Geometry -1

Point: A fine dot made by a sharp pencil, or the prick made by a fine pin on a sheet of paper is close to a point. It is dimensionless.

Collinear points: If points lie on the same line.

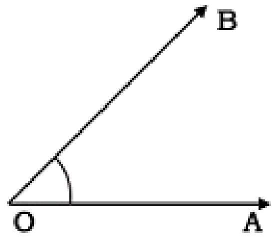
Line: Collection of infinite collinear points without any end point.

Line-Segment: Part of a line with 2 end points.

Ray: Part of a line with 1 fixed point.

Plane: The surface of a smooth wall or table-top is close to a portion of plane. In short, a plane is a flat surface. It has length and width but no thickness. A plane extends infinitely in all directions.

Angle: Two rays with a common initial point form an angle.



0 <Acute angle < 90

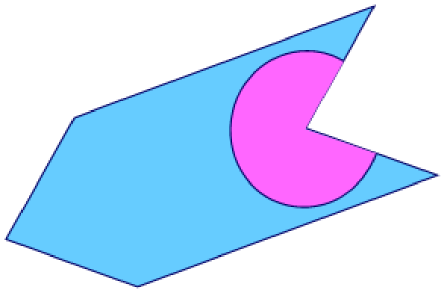
Right = 90

90< Obtuse < 180

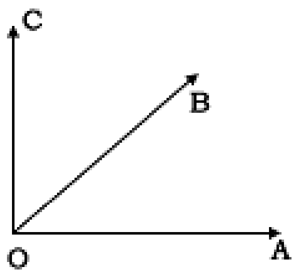
Straight line = 180

180< Reflex Angle< 360

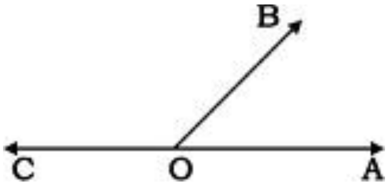
Complete Angle = 360



Adjacent Angles: In the given figure, ∠AOB and ∠BOC are adjacent angles.



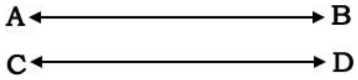
Linear Pair: In the given figure ∠ COB and ∠ AOB is a linear pair



Supplementary Angles : Sum of 2 angles is 180

Complementary Angles: Sum of 2 angles is 90.

Parallel Lines: Two distinct lines in the same plane are said to be parallel if they do not intersect at any point.



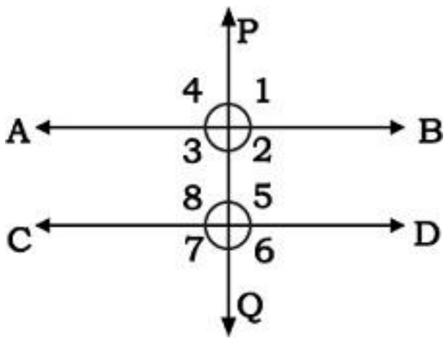
Or

If distance between 2 lines is same at all the points then lines are ||.

Here line AB is parallel to line CD. This is denoted as AB||CD

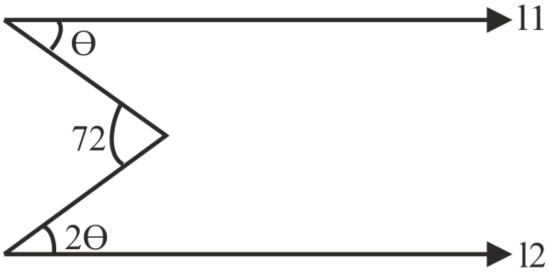
Angles Made by a Transversal with Two Lines:

PQ is a transversal on two parallel lines AB and CD

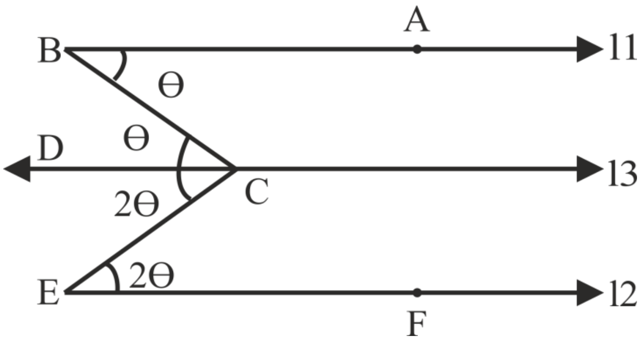


- $\angle 3 = \angle 5$ and $\angle 2 = \angle 8$ (Pairs of alternate interior angles)
- $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 4 = \angle 8$ and $\angle 3 = \angle 7$ (Pairs of corresponding angles)
- $\angle 2 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 8 = 180^\circ$ (Interior angles on the same side of the transversal)
- $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 6 = \angle 8$ and $\angle 5 = \angle 7$ (vertically opposite angles)

Problem:



Find **θ** if l1 || l2 ?



