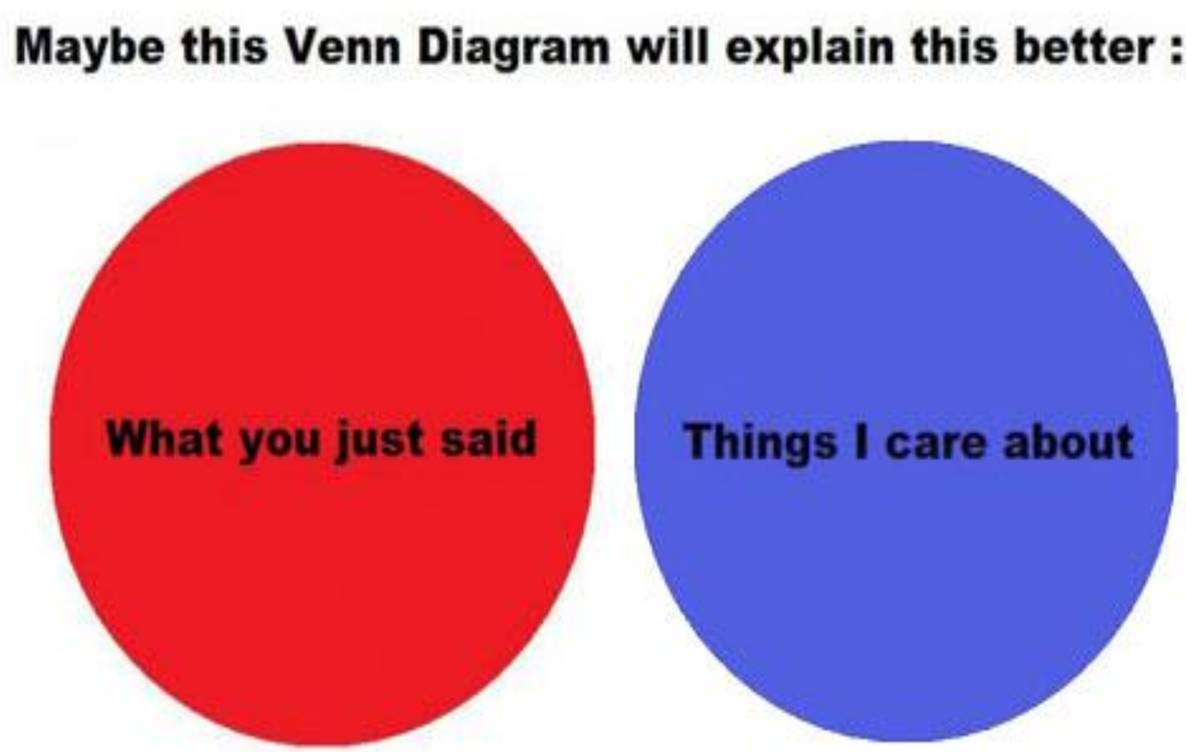
SET THEORY


Maybe this Venn Diagram will explain this better :
What you just said
Things I care about

A collection of well-defined objects is called a **set**.

For example,

'the set of former Nobel Prize winners'

is a well-defined set

'the set of tall students in our university'

is not a well-defined set

- Definition of a set:
  - A set is simply a collection of objects or elements or members.
    - E.g. A={1,2,3,4} describe a set *A* made up of four elements 1, 2, 3, and 4.
  - A set is determined by its elements and not by any particular order in which the elements might be listed. Hence above mentioned set can be expressed as:
    - E.g. A={1,3,4,2}
  - Elements making up a set as assumed to be distinct
  - Only one occurrence of each element even if there are duplicates in the set:
    - E.g. A={1,2,3,3,4}

EQUAL SET :-

Two sets are equal if they contain exactly the same elements. That is, set A is equal to set B if every element of A is also an element of B, and every element of B is also an element of A.

The order in which the elements of a set are listed in its definition is irrelevant.

For example, the sets {1,2,3} and {3,2,1} are equal.

