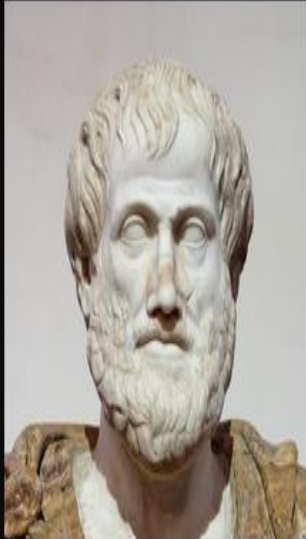


## Probability



It is a part of probability that many improbable things will happen.

~ Aristotle

Random Experiment:

An experiment, where we know the set of all possible results but find it impossible to predict one at any particular execution, is a random experiment.

Sample space: Set of all the possible outcomes associated with an experiment.

Event: Subset of sample space.

Fair/ Unbiased coin or dice:

$$P(\text{Head}) = P(\text{Tail}) = \frac{1}{2}$$

$$P(1) = P(2) = \dots = P(6) = \frac{1}{6}$$

## Definition of Probability

- ♥ Probability is the measure of how likely it is the some event will occur, a number expressing the ratio of favorable cases to the whole number or cases possible.
- ♥ For example the chance of a coin landing on heads is 50%. This is because a coin has two sides so there is a 50% chance that the coin will land on heads and 50% that a coin will land on tails.

$$P(E) = \frac{\text{fav outcomes}}{\text{Sample space}}$$

$$0 \leq P(E) \leq 1$$

If  $P(E) = 0$ , then it is impossible event.

If  $P(E) = 1$ , then it is definite event.

If  $P(E) = x$  then

$$P(\text{Not } E) = P(\bar{E}) = P(E') = P(E^c) = 1-x$$

Q. If 2 fair coins are tossed at random then find the probability of getting 1 head?

Soln: {HH,TT,HT,TH}

Total outcomes = 4

Fav outcomes = {HT,TH} = 2

$$P(E) = \frac{\text{fav outcomes}}{\text{Sample space}} = \frac{2}{4} = \frac{1}{2}$$

The chances of winning the lottery is 50%. You either win or don't



Q. If 5 fair coins are tossed at random then find the probability of getting 3 heads.

Soln:

Sample space =  $2^5$

5 coins: 3 heads and 2 tails

{HHHTT}

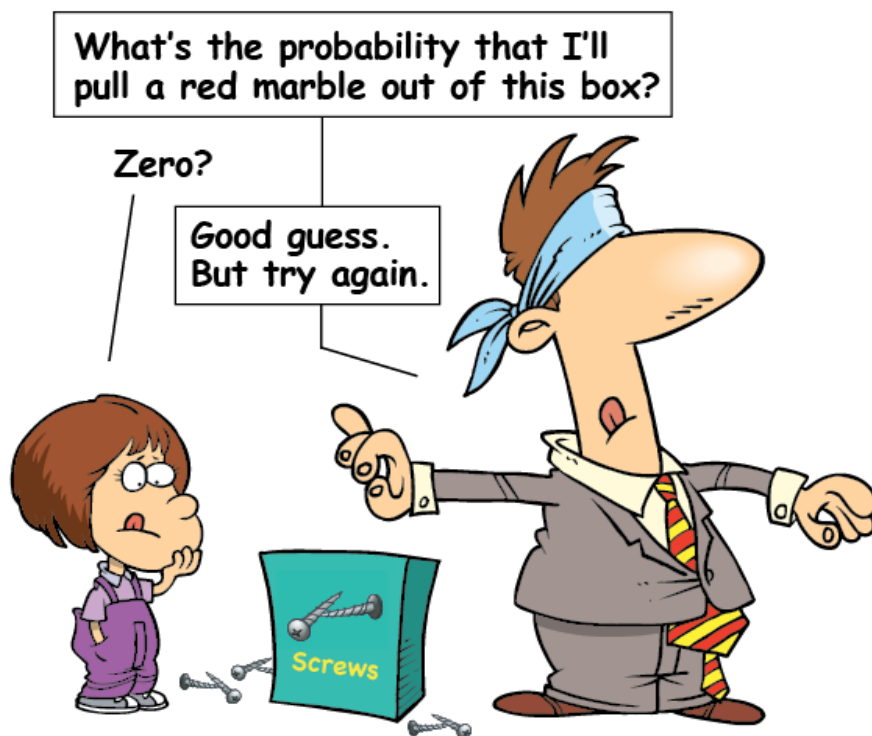
Fav outcomes = Select 3 places for Heads out of 5 =  ${}^5C_3$

$$\frac{{}^5C_3}{2^5}$$

Q. If 10 fair coins are tossed at random then find the probability of getting 7 heads and 5 tails?

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Soln: This is impossible event and therefore **probability is 0**.  
10 coins are tossed so number of heads and tails should be equal to 10.