If we form a line $l_3 \parallel l_1 \parallel l_2$ then

$\angle ABC = \angle BCD = \theta$ and

$\angle DCE = \angle CEF = 2\theta$

[Alternate angles]

So, $3\theta = 72$

$\theta = 24°$

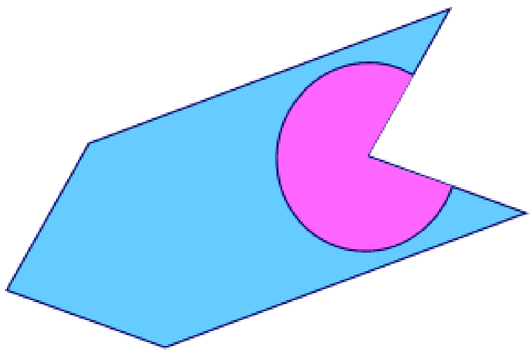
Polygon:

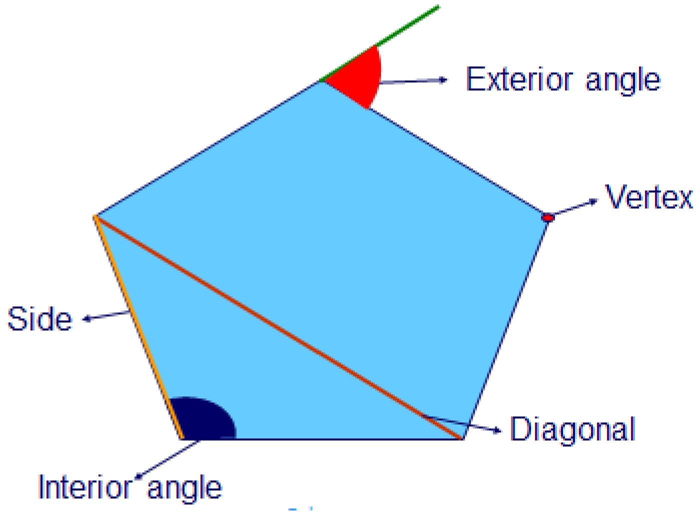
It is a closed plane figure bounded by some straight lines. Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Octagon, Nonagon and Decagon are polygons with 3, 4, 5, 6, 7, 8, 9, 10 sides respectively.

Convex & Concave Polygon:

A polygon, in which none of the Interior angles is more than 180°, is a convex polygon.

On the other hand, if at least one angle of a polygon is more than 180°, then it is a concave polygon.





I. Sum of all interior angles = $(n-2)\times180$

II. Sum of all exterior angles = 360°

Diagonal: line segment joining any 2 non- adjacent vertices.

Total no. of diagonals = $\dfrac{n(n-3)}{2}$

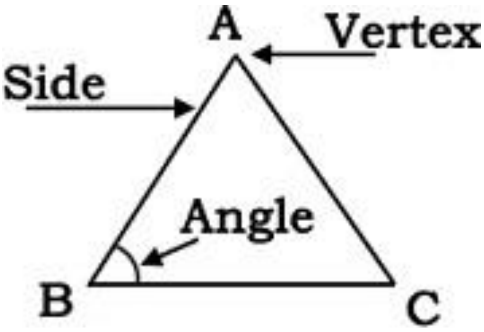
Regular Polygon :

A polygon having all sides equal and all angles equal is called a regular polygon.

Each exterior angle $= \dfrac{360}{n}$

Each interior angle = 180° - (exterior angle)

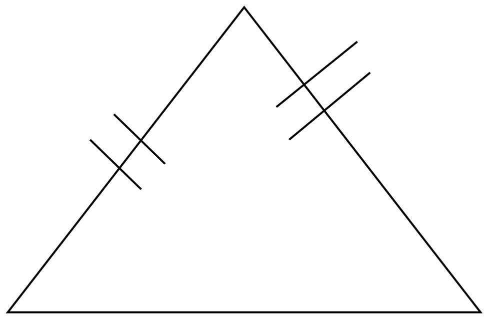
Triangle:



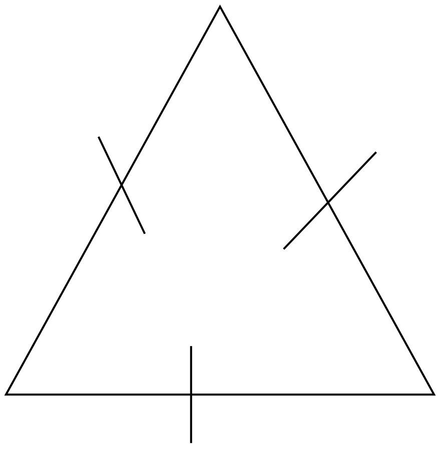
Classification of Triangles

I. According to sides:

1. A triangle having no two sides equal is called a scalene triangle.
2. A triangle having two sides equal is called an isosceles triangle.



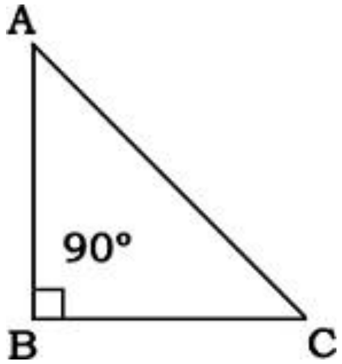
3. A triangle having all sides equal is called an equilateral triangle.



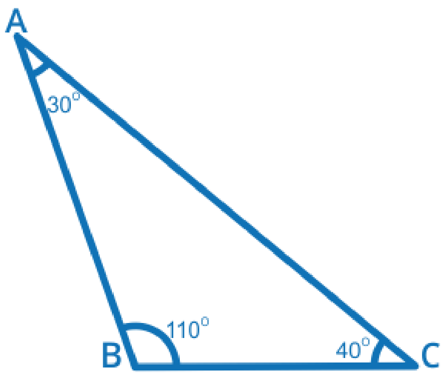
II. According to Angles:

1. A triangle with 3 acute angles.

2. A triangle one of whose angle is a right angle is called a right-angled triangle.



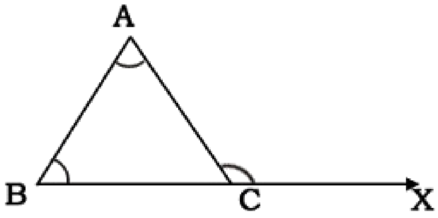
3. A triangle one of whose angle is obtuse is called an obtuse angled triangle or simply an obtuse triangle.