EMPTY SET :-

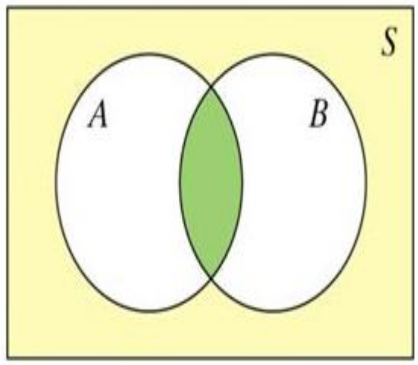
A set that contains no elements is called the empty set, and is represented by the symbol ∅.

ORDERED PAIRS :-

An ordered pair is a set of two elements in a specified order. An ordered pair is usually written (a,b) where a is the first element and b is the second element. Reversing the elements of an ordered pair produces a different ordered pair if the elements are not the same. For example, the ordered pair (1,2) is not equal to the ordered pair (2,1).

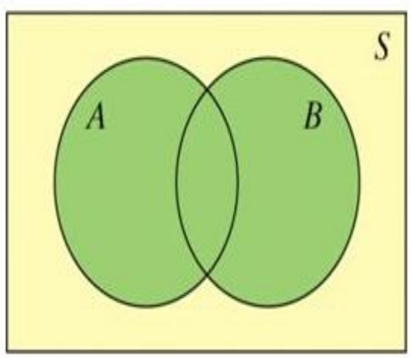
The intersection of events *A* and *B* ($A \cap B$) is the set of all outcomes in both events *A and B*.

$A \cap B$



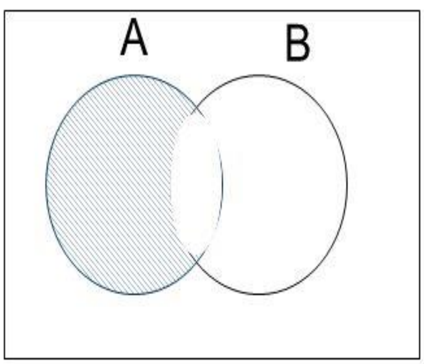
The union of events *A* and *B* ($A \cup B$) is the set of all outcomes in either event *A or B*.

$A \cup B$

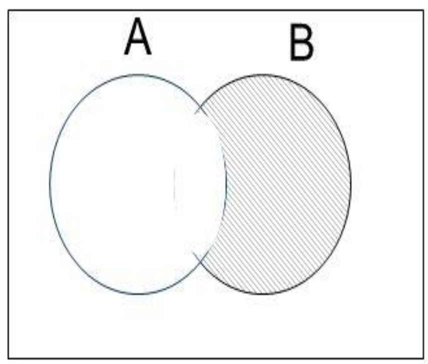


Complement

- If A and B are two sets, the complement of B with respect to A is defined as the set of all elements that belong to A but not to B.
- It is denoted by A – B.
- $A - B = \{x \mid x \in A \text{ and } x \notin B$
- $B - A = \{x \mid x \in B \text{ and } x \notin A$



A - B



B - A

Subsets

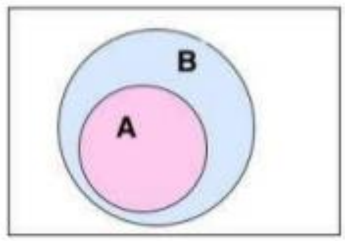
A is a **subset** of *B* if every element of *A* is also contained in *B*. This is written

