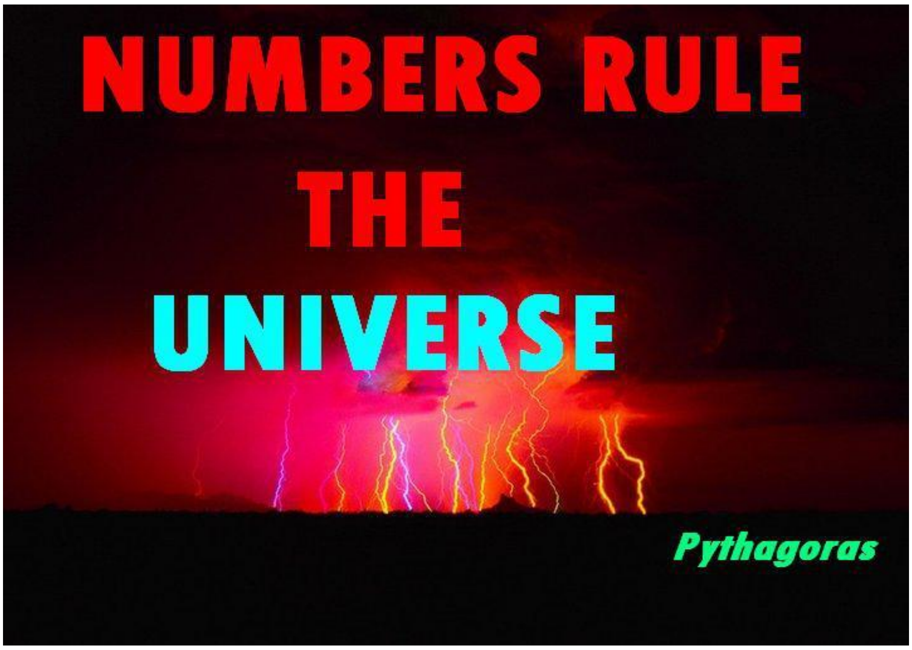
Math may not teach me how to add love or subtract hate but it gives me hopes that every problem has a solution.



Classification of numbers

Natural numbers (Counting numbers): 1,2,3,4…

Whole numbers: 0, 1, 2, 3,….

Integers: ….. -2,-1, 0, 1, 2, 3……

+ve Integers (Natural numbers) : 1,2,3…..

-ve Integers: ……-3,-2,-1

Non –ve Integers(Whole Numbers): 0,1,2,3…….

Note: 0 is an integer but it is neither +ve nor –ve.

Place value


thousands
hundreds
tens
ones
1,234.56789
tenths
hundredths
thousandths
ten thousandths
hundred thousandths

Rational Numbers

Which can be represented in the form of $\frac{p}{q}$ (where $q \neq 0$) and p & q are integers.

Example: 5, -7, 0, $\frac{1}{2}, \frac{7}{3}, -\frac{25}{8}, \frac{22}{7}$ etc.

Terminating : $\frac{M}{N}$ is terminating if $N = 2^a \times 5^b$,

Where a & b are whole numbers and M & N do not have any common factor.

Example: $\frac{7}{80}, \frac{33}{200}$

$80 = 2^4 \times 5^1$,

$200 = 2^3 \times 5^2$

Recurring:

Example: $\frac{11}{60}, \frac{9}{14}, \frac{101}{34}$

$60 = 2^2 \times 3 \times 5$

$14 = 2 \times 7$

$34 = 2 \times 17$

How to convert recurring to fraction

Example: 0.2323232323…= $\frac{a}{b}$

0.2323232323…

$= 0.\overline{23}$

$= \frac{23}{99}$

2 digits are repeating and therefore two times 9.

Example: 0.1255555555555555555… = $\frac{a}{b}$

$0.12555555555555555555\ldots$

$= 0.12\bar{5}$

$= \frac{125-12}{900} = \frac{113}{900}$

One digit is repeating therefore once 9 and 2 digits are not repeating therefore two times 0.

Irrational Numbers: Which are non-recurring and non-terminating.