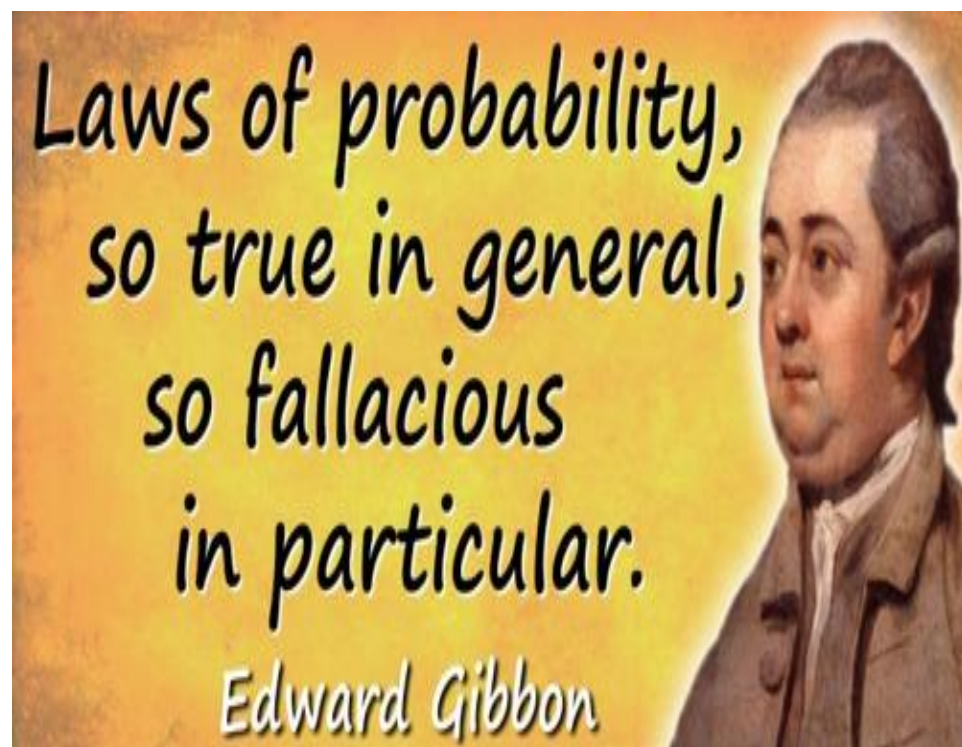


Q. If 10 fair coins are tossed at random then find the probability of getting at least 9 heads?

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Soln: At least 9 heads = 9 heads or 10 heads

$$= \frac{{}^{10}C_9 + {}^{10}C_{10}}{2^{10}} = \frac{11}{1024}$$



Q. When any natural number is divided by 5 then find the probability of getting odd remainder?

Soln: When any natural number is divided by 5 possible remainders are  $\{0,1,2,3,4\}$

Sample space = 5

Fav outcomes =  $\{1,3\} = 2$

$$P(E) = \frac{2}{5}$$

Q. Find the probability of 53 Sunday in a leap year?



Q. Find the probability of 53 Sunday in a leap year?

Soln:  $366 = 52 \times 7 + 2$

2 extra days can be {MT,TW,WT,TF,FS,SS,SM}

Sample space = 7

Fav outcomes = {SS,SM} = 2

$$\frac{2}{7}$$

Q. 10 coins are tossed at random such that probability of getting a head on a single throw is  $\frac{3}{5}$ . Then find the probability of getting 7 heads?

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$$\text{Soln: P(Head on a single throw)} = \frac{3}{5}$$

$$\text{P(Tail on a single throw)} = 1 - \frac{3}{5} = \frac{2}{5}$$

$$\left(\frac{3}{5}\right)^7 \times \left(\frac{2}{5}\right)^3 \times {}^{10}C_7$$

$$P(E) = \frac{\text{Fav outcomes}}{\text{Total Outcomes}}$$

$$\text{Odds against the event} = \frac{\text{Unfav outcomes}}{\text{Fav Outcomes}}$$

$$\text{Odds in favour of event} = \frac{\text{Fav outcomes}}{\text{Unfavo utcomes}}$$