$$A \subset B.$$

For example, the set of integers

$$\{\ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

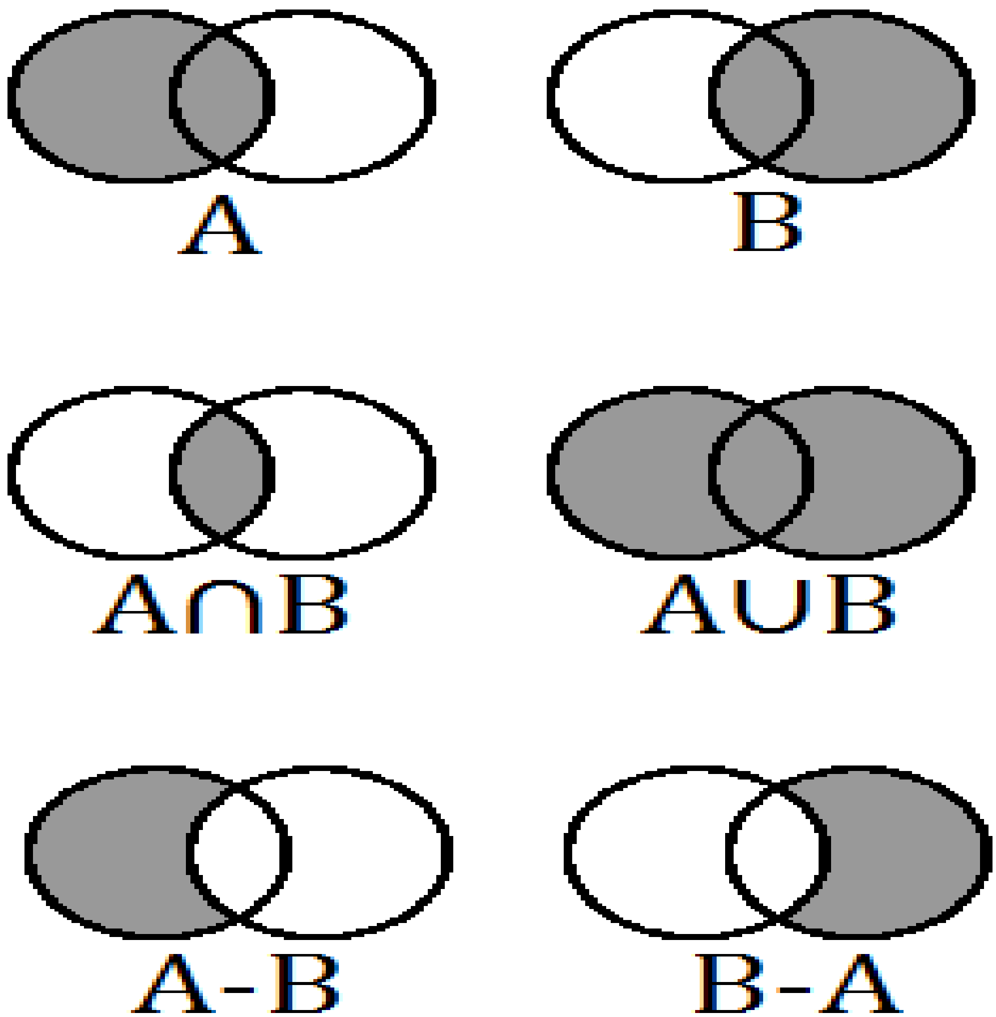
is a subset of the set of real numbers.



