# Numbers-2

## Number of factors:

Factors of 12 are 1,2,3,4,6 & 12.

$N = a^{p} \times b^{q} \times c^{r} \times .....$

a, b, c,…… are prime numbers and p, q ,r …. are natural numbers

Total no. of factors = (p+1)(q+1)(r+1)……..

$12 = 2^{2} \times 3^{1}$

Total number of factors = (2+1)×(1+1) = 6

Total prime factors = addition of powers = 2+1 = 3

Note: 3 prime factors are 2,2 & 3.

Distinct prime factors = 2

Note: Distinct prime factors are 2 & 3 and it is independent of power of prime factors.

Example: Find all the 3 answers for 168.

Find all the 3 answers for 168.

$168 = 2^3 \times 3^1 \times 7^1$

Total number of factors = (3+1)(1+1)(1+1) = 16

Total prime factors = 3+1+1 = 5

Distinct prime factors = 3

$$1+2+3+\ldots..+N = \frac{N(N+1)}{2}$$

$$1^2+2^2+\ldots..+N^2 = \frac{N(N+1)(2N+1)}{6}$$

$$1^3+2^3+3^3+\ldots+N^3=\left(\frac{N(N+1)}{2}\right)^2$$

**Sum of odd numbers & even numbers**

$1 + 3 + 5 + \ldots + (2n-1) = n^2$

Example: $1+3+5+7 = 4^2 = 16$

$2+4+6+\ldots\ldots + 2n = n^2 + n$

Example: $2+4+6+8+10 = 5^2 + 5 = 30$

Example:

(i) $1+3+5+\ldots + k = 441$

(ii) $2+4+6+\ldots + 22 = x$

(i) $1+3+5+\ldots..+k = 441$

$441 = 21^2$

Therefore k is 21st odd natural number

$k = 2\times21-1 = 41$

(ii) $2+4+6+\ldots..+22 = x$

$22 = 2\times11$

22 is 11th even natural number.

Therefore $x = (11^2+11) = 132$

**Last digit/Unit digit**

**Example:** Find the last digit of (378×453×289).

Last digit depends on last digit only.

Therefore last digit of ($37\boxed{8}\times45\boxed{3}\times28\boxed{9}$)

= last digit of (8×3×9)

= 24×9

Considering last digit only.

4×9 = 36.

Last digit of (37**8**×45**3**×28**9**) = 6

Also, last digit of $(1243)^{55}$ = last digit of $3^{55}$

| | | | |
|---|---|---|---|
| $2^1 = 2$ | $2^2 = 4$ | $2^3 = 8$ | $2^4 = 1\boxed{6}$ |
| $2^5 = 3\boxed{2}$ | $2^6 = 6\boxed{4}$ | $2^7 = 12\boxed{8}$ | $2^8 = 25\boxed{6}$ |

Last digit repeats after every 4 powers and this is not only for 2 but for all the digits.

Rules for finding unit digit:

1. Divide power by 4 and find the remainder.

2. If remainder > 0, then

   Unit digit = Unit digit of $(\text{base})^{\text{remainder}}$

3. If remainder = 0 , then

   Unit digit = Unit digit of $(\text{base})^{4}$

Example: Find the last digit of $3242^{247}$

Find the unit digit of $3242^{247}$

Unit digit of $3242^{247}$ = Unit digit of $2^{247}$

Divide power by 4

$\frac{247}{4}$ remainder is 3

Unit digit = Unit digit of $2^3 = 8$

Example: Find the unit digit of $(126^{36} + 523^{88})$

Find the unit digit of $(126^{36}+523^{88})$

Unit digit of $(126^{36}+523^{88})$ = Unit digit of $(6^{36}+3^{88})$

Unit digit of $6^{36} = 6$

$\frac{88}{4}$ Remainder = 0

Unit digit of $3^{88}$ = Unit digit of $3^4 = 1$

Therefore

Unit digit of $(126^{36}+523^{88})$ = 6+1 = 7

**Last 2 digits depends on the last 2 digits**

Find the last 2 digits of $3266^2$

Find the last 2 digits of $3266^2$

Last 2 digits of $3266^2$ **=** last 2 digits of $66^2$

$N^2$,$(50\text{-}N)^2$,$(50+N)^2$,$(100\text{-}N)^2$,$(100+N)^2$,$(150\text{-}N)^2$,$(150+N)^2$,$(200\text{-}N)^2$,$(200+N)^2$ ......... all these numbers end with same last 2 digits.

