= (6\sqrt{3})^2 = 108$

**Median:** Line segment joining mid-point of the side to the opposite vertex.(AD is median in the diagram)

Median divides the triangle into two equal areas, $A(\Delta ABD) = A(\Delta ADC)$



D, E & F are midpoints of the sides

3 Medians divide triangle into 6 equal areas.

**Centroid:**

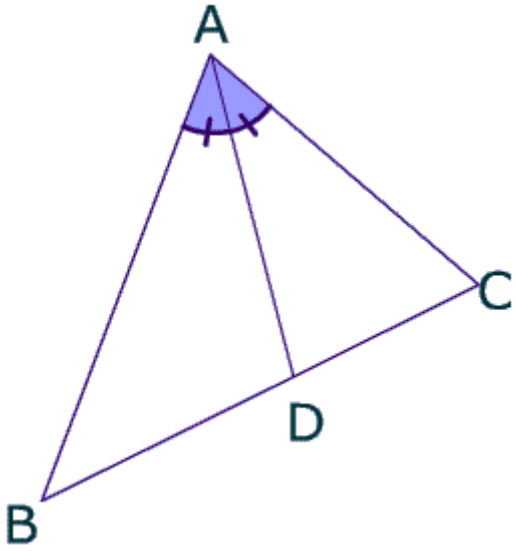
Concurrency point of the medians(point G in the diagram).

The centroid divides the median in the ratio 2:1, with the larger part being towards the vertex. Thus, AG : GD is 2: 1.

**Apollonius Theorem**

$AB^2 + AC^2 = 2 \times (AD^2 + BD^2)$

**Angle bisector:**

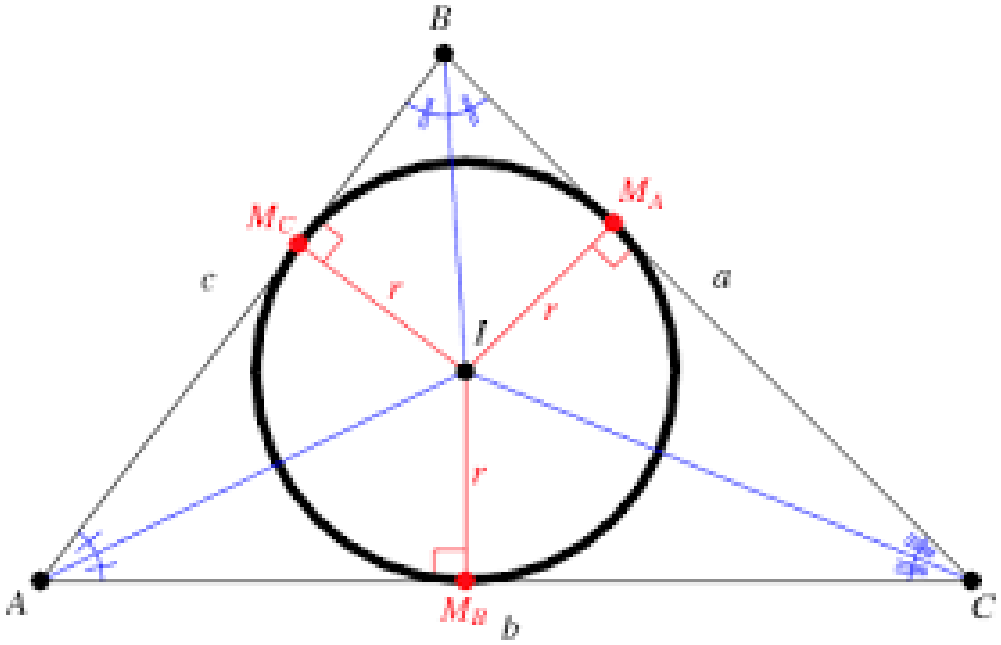



AD is angle bisector then

$$\frac{AB}{AC} = \frac{BD}{DC}$$

**In-center:**

Concurrency point of the angle bisectors (I in the figure). It is called an In-center because with this point as a center a circle can be drawn which lies ‘in’ the triangle i.e. it touches each of the three sides. The radius to the circle is the perpendicular distance from I to any of the sides, shown as dotted lines in the figure.




