Homework 3

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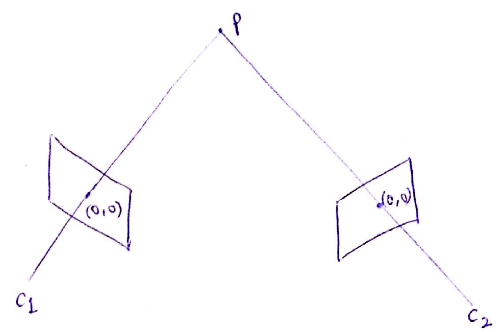
## Part I

1.1.




By epipolar constraint

$$x_2^T F x_1 = 0$$

where F is the fundamental matrix

Now, given

$x_2$ & $x_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ since they are at origin

Now, from the $x_2^T F x_1 = 0$ equation:-

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

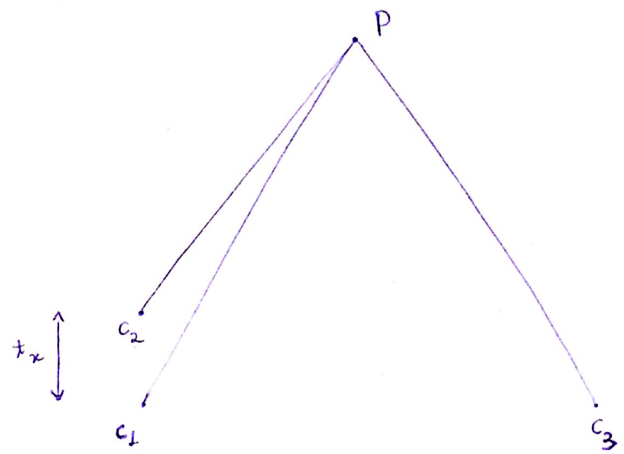
$$\left( \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & " & \end{bmatrix} \right) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$F_{33} = 0.$$

Hence, proved.

1.2.




now,

$$E = t \times R$$
$$= [T]_\times R$$

given, since there is no rotation $R = I_3$ (identity)
and given translation is parallel to $x$ axis
so.

$$T = \begin{bmatrix} t_x & 0 & 0 \end{bmatrix}^T$$

$\Rightarrow$ now, by property of cross multiplication

$$v \times x = \begin{pmatrix} v_2 x_3 - v_3 x_2 \\ v_3 x_1 - v_1 x_3 \\ v_1 x_2 - v_2 x_1 \end{pmatrix} = [v]_\times x.$$

where.
$$[v]_\times = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

so, comparing
$[T]_\times R$ and $[v]_\times x$.

$$[T]_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$P_2^T E P_1 = 0$$

$$P_2^T = [x_2 \quad y_2 \quad 1]$$
$$P_1^T = [x_1 \quad y_1 \quad 1]$$

$$\left( [x_2 \quad y_2 \quad 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & +t_x & 0 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

$$[0 \quad t_x \quad -y_2 t_x] \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

$$t_x y_1 - y_2 t_x = 0$$

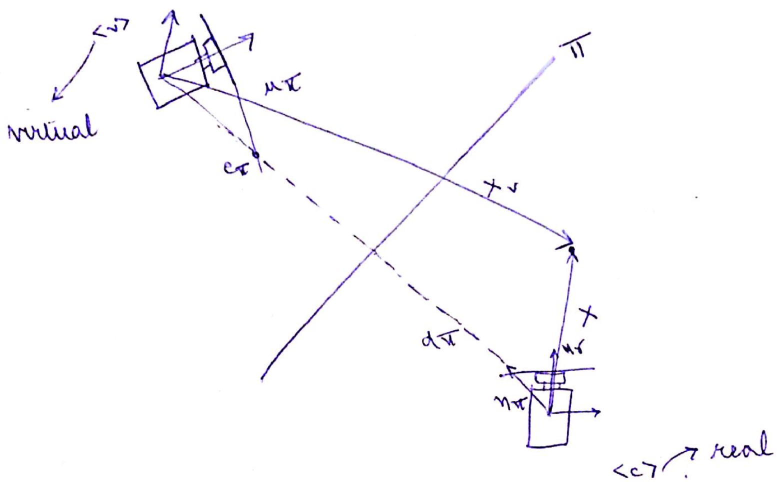
$$\cancel{t_x} y_1 = y_2 \cancel{t_x}$$

$$y_1 = y_2$$

for a point given in image in $C_2$ for a particular $y_2$, the corresponding point will lie on a line $y_1 = y_2$ which is parallel to the $x$ axis. This is the definition of epipolar line.

Hence, proved.

1.3 Taken from :-
Catadioptric Stereo with planar mirrors: Multiple view geometry and camera localization - Gian Lucca Mariottini et al.




