Homework 3
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1.1.

HW3

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1.1. $$E(u,v) = \sum_{(x,y)\in R_t} \left(I_{t+1}(x+u, y+v) - I_t(x,y)\right)^2$$

$$\approx \sum \left[I_{t+1}(x,y) + u I_x(x,y) + v I_y(x,y) - I_t(x,y)\right]^2$$

$$E(u,v) = \sum \left[u I_x(x,y) + v I_y(x,y) + D(x,y)\right]^2$$

$$\frac{d\, E(u,v)}{du} = \sum 2\left(u I_x(x,y) + v I_y(x,y) + D(x,y)\right)(I_x)$$

$$= \sum u I_x^2 + v I_x I_y + I_x D$$

$$\frac{d\, E(u,v)}{dv} = \sum 2(I_y)\left(u I_x + v I_y + D\right)$$

$$= \sum u I_x I_y + v I_y^2 + D I_y$$

Writing it as matrices

$$\underset{A}{\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}} \underset{\Delta p}{\begin{bmatrix} u \\ v \end{bmatrix}} = \underset{b}{-\sum \begin{bmatrix} I_x D \\ I_y D \end{bmatrix}}$$

$$\min_{\Delta p} (A \Delta p - b)^2$$

$$= \min_{\Delta p} \left(\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \sum \begin{bmatrix} I_x D \\ I_y D \end{bmatrix} \right)^2$$

For the equation to be solvable:
Ref: MIT Computer Vision Slides

- $\mathbf{A}^T\mathbf{A}$ should be invertible
- $\mathbf{A}^T\mathbf{A}$ should not be too small due to noise
  - eigenvalues l_1 and l_2 of $\mathbf{A}^T\mathbf{A}$ should not be too small
- $\mathbf{A}^T\mathbf{A}$ should be well-conditioned
  - l_1/ l_2 should not be too large (l_1 = larger eigenvalue)

1.3.

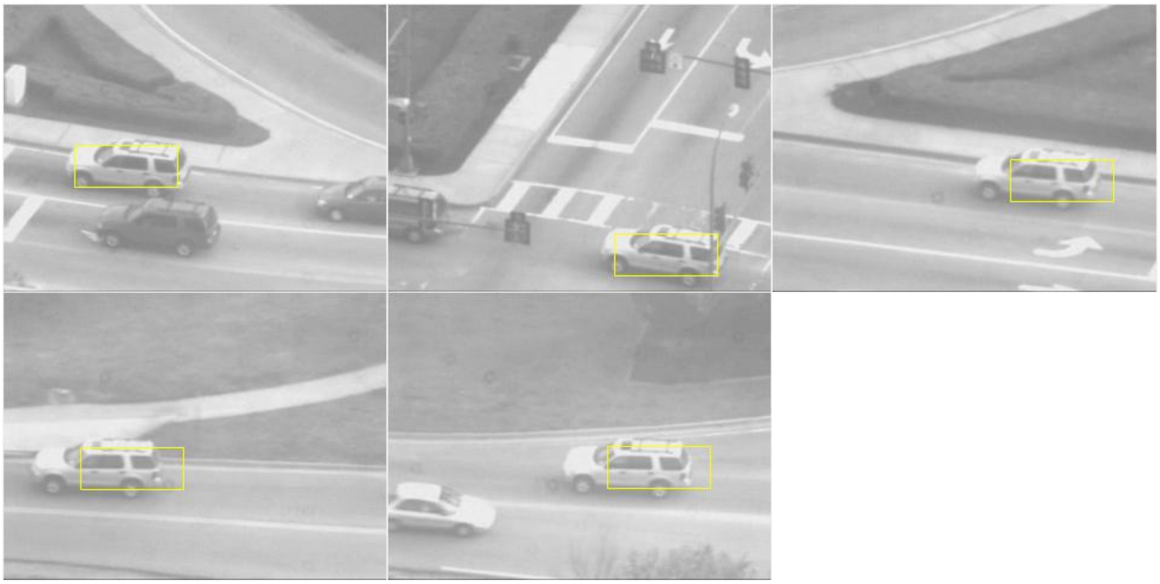

Fig 1: Frames for Lucas Kanade for frames 1, 100, 200, 300, 400

1.4.