Example: Let N =16 Then

$N^2 = 16^2 = 2\mathbf{56}$

$(50\text{-}N)^2 = (50\text{-}16)^2 = 34^2 = 11\mathbf{56}$

$(50+N) = (50+16)^2 = 66^2 = 43\mathbf{56}$

$(100\text{-}N)^2 = (100\text{-}16)^2 = 84^2 = 70\mathbf{56}$

$(100+N)^2 = (100+16)^2 = 116^2 = 134\mathbf{56}$

Answer is 56.

Find the last 2 digits of $598^2$?

Find the last 2 digits of $598^2$?

Last 2 digits of $598^2$ = Last 2 digits of $98^2$

98 = 100-2

Last 2 digits of $98^2$ = Last 2 digits of $2^2$ = 04

## Arithmetic Progression

In an A.P., the successive term is got by adding a constant value to the last term. Each A.P. is characterized by its first term and by the constant that is added.

Example: If the first term is 7 and the constant to be added is 6, we get the following series:

7, 13, 19, 25, 31,……..

The first term of an A.P. is denoted by **a** and the constant difference between the successive terms is called 'common difference' and is denoted by **d**.

The value of any $n^{th}$ term is denoted by $T_n$

$$T_n = a + (n - 1)d$$

Example: Find $t_{20}$ in the series

7,13,19,……..,

Find $t_{20}$ in the series

7,13,19,25,31……..,

$t_{20} = 7+19\times6 = 121$

Alternate

All the terms are 1 more than the multiple of 6.

$20^{th}$ term is 1 more than the $20^{th}$ multiple of 6 i.e. 121.

In an AP

$7+31 = 13+25 = 19+19 = 2\times19$

$(t_1+t_n)=(t_2+t_{(n-1)}) =...= 2\times$Middle term $= 2\times$Average of AP

**Sum of first n terms of an A.P.**

$$S_n=\frac{n}{2}\{2a+(n-1)d\}$$

or

$$S_n = \frac{n}{2}(\text{First Term} + \text{Last Term}) = n \times \text{Middle term}$$

Middle term = Average of AP

Example: If sum of first 3 terms of an AP is equal to sum of first 4 terms then find sum of first 7 terms?

If sum of first 3 terms of an AP is equal to sum of first 4 terms then find sum of first 7 terms?

$S_4 = t_1 + t_2 + t_3 + t_4$

$S_3 = t_1 + t_2 + t_3$

$t_4 = S_4 - S_3 = 0$

$S_7 = 7 \times t_4 = 7 \times 0 = 0$

Answer is 0.

Note: In an AP if $S_p = S_q$, then $S_{(p+q)} = 0$

## Geometric Progression

An A.P. was formed by adding a constant to the last term to get the successive terms. A G.P. is formed by multiplying the last term by a constant to get the next term.

In a G.P. constant with which each term is multiplied to get the next term is called common ratio and is denoted by r.

If a = 2 and r = 3

we get the G.P: 2, 6, 18, 54, 162, ....

$$T_n = ar^{(n-1)}$$

Example: $125, 25\sqrt{5}, 25, 5\sqrt{5}, \ldots\ldots\ldots\ldots t_{17}$

Example: $125, 25\sqrt{5}, 25, 5\sqrt{5}, \ldots\ldots\ldots. t_{17}$

$$r = \frac{25\sqrt{5}}{125} = \frac{1}{\sqrt{5}}$$

$$t_{17} = 125\times\left(\frac{1}{\sqrt{5}}\right)^{16} = \frac{5^3}{\left(\sqrt{5}\right)^{16}} = \frac{5^3}{5^8} = 5^{3-8} = 5^{-5}$$