The basic thinking behind this approach is that the camera projections of the reflected points corresponds to the virtual camera projection of the real point.

$$X_v = X^{[\pi]}.$$

where $X_v$ is the point $X$ in <v>.

The reflective epipolar constraint is given by:

$$\tilde{u}_\pi^T E^{[\pi]} \tilde{u}_r = 0, \qquad (1)$$

where the reflective ~~epipolar~~ essential matrix,

$$E^{[\pi]} = 2 d_\pi [n_\pi]_\times, \qquad (2)$$

being $[n_\pi]_\times$ the skew symmetric matrix associated with the vector $n_\pi$.

Let $x$ and $x_v$ be the 3D coordinates of a point in camera frames <c> and <v>. We can say that

$$x_v = S^{[\pi]} x + 2d_\pi n_\pi , \qquad -③$$

if camera calibration $K$ is identity, then $x_v = \lambda_\pi \tilde{u}_\pi$, $x = \lambda_r \tilde{u}_r$.

$$\lambda_\pi \tilde{u}_\pi = \lambda S^{[\pi]} \check{u}_r + 2d_\pi n_\pi . \qquad -④$$

where $\lambda_\pi$, $\lambda_2$ are unknown depths. From ④, we get the epipolar constraint

$$\check{u}_\pi^T \left(2d_\pi [n_\pi]_\times S^{[\pi]}\right) \tilde{u}_r = 0 .$$

By definition,

$$E^{[\pi]} \triangleq 2d_\pi [n_\pi]_\times S^{[\pi]} = 2d_\pi [n_\pi]_\times (I - 2n_\pi n_\pi^T)$$
$$= 2d_\pi [n_\pi]_\times .$$

given some $K$, fundamental matrix $F^{[\pi]}$

$$\Leftarrow F^{[\pi]} \triangleq K^{-T} E^{[\pi]} K^{-1}$$

now,

$$F^{[\pi]} + (F^{[\pi]})^T = 2d_\pi \left(K^{-T}[n_\pi]_\times K^{-1} - K^{-T}[n_\pi]_\times K^{-1}\right)$$
$$= 0 .$$

Hence, $F^{[\pi]}$ the fundamental matrix is skew-symmetric

Hence, proved.

What we can say from this result is that the epipoles for the two cameras would be equal. which is intuitive given ~~only~~ translation towards the camera would not affect the epipoles, which would lie on the line joining the real and the imaginary camera.

Part II

2.1. The fundamental matrix and the displayEpipolarF output are as follows:

F =

```
  -0.0000   -0.0000    0.0011
  -0.0000    0.0000   -0.0000
  -0.0011    0.0000   -0.0042
```

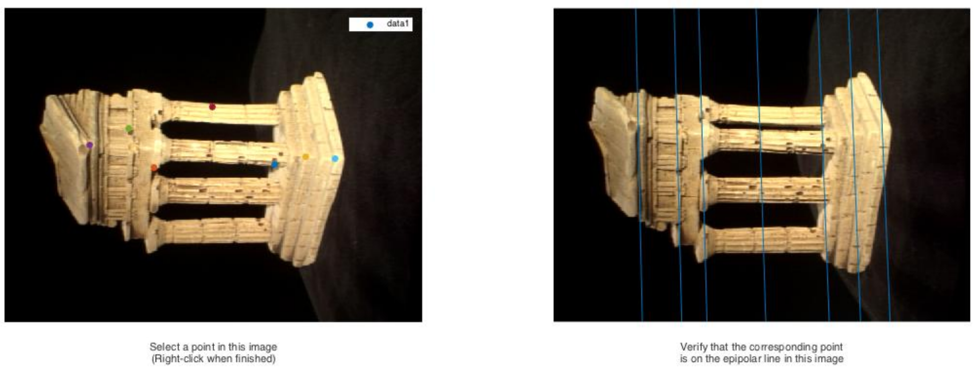



*Figure 1:* Output of displayEpipolarF

2.2. F is a cell of 3 elements. Only one of them gave good results. The different F are then visualized by displayEpipolarF and the correct one is taken for this approach.

```
F =

   [3x3 double]    [3x3 double]    [3x3 double]

F{1} =

    0.0000    0.0000   -0.0020
   -0.0000   -0.0000    0.0005
    0.0019   -0.0005    0.0138
```