In center is the only point which is equidistant from all the 3 sides.

$$\angle BIC = 90 + \frac{\angle A}{2}$$

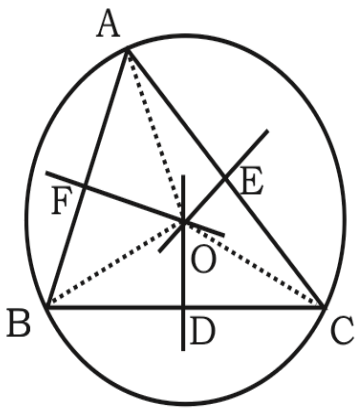
**Circum-center:**

Concurrency point of the perpendicular bisectors of the side (point O in the figure)

Circum center is the only point which is equidistant from all the 3 vertices.

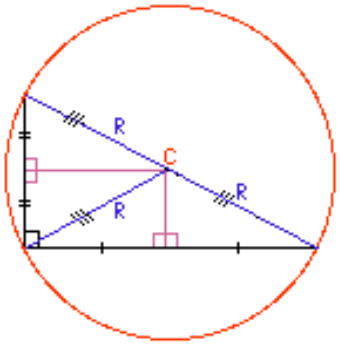
The radius of the circle = AO = BO = CO =R

$$\angle BOC = 2 \times \angle A$$




OD, OE & OF are perpendicular bisector of sides

- In Acute angle triangle :Inside the triangle.
- In Right angle triangle : Mid pint of Hypotenuse.



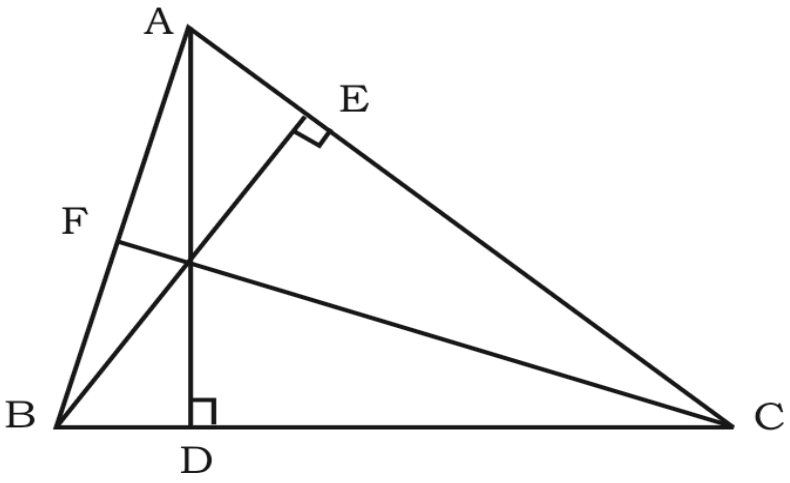

$$R = \frac{\text{Hypotenuse}}{2}$$

- In obtuse angle triangle: outside the triangle

**Orthocenter:**

Concurrency point of the Altitudes (point H in the figure)

**Property 1:** $\angle BHC + \angle A = 180°$




AD, BE & CF are altitudes

- In Acute angle triangle Orthocentre is inside the triangle.
- In Right angle triangle Orthocentre is the point(Vertex) where angle is 90°
- In obtuse angle triangle Orthocentre is outside the triangle.

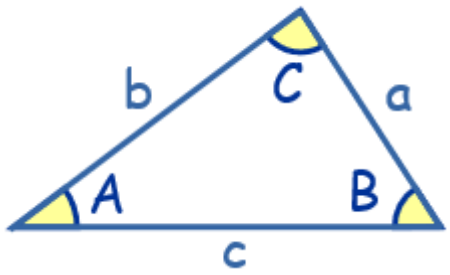
**1.** $\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$

**2.** Heron's Formulae: Area = $\sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter and a, b and c are the sides of the triangle.