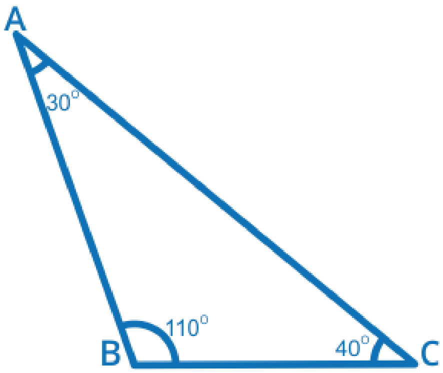
Properties of a Triangle:

1. The sum of three angles of a triangle is 180°.

2. In a triangle, an exterior angle equals the sum of the two interior opposite angles. For example in the given figure $\angle ACX = \angle ABC + \angle BAC$.



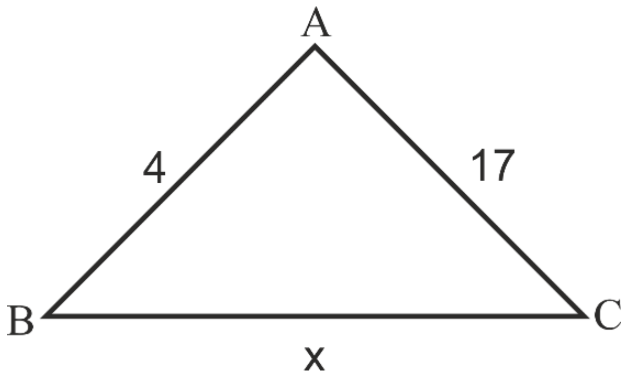
3. Side opposite to the greatest angle is the longest side.



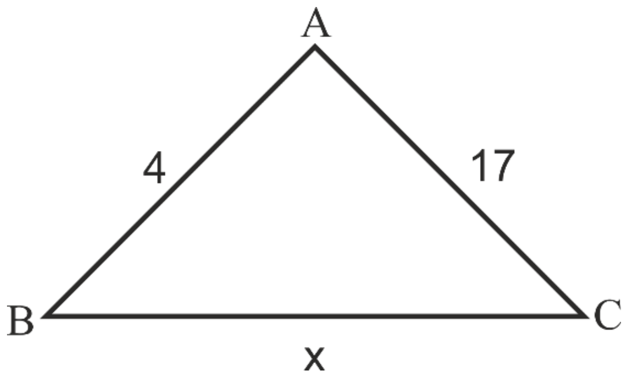
4. The sum of any two sides of a triangle is greater than the third side. So in the above figure
   AB + BC > AC, AB + AC > BC, BC + AC > AB

   The difference of any two sides of a triangle is less than the third side. Hence, |AB - BC| < AC.

Problem:



Quantity A	Quantity B
No. of values that x can take	20



For a triangle

$4 + 17 > x \Rightarrow x > 21$

$4 + x > 17 \Rightarrow x > 13$

Therefore, $13 < x < 21$

x can take any value between 13 & 21.

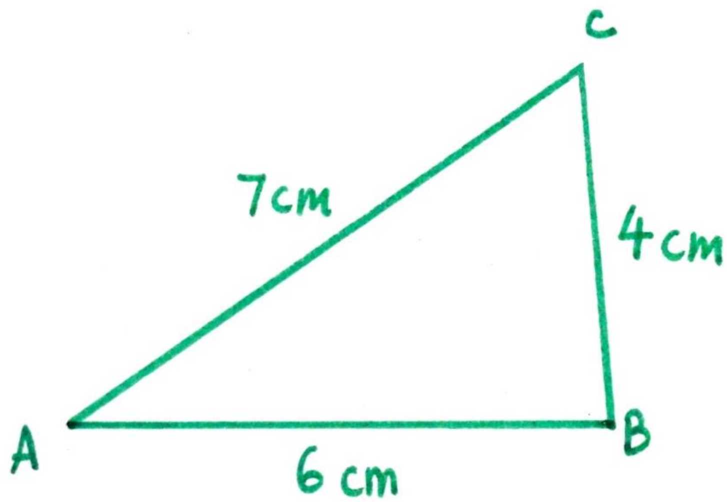
So, answer is A.

Note: - If x is an integer then x can take only 7 values

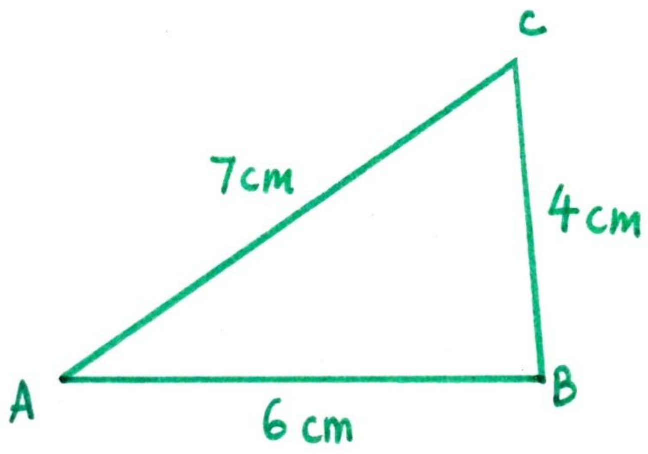
from 14 to 20. Then answer is B.