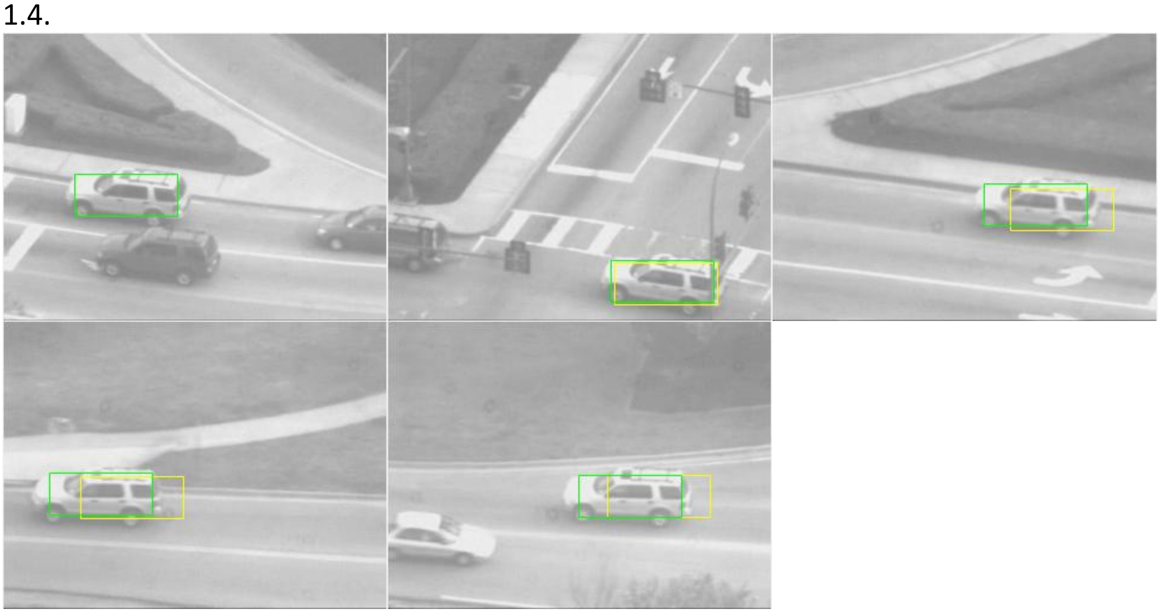

Fig 2: Frames for Lucas Kanade with template correction for frames 1, 100, 200, 300, 400

2.1.

2.1.

$$I_{t+1} = I_t + \sum_{c=1}^{k} w_c B_c$$

We want to minimize the error

$$I_{t+1} - I_t - \sum_{c=1}^{k} w_c B_c$$

The squared error =

$$E = \left(I_{t+1} - I_t - \sum_{c=1}^{k} w_c B_c \right)^2$$

~~differentiating~~ ~~w.r.t. w~~c We need to simultaneously minimize it for the error in images and $\frac{dE}{dw} = 0$ for minimum w_c as,

We need to minimize

$$\left\| I_{t+1} - I_t - \sum_{c=1}^{k} w_c B_c \right\|^2_{span(B_c)} + \left\| I_{t+1} - I_t \right\|^2_{span(B_c)^{\perp}}$$

Using the minimum value of the second term to minimize the first term, we get the closed form solution of w_c as :-

$$w_c = \sum_{c} B_c \cdot \left[I_{t+1} - I_t \right]$$

2.3.

Fig. 3 – Lucas Kanade with Basis Templates at frames - 1, 200, 300, 350, 400

3.3.

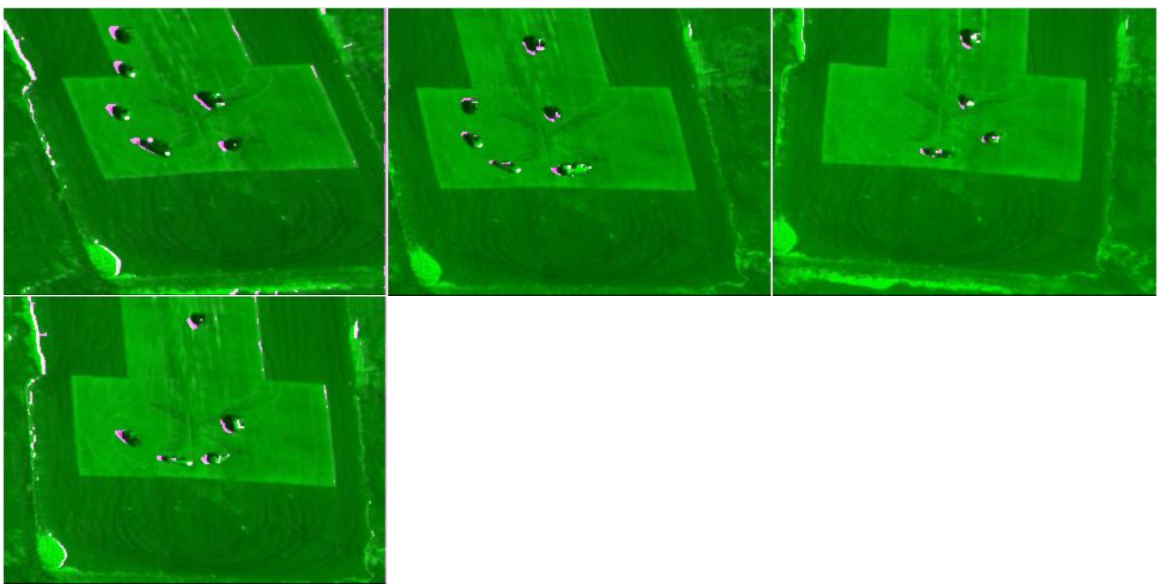

Fig 4 – Background Subtraction with Lucas Kanade Affine Subtraction