Example: e, π, $2^{1/2}$,$5^{1/3}$,$10^{1/5}$

Note: $\frac{22}{7}$ is rational whereas π is irrational.

$\frac{22}{7} = 3.\overline{142857}$

$\pi = \frac{22}{7}$ is approx. value of pi.

Real Numbers: Numbers which can be represented on number line.

Or

Rational + Irrational

Example: 5, -7, 0, e, π, $2^{1/2}$,$5^{1/3}$,$10^{1/5}$,$\frac{1}{2}$,$\frac{7}{3}$,$-\frac{25}{8}$,$\frac{22}{7}$

Even Number

A number which is divisible by 2

…..,-4, -2, 0, 2, 4, 6, 8.....

Even numbers are represented by 2n.

Odd Number

A number that is not divisible by 2

….-3,-1,1, 3, 5, 7, 9.......

Odd numbers are represented by (2n - 1).

Odd±Odd = Even

Odd± Even = Odd

Even ±Even = Even

- If product of some integers is odd then all the integers are odd.
- If product of some integers is even then at least one of them is even.
- $\frac{\text{Odd}}{\text{Even}} \neq \text{Integer}$

Problem

If y is an integer then which of the following must be an odd number?

(A) y^2+4y+4 (B) y^2+3y+8 (C) y^2-7y+3

(D) $y^2-11y-10$ (E) y^2+8y-3

(A) y^2+4y+4 (B) y^2+3y+8 (C) y^2-7y+3

(D) $y^2-11y-10$ (E) y^2+8y-3

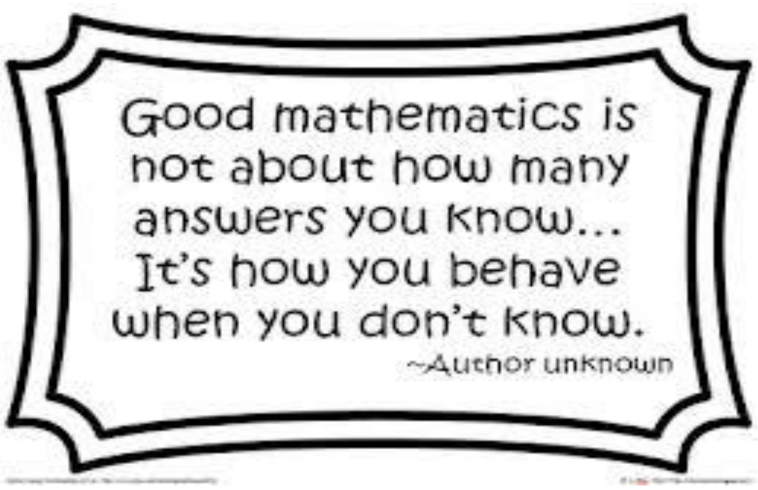
Check option A

Put y = 0, then answer is 4 i.e. even.

Put y = 1, then answer is 1+4+4 = 9 i.e. odd.

Thus option A depends on y.

Answer is C.



Prime numbers:

Natural numbers that have exactly two factors.

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29 ...}

Points to note……

- 2 is the only even prime number.
- 5 is the only prime number ending with 5.
- All the prime numbers greater than 3 are either (6k-1) or (6k+1) but reverse is not true.

  $17 = 6\times3-1$

  $97 = 6\times16+1$

- There are 25 prime numbers less than 100

Composite numbers:
Natural numbers with more than two factors.

Example: 4,6,8,9,10 and so on.

Co–Prime Or Relative prime Numbers:

Two numbers are said to be co–prime to each other if they do not have any common factor, other than 1.

Or

Any 2 numbers with HCF 1.

Example: 8 & 9
8: 1, 2, 4, 8
9: 1, 3, 9
Thus 8 and 9, though they themselves are not primes, are co–prime to each other

Example: 4 & 34

4: 1, 2, 4
34: 1, 2, 17, 34
Common factor is 2 and therefore 4 & 34 are not co-primes.

Also note that......