5. Let a is the longest side of the triangle then
    1. If $a^2 > b^2 + c^2$ then triangle is obtuse.
    2. If $a^2 < (b^2 + c^2)$ then triangle is acute.

Problem:



What is the type of the triangle based on the angles?



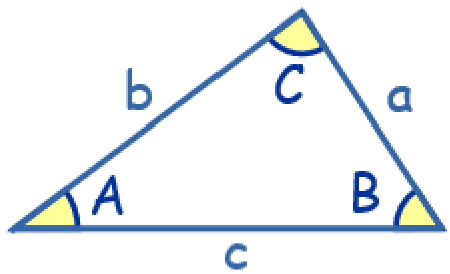
$7^2 < 4^2 + 6^2$

Therefore acute triangle.

Sin rule:

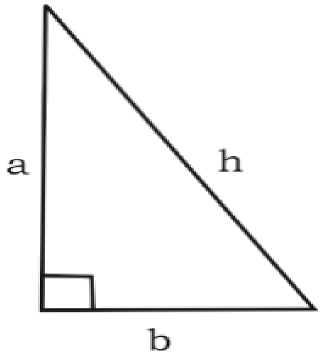
$$\frac{a}{\text{SinA}} = \frac{b}{\text{SinB}} = \frac{c}{\text{SinC}} = 2R$$

Cosine Rule:



$$\text{CosA} = \frac{b^2 + c^2 - a^2}{2bc}$$

Right Angle Triangles



Area = $\frac{1}{2} \times ab$

Pythagoras Theorem

$a^2 + b^2 = h^2$

(3, 4, 5) ,(5, 12, 13),(7, 24, 25),(8, 15, 17),(9,40,41),(11,60,61)

2 × (3, 4, 5) = (6, 8, 10)

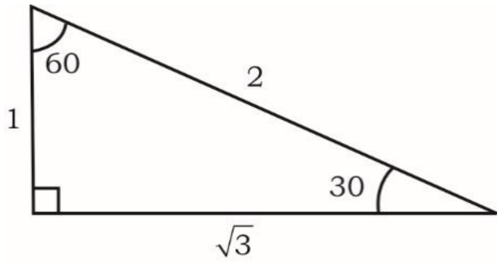
3 × (3, 4, 5) = (9, 12, 15)

4 × (3, 4, 5) = (12, 16, 20)

2 × (5, 12, 13) = (10, 24, 26)

30-60-90 triangle

In a 30-60-90 triangle, the ratio of the lengths of the sides opposite to 30, 60 and 90 degrees is $1:\sqrt{3}:2$. This is also memorized as side opposite to 30 degrees is half the hypotenuse and side opposite to 6 degrees is $\frac{\sqrt{3}}{2}$ times the hypotenuse.



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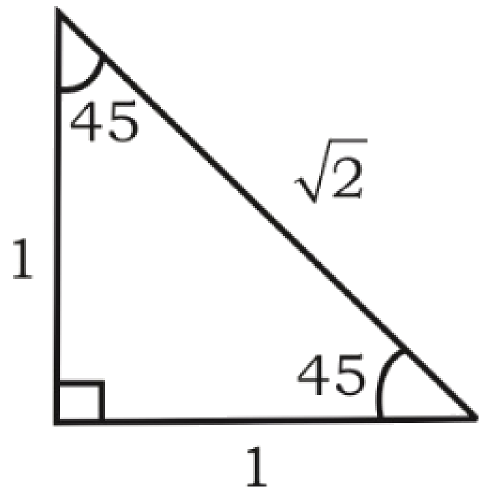
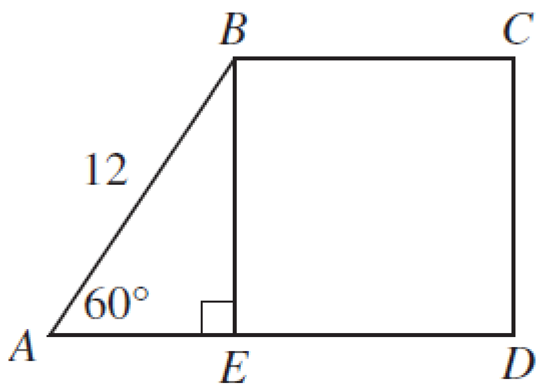
Isosceles Right angle triangle, 45-45-90 triangle

In an isosceles right angle triangle, if the lengths of the perpendicular sides is a, the hypotenuse is $\sqrt{2}a$. Accordingly if the hypotenuse is h, length of each perpendicular side is $\frac{h}{\sqrt{2}}$.



Problem:

In the figure below, BCDE is a square and AB = 12. What is the area of square BCDE?



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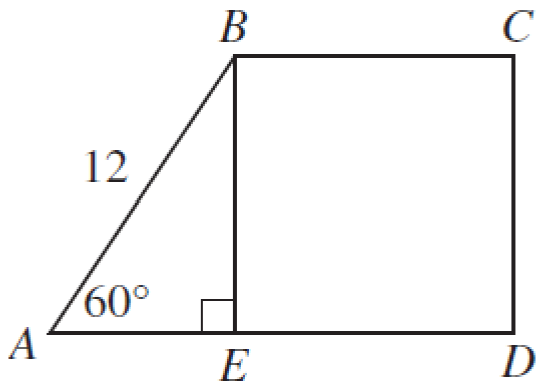


In the figure below, BCDE is a square and AB = 12. What is the area of square BCDE?