In a GP

$t_1 \times t_n = t_2 \times t_{(n-1)} = t_3 \times t_{(n-2)} = \ldots.. = (\text{Middle term})^2 =$

$(\text{Geometric Mean})^2$

Product of n consecutive terms = $(\text{Middle Term})^n$

$= (\text{Geometric Mean})^n$

**Sum of n terms in a GP**

$$S_n = a \times \frac{r^n - 1}{r - 1}$$

**Infinite GP (|r|<1)**

Example: $6, 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots.$

Since we are multiplying by r, which is less than 1, the terms of this G.P. become smaller and smaller as the number of terms keeps increasing.

$$S_{\infty} = \frac{a}{1-r}, \ |r|<1$$

**Example:** In an infinite GP every term is equal to the sum of its following terms. Find common ratio of the GP?

In an infinite GP every term is equal to the sum of its following terms. Find common ratio of the GP?

Let a is the 1st term & r is common ratio

Then GP is a, ar, $ar^2$, $ar^3$,....

Consider any term and that term is equal to sum of its following terms.

$$ar^2 = ar^3 + ar^4 + ar^5 + \ldots\ldots$$

⇨ $ar^2 = \dfrac{ar^3}{1-r}$

⇨ $r = \dfrac{1}{2}$

Answer is $\dfrac{1}{2}$

> ***An infinite number of mathematicians walk into a bar. The first one orders a beer. The second orders half a beer. The third, a quarter of a beer. The bartender says "You're all idiots", and pours two beers.***
>
> — Randgruppenhumor cont'd

**Arithmetic Mean (AM)**

Arithmetic mean is the average of all the given numbers.

Average = (sum of all the given numbers)/n

where 'n' is the number of observations in the given data set.

For example: let the data set be {3, 43, 10, 21, 89, 9, 17}.

Then the average = (3+43+10+21+89+9+17)/7

**Geometric Mean (GM)**

The geometric mean of 'n' numbers $x_1$, $x_2$, $x_3$, …., $x_n$ is the $n^{th}$ root of the product of the given numbers.

$$GM = \left(x_1 \times x_2 \times x_3 \times ......x_n\right)^{\frac{1}{n}}$$

For example:

Geometric mean of {4, 7, 9, 10, 20} = $(4\times7\times9\times10\times20)^{1/5}$

**Harmonic Mean (HM)**

The harmonic mean of 'n' numbers $x_1$, $x_2$, $x_3$, …., $x_n = \dfrac{n}{\sum \frac{1}{x_i}}$

For example: the harmonic mean of {4, 5,10} = $\dfrac{3}{\frac{1}{4}+\frac{1}{5}+\frac{1}{10}}$

**Relations between AM, GM and HM for 2 numbers**

AM of two numbers a & b = $\frac{(a+b)}{2}$

GM = $\sqrt{ab}$

HM = $\frac{2ab}{a+b}$

$GM^2 = AM \times HM$

For positive numbers AM $\geq$ GM $\geq$ HM

AM (5,5,5) = GM(5,5,5) = HM(5,5,5)

AM (5,12,14) > GM(5,12,14) > HM(5,12,14)

**Example:** Find the minimum value of sum of any positive real number and its reciprocal?

Find the minimum value of sum of any positive real number and its reciprocal?

Let natural number is x and its reciprocal is $\frac{1}{x}$

As we know for positive numbers AM≥GM

$$\frac{x+\frac{1}{x}}{2} \geq \sqrt{x \times \frac{1}{x}}$$

⇨ $x+\frac{1}{x} \geq 2$

Therefore minimum value is 2.

Example: If $a+b = 25$, then find maximum value of $(a \times b)$

(i) $a,b > 0$ (ii) a & b are natural numbers

If $(a+b) = 25$, then find maximum value of $(a\times b)$

(i) $a, b > 0$ (ii) a & b are natural numbers

(i) For maximum value $a = b = \frac{25}{2} = 12.5$

Maximum value of $(a\times b) = 12.5\times 12.5 = 156.25$

(ii) a & b are natural numbers

Therefore a & b both cannot be equal.