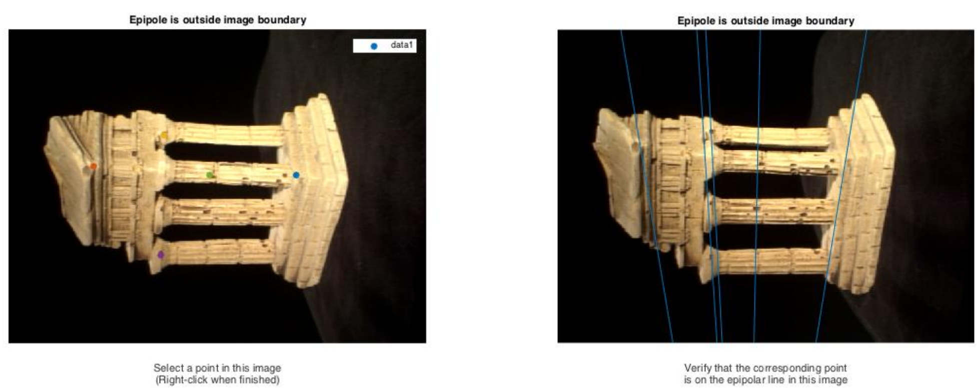



*Figure 2:* Output of displayEpipolarF

```
>> F{2}

ans =

    0.0000   -0.0000    0.0011
   -0.0000    0.0000    0.0000
   -0.0011    0.0000   -0.0046
```

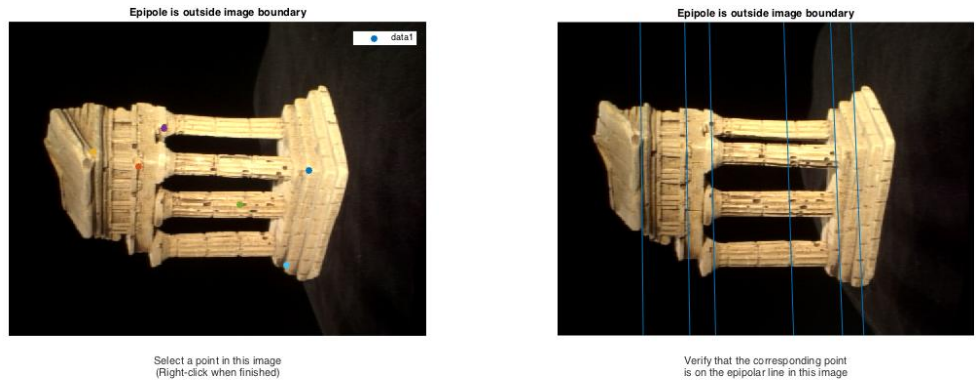



*Figure 3:* Output of displayEpipolarF

```
>> F{3}

ans =

   -0.0080    0.0264   41.7427
   -0.0331   -0.0011    8.0858
  -35.8036   -7.4308 -611.4318
```

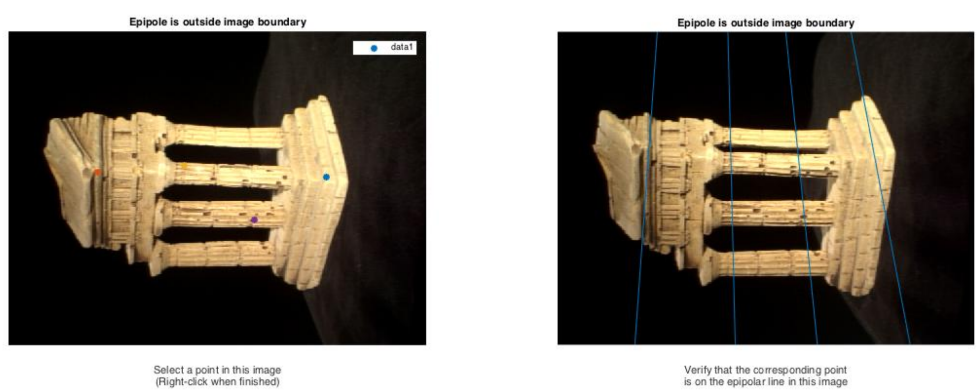



*Figure 4:* Output of displayEpipolarF

2.3. The essential matrix is as follows:

E =

$$
\begin{matrix}
-0.0030 & -0.2862 & 1.6609 \\
-0.1368 & 0.0083 & -0.0507 \\
-1.6653 & -0.0056 & -0.0006
\end{matrix}
$$

2.6. For the epipolar correspondences I have taken a Gaussian window for given more emphasis for the central pixels and then taken two measures which are the Euclidean distance between the two image patches and then the distance between the projected point and the actual point with weights 0.75 and 0.25 respectively. Following are the results from epipolarMatchGUI:

