Permutation and Combination

Combination:
Only selection (Order is not important).
My salad is combination of Tomato, carrot and cucumber.

Permutation:
Selection as well as arrangement (order is important).
Example: My mobile pin consist of 4 digits.
Here, which digits and their order both are important.
If Pin is 1256 then any other order of the digits won't work.

Q. In a class there are 20 boys and 30 girls then in how many ways one can select a boy and a girl?

1 boy out of 20 can be selected in 20 ways

1 girl out of 30 can be selected in 30 ways

$20 \times 30 = 600$

N! = 1×2×3×…….N, for all natural number N

4! = 1×2×3×4
5! = 1×2×3×4×5

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Number of ways of selecting r distinct things out of n distinct things.

$${}^{16}C_{3} = \frac{16\times15\times14}{3!}$$

$${}^{n}C_{r} = {}^{n}C_{(n-r)}$$

$${}^{12}C_{2} = {}^{12}C_{10}$$

Problem: 15 Members Indian cricket team has to go for the world cup

(i) In how many ways playing 11 can be selected?

Soln: We've to select 11 out of 15

$^{15}C_{11}$

(ii) In how many ways playing 11 can be selected if captain Kohli is always included?

Soln:One player is already selected . Now 14 players are remaining and we've to select 10 players now.

$^{14}C_{10}$

(iii) In how many ways playing 11 can be selected if Vijay Shankar is always excluded?

Soln: Vijay Shankar is excluded implies we've only 14 players for selection. Select 11 out of 14

$^{14}C_{11}$

(iv) In how many ways playing 11 can be selected if captain Kohli is always selected and Vijay Shankar is always excluded?

Out of 15 players only 13 are remaining and we've to select 10players now.

$^{13}C_{10}$

Q. There are 4 IT engineers, 5 mechanical engineers and 3 Electrical engineers.
A team of 3 engineers to be selected

(i) Total number of ways

Select 3 out of total 12.

$^{12}C_3$

(ii) Team includes at least one engineer from each branch

Select 1 engineer from each branch. For mechanical 1out of 5,1 out of 4 IT and 1 out of 3 electrical

$3 \times 4 \times 5 = 60$

(iii) One mechanical engineer should be there

One mechanical out of 5 and 2 out of 7(IT+EE)

$^{7}C_2 \times {}^{5}C_1$

(iv) No mechanical engineer should be there

All the 3 engineers should be from IT & EE i.e. 7

$^{7}C_{3}$

(v) At least one mechanical engineer should be there

At least one = Total – None

Here, total possibilities $^{12}C_{3}$ and in $^{7}C_{3}$ cases there is no mechanical.

$^{12}C_{3}$-$^{7}C_{3}$

Q. In a class there 25 students. Every student shakes hand with every other student exactly once. Find the total number of handshakes

Between every 2 students there is only 1 handshake. Indirectly question is in how many ways one can select 2 out of 25.

$^{25}C_2$

Or

First student shakes hand with remaining 24, 2nd shakes with 24 but 1 is already counted so 23. For 3rd again 24 handshakes but 2 are already counted so 22 and so on.

1+2+3+….+24 = 300

Q. There are 20 railway stations from Ahmedabad to Mumbai. Find the number of total different journey tickets should be printed so that a person can travel anywhere?

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Soln:

For a ticket one need to select 2 stations source and destination and order is also important.

$^{20}C_2 \times 2$

Or

From each station one can go to 19 places so we need 19 different tickets from each station.

Therefore, $20 \times 19 = 380$

Q. In how many ways 10 people can seat in a row?

N people can seat in N! ways.
10!

Q. In how many ways 4 people can seat in row if there are 10 chairs?

For a first person 10 choices, 2^{nd} 9 choices and so on.

$10 \times 9 \times 8 \times 7$

Or

${}^{10}C_4 \times 4!$

Q. 5 Boys and 5 girls are seating in a row

(i) Total ways

No restriction.
Therefore, 10!

(ii) All the boys are together

Make a group of all the boys and consider it as a single unit. Now we've 6 people(5 girls and 1 group of boys) which can be arranged in 6! Ways and 5 boys can be arranged in 5! ways.

$5! \times 6!$

(iii) All the boys should not be together

All should not be together = Total – All together

$10!-5!\times6!$

(iv) No 2 boys should be together

No 2 boys should be together means between every 2 boys there should be at least a girl i.e. condition is on boys and not on girls. 1^{st} arrange girls – 5! ways.