$a = 12$ & $b = 13$

maximum value of $(a\times b) = 12\times 13 = 156$

**Note:** If the sum of positive variables is a constant, the product of the variables is maximum when all of them are equal or very close.

HCF/GCD:

Highest Common Factor/Greatest Common Divisor

HCF (15,35)

$15 = 3 \times 5$

$35 = 5 \times 7$

HCF (15,35) = 5

Example: Find the HCF of (A,B)

if $A = 2^3 \times 3^5 \times 7^2$ & $B = 2^8 \times 5^{10} \times 3^9$

$A = 2^3 \times 3^5 \times 7^2$ & $B = 2^8 \times 5^{10} \times 3^9$

HCF = $2^3 \times 3^5$

LCM: LEAST COMMON MULTIPLE

LCM (15,35)

$15 = 3 \times 5$

$35 = 5 \times 7$

LCM (15,35) = $5 \times 3 \times 7 = 105$

Find the LCM of (A,B)

if $A = 2^3 \times 3^5 \times 7^2$ & $B = 2^8 \times 5^{10} \times 3^9$

$A = 2^3 \times 3^5 \times 7^2$ & $B = 2^8 \times 5^{10} \times 3^9$

$LCM(A,B) = 2^8 \times 3^9 \times 5^{10} \times 7^2$

Note:

1. $LCM \geq HCF$
2. HCF is always factor of LCM.
3. For any 2 numbers a,b

   $HCF \times LCM = a \times b$

Example: HCF of 2 numbers is 14 and their sum is 210. Find the number of pairs satisfying both the conditions?

HCF of 2 numbers is 14 and their sum is 210. Find the number of pairs satisfying both the conditions?

HCF of 2 numbers is 14 & their sum is 210.

Let numbers are 14x & 14y where x & y are co-prime.

$14x+14y = 210$

⇨ $x+y = 15$

(1,14),(2,13),(4,11)(7,8)

Therefore, total  4 pairs.

Example: When 489 and 604 are divided by n remainders are 9 & 4 respectively then find maximum value of n?

Example: When 489 and 604 are divided by n remainders are 9 & 4 respectively then find maximum value of n?

$489 - 9 = 480 = n \times k$

$604 - 4 = 600 = n \times m$

Here, n is factor of 480 as well as 600 and we've to find maximum value of n.

Therefore,

Maximum value of n = HCF (480,600) = 120

$480 = 4 \times 120$ & $600 = 5 \times 120$

Example: Find the 3 digit number which when divided by 5,6,7 & 8 leaves remainder 3.

Find the 3 digit number which when divided by 5,6,7 & 8 leaves remainder 3.

Let number is $n$

Then, $n = 5k+3 = 6q+3 = 7m+3 = 8p+3$

Here, $n$ is 3 more than common multiple of 5, 6, 7 & 8.

Therefore $n = a \times \text{LCM of divisors} + \text{same remainder}$

$n = a \times LCM(5,6,7,8) + 3$

$n = 840a + 3$, $a$ is whole number

3 digit value of $n = 840 \times 1 + 3 = 843$

Example: Find the least +ve integer which when

divided by 2 gives remainder 1

divided by 3 gives remainder 2

divided by 4 gives remainder 3

divided by 5 gives remainder 4

(A) 33 (B) 47 (C)59 (D) 61

Find the least +ve integer which when

divided by 2 gives remainder 1

divided by 3 gives remainder 2

divided by 4 gives remainder 3

divided by 5 gives remainder 4

(A) 33 (B) 47 (C)59 (D) 61

Let least number is n

When n is divided by 2 remainder is 1 means it is 1 less than multiple of 2.

Similarly, n is 1 less than multiple of 3,4 & 5

Therefore n is 1 less than common multiple of 2,3,4 & 5.