$\angle ABE = 30$, Let AE = x

Then BE = $\sqrt{3}\,x$ and

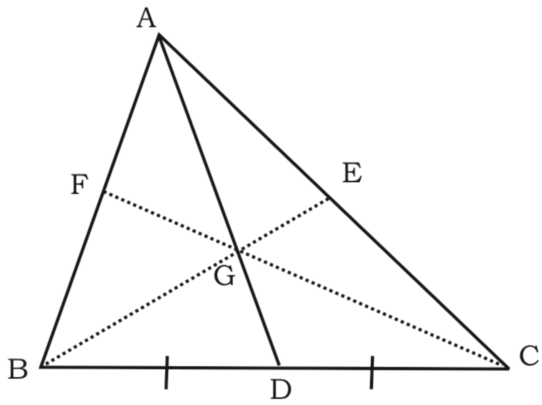
AB = 2x = 12 => x = 6

BE = $\sqrt{3}\,x = 6\sqrt{3}$

Area of square = $(6\sqrt{3})^2 = 108$

Median: Line segment joining mid-point of the side to the opposite vertex.(AD is median in the diagram)

Median divides the triangle into two equal areas, $A(\Delta ABD) = A(\Delta ADC)$



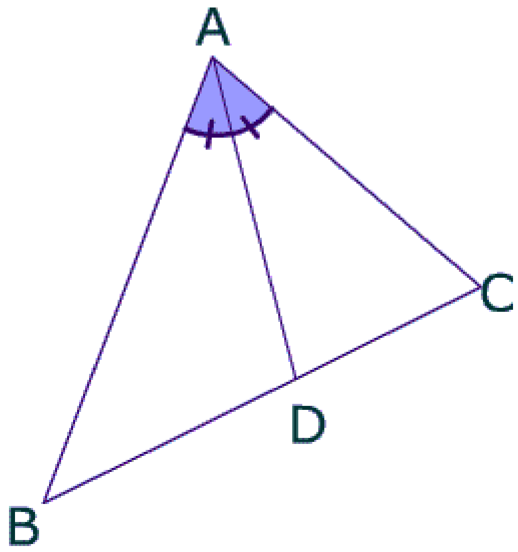
D, E & F are midpoints of the sides

3 Medians divide triangle into 6 equal areas.

Centroid:
Concurrency point of the medians(point G in the diagram).
The centroid divides the median in the ratio 2:1, with the larger part being towards the vertex. Thus, AG : GD is 2: 1.

Apollonius Theorem
$AB^2 + AC^2 = 2 \times (AD^2 + BD^2)$

Angle bisector:

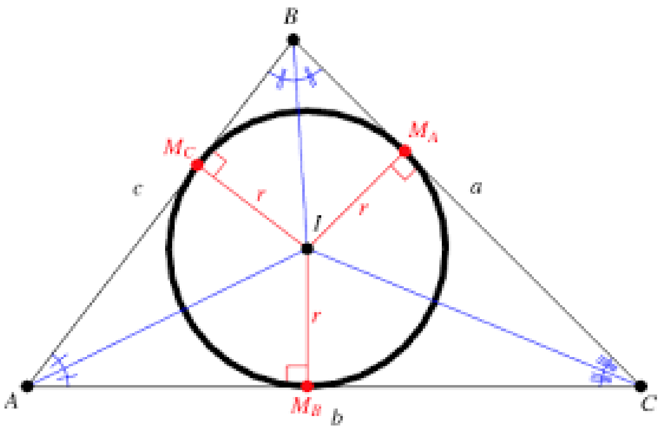


AD is angle bisector then

$$\frac{AB}{AC}=\frac{BD}{DC}$$

In-center:

Concurrency point of the angle bisectors (I in the figure). It is called an In-center because with this point as a center a circle can be drawn which lies 'in' the triangle i.e. it touches each of the three sides. The radius to the circle is the perpendicular distance from I to any of the sides, shown as dotted lines in the figure.



In center is the only point which is equidistant from all the 3 sides.

$$\angle BIC = 90 + \frac{\angle A}{2}$$

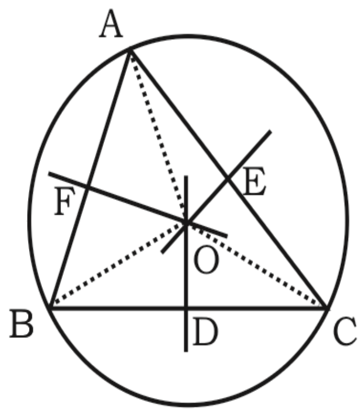
Circum-center:

Concurrency point of the perpendicular bisectors of the side (point O in the figure)

Circum center is the only point which is equidistant from all the 3 vertices.

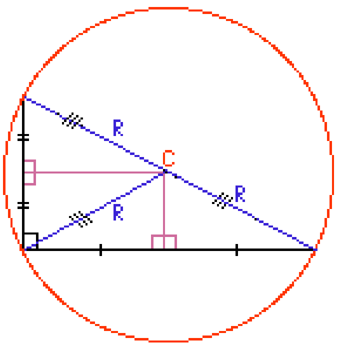
The radius of the circle = AO = BO = CO =R

$$\angle BOC = 2 \times \angle A$$



OD, OE & OF are perpendicular bisector of sides

- In Acute angle triangle :Inside the triangle.
- In Right angle triangle : Mid pint of Hypotenuse.