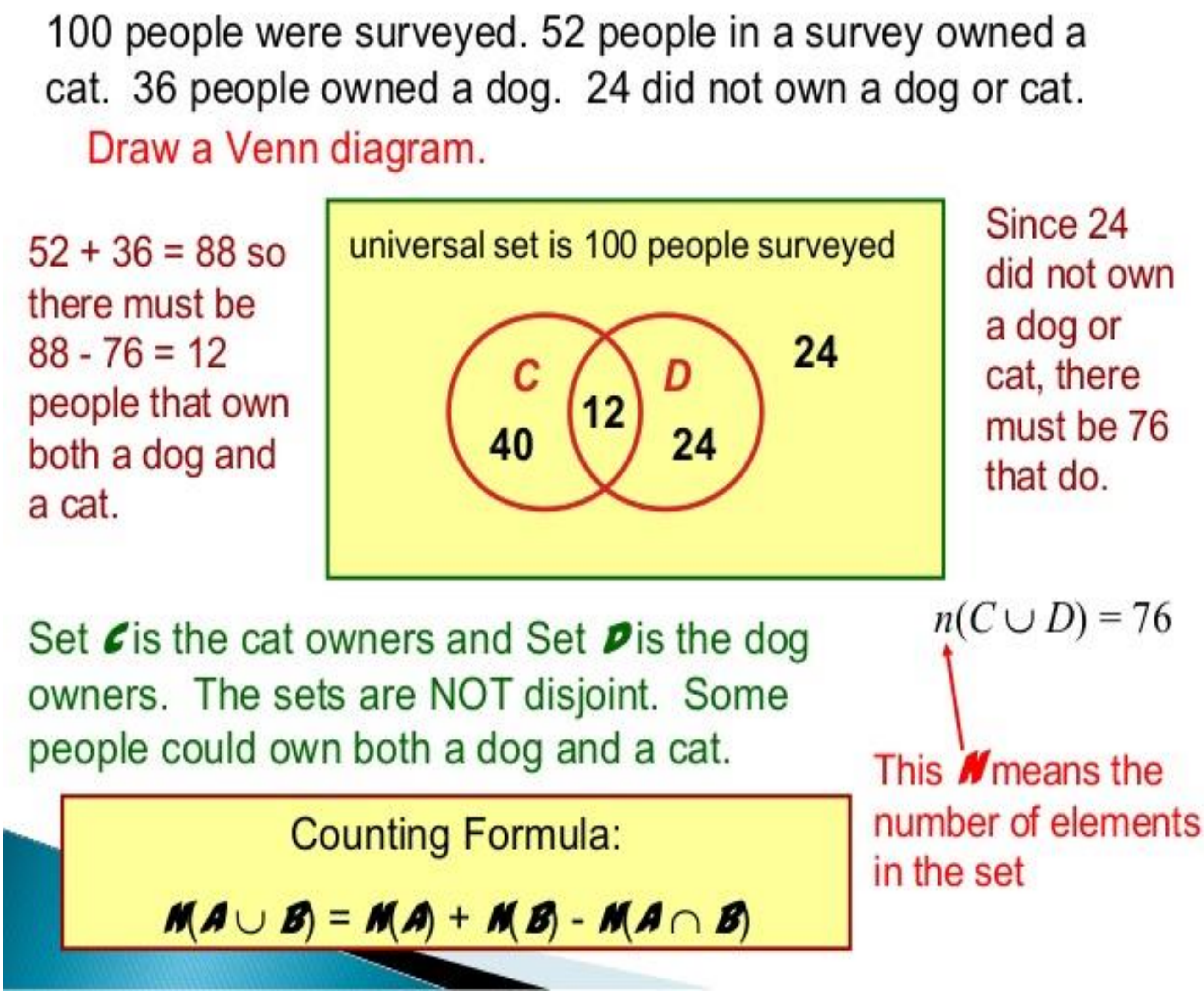
Formal Definition:

$A \subset B$ means "if $x \in A$, then $x \in B$."