$n = k \times \text{LCM}(2,3,4,5) - 1$, k is natural number

$= k \times 60 - 1$

Minimum value of $n = 1 \times 60 - 1 = 59$

Answer option is (C)

Or

When N is divided by 5 remainder is 4 or (-1)

When N is divided by 4 remainder is 3 or (-1)

When N is divided by 3 remainder is 2 or (-1)

When N is divided by 2 remainder is 1 or (-1)

**1.** Find the last digit/unit digit of
(i) $245^{86} \times 3696^{239}$
(ii) $2687^{238} + 23^{264}$
(iii) $81a52^{68a48}$
(iv) Product of first 10 prime numbers
(v) Find the unit digit of (0!+1!+2!+.....+368!)

**2.** Find the last 2 digits of
(i) $4586^2$
(ii) $12458 \times 3652$

**3.** Find the number of factors, total prime factors and distinct prime factors of
(i) 168
(ii) $N = p^3q^6$, where p and q are prime numbers

**4.** How many factors of N are multiple of 10 if $N = 20 \times 3^4 \times 125$ ?

**5.** Find the term in
(i) 7,13,19,........,$t_{20}$
(ii) 125, $25\sqrt{5}$ ,25, $5\sqrt{5}$ ,... $t_{17}$

**6.** If sum of first 3 terms of an AP is equal to sum of first 4 terms then find sum of first 7 terms ?

**7.** If the sum of the first n terms of an arithmetic progression in 2400 and the sum of next n terms as 7200, then find the ratio of first term and common difference.
(A) 3 : 2
(B) 2 : 1
(C) 1 : 2
(D) 2 : 3

**8.** Find the average of all two-digit numbers that give a remainder 4 when they are divided by 5?

**9.** If a+b = 25, then find maximum value of (a×b)
(i) a,b > 0
(ii) a & b are natural numbers

**10.** If 5th term of a GP is 10 then find the product of the first 9 terms and also find product of 2nd term and 8th terms?

**11.** Find the GCD & LCM of the following
(i) (12,44)
(ii) (25,9)
(iii) $\left(\frac{4}{7}, \frac{8}{21}\right)$

**12.** When 231 is divided by n remainder is 11 then which of the following could be the value of n?(Mark all the correct answer)
(A)44 (B)10 (C)22
(D)30 (E)4 (F) 55

**13.** When 489 and 604 are divided by n remainders are 9 & 4 respectively then find maximum value of n?

**14.** (i) Find the sum of first 13 odd natural numbers
(ii) 1+3+5+.....+k = 441
(iii) 2+4+6+.....+22 =
(iv) 2+4+6+.....+k = 600

**15.** Which of the following could be the average of 20 distinct natural numbers ?(Mark all the correct answers)
(A)12
(B)13.6
(C)7
(D)43.82
(E)89.35
(F) 81.37

**16.** If a, b & c are positive real numbers and a+b+c = 21

| Quantity A | Quantity B |
|---|---|
| Maximum value of (a+1)×(b+2)×(c+3) | 721 |

**17.** Find the largest 3 digit number which when divided by 6,9,10 & 15 will leave a remainder 2?

**18.** Find the number which when divided by 8 leaves remainder 6, when divided by 7 leaves remainder 5, when divided by 6 leaves remainder 4, when divided by 5 leaves remainder 3 ?(Mark all the correct answers)
(A) 118
(B)418
(C)838
(D)1678
(E) 2008

**19.** Find the greatest number that will divide 43,91 & 183 so as to leave the same remainder in each case?

**20.** HCF of 2 numbers is 14 and their sum is 15. How many such pairs are possible?

**21.** ABCD is a square with side 10cm. By joining the mid points of this square again a square is formed and this process is repeated till infinite. Find the sum of area of all the squares?

**22.** There are 2 Arithmetic progressions : 3,7,11,.......219 & 1,7,13,........ 241. Find the number of common terms in both the AP?

**23.** A ={ n,n+1,n+2,n+3,n+4,n+5} for any natural number n ≤ 98. For how many values of n A contains at least one multiple of 10?

**24.** $t_n = t_{(n-1)} - t_{(n-2)} + t_{(n-3)}$, for $n \geq 4$, then find the sum of first 102 terms of the given series?