**3.** Area = $r \times s$, where r is the in-radius and s is the semi-perimeter.

**4.** Area = $\frac{abc}{4R}$, where R is the circum-radius and a, b and c are the sides of the triangle.

**5.** $\text{Area} = \frac{1}{2} ab \sin C$



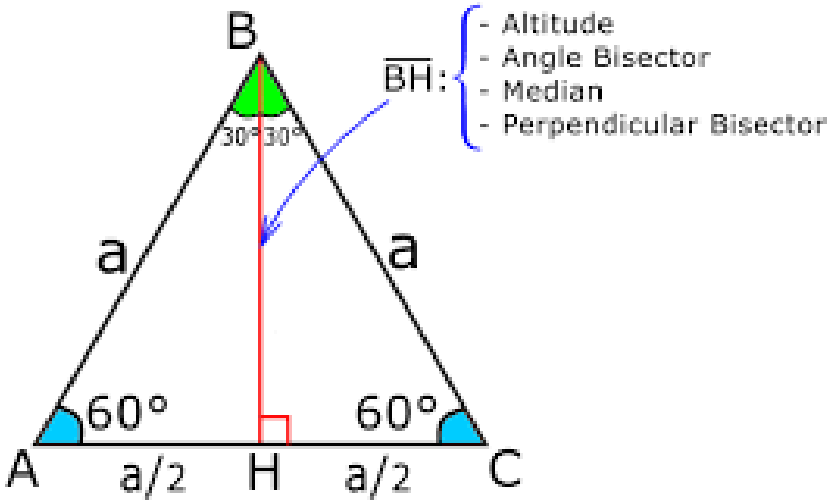
In an equilateral triangle all the 4 geometric centers are concentric.

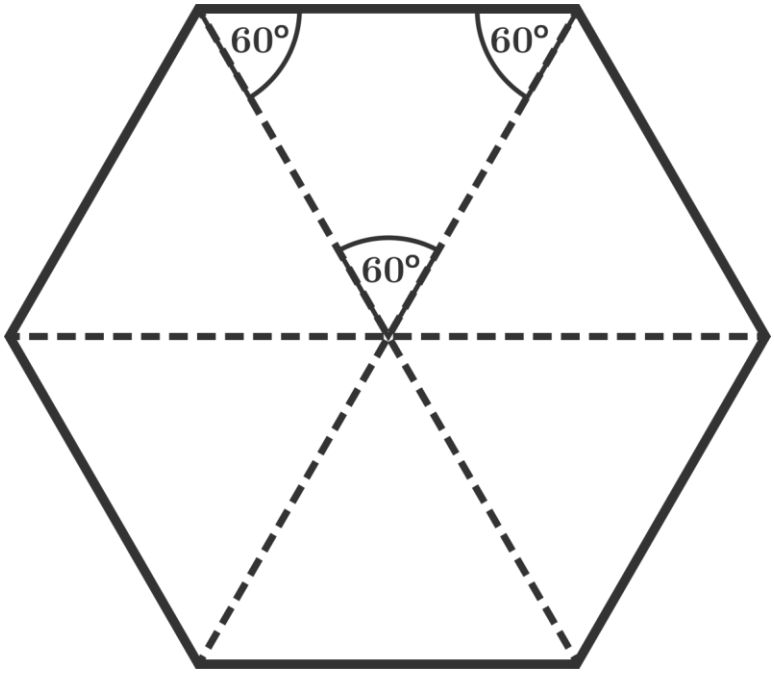
In equilateral triangle median, altitude, angle bisector and perpendicular bisector all are same i.e. single line.

Height of Equilateral triangle of side a units $= \frac{\sqrt{3}}{2}a$

Area of Equilateral triangle of side a units $= \frac{\sqrt{3}}{4}a^2$

$R = \frac{a}{\sqrt{3}}$ & $r = \frac{a}{2\sqrt{3}}$

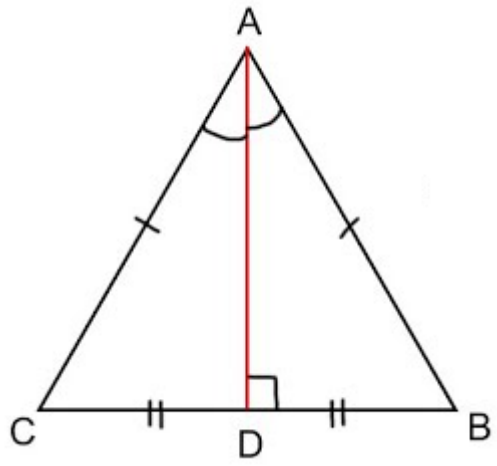





Area=6×Area of equilateral triangle

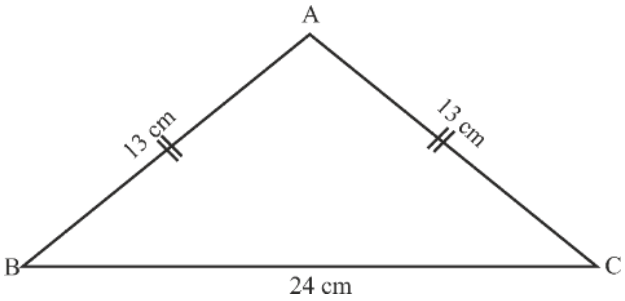
In an isosceles triangle all the 4 geometric centers are collinear.

Perpendicular drawn on non-equal side is median, angle bisector and perpendicular bisector.