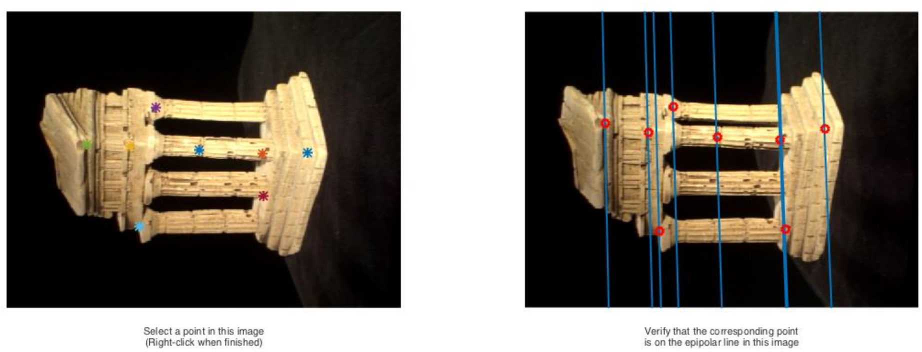
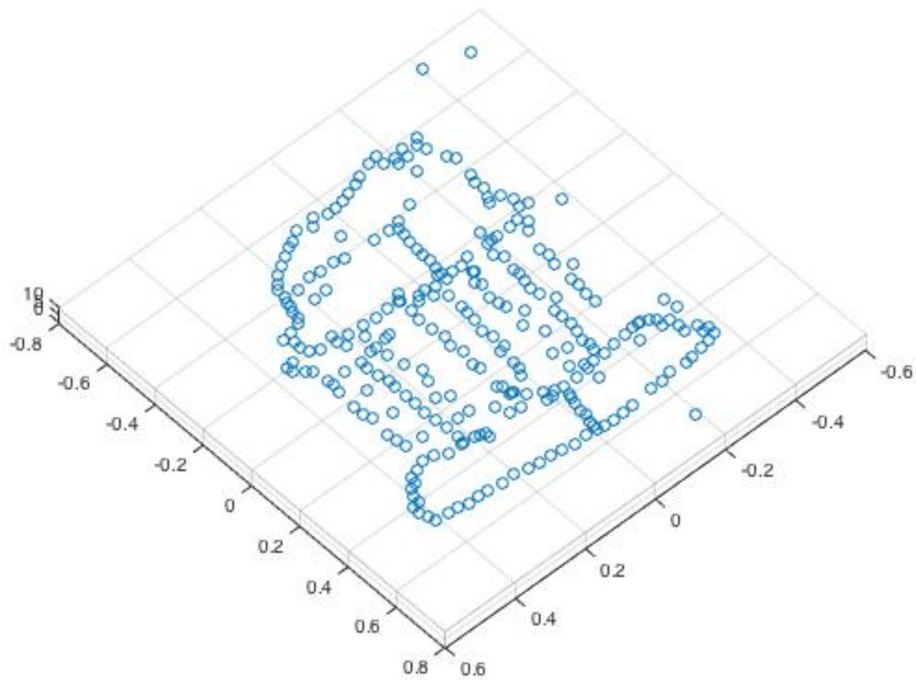



*Figure 5:* Output of epipolarMatchGUI

2.7. The 3D scatter plot of the reconstructed 3D points:



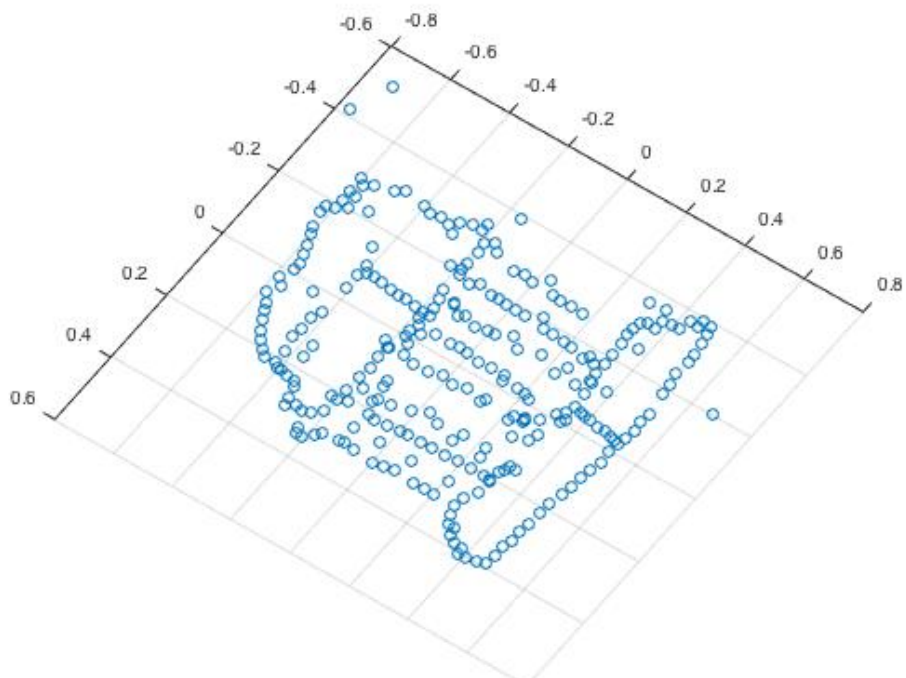




Figure 6, 7: Screengrab of 3D scatter plot