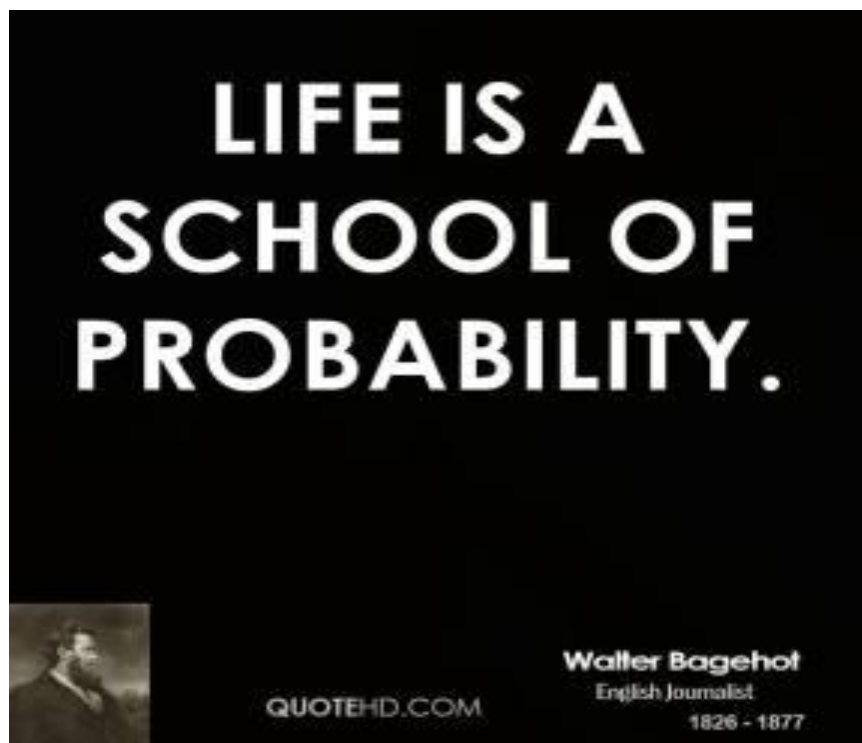
Q. If probability of an event is  $\frac{5}{7}$ , then find the odds in favour of that event?

Soln: Fav outcomes = 5

Total outcomes = 7

Unfav outcomes =  $7 - 5 = 2$

Odds in fav of the event =  $\frac{5}{2}$



Q. 2 Fair dice are tossed at random then find the probability of getting a score 5 or sum 5?



Soln: Total outcomes =  $6 \times 6 = 36$

Fav outcomes =  $\{(1,4)(4,1)(2,3)(3,2)\} = 4$

$$\frac{4}{6^2}$$

Q. In a room there are 3 lamp sockets. There are 12 bulbs of which 4 are defective if 3 bulbs are selected at random, what is the probability that room will be lighted?

Soln:  $P(\text{At least 1}) = \text{Total} - P(\text{None})$   
 $= 1 - P(\text{None is good})$

$$1 - \frac{{}^4C_3}{{}^{12}C_3}$$

**A PINCH OF  
PROBABILITY IS  
WORTH A POUND  
OF PERHAPS**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \text{ or } B) \qquad P(A \text{ and } B)$$

Independent Events: 2 events are said to be independent if occurrence or non- occurrence of an event does not affect the probability of the other event.

If A and B are 2 independent events then

$$P(A \cap B) = P(A) \times P(B)$$

Reverse is also true.

Mutually exclusive events: 2 Events are exclusive if they cannot occur simultaneously i.e.  $P(A \cap B) = 0$

Mutually exhaustive events: 2 Events are exhaustive if at least one of them occurs.

If 2 events A and B are both exclusive and exhaustive then  $P(A) + P(B) = 1$

Playing cards  
Total 52



Honour cards: (Ace, King, Queen, Jack)  
 $4 \times 4 = 16$

Picture cards: 3 (King, Jack, Queen)  
 $4 \times 3 = 12$



Q. 2 cards are drawn at the random from a pack of 52 playing cards then find the probability of

(i) Both red

(ii) at least 1 spade

(iii) both red or both king

$$(i) \quad \frac{{}^{26}C_2}{{}^{52}C_2}$$

$$(ii) \quad 1 - \frac{{}^{39}C_2}{{}^{52}C_2}$$

$$(iii) \quad \frac{{}^{26}C_2 + {}^4C_2 - {}^2C_2}{{}^{52}C_2}$$

Q. 2 cards are drawn at the random from a pack of 52 playing cards one after the other with replacement then find the probability of

(i) Both red

(ii) at least 1 spade

(iii) both from same suit

$$\frac{26 \times 26}{52 \times 52}$$
$$1 - \frac{39 \times 39}{52 \times 52}$$

$$\frac{4 \times 13 \times 13}{52 \times 52}$$

Q. The probabilities of A, B and C solving a problem are  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$  respectively. Find the probability that the problem is solved?

Soln: Indirectly question is at least 1 should solve the problem.

$P(\text{At least 1 should solve the problem}) = 1 - (\text{None solves})$

$$= 1 - P(A')P(B')(C')$$

### *Conditional Probability*

Q. A fair die is thrown at random and it shows a prime number then find the probability of getting an odd number?

Soln: A die is thrown and it shows a prime number means

total outcomes =  $\{2,3,5\} = 3$

Fav Outcomes =  $\{3,5\}$

$$\frac{2}{3}$$

Q. A circular coin of unit radius is flipped and it falls on a square table of length 10units. What is the probability that the coin lies completely inside the table?

16/25

Q. A box contains 40 numbered cards from 1 to 40. 5 cards are drawn at random then find the probability that their range is 37?

Soln:

Minimum	Maximum	Definitely not allowed
1	38	39,40
2	39	1,40
3	40	1,2

$$\frac{3 \times {}^{36}C_3}{{}^{40}C_5}$$

The probability of success is difficult to estimate; but if we never search the chance of success is zero.

— Philip Morrison —