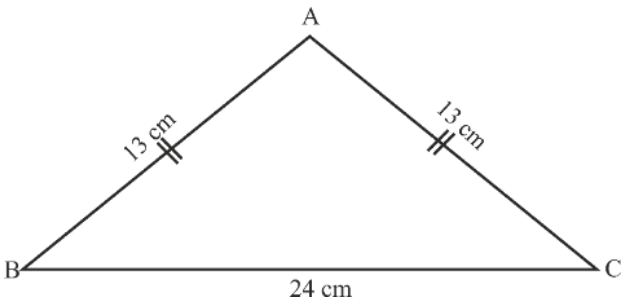


## Problem

| Quantity A | Quantity B |
| --- | --- |
| Area of the triangle | Area of triangle with 2 sides 10cm &15cm |



| Quantity A | Quantity B |
| --- | --- |
| Area of the triangle | Area of triangle with 2 sides 10cm &15cm |




Quantity A

Triplet (5,12,13)

Area = $\frac{1}{2} \times 24 \times 5 = 60$

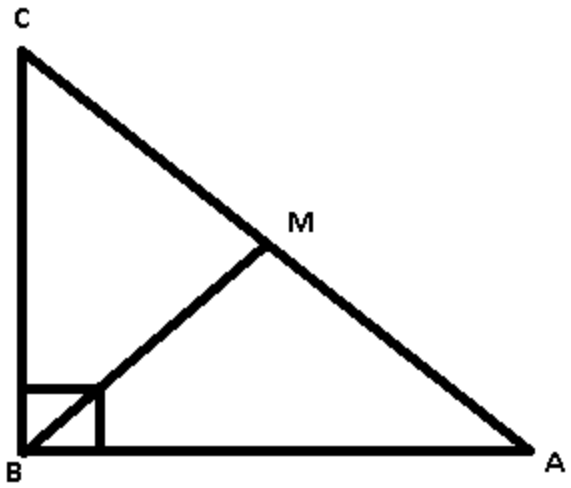
Quantity B

Maximum Area = $\frac{1}{2} \times 10 \times 15 = 75$

Therefore answer is (D).

Problem: In $\Delta ABC$, find median BM if AB = 3.6cm, BC = 4.8cm and $\angle B = 90°$?

In ΔABC, find median BM if AB = 3.6cm, BC = 4.8cm and ∠B = 90°




As we know (3,4,5) is a triplet .

3.6 = 1.2×3, 4.8 = 1.2 ×4

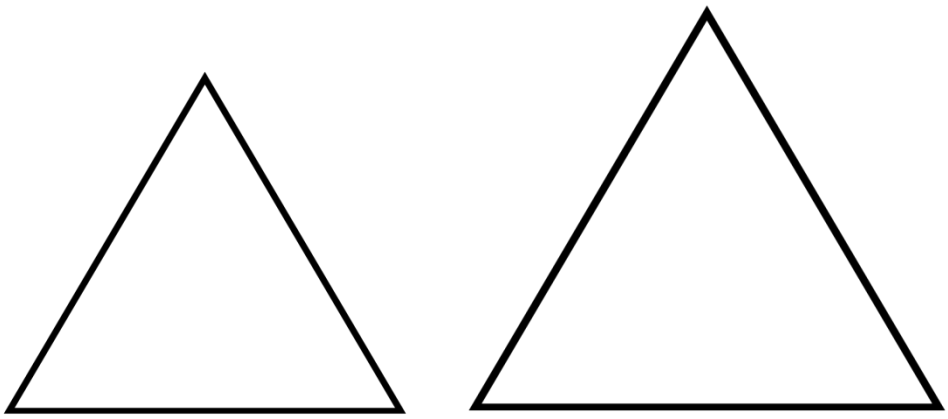
Therefore

Hypotenuse = 5×1.2 = 6

AC = 6.

$$BM = \frac{AC}{2} = \frac{6}{2} = 3$$

**Similar Triangle**



If 2 triangles are similar then ratio of their corresponding sides is same and reverse is also true.

$$\left(\frac{AB}{PQ}\right)=\left(\frac{AC}{PR}\right)=\left(\frac{BC}{QR}\right)=\frac{h1}{h2}=\frac{m1}{m2}=\frac{p1}{p2}=\sqrt{\frac{A1}{A2}}\ \ldots\ldots$$

**Congruent triangles**: Congruent triangles are same in every respect i.e. sides, angles, medians, altitudes, area, R, r etc.