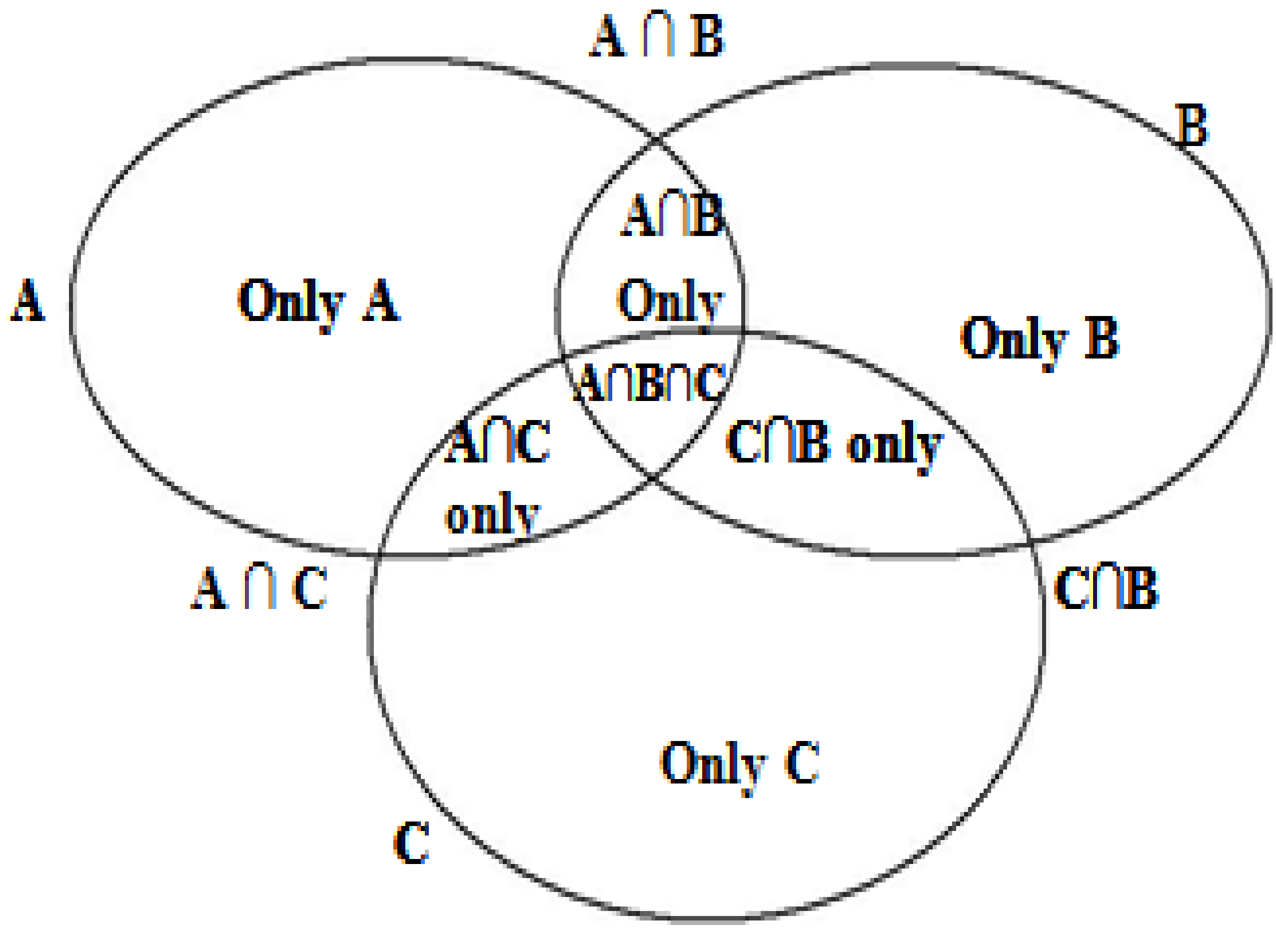
Note: Total no. of subsets of a set A having n elements is 2^n