- 1 is said to be co–prime to all the other numbers.
- 2 Consecutive natural numbers are always co-prime.
- 2 Prime numbers are always co-primes.
- 2 Even numbers cannot be co-prime.

Perfect Square

All the powers of prime factors should be even.

Example:

$16 = 2^4$

$100 = 2^2 \times 5^2$

Example: If x & y are perfect squares, then which of the following is not necessarily a perfect square?

(A) xy (B) $4x$ (C) x^5 (D) $x+y$

If x & y are perfect squares, then which of the following is not necessarily a perfect square?

(A) xy (B) $4x$ (C) x^5 (D) $x+y$

Product of 2 perfect squares is again perfect square and

therefore (A) & (B) are perfect squares.

Any natural power of perfect square is also a perfect square

And therefore (C) is also square.

Answer is **(D)**

Perfect cube:

All the powers of prime factors should be multiple of 3.

Example:

$64 = 2^6$

$343 = 7^3$

Factorial

For any natural number N

$N! = 1\times2\times3\ldots..\times N$

$3! = 1\times2\times3$

$4! = 1\times2\times3\times4$

Note: $0! = 1$.

Example:

Find the highest power of 3 in 78!?

Or

$$\frac{78!}{3^x} = \text{Integer}$$

Find the maximum value of x?

$$\frac{78!}{3^x} = \text{Integer}$$

$$\frac{78}{3} = 26$$

$$\frac{26}{3} = 8\frac{2}{3}$$

$$\frac{8}{3} = 2\frac{2}{3}$$

Maximum value of x is summation of all the quotients = 26+8+2 = 36

Highest power of 3 in 78! is 36.

Remainder

$37 = 7\frac{2}{5}$

Dividend = Quotient × divisor + remainder

$37 = 5 \times 7 + 2$

Here, 5 is the divisor, 7 is quotient and 2 is the remainder.

Remainder is the extra number in the dividend if it is subtracted then result is divisible by divisor.

Remainder is always less than the divisor.

Example: When any natural number N is divided by 8 then what can be said about the remainder?

As we know remainder is always less than divisor therefore remainder is any number from 0 to 7.

Example: Find the remainder when 2 is divided by 6 i.e. $\frac{2}{6}$

Here, remainder is 2. Whenever numerator (dividend) is less than denominator (divisor) then remainder is numerator.

$$\frac{2}{6}=\frac{1}{3}$$

Remainder is 1, which is wrong.

Hence, avoid cancellation while finding the remainder.

Remainder is always non- negative(zero or positive) but in some cases we use –ve remainder for our convenience but while answering your final answer should be zero or positive.

When 47 is divided by 8 remainder is 7.

Remainder 7 means 47 is 7 more than the nearest multiple(less than 47) of 8.

When 47 is divided by 8 remainder is (-1) .

Remainder is (-1) means 47 is 1 less than the nearest multiple (greater than 47) of 8.

Example:
When 58 is divided by 10 remainder is 8 or (-2)

Example: When any natural number N is divided by 32 remainder is -4 then what is +ve remainder .

Here, given remainder is (-4) and therefore +ve remainder is 32-4 = 28.

Example: Find the remainder $\frac{61 \times 64}{15}$

Consider individual remainders.

When 61 and 64 are divided by 15 then remainders are 1 and 4 respectively.

Therefore final remainder is $1 \times 4 = 4$.

Example: Find the remainder

$$\frac{121 \times 122}{60}$$

$$\frac{121\times122}{60}$$

When 121 & 122 are divided by 60 respective remainders are 1 & 2 respectively.

Therefore final remainder is $(1\times2) = 2$

Example: Find the remainder

$$\frac{118\times117}{60}$$

$$\frac{118\times117}{60}$$

When 118 & 117 are divided by 60 respective remainders are 58 & 57 respectively.

Final remainder (58×57) and again divided it by 60.

Or

Use –ve remainder.

When 118 & 117 are divided by 60 respective remainders are (-2) & (-3).

Therefore final remainder is (-2×-3) = 6.