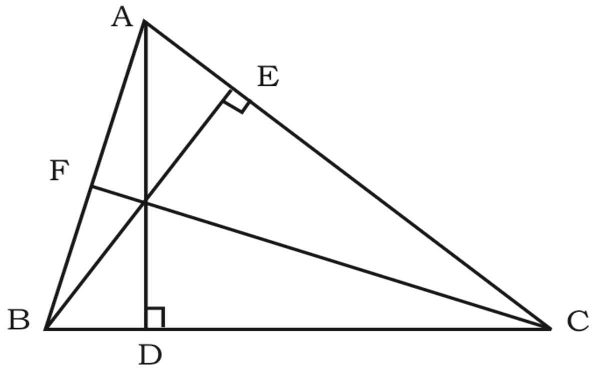
$$R = \frac{\text{Hypotenuse}}{2}$$

- In obtuse angle triangle: outside the triangle

Orthocenter:

Concurrency point of the Altitudes (point H in the figure)

Property 1: $\angle BHC + \angle A = 180°$



AD, BE & CF are altitudes

- In Acute angle triangle Orthocentre is inside the triangle.
- In Right angle triangle Orthocentre is the point(Vertex) where angle is 90°
- In obtuse angle triangle Orthocentre is outside the triangle.

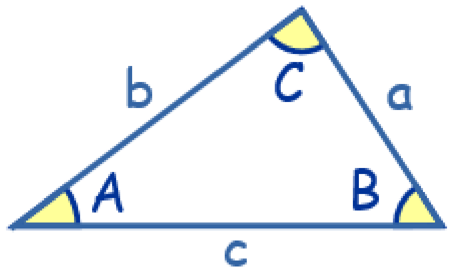
1. Area $= \frac{1}{2} \times \text{Base} \times \text{Height}$

2. Heron's Formulae: Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter and a, b and c are the sides of the triangle.

3. Area $= r \times s$, where r is the in-radius and s is the semi-perimeter.

4. Area $= \frac{abc}{4R}$, where R is the circum-radius and a, b and c are the sides of the triangle.

5. Area $= \frac{1}{2} ab \sin C$

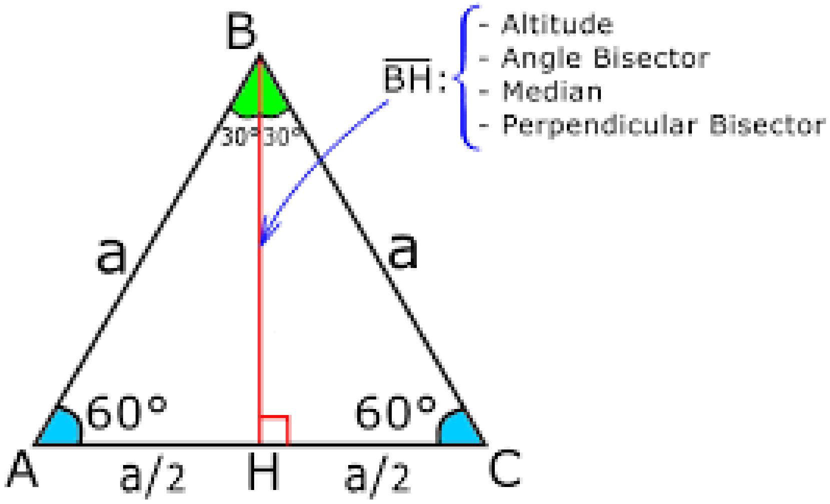


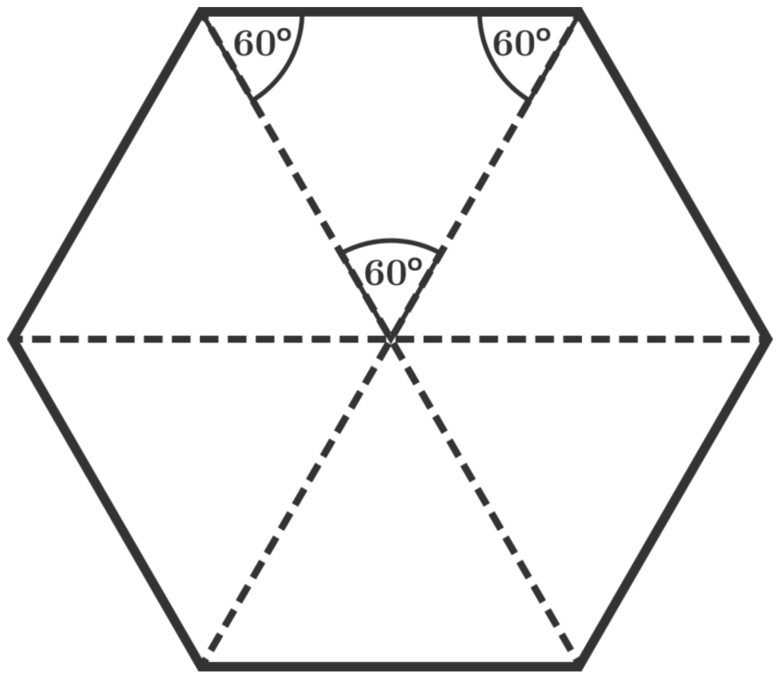
In an equilateral triangle all the 4 geometric centers are concentric.
In equilateral triangle median, altitude, angle bisector and perpendicular bisector all are same i.e. single line.