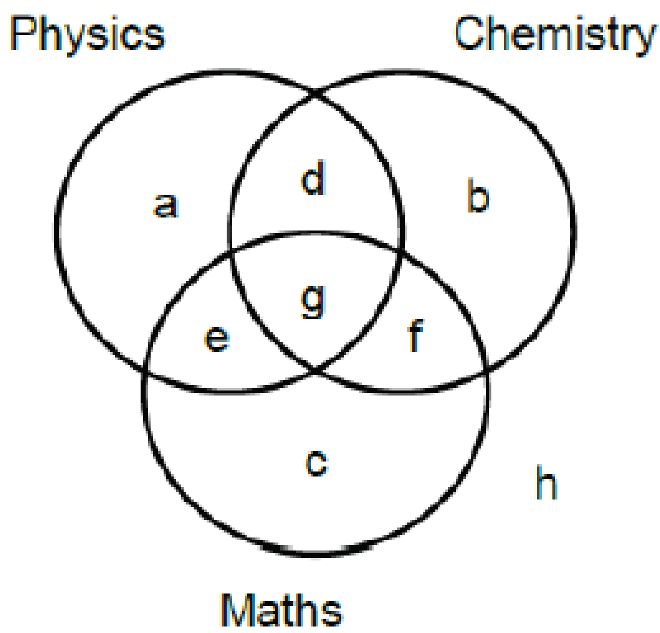
EXAMPLE:-


100 people were surveyed. 52 people in a survey owned a cat. 36 people owned a dog. 24 did not own a dog or cat.
Draw a Venn diagram.
52 + 36 = 88 so there must be 88 - 76 = 12 people that own both a dog and a cat.
universal set is 100 people surveyed
C
D
40
12
24
24
Since 24 did not own a dog or cat, there must be 76 that do.
Set C is the cat owners and Set D is the dog owners. The sets are NOT disjoint. Some people could own both a dog and a cat.
$n(C \cup D) = 76$
This n means the number of elements in the set
Counting Formula:
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

EXAMPLE:-



A ∩ B
B
A
Only A
A∩B
Only
Only B
A∩B∩C
A∩C
only
C∩B only
A ∩ C
C∩B
Only C
C



Physics → a + d + e + g

Chemistry→ b + d + f + g

Maths→ c + e + f + g

Only Physics→ a

Only Chemistry→ b

Only Maths→ c

Physics and Chemistry→ d + g

Chemistry and Maths→ f + g

Maths and Physics→ e + g

Physics and Chemistry and Maths→ g

Only Physics and Chemistry→ d

Only Chemistry and Maths→ f

Only Maths and Physics→ e

None → h

Exactly one subject (**I**)→ a + b + c

Exactly two subject (**II**)→ d + e + f

Exactly three subject (**III**)→ g

Total → a + b + c + d + e + f + g + h ------------ **(i)**

Physics + Chemistry + Maths → $\underbrace{a + b + c}_{\textbf{(I)}} + 2(\underbrace{d + e + f}_{\textbf{(II)}}) + \underbrace{3g}_{\textbf{(III)}}$

Physics + Chemistry + Maths→ **I + 2II + 3III** ----------- **(ii)**

Ex:- In a class there are 100 students. Every student play at least one sports . 40 play football, 60 play cricket and 50 play hockey. What will be the maximum no. of student who play exactly one sport ?

Que: In a competition, a school awarded at least one medal in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

Ans: Here, Total = 45 = **I + II + III** ------- **(1)**

Exactly one – **I**

Exactly two – **II**

Exactly three – **III**

Dance + Dramatics + Music = **I** + **2II** + **3III** **----- (2)**

Equ(2) – Equ(1)

(36 + 12 +18) – 45 = **II + 2III**

II = 21 – 2(4) = 13

II = 13

Directions for questions 17 to 20: These questions are based on the following data.

In the summer of last year, a survey was conducted in a colony to know how many houses have refrigerators, ACs and water coolers. 112 houses have water coolers. The number of houses which have none of the three devices is five times of that have with all the three. 10% of the houses have water coolers and refrigerators. 42% of the houses have ACs. 44% of the houses have refrigerators. 20% of the houses have none of the three. 22 % have only refrigerators and 12% of the houses have ACs and water coolers

17. How many houses have refrigerators only?

(A) 176 (B) 72 (C) 88 (D) 160

18. How many houses have at most two devices among ACs, refrigerators and cooler?

(A) 384 (B) 400 (C) 200 (D) 304

19. How many houses were surveyed?

(A) 320 (B) 384 (C) 300 (D) 400

20. How many houses do not have refrigerators?

(A) 88 (B) 176 (C) 224 (D) 144

Directions for questions 13 to 16: These questions are based on the following data.

A survey was conducted among 500 families regarding the types of fuel that they use for cooking purpose. It is known that 200 people use Kerosene, 270 people use LPG and 240 people use Electricity, 72 people use both Kerosene and LPG. 126 people use both LPG and Electricity, 62 use both Kerosene and Electricity. It is also known that, 20 people use none among these three.