Example: Find the remainder

$$\frac{121\times118\times124}{60}$$

$$\frac{121 \times 118 \times 124}{60}$$

When 121, 118 & 124 are divided by 60 respective remainders are 1,(-2) & 4 respectively.

Therefore final remainder is (1×-2×4) = -8

As we know final remainder is always +ve .

Therefore 60-8 = 52

Example: Find the remainder

$$\frac{82^{75}}{27}$$

$$\frac{82^{75}}{27}$$

$82^{75} = 82\times82\times82\times......75$times

In each case remainder is 1.
Therefore final remainder is $1^{75} = 1$.

Example: Find the remainder

$$\frac{84^{55}}{17}$$

$84^{55} = 84 \times 84 \times ....55$times

In each case remainder is 16 or (-1).

Therefore final remainder is $(-1)^{55} = -1$

Remainder cannot be –ve and therefore final remainder is 17-1 = 16.

Example: $\frac{N}{100}$ remainder is 1 then

(i) Remainder in $\frac{N}{5}$

(ii) Remainder in $\frac{N}{7}$

$N = 100k+1$

(i) Since 5 is a factor of 100 therefore, when N is divided by 5 remainder is 1.

100k is divisible by 5 and therefore remainder is 1.

(ii) 7 is not a factor of 100 therefore, remainder is any number from 0 to 6.

One cannot say what is the remainder in 100k.

Divisibility Rules:

Note: Divisibility rules of prime numbers and their powers can also be used to find the remainder.

Rules for prime numbers and their powers

Rule for 2

If a number is even then it is divisible by 2.

Rule for 4

The two–digit number formed by the last two digits should be divisible by 4.

If not, the remainder when the number is divided by 4 is same as the remainder when the last two digits is divided by 4.

Example: The number 34700 is divisible by 4 because 00 is divisible by 4.

Rule for 8

The three–digit number formed by the last three digits should be divisible by 8.

If not, the remainder when the number is divided by 8 is same as the remainder when the last three digits is divided by 8.

Rule for 5
If a number ends with 0 or 5 then it is divisible by 5.

Rule for 25
If last 2 digits of the number are 00, 25, 50 or 75 then it is divisible by 25.

Rule for 3
Sum of digits should be divisible by 3.

If not, the remainder when the number is divided by 3 is same as the remainder, when the sum of digits is divided by 3.

Example: The number 34728 is divisible by 3 because 3 + 4 + 7 + 2 + 8 = 24 is divisible by 3.

Rule for 9
Sum of digits should be divisible by 9.

If not, the remainder when the number is divided by 9 is same as the remainder when the sum of digits is divided by 9.

Example: 452

Sum of digits is 4+5+2=11

11 is not divisible by 9 hence, 452 is not divisible by 9.

Here, remainder when 452 is divided by 9 is same as when 11 is divided by 9 i.e. 2.

Note: Difference of any natural number and its reverse is always multiple of 9.
Any natural number – Sum of its digits = Multiple of 9

Example:
400
400-(4+0+0) = multiple of 9

Rule for 11
Difference of sum of digits at odd places and even places should be multiple of 11 i.e. 0, 11, -11, 22, -22,..….
(Here, odd place and even place should be considered from right to left).

Example: 125546
Sum of digits at odd places = 6+5+2= 13 and Sum of digits at even places = 4+5+1 =10.
13-10 = 3
Since 3 is not divisible by 11 the given number is not divisible by 11.
When the number 125546 is divided by 11 remainder is 3.

Rules for composite numbers:

One can easily form rules for composite number by taking its 2 co-prime factors such that their product is equal to the number.

Rule for 6

$6 = 2 \times 3$

If a number is divisible by 2 and 3 both then number is divisible by 6.

Rule for 12

$12 = 2 \times 6 = 3 \times 4$

Here, 3 and 4 are co-prime therefore if a number is divisible by 3 and 4 both then number is divisible by 12.

Rule for 36

$36 = 4 \times 9$

If a number is divisible by 4 and 9 both.

Rule for 75

$75 = 25 \times 3$

If a number is divisible by 25 and 3 both.