O G O G O G O G O G O

Now there are 6 gaps(represented by O) . Select 5 gaps out of 6 in ${}^{6}C_{5}$ ways and arrange 5 boys on 5 places in 5! ways.

$5! \times {}^{6}C_{5} \times 5! = 5!\times6!$

Q. How many words can be formed with the rearrangement of the letters of the word
(i) SEA

Consider it as a 3 people are sitting in a row.

$3!$

(ii) SEE

3 alphabets 3! but 2 are repeating therefore divide by 2!

$$\frac{3!}{2!}$$

(iii) MATHEMATICS
Total 11! And M,A & T are repeating therefore 2! thrice.

$$\frac{11!}{2!\times 2!\times 2!}$$

‘EQUATION’

(i) Total words

8 alphabets = 8!

(ii) Starts with E and ends with N

E*QUATIO*N

Alphabets between E and N can be arranged in 6! ways.

(iii) E and N are at extreme end

Now E and N can interchange therefore include 2!

$6! \times 2!$

0,1,2,3,4,5,6

(i) How many 5 digit numbers can be formed?

For 1^{st} place there are 6 choices (1 to 6). From 2^{nd} place onwards 7 places since repetition is allowed.

$6 \times 7 \times 7 \times 7 \times 7$

(ii) How many 5 digit numbers with distinct digits can be formed?

For 1^{st} place 6 choices (1 to 6).For 2^{nd} place again 6 choices because 0 can come on 2^{nd} place onwards. For 3^{rd} place 5 choices and so on.

$6 \times 6 \times 5 \times 4 \times 3$

(iii) How many 5 digit even numbers can be formed with distinct digits?

Can’t answer directly.
Case I: Unit digit is 0
$6 \times 5 \times 4 \times 3$
Case II: Unit digit is 2 or 4 or 6
$5 \times 5 \times 4 \times 3 \times 3$

Q. How many 4 digit numbers can be formed such that digits are 1,2,5,9?

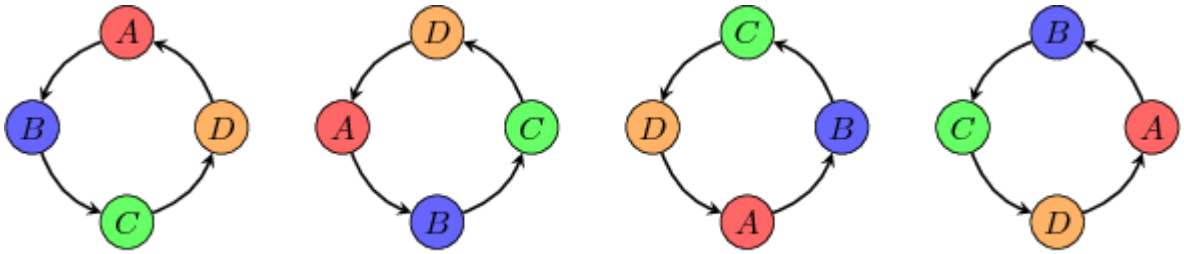
Whenever you get *such that digits are* means use all the digits.
4!

Q. A number lock contains 4 rings and each ring contains 6 digits from 0 to 5. How many 4 passwords can be made? (A password is a 4 digit number formed from all the 4 rings).

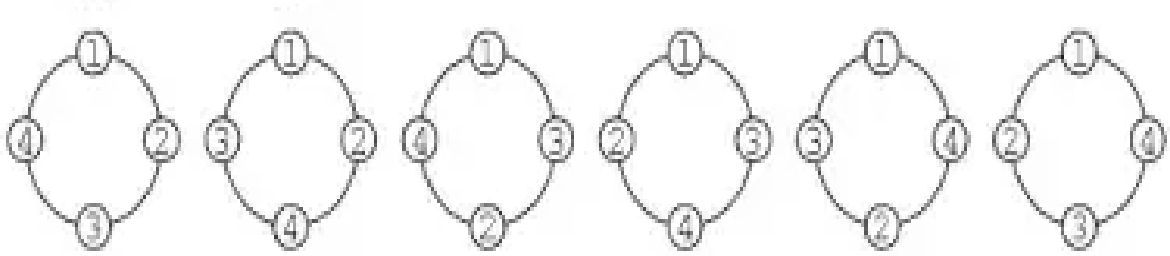
Here, 0 can come on 1^{st} place also (**Note**:-0000 is also a password). For each place there are 6 choices.