13. How many people use fuel of all the three types?

(A) 30 (B) 40 (C) 10 (D) 20

14. How many people use at most one type of fuel?

(a)300 (B) 320 (C) 280 (D) 340

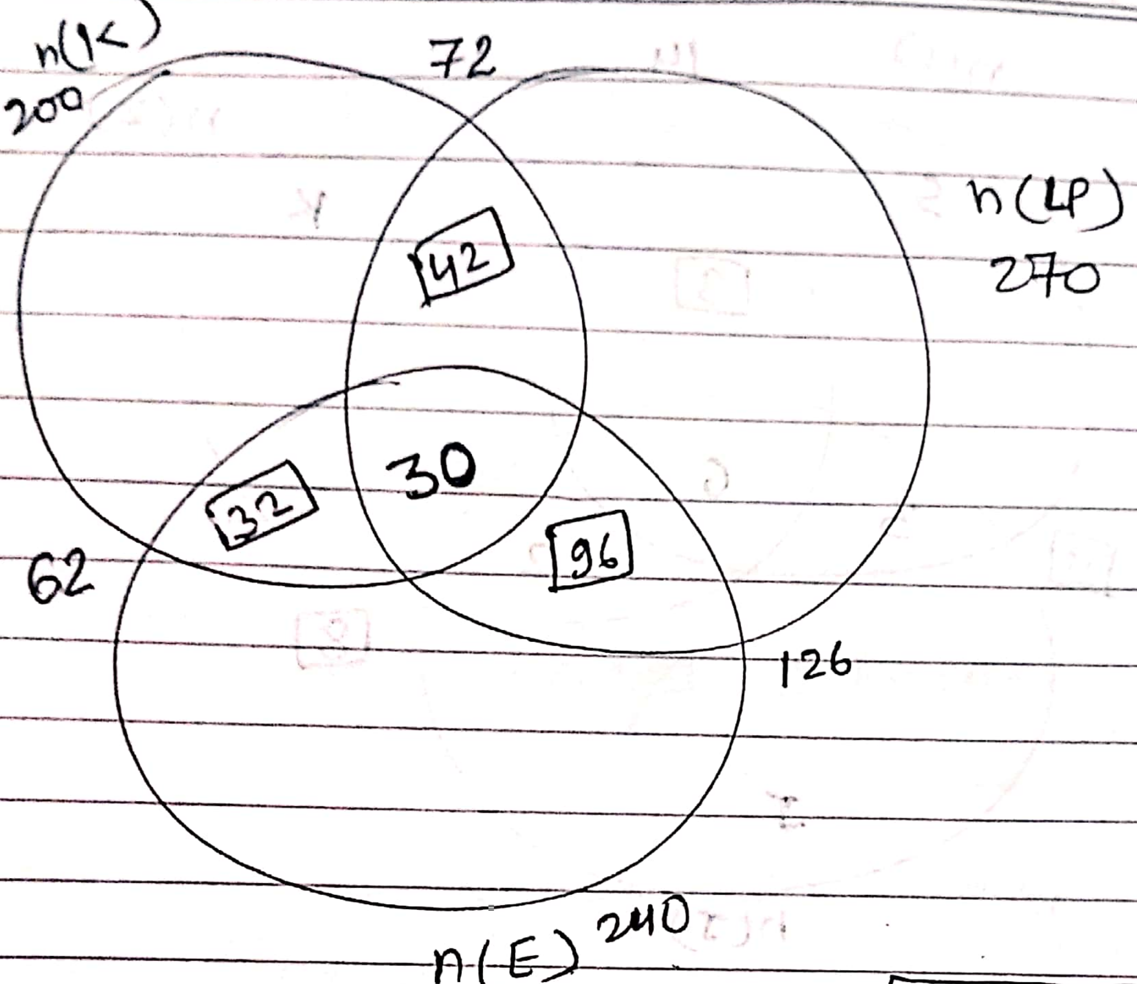
15. How many use either Kerosene or LPG?