Height of Equilateral triangle of side a units $= \dfrac{\sqrt{3}}{2}a$

Area of Equilateral triangle of side a units $= \dfrac{\sqrt{3}}{4}a^2$

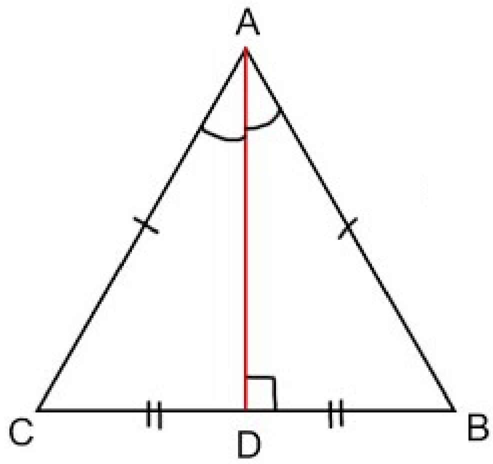
$R = \dfrac{a}{\sqrt{3}}$ & $r = \dfrac{a}{2\sqrt{3}}$





Area=6×Area of equilateral triangle

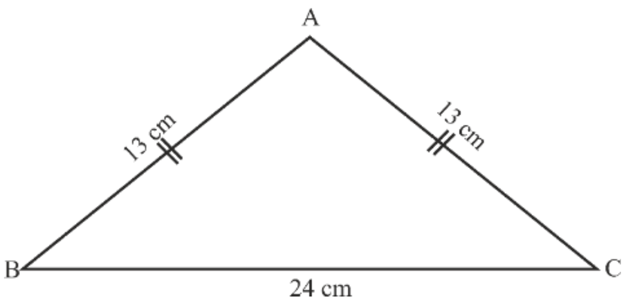
In an isosceles triangle all the 4 geometric centers are collinear.
Perpendicular drawn on non-equal side is median, angle bisector and perpendicular bisector.



Problem

Quantity A | Quantity B

Area of the triangle | Area of triangle with 2 sides 10cm &15cm

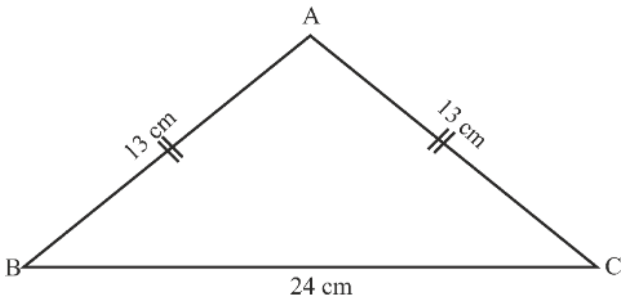


Quantity A

Area of the triangle

Quantity B

Area of triangle with 2 sides 10cm &15cm



Quantity A

Triplet (5,12,13)

Area = $\frac{1}{2} \times 24 \times 5 = 60$

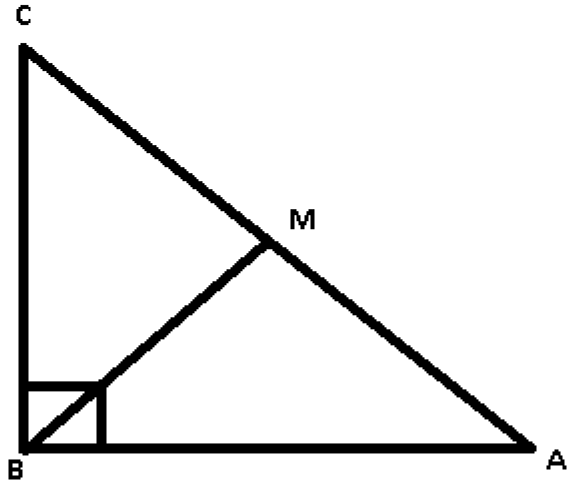
Quantity B

Maximum Area = $\frac{1}{2} \times 10 \times 15 = 75$

Therefore answer is (D).

Problem: In ΔABC, find median BM if $AB = 3.6$cm, $BC = 4.8$cm and $\angle B = 90°$?

In ΔABC, find median BM if AB = 3.6cm, BC = 4.8cm and $\angle B = 90°$



As we know (3,4,5) is a triplet .

3.6 = 1.2×3, 4.8 = 1.2×4

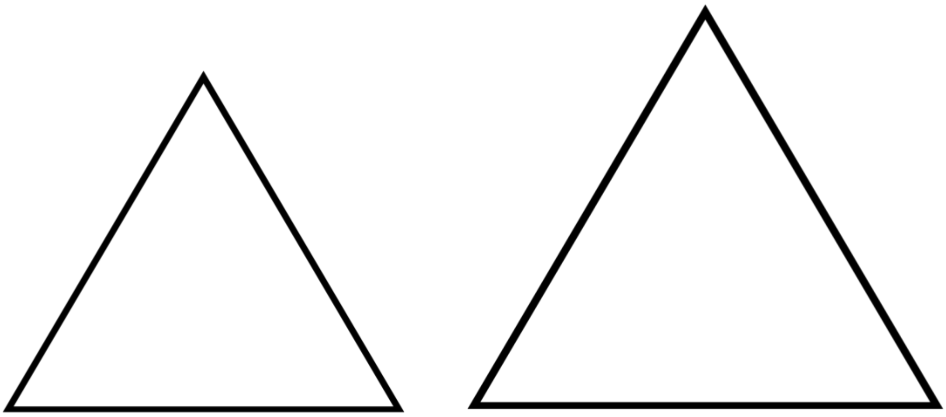
Therefore

Hypotenuse = $5 \times 1.2 = 6$

AC = 6.