$6\times6\times6\times6$

Circular permutation:



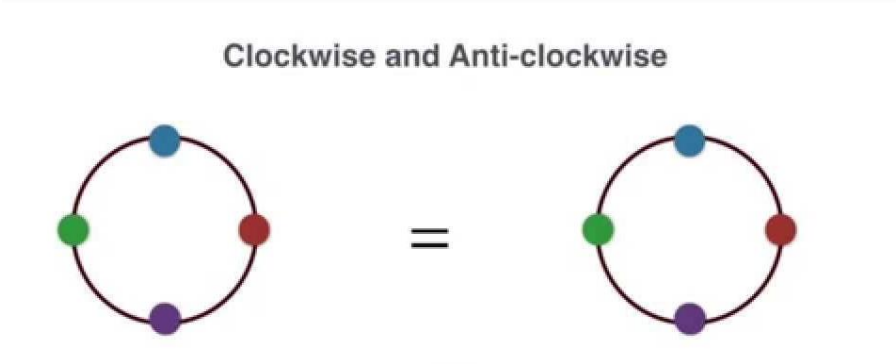
N people around a circle can seat in : (N-1)! Ways



In how many ways 10 people can seat around a circular table?

9!

N distinct beads or pearls in necklace: $\frac{(N-1)!}{2}$ ways



Q. In how many ways one can select at least one apple from 5 identical apples?

Here, only quantity is important.
1 apple-1 way, 2 apples- 1 way,…..5 apples-1 way
Total = 5 ways

Q. In how many ways one can select at least one apple from 5 distinct apples?

Here, how many and which apples both are important.

$^{5}C_{1}+^{5}C_{2}+....+^{5}C_{5}=2^{5}-1=31$

$^{n}C_{0}+^{n}C_{1}+^{n}C_{2}+.....+^{n}C_{n}=2^{n}$

Q. In how many ways 5 distinct balls can be put into 3 distinct boxes?

For each ball we've 3 choices.

3^5

Q. In how many ways 5 distinct balls can be put into 3 identical boxes so that no box should remain empty?

Soln: Just divide the balls into 3 groups and no need of further arrangement since boxes are identical.

(2,2,1) & (3,1,1)

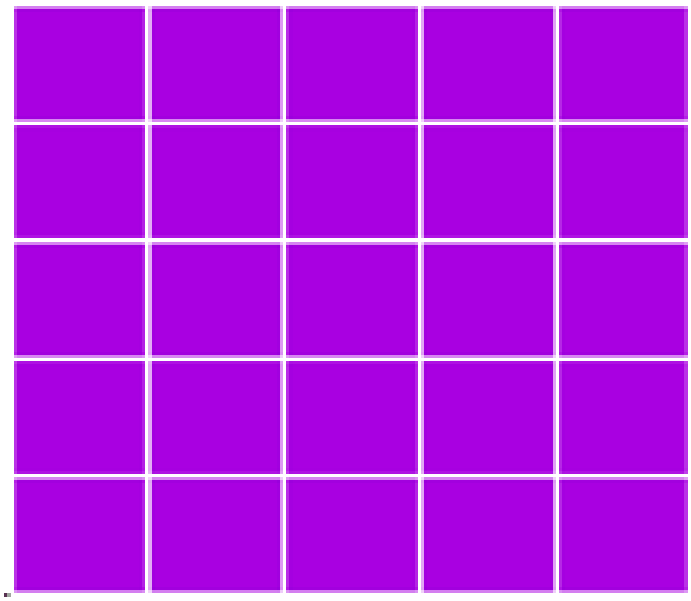
Q. In how many ways 10 identical balls can be put into 3 distinct boxes?

$^{12}C_2$

‘n’ identical thing to be divided into ‘r’ distinct

$= {}^{(n+r-1)}C_{(r-1)}$

How many squares are there?



Solution:

Here, total number of squares

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

In any n×n square grid

Total number of squares = $1^2 + 2^2 + 3^2 + \ldots + n^2$

Total number of rectangles = $1^3 + 2^3 + \ldots + n^3$

$${}^nP_r = \frac{n!}{(n-r)!} = {}^nC_r \times r!$$

Number of permutations of r distinct things out of n distinct things.