(A) 470 (B) 198 (C) 398 (D) 240

16. How many people use at least two types of fuels?

(A) 190 (B) 200 (C) 210 (D) 170

13 to 16



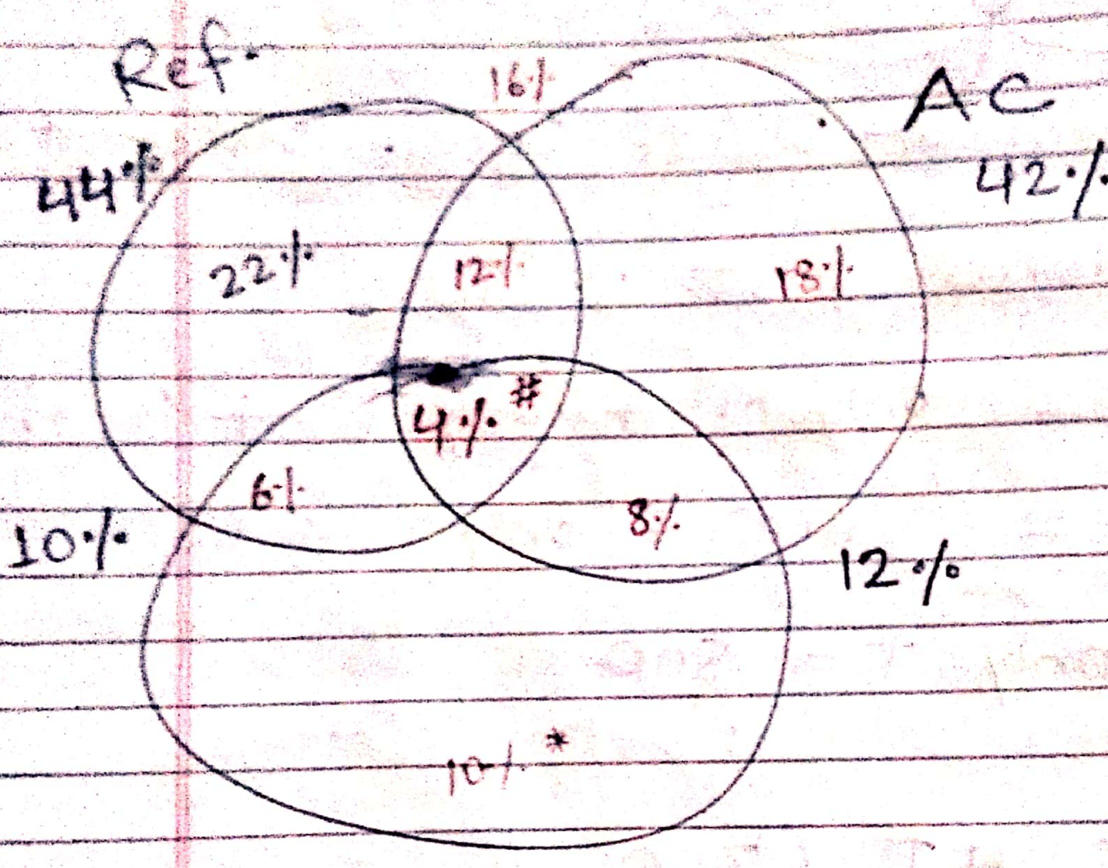
None = 20

Total = n(A) + n(B) + n(C) − n(A∩B) − n(B∩C) − n(C∩A) + n(A∩B∩C) + None

500 = 200 + 270 + 240 − 72 − 126 − 62 + n(A∩B∩C) + 20

n(A∩B∩C) = 30

Q. [Set 17 to 20]



WC → 112 houses

None = 5 × all III

20% = 5 × all III

all III = $\frac{20\%}{5}$ = 4%

* $\text{Total} = A + B + C + e + f + g + d + \text{None}$

$100\% = 22\% + 18\% + \text{only 'wc'} + 12\% + 8\% + 6\% + 4\% + 20\%$

~~only~~ 'wc' $= 10\%$

$n(wc) = 6\% + 4\% + 8\% + 10\%$

$= 28\%$

$28\% = 112$ houses (Given)

$100\% = ?$

400 houses.

Total house $= 400$.