**25.** A = {7,12,17,22,............,182} . Maximum how many terms from A can be selected such that sum of any 2 terms is less than 189?

**1.** Quantity A Quantity B

$0.1\overline{2}$ $0.\overline{121}$

**2.** In an organization total number of employees is a 3 digit prime number then which of the following could be the ratio of graduate and under-graduate employees?

(A)100:89 (B)100:121
(C)100:223 (D) 100:99

**3.** If 4N is divisible by 32 but not by 64 then which of the following could be divisible by 48?(Mark all the correct answers)

(A)6N (B)20N
(C)100N (D)3N (E)7N

**4.** Find the remainder

(I) $\frac{121\times122}{60}$ (II) $\frac{118\times117\times131}{60}$

(III) $\frac{121\times118\times124}{60}$ (IV) $\frac{82^{75}}{27}$

(V) $\frac{84^{55}}{17}$ (VI) $\frac{31^{123}+33^{86}}{16}$

(VII) $\frac{35^{62}}{11}$ (VIII) $\frac{4^{56}}{15}$

**5.** If x is a positive even integer and y is a positive odd integer, which of the following statements CANNOT be true? (Indicate all such statements)

(A) x + y is divisible by 3
(B) 3x + 2y is divisible by 4
(C)2x+ 3y is divisible by 6

**6.** When N is divisible by 12 remainder is 3.Then which of the following are definitely divisible by 3?(Mark all the correct answers)

(A) 5N+726 (B)13N+79
(C)22N+891 (D) $20N^2+561$

**7.** Find the remainder when (N+M) is divided by 10 if N is divisible by 8 and
N = 2358993a & M = 6930

**8.** If (180!+1) < P < (180! + 180), P is natural number. How many prime values P can take?

(A)4 (B) 7
(C) more than 20 (D) 20

**9.** If $n$ is an odd integer, which of the following must be an even integer?

(A) $\frac{n}{2}$ (B) $5n + 3$

(C) $3n$ (D) $\frac{n}{4}$ (E) $\sqrt{n}$

**10.** Find the highest power of 7 in 178!?

**11.** If $\frac{57!}{10^x}$=Integer , then find maximum value of x?

**12.** If $x$ and $y$ are perfect squares, then which of the following is not necessarily a perfect square?

(A) $x^2$ (B) xy
(C) 4x (D) x + y (E) $x^5$

**13.** If the integer $x$ is divisible by 3 but not by 2, then which one of the following expressions is NEVER an integer?

(A) $\frac{(x+1)}{2}$ (B) $\frac{x}{7}$

(C) $\frac{x^2}{3}$ (D) $\frac{x^3}{3}$ (E) $\frac{x}{24}$

**14.** When a natural number N is divided by 40 remainder is 26 then

Quantity A Quantity B
Remainder when N is divided by 7 4

**15.** Find the minimum natural number which should be multiplied to 120 so that it becomes perfect cube?

**16.** For any natural numbers a, b, & c : (a+b) (b+c) (c+a) is

(A)Always even
(B) Always odd
(C) Depends on numbers

**17.** Smallest n! Such that it is divisible by 242?

**18.** N is the smallest perfect square which is divisible by 6, 8 & 15. Find N?

**19.** From all-natural numbers from 2 to 2500, first all the perfect squares are erased, then all the perfect cubes are erased, then all the perfect 4th powers and so on, in successive rounds. How many rounds are there such that at least one number is erased in the round.

**20.** How many multiples of 9 are between 100 & 450?

**21.** Quantity A Quantity B

$2^{100}$ $3^{60}$

**22.** The remainder when the positive integer m is divided by n is r. What is the remainder when 2m is divided by 2n ?

(A) r (B) 2r
(C) 2n (D) m – nr (E) 2(m – nr)

**23.** What is the value of 3!(7 – 2)! ?

**24.** If $2^{5^3}=4^x$, then find x?

**25.** When 1000 is added to 459x251 and the resulting number is divided by 11, the remainder is 8. Find x.

(A)3 (B) 5
(C) 7 (D) 8 (E) 9