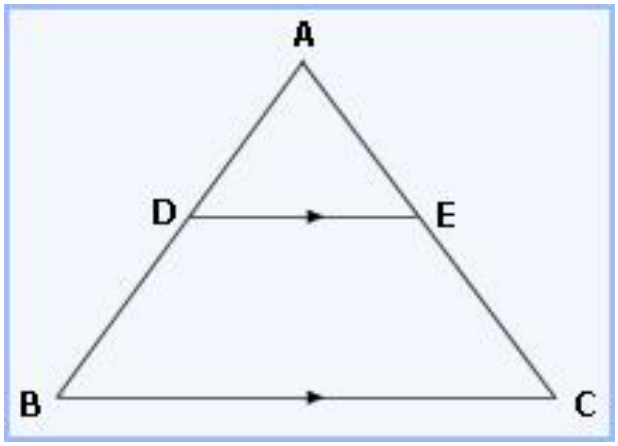
i.e. carbon copy of each other.

All the congruent triangles are similar but reverse is not true.

**Basic Proportionally Theorem:**

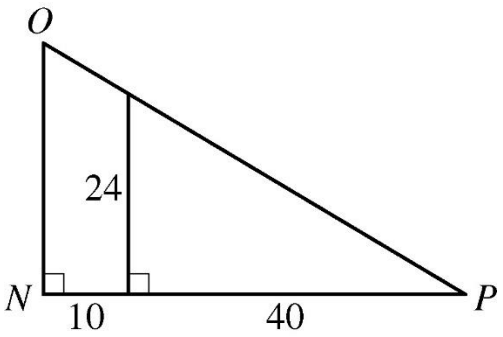
If a line is drawn parallel to one side of a triangle and intersects the other sides in two distinct points then the other sides are divided in the same ratio by it.

If DE is parallel to BC, then $\frac{AD}{DB} = \frac{AE}{EC}$

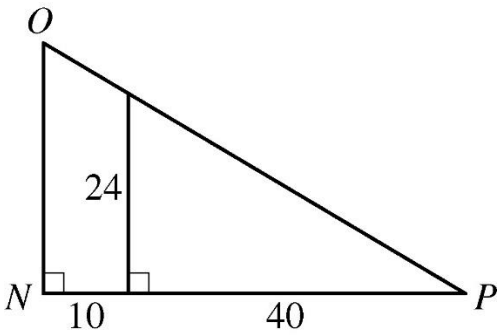



Also $\triangleright ADE \sim \triangleright ABC$

Problem:




Find ON?




Find ON?

Here 2 triangles are similar

Therefore

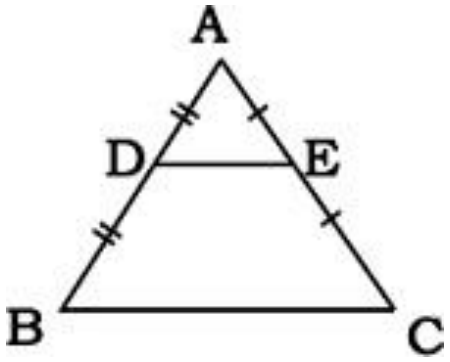
$$\frac{40}{50} = \frac{24}{ON}$$

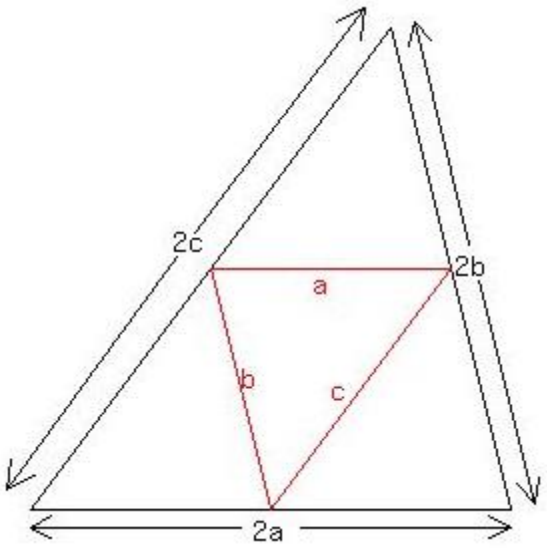
ON = 30

**Midpoint theorem:**

The segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of the third side.

If AD = DB, AE = EC, then DE is parallel to BC and DE = ½ BC



## Important points:

When a polygon is inscribed in a circle or perimeter of a polygon is constant then its area is maximum when it is a regular polygon.