$$BM = \frac{AC}{2} = \frac{6}{2} = 3$$

Similar Triangle



If 2 triangles are similar then ratio of their corresponding sides is same and reverse is also true.

$$\left(\frac{AB}{PQ}\right)=\left(\frac{AC}{PR}\right)=\left(\frac{BC}{QR}\right)=\frac{h1}{h2}=\frac{m1}{m2}=\frac{p1}{p2}=\sqrt{\frac{A1}{A2}}\ \ldots\ldots$$

Congruent triangles: Congruent triangles are same in every respect i.e. sides, angles, medians, altitudes, area, R, r etc.

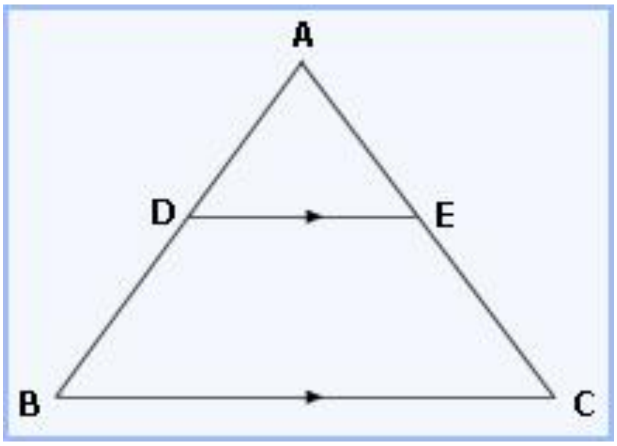
i.e. carbon copy of each other.

All the congruent triangles are similar but reverse is not true.

Basic Proportionally Theorem:

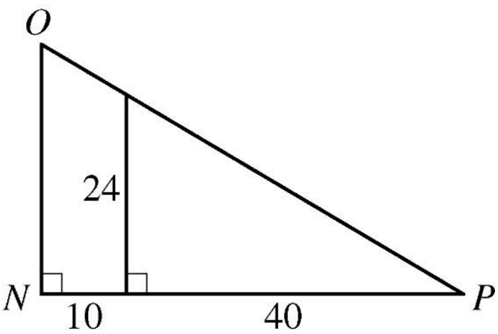
If a line is drawn parallel to one side of a triangle and intersects the other sides in two distinct points then the other sides are divided in the same ratio by it.

If DE is parallel to BC, then $\frac{AD}{DB} = \frac{AE}{EC}$

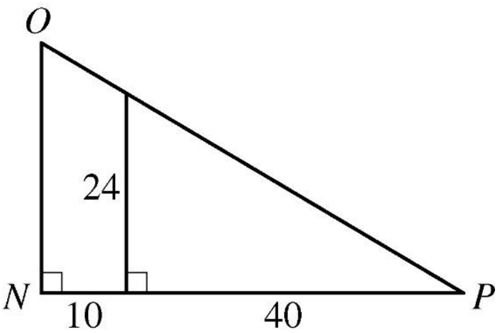


Also ▷ADE ~ ▷ABC

Problem:



Find ON?



Find ON?

Here 2 triangles are similar

Therefore

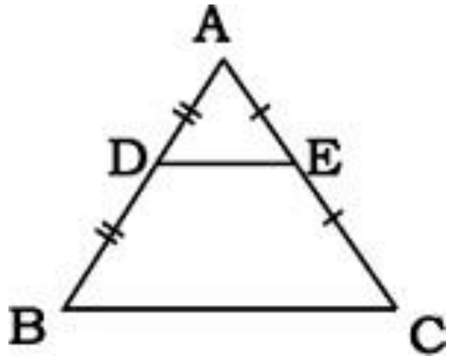
$$\frac{40}{50} = \frac{24}{ON}$$

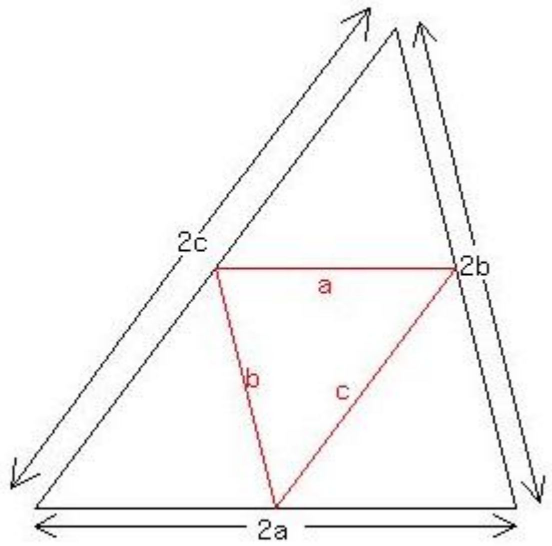
ON = 30

Midpoint theorem:

The segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of the third side.

If AD = DB, AE = EC, then DE is parallel to BC and DE = ½ BC





Important points:

When a polygon is inscribed in a circle or perimeter of a polygon is constant then its area is maximum when it